BOX NUMBER 124

TECHNICAL INSTRUCTION ON PLOTTING TARGETS AND BURSTING POINTS FOR ADJUSTMENT OF FIRE

CONTENTS: TRANSLATION OF A FRENCH OFFICIAL DOCUMENT OF OCTOBER, 1916

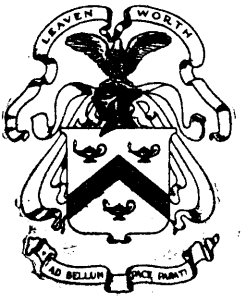
ARMY WAR COLLEGE
SEPTEMBER, 1917

41822

WASHINGTON
GOVERNMENT PRINTING OFFICE
1917

THE GENERAL SERVICE SCHOOLS

LIBRARY



CLASS NUMBER M 9403-B15

ACCESSION NUMBER 41822

neral.

WAR DEPARTMENT,
WASHINGTON, *September 26, 1917.*

The following pamphlet on "Technical Instruction on Plotting Targets and Bursting Points for Adjustment of Fire," is published for the information of all concerned.

[062.13, A. G. O.]

By ORDER OF THE SECRETARY OF WAR.

TASKER H. BLISS,
Major General, Chief of Staff.

OFFICIAL:
H. P. McCAIN,
The Adjutant General.

WAR DEPARTMENT,
THE ADJUTANT GENERAL'S OFFICE,
Washington, June 19, 1917.

To all officers of the Army:

You are advised that this and all subsequent documents of a similar character, which may be furnished to you from this office, are to be regarded as strictly confidential. They are to be kept at all times in your personal possession, and are not to be copied, nor are any parts of their contents to be communicated either directly or indirectly to the press, nor to any persons not in the military or naval service of the United States. In Europe these documents are not to be carried into the front-line trenches, nor farther to the front than the usual post of the officers to whom issued.

Strict compliance with this injunction is enjoined upon every officer into whose hands any of these confidential documents may come.

By ORDER OF THE SECRETARY OF WAR:

H. P. McCAIN,
The Adjutant General.

TABLE OF CONTENTS.

Page.

Technical instruction on plotting and bursting points for adjustment of fire 5

1. Composition of a section and organization of the observation stations 5

CHAPTER I.—Determination of an unknown point by bearings 7

2. Definition 7

3. Influence of an error in the angle on the position of the geometrical locus 8

4. Influence of errors of position of the reference point and of the observation station on the position of geometric locus 9

5. Mean errors of the geometric loci.—Choice of a solution 11

6. Use of calculation and of a graphic construction on a large scale 12

7. Precision of the determination 13

8. Special case of two bearings 14

CHAPTER II.—Plotting the targets 16

9. Construction of the bearing 16

10. Utilization of the observations 19

11. Case in which the position of the observation stations can not be determined with precision 22

CHAPTER III.—Adjustment of percussion fire 23

12. Problem of the adjustment by the observation of the bursts 23

13. Adjustment on an invisible target 23

14. Preparation of the construction sheets of the bursting points, etc 25

15. Completion of the preparation, etc 27

16. Adjustment on a target, etc 28

Example I 30

Example II 31

Calculation of the adjustments, etc 32

UNCLASSIFIED

TECHNICAL INSTRUCTION ON PLOTTING TARGETS AND BURSTING POINTS FOR ADJUSTMENT OF FIRE.

[Translation of a French official document of October, 1916, by the Army War College, September, 1917.]

(*Secret and confidential, not to be taken into front-line trenches.*)

TERRESTRIAL OBSERVATION SECTIONS.

1. *Composition of a section and organization of the observation stations.*—A terrestrial observation section generally comprises four observation stations, O_1, O_2, O_3, O_4, and a central station C.

Each observation station should be provided with observation instruments (prismatic monocular telescope, glasses for nocturnal observations, etc.) mounted on goniometers which permit the measurement of the horizontal angles in mils. The limbs of the goniometers are oriented in such a manner that their readings are zero when the optical axes of the corresponding glasses are directed on a very distinct, distant point, of known position, included in the observation sector of the station—preferably a geodetic point. As substitute for this reference point R, by night or day, when fog or obstacles hide it, it is indispensable to have an auxiliary reference point R′, near enough to be seen in almost any weather by day and capable of being illuminated at night by a little electric lamp turned toward the station. The reading $L_{R'}$ of the goniometer corresponding to R′ generally (see Sec. 4, 2°) varies from instrument to instrument; and, if it is desired to use R′ for orienting a limb of the goniometer or verifying its location, it is necessary that the instrument should occupy exactly the position that it had at the moment that the reading $L_{R'}$ was made on the oriented limb.

The observer at a station has thus the means of reading, almost to a mil, the relative direction of an object which he has sighted.

The special telegraphic material, called Military Telegraph (for a description, see the notice published by the Geographical Service), with which the sections are provided, enable the observer:

1. To light a lamp at the central for signaling the exact instant of the sighting of a target or of a burst.

2. To announce, or cause to be announced to the central, the division of the goniometer corresponding to the sighting, as well as all the indications likely to facilitate the utilization of the observation (see Sec. 10).

The central knows exactly, therefore, at what instant an observation is made at any of the stations of the section, and receives the corresponding goniometrical reading without any delay. By the simultaneity of the observations he can easily recognize those of the readings which can be referred to a single objective—flash of an enemy gun or flash or smoke ball of a friendly shell.

The problem for the central consists therefore in making the most of the elements thus obtained singly or together. It is a well-known problem of topography, which is generally solved by the process known as intersection. But this purely graphic process, especially adapted to the plane-table, can not be used here; it must be replaced by the general method of the geometrical loci, which must first be explained so that its principles can be evolved.

CHAPTER I.

DETERMINATION OF AN UNKNOWN POINT X BY BEARINGS.

2. *Definition.*—The horizontal angle α at the known point O (observation station), fig. 1, made by the direction of an unknown point X (target) with that of the known point R (reference point) gives a geometrical locus of X which is the bearing [1] starting from O and forming with OR the angle α.

In what follows, the angle α is considered as measured in mils and reckoned from O to 6,400, starting from OR in the clockwise direction.

Two bearings suffice, in a geometrical sense, to determine X, but generally three are necessary for a verification. If there were no errors of measurement and if the positions of the observation stations O and of their reference points R were known exactly, all these bearings would intersect at the same point. But, in consequence of the errors by which the angles α and the positions of the points O and R are affected, their meeting point is only approximate. To choose rationally the point to be adopted, it is necessary to know the precision of each locus, which depends, on the one hand, on the precision of this locus and that of the angle which supplied it, and, on the other hand, on the precision of the positions of the points which define the direction from which the angle was measured.

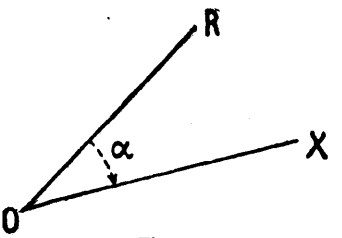


Fig. 1.

[1] This term bearing must not be confused with that used in topography. In this connection it designates the geometrical locus given by sighting from a known point on an unknown point referred to another known point. In topography, bearing means the process which consists in determining an unknown point by the station, by taking sights on at least three known points. This is what is called in military topography the *map problem*. The word "intersection" is frequently, although inaccurately, used to designate the geometrical locus here called bearing.

There is reason, therefore, to study, first, the influence on the position of the locus:

1. Of an error in the angle α.
2. Of errors in the positions of the points O and R.

By reason of the smallness of these errors relatively to the distance of the observation stations from the point to be determined, they may be considered as differentials, and the influence of each may be determined separately

3. *Influence of an error in the angle on the position of the geometrical locus.*—Let α be the erroneous value of the angle ROX, and $\alpha+d\alpha$ its real value. The error $d\alpha$ of the angle has the effect of giving the bearing OX in place of OX′ (fig. 2). The displacement $p=XX'$ to

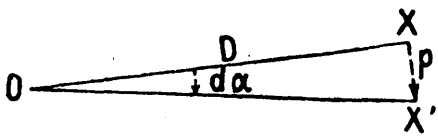


Fig. 2.

be applied at X to the erroneous locus, in order to obtain the real locus, has therefore as expression

$$(1)\quad p=D d\alpha\frac{3.1416}{3200}=KD d\alpha, \text{ with } \begin{cases} K=\frac{3.1416}{3200}=\frac{1}{1000}\left(1-0.0183\right) \\ (\log K=\bar{4}.99200), \end{cases}$$

D designating the distance OX, and p being reckoned positively, with reference to the point O, from left to right.

Therefore the error of the geometric locus is proportional to the error of the angle, which results from the fact that we have assumed that the error $d\alpha$ could be considered as a differential. *It is, moreover, proportional to the distance of the point sighted.*

The inverse of p can be taken as measure of the precision of the locus. The factor $\frac{1}{D}$ characterizes, therefore, the real precision of the locus and $\frac{1}{Kd\alpha}$ that of the angle. Starting from these definitions, we may formulate the following rule, which is applicable whatever the nature of the angle may be:

The precision of a locus is the product of its real precision and of that of the angle which has supplied it.

The value of $d\alpha$ in mils makes it possible to obtain p very simply. When $d\alpha=1$, the value of p in meters is equal to the distance D expressed in kilometers, with approximately the factor $(1-0.0183)$. To have the value of p in meters corresponding to any value of $d\alpha$, it is only necessary to multiply the distance D in kilometers by $d\alpha$.

4. *Influence of errors of position of the reference point and of the observation station on the position of geometric locus.*—1. Influence of an error in the reference point.

Let (fig. 3) R be the assumed position of the reference point which, alone, is supposed to be erroneous, and R′ its exact position, $\rho=RR'$ and $\omega=ORR'$, the polar coordinates of R′ referred to the

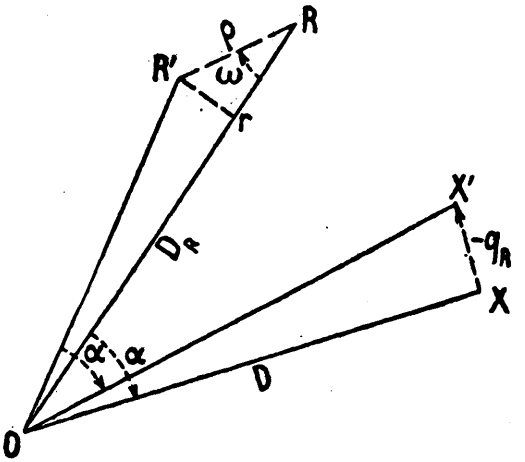


Fig. 3.

origin R and to the axis RO. ω is reckoned positively from RO in the clockwise direction.

It is apparent on the figure that the effect of the error of the position RR′ is the same as that of an error of angle $d\alpha=-\frac{R'r}{KD_R}$, r being the projection of R′ on OR, and D_R the distance DR. As $R'r=\rho\sin\omega$, $d\alpha=-\frac{\rho\sin\omega}{KD_R}$. The displacement $q_R=-XX'$ to be substituted at X for the erroneous locus, in order to have the exact locus, is consequently, according to (1):

$$q_R=-\frac{D}{D_R}\rho\sin\omega, \tag{2}$$

q_R being reckoned positively in the same direction as ρ.

The formula (2) shows, first, *that the component of the error of the position of R normal to the direction of the reference point intervenes alone;*

Second, *that its influence is proportional to the distance of the point sighted and inversely proportional to that of the reference point.*

It is therefore advantageous, in so far as visibility permits, to take as reference point a point as far away as possible.

The absolute value of q_R is easily calculated when the normal component $\rho \sin \omega$ is known, since it is only necessary to multiply it by the ratio $\frac{D}{D_R}$.

2. *Influence of an error of position of the observation station.*—Let (fig. 4) O be the given position of the observation station, which is, alone, supposed to be erroneous, and O′ be its exact position.

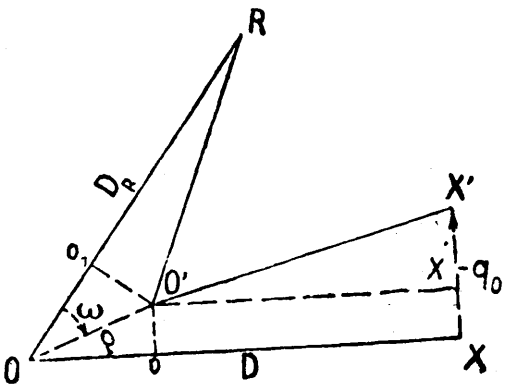


Fig. 4.

ρ=OO′ and ω=ROO′, the polar coordinates of O′ referred to the origin O and the axis OR.

ω is always reckoned positively from OR in the clockwise direction.

The normal displacement $q_0 = -XX'$, to be substituted at X for the geometrical locus, in order to have the exact locus, consists of two parts $-XX_1$ and $-X_1X'$, X_1 being the point of intersection of XX′ with the parallel to OX drawn through O′. The first is equal to the component O′ of OO′ normal to OX; it has as its expression

$$-XX_1 = -\rho \sin (\alpha - \omega).$$

The second $-X_1X'$ is equal to the error of the locus at the point X for an error of angle $da=-X_1O'X'=-O'RO$ since $RO'X'=ROX=a$. Now $O'RO=\frac{O'o_1}{D_R}O'o$, being the component normal to OR of the error of position of O,—that is, $\rho \sin \omega$.

Consequently, $da=-\frac{\rho \sin \omega}{K D_R}$ and, according to the formula (1),

$$-X_1X'=-\frac{D}{D_R}\rho \sin \omega.$$

Therefore, finally,

$$(3)\quad q_0=-\rho\left[\sin (a-\omega)+\frac{D}{D_R}\sin \omega\right],$$

q_0 being reckoned positively in the same direction as p and q_R.

The second term of the expression of q_0 is the same as q_R, as might be foreseen geometrically, a variation of orientation of the reference direction producing the same effect on the geometrical locus that arises from the displacement of one or the other of its extremities. The conclusions obtained above from the formula (2) are therefore applicable. As for the first term, it is always inferior to ρ in absolute value, and it is proportionately less as the error of position of O is nearer the direction of the point sighted X.

When $D=D_R$, the factor in brackets of the second term of (3) tends toward zero at the same time as a. Therefore *the influence of the error of position of the observation station on the position of the geometric locus is perceptibly nil when a well-determined point near the point sighted is taken as reference point.* This remark is of great importance from the practical point of view; it will be put to advantage later.

5. *Mean errors of the geometric loci—Choice of a solution.*—If the real errors da, $\rho \sin \omega$ and $\rho \sin (a-\omega)$ are replaced in the formulæ (1), (2), and (3) respectively by the mean error of angle δa and the mean errors u_R and u_o of the position of R and O, the second terms represent the mean errors of the geometric locus at the point X resulting from δa_1 of u_R', and u_o. Assuming that the points R and O have the same mean error u, we have for the mean total error of the locus at the point X

$$(4).\quad \Delta=D\sqrt{\left(\frac{\delta a}{1000}\right)^2+u^2\left[\frac{2}{(D_R)^2}+\frac{1}{D^2}-\frac{2\cos a}{DD_R}\right]}.$$

δa being expressed in mils.

Knowing the mean errors of each of the loci traced on the construction sheet, and consequently, their values which are inversely proportional to the squares of the mean errors, we choose the point X on

condition that Σpd^2 be the minimum, d designating the distance of X to the locus of the value p, and the sign Σ extending to all the geometrical loci.

The application of this rule in practice would require too much time, and time spent on a rather illusory increase in precision, since the mean errors of the loci are not generally well known. We content ourselves in being guided by it in choosing at sight, on the construction sheet, the most probable point. It should be remarked that, in case it may be assumed that the angles have the same precision and that the positions of the observation stations and of their reference points are equally well determined, the precision $\frac{1}{\Delta}$ of each locus, if the ever feeble influence of the mean error of the normal 0 of the sight is overlooked, is inversely proportional to D. Its value is accordingly proportional to $\frac{1}{D^2}$, or to the square of its real precision.

6. *Use of calculation and of a graphic construction on a large scale.*— It may be important, in certain cases which will be indicated later (sec. 10), to have recourse to calculation to obtain the position P with all the precision that the data of the problem permits.

The method to be employed consists in taking the position X, obtained graphically on the construction sheet on the ordinary scale, as the approximate position, and in looking for the displacement to be applied to each of the geometric loci beginning with X, so that at the corresponding observation station the angle of the bearing with the direction of the reference point may be equal to the measured angle α.

The corrected loci are transferred to a graphic sketch on a large scale with reference to two rectangular axes representing the parallels to the axes of the coordinates of the projection which pass through the point X. The point to be adopted, X′, is chosen according to the considerations discussed in the preceding section; its coordinates are the corrections to be applied to the coordinates of X.

Let x_o, y_o, x_R, y_R be the coordinates of the observation station 0 and of its reference point R, and x and y the coordinates of X represented graphically on the construction sheet on a small scale. By means of the formulæ

$$\tan U_c = \frac{x-x_o}{y-y_o}, \quad Dc = \frac{X-X_o}{\sin U^c} = \frac{y-y_o}{\cos U^{c'}}, \tag{5}$$

$$\tan U_R = \frac{x_R-x_o}{y_R-y_o}, \quad D_R = \frac{x_R-x_o}{\sin U_R} = \frac{y_R-y_o}{\cos U_R}, \tag{6}$$

we calculate for each observation station where the sighting of X has been carried out, the reference angles U^c and U_R of the vectors OX and OR; that is, the angles of these vectors with the semiaxis of the $y>0$ reckoned from 0 to 6,400 mils in the clockwise direction, starting from this semiaxis. We also calculate the lengths of the vectors. The calculated reference angle U^c is compared with the reference angle $U_R+\alpha$ supplied by measure; the difference,

$$dU^c=U_R+\alpha-U^c \tag{7}$$

introduced in the place of $d\alpha$ in the equation (1) gives the displacement p to be applied to the locus, starting from X.

To trace the corrected locus on the large scale[1] graphical chart, the length p is laid off from the origin in a positive sense in the direction $U^c+1{,}600$ mils, and a perpendicular is drawn at its extremity; it is an element of the locus corrected in its determining part.

In the same way an element of each of the other loci is traced near the origin, and the point X′ is selected as being the most probable, as has been indicated in the preceding section. Let dx and dy be its coordinates on the chart. The coordinates of X′ are $x+dx$ and $y+dy$.

7. *Precision of the determination.*—If the number of the loci is great enough, and if the point X′ is equally well determined in every sense, the expression $\sqrt{\frac{\Sigma\rho d^2}{\Sigma\rho}}$, in which d represents the deviation with reference to the selected point X′ of the locus of the value p, and Σ extends to all the geometric loci, may be taken as the value of the mean error of position of P. But these conditions are practically never realized in the actual practice of locating and adjusting. Generally the number of loci does not exceed three, and the extreme loci intersect at an angle which rarely reaches 1,600 mils and is often inferior to 800. There is consequently a vast azimuthal region around X′ not comprising any locus direction. The idea of error of position loses, therefore, all significance. The mean error of X′ can no longer be defined by a unique quantity, and it is necessary to calculate the mean errors of the coordinates of this point. In order that these mean errors may have a concrete sense, it would be necessary to start by taking as axes of the coordinates the balanced mean of the directions of the loci and its perpendicular, so as to have the maximum and minimum mean errors. It is evident

[1] The scale is adopted with reference to the precision which the angles give reason to expect for X. It is useless, in the actual case, to go beyond $\frac{1}{1000}$.

that the determination of the precision of the solution by the method of least squares would lead to quite protracted calculations for an illusory result, in consequence of the impossibility of avoiding an arbitrary proceeding in the attribution of the values to the different loci.

We may content ourselves, to avoid loss of time, by taking the maximum deviations of the intersections, in the sense of the mean direction of the loci and in the perpendicular direction, as measure of the uncertainty of the position adopted in the two directions. Furthermore, *the number of the loci must be at least four.*

8. *Special case of two bearings.*—Before passing to the application to the plotting of targets and to the adjustment of fire, we should examine a special case frequently occurring in practice—the case

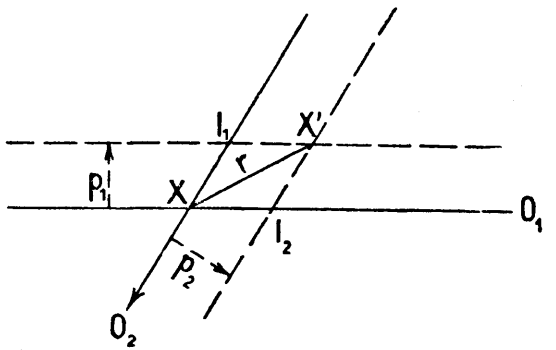


Fig. 5.

in which the number of bearings is reduced to two. The problem is then determinate, and offers no occasion for a choice of a soultion. But it is important to study *the influence of the angle at which the two loci intersect on the precision of their intersection.*

Let (fig. 5) O_1 X, and O_2X be the two erroneous loci starting from the points O_1 and O_2, the positions of the observation stations 1 and 2, and let p_1 and p_2 be the displacements to be applied to them to obtain the correct loci I_1X' and I_2X', so that the correct point is X′ and so that $r=XX'$ represents the vector of error of the position of X. The problem is to find the value of r.

In the triangle XI_1X'

$r^2=XI^2+XI'-2XI_1\times X'I_1\times\cos\ XI_1X.$ $r^2=XI^2,\ +XI_1-2XI_1\times X'I_1\times\cos XI_1X'.$

The angle XI_1X' is evidently the one of which the bearing O_1X must be turned to a position parallel with O_2X, and in the same direction. It is then equal to U_2-U_1, the difference of the reference angles of the two bearings.

On the other hand, we have

$$XI_1=\frac{\rho_1}{\sin(U_2-U_1)}, \text{ and } XI_1=\frac{\rho_2}{\sin(U_2-U_1)}.$$

Consequently,

$$(8)\quad r^2=\frac{\rho_1^2}{\sin^2(U_2-U_1)}\left[1+\frac{\rho_2^2}{\rho_1^2}-\frac{2\rho_2}{\rho_1}\cos(U_2-U_1)\right].$$

The following table gives the values of $\frac{r}{\rho_1}$, for $\frac{\rho_2}{\rho_1}=0$, 5, and 10, which correspond to different values of (U_2-U_1) in the case where $\frac{\rho_2}{\rho_1}$ and $\cos(U_2-U_1)$ are of contrary signs and where, consequently, r is the longer diagonal of the parellelogram $XI_1X'I_2$.

Table of the values of $\frac{r}{\rho}$.

$\frac{\rho_2}{d_1}$	Angles (U_2-U_1) in mils.					
	1,600	2,400	28,000	3,000	3,100	3,200
0	1.0	1.4	2.6	5.1	10.2	00
5	5.1	8.1	15.5	30.7	61.2	00
10	10.1	15.2	28.6	56.3	112.2	00

It appears from the numbers of this table how rapidly the error of position r augments when the smaller angle at which the two loci intersect falls below 400 mils.

CHAPTER II.

PLOTTING THE TARGETS.

Although the plotting of the targets and that of the bursts of friendly shells are simply the application of what precedes, the two problems, regarded from the practical point of view, differ sufficiently to warrant a separate treatment.

9. *Construction of the bearing.*—At the observation station O_1, a target X has been sighted under the division m_1 of the micrometer of the sight, the goniometric reading corresponding to the position of this sight being l_1. The verification of the orientation of the limb of the goniometer has been made immediately afterwards, by laying the reference point R_1 under the zero division of the micrometer, and it has been found that the reading, which should have been zero if the instrument had been correctly oriented, was r_1. The horizontal angle $\alpha_1 = l_1 + m_1 - r_1$ between the reference point and the target X has therefore been measured. The problem is to construct the bearing of X from this measurement.

a. Direct methods.—According to their rectangular coordinates in connection with a kilometric cross-sectioning in the system of coordinates used by the Army to which the section belongs, transfer the points O_1 and R_1 to the construction sheet on the scale of $\frac{1}{20000}$. The bearing $O_1 X_1$ may then be traced directly, if we have an alidade protractor with limb graduated in mils and a vernier for reading the mils.

In the absence of this instrument, which is not generally possessed by the sections, a graphic sketch of angles in mils on tracing paper may be used. The center is placed at O_1 and the origin radius on $O_1 R_1$, so that a second point for $O_1 X_1$ can be pricked by interpolation for α_1. The Geographic Service has had a graphic sketch of this sort prepared, giving all the angles from 0 to 1,600 mils. It is printed on tracing paper and delivered to the sections. By reason of the inaccuracies in printing, this process is less exact than that of the protractor.

That the bearing thus obtained may have the exactitude required in the vicinity of the point X, it is requisite that the distance $O_1 X$, reduced to the scale of the construction sheet, should not exceed by too much the length of the branches of the protractor when this instrument is used. It is above all important, whatever method is used, that the direction $O_1 R_1$, the origin of the angle α_1, should be graphically defined without appreciable error at the distance $O_1 X$

from O_1. This is the condition when O_1X is, at most, equal to O_1R_1. It is no longer true if O_1X is notably superior to O_1R_1, and it is then necessary to prolong this line beyond R_1 with the aid of the calculation. This is done by determining the intersection of the prolongation with one of the lines of the cross-sectioning which are most normally cut by it. The formulæ to be employed are the following:

To obtain the x of the intersection of O_1R_1 with the parallel to the axis of the x's $y=k$,

$$(9) \qquad x=x_{o1}+\frac{x_{R_1}-x_{o1}}{y_{R_1}-y_{o1}}(k-y_{o1})=x_{o1}+(k-x_{o1})\tan U_{R_1};$$

to obtain the y of the intersection of O_1R_1 with the parallel to the axis of the y's $x=k$,

$$(10) \qquad y=y_{o1}+\frac{y_{R_1}-y_{o1}}{x_{R_1}-x_{o1}}(k-x_{o1})=y_{o1}+(k-x_{o1})\cot R_1,$$

x_{o1}, y_{o1}, x_{R_1}, y_{R_1}, designating the coordinates of O_1 and R_1 and U_{R_1}, the reference angle of O_1R_1. The logarithm tables with four decimals are sufficient for this calculation.

b. Use of a graduation.—The above-mentioned methods are appropriate for tracing bearings taken from a mobile observation station. In installations remaining stationary for a long time the labor of graphical construction can be much reduced, and an equal, if not superior, precision can be obtained by using a graduation in mils, traced once for all for each station on the construction sheet. Each graduation starts from the direction of the reference point of the corresponding station and covers its entire sector of visibility. The divisions are numbered from 0 to 6,400 in the clockwise direction, so as to correspond to the angles α defined above. To trace a bearing taken from O_1, for example, it is only necessary to join the point O_1 to the real or interpolated division which has the angle α_1 as number. If a precise determination is not required, one need only *stretch a wire* attached to a knob fastened on a special point at O_1, making it pass through the division α_1, to have the bearing. Thus many erasures of pencil marks on the construction sheet can be avoided, and the latter can be rendered serviceable for a long time. Care should be exercised in inserting the point not to make the hole at O_1 too large, if this point is to be used in tracing very precise bearings. But, since it is difficult not to enlarge the hole, if the point is removed too often, it is expedient to have for the use of the wire a special construction sheet fastened to a board, and to reserve the main construction sheet, which had better be fastened to a surface of zinc, for tracing bearings with pencil.

The graduations corresponding to the different observation stations O_1, O_2, O_3, O_4 are arranged in arcs of circles, or, better to avoid overlapping, the construction sheet is removed and the desirable precision is had on its entire extent on the margins by means of parallel bands about 10 millimeters in width. These are distinguished by different colors.

The quickest way to trace them is to use the graphic sketch of the Geographic Service on tracing paper, pricking a point or two of each division near the margin of the sheet. The angular spacing of the radii of this graphic sketch at the different distances from the center has been chosen in such a way that the interpolation at sight is everywhere possible without appreciable error, and that their linear spacing never descends below 1.5 millimeters; it is only necessary then, whatever part of the graphical sketch is used, to prick one or two points of each of the radii successively without adding or cutting off.

Nevertheless, by reason of the inaccuracies of the graphical sketch due to printing, it may happen that a graduation obtained by this means would present inadmissible errors. This difficulty is avoided by determining some divisions by calculation, for instance, those of 800, 1,600, ... 4,800, 5,400 mils, besides that of the reference point. Once these principal divisions have been traced on the construction sheet, the graphical sketch makes it possible to obtain the intermediary divisions without appreciable error. For this purpose it is placed over the construction sheet, with the center at O_n, n being the number of the station corresponding to the graduation, and a principal radius on one of the extreme divisions calculated, that on the left, for instance, of the sector of 800 mils that is to be graduated. If the radius laid off at 800 mils from the first does not fall on the limiting division on the right, the graphical sketch is shifted gently with respect to the point O_n following the bisector of the angle of the sector, the first radius being kept on its division until this result is obtained.

The formulæ to be used for determining by calculation the intersection with one of the lines of the cross-sectioning of the division d, reckoned from the aiming line, are the following:

To obtain the x of the intersection of $O_n d$ with the parallel to the axis of the x of the cross-sectioning farthest from O_n $y=k$,

$$x=xo_n+(k-y_{on}) \tan (U_{R_n}+d); \tag{11}$$

to obtain the y of the intersection of $O_n d$ with the parallel at the axis of the y of the cross-sectioning farthest from O_n $x=k$,

$$y=yo_n+(k-xo_n) \cot (U_{R_n}+d). \tag{12}$$

U_{R_n} is given by the first of the formulæ (6). That one of the formulæ (11) and (12) which corresponds to the most normal intersection is of course employed.

A type of the calculation by the formula (12) will be found in Example I at the end of the pamphlet.

In the absence of the graphical sketch of the angles in mils, it is necessary to graduate entirely by calculation. The same formulæ are used; it is only necessary to make d in them equal successively to 5, 10, 15, ... mils, or to 10, 20, 30, ... mils, etc., according to the distance of the intersection from the point O_n. The calculations in this case should be made in series, so as not to require too much time, and should be verified by the differences.

Tables of logarithms of four decimals are sufficient for all these calculations.

10. *Utilization of the observations.*—A single observation supplying only one geometric locus of the position of the target is not enough to determine it; but it furnishes an indication capable of being used in many cases.

To turn it to account, an investigation is first made to see whether it corresponds to a known target. The bearing is applied to the construction sheet on which the positions of all the previously determined targets has been marked. The use of the sheet pasted to the board and provided with wires is particularly adaptable for this use. Several cases may occur.

(*a*) *The bearing passes through a target at the usual degree of approximation.*—Let A be this known target, O_2 being the station which has given the observation. The central pricks on the construction sheet the directions α_1, α_3, α_4, in which the observers of O_1, O_3, O_4, should sight to see A, and he has them watch in these directions respectively, until O_1 signals a new observation in the same direction as the first. If one or more of them get an observation at the same time as O_1 the operation is conducted as described later (case of two or more simultaneous observations). In the contrary case, it is decided, either that the target seen from O_2 is not A, or that, if it is A, the circumstances do not permit sighting it from O_1, O_3, O_4.

If the bearing from O_1 encounters two or more targets they are tried successively, in the order of probability, until at least two simultaneous observations are obtained.

(*b*) *The bearing does not pass through any known target.*—It happens quite often, at least by day, that the observer who has made an observation of a flash or smoke ball is able to locate approximately

this flash or smoke ball. The Central adopts provisionally, a location of the target on the bearing in the region indicated by the observer and proceeds, with reference to this supposed location, as with reference to a known target through which the bearing might pass, to try to obtain at least two simultaneous bearings.

In the absence of supplementary information supplied by the observer who has made the observation, application is made to the neighboring terrestrial observation sections, or sound locating sections, or balloons or first-line trench stations of the Artillery Intelligence Service, etc., and the Central, in this way, may often obtain data which will enable him to determine somewhat more accurately the position of the target on the single bearing, sufficiently at least to try to obtain simultaneous observations.

Two simultaneous observations are theoretically sufficient for the determination of a target; but, generally, every determination which is not verified should be regarded as doubtful. The best verification is a third concurrent observation supplying a bearing which passes very close to the point of intersection of the two others. For the probability that three flashes, for instance, should be produced at the same instance and should give concurrent directions with three stations from which they have been observed is very slight, and the probability that the three stations have sighted the same flash is practically equivalent, under these conditions, to a certainty. It is this verification which we must try to get, especially for new targets. But, since it is quite often impossible, for reasons of visibility or clearness, that three stations should sight the same flash or smoke ball, we must often be satisfied with other verifications.

The fact that the two observations are simultaneous, especially when it is a case of an instantaneous phenomenon like a flash, is in itself a partial proof that they relate to the same target. If the same fact is repeated a certain number of times, while the directions of the observations remain apparently constant, the presumption is strengthened, without absolute certainty being produced, since, in the case of firing, for instance, the enemy might discharge two guns at a time, or make false flashes.

A much more weighty confirmation results from the coincidence of the intersection with a known target. The concurrence of the result with the approximate position given by one of the observers on a bearing may also be regarded as a satisfactory verification. Finally, the intelligence obtained outside the section, in case that of the section is insufficient, sometimes reveals the degree of confidence that can be placed in the result.

The use of wires representing the bearings is very convenient when there are only two bearings to be applied, on condition that the angle at which they intersect is large enough. The tracing of the loci on the construction sheet becomes necessary as soon as the angle is less than 400 mils, especially if the point of intersection is very distant (Sec. 8). It is even necessary to resort to calculation when the angle is less than 300 mils.

The determination by two bearings only should be turned over to the Artillery with all the necessary reserve, according to the confirmations obtained. *In no case should it be regarded as definite.*

With three or more simultaneous observations the problem is more than determined and, as the geometric loci generally do not intersect at the same point, the position to be adopted is open to a choice. *If the observations are equally good,* the rule given in section 5 leads to choosing a point nearest the loci whose value is greatest. Furthermore, as is known, the value of a locus may be regarded as inversely proportional to the square of the distance from the point sought to the corresponding station, or, in other words, to the square of the displacement of the locus for the same error of observation, 10 mils for instance. It is sufficient, then, to trace near the *cap* (the figure having the intersections of the loci as summits) bearings corresponding to the same angular deflection of the first, in order to have a very clear conception of the values which are inversely proportional to the squares of the linear spacings.

But the problem is rarely presented with this degree of simplicity. *Generally the observations are of unequal precision* and the difficulty consists in observing the inequalities of precision in the attribution of value to the bearings. Thus, in the case where two observers have been able to observe the flash of the discharge of a gun, and the third the dust or the smoke only, the observation of the third is, all other things being equal, less precise than that of the first two; but the central has not the data by which he may determine in what measure. Only the observers, if they are conscientious and well trained, can indicate to him the degree of uncertainty of their observations and enable him to reduce to a minimum the arbitrary part in the attribution of the values. The chiefs of sections can not use too much care, therefore, in the selection, development, and supervision of the observers.

The use of wires is not recommended for very precise determinations, especially if the loci intersect at rather acute angles, for if the wires rub on one another it is to be feared that they will not be

perfectly straight. Moreover, not being on the same plane, they produce the effect of a parallax. For tracing the bearings on the construction sheet fastened on zinc, a hard pencil sharpened to a very fine point must be used. Finally, if the loci intersect at very acute angles, it is necessary to resort to a calculation and to a graphic construction on a large scale. (Sec. 6.)

When the dimensions of the cap exceed notably those which correspond to the mean errors of the observations, the determination should be regarded as nul. *Every correction of observation* tending to reduce the cap which has not been signalled at once by the observer himself *must be rigidly avoided.*

Unless there is absolute certainty that the sightings of the different stations relate to a single and identical target, *it is never permissible to proceed by mean observations*, especially if the flashes or smoke balls have not all been sighted by the same observation stations. The variations of the positions corresponding to the different series of simultaneous sightings show whether several targets are involved, and, in the contrary case, they afford a much more exact idea of the approximation really obtained than that which can be formed from the variation of the loci in respect to the point chosen in a single determination.

11. *Case in which the position of the observation stations can not be determined with precision.*—The preceding is based upon the implicit assumption that the positions of the observation stations are so well known that the influence of their errors on the determination can be regarded as negligible in regard to that of the errors of measure. It is not always so, even in position warfare, since some stations difficult to establish have to be used before the range triangulation group has had the time or opportunity to determine their locations. In open warfare it is often necessary to rely on positions taken at sight on the battle map or even on the ordinary map.

The remark at the end of section 4 shows that it is still possible in this case to have exact geometric loci if a reference point sufficiently near the target is available for each station. But, then, since the reference points change with the targets, there is no occasion to trace the graduation on the construction sheet. By way of compensation, the graphical sketch on tracing paper of the angles in mils can be used with advantage to prick the bearings, since, the angles being small, its deviations are inconsequential.

CHAPTER III.

ADJUSTMENT OF PERCUSSION FIRE.

12. *Problem of the adjustment by the observation of the bursts.*—A battery may ask the terrestrial observation service located in the same sector to determine for it either the absolute positions (the rectangular coordinates) of the bursts of its projectiles, or the positions of these bursts with reference to the target, which should, in this case, be a clearly defined point of the battle map.

The first problem does not differ from the one considered in the preceding chapter; it is, in general, even easier, both for the observers, who can be informed in advance of the time and the direction in which they should watch, and for the central, who works on the basis of observations unusually concordant.

The second is distinguished by the fact that the position sought is a *relative* position, which can be obtained by differential calculus, and consequently constructed on a geographical sketch on a large scale with all the desirable precision and rapidity. It constitutes the real problem of fire adjustment, which may be formulated thus:

A friendly gun P fires a percussion shell at a target X; the burst is heard by the terrestrial observation section. Determine the errors in range and in lateral deviation.

There are two cases to be distinguished, according as the target is invisible and known only by its rectangular coordinates, or as it can be sighted by the stations during the adjustment.

13. *Adjustment on an invisible target.*—Let (fig. 6) O_1, O_2, O_3, O_4 be the four observation stations of the section, α_1, α_2, α_3, α_4, *the angles* observed in each of them between the reference point and the bursting point X′ of a projectile.

To have the errors of X′ in respect to X in range and direction, there are traced on a graphic sketch on the scale $\frac{1}{5000}$ two rectangular axes Xx and Xy (fig. 7), representing the parallels to the axes of the rectangular coordinates passing through the target X, and the position of the burst X′ is determined with respect to these axes by constructing its four geometric loci, one of which may be lacking.

If the point X′ is near enough to the point X, the directions of the loci are very noticeably parallel to O_1X, ..., O_4X, of which the reference angles U^c_1, ..., U^c_4 (fig. 6) are calculated by means of the

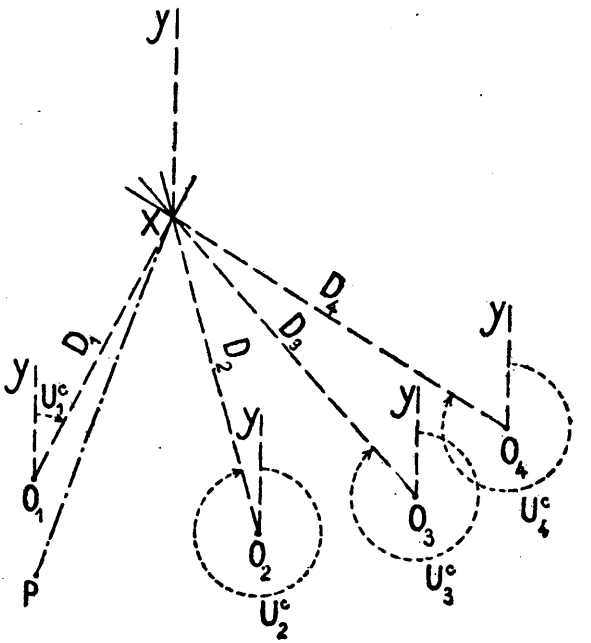


Fig. 6.

first of the formulæ (5) of section 6; their distances at the origin have as expressions (formula (1) of Sec. 3):

$$p_1 = KD_1 dU^c_1, \ldots\ldots, p_4 = KD_4 dU^c_4$$

with [formula (7)]:

$$dU^c_1 = U_{R1} + \alpha_1 - U^c_1, \ldots\ldots, dU^c_4 = U_{R4} + \alpha_4 - U^c_4.$$

D_1, ..., D_4 are the distances O_1X, ..., O_4X measured on the construction sheet on the scale $\frac{1}{20000}$; U_{R1}, ..., U_{R4} designate the reference angles of the lines O_1R_1, ..., O_4R_4, running from the observation stations to their reference points, the reference angles being calculated once for all by the first of the formulæ (6).

The direction in which these distances should be pricked is seen on the construction sheet, as is shown in figure 6. Thus, in figure 7, the trace of the first locus corresponds to a difference $dU^c_1 > 0$.

All the loci should intersect at the same point if the sightings are free of errors and if the positions of the stations and of the reference points are exactly known. Generally they form a little cap in which the bursting point X′ is selected, as shown in sections 5 and 10.

It is then only necessary to project X′ in ξ on the direction XP of the gun or of the battery P, which forms with the axis Xy an angle

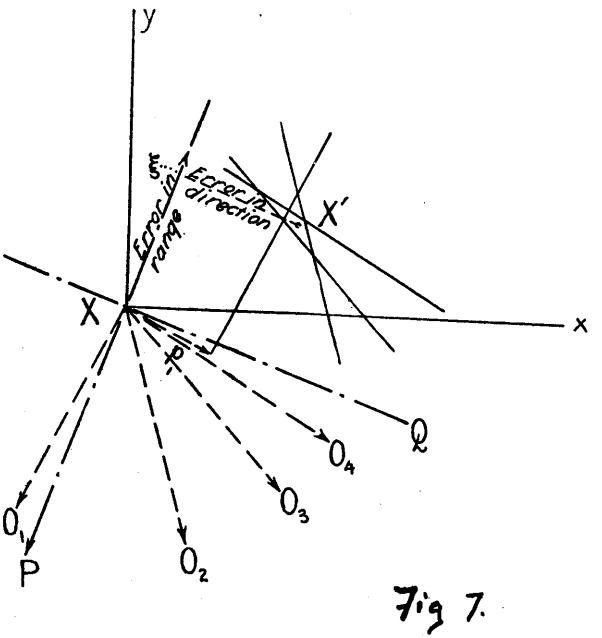


Fig 7.

given by the construction sheet, to have in Xξ the error of range and in ξX′ the error in direction.

14. *Preparation of the construction sheets of the bursting points on the scale of* $\frac{1}{5000}$.—The formulæ (1) and (7) are independent of the gun or battery P. The section may therefore have in advance, for each of the targets on which it will have to make the adjustment, a construction sheet on the scale of $\frac{1}{5000}$, like the one represented in

figure 7, on which are pricked the directions of the loci starting from X with respect to the axes Xx, Xy and, normally to these directions, of the scales of the corresponding p's for the successive values of dU^c in multiples of 5 mils or in multiples of 10 mils.

The graphical sketch of the angles in mils may be used to graduate these scales in pricking the divisions through the tracing paper when the distance OX from the station to the target is less than 5 kilometers. For distances greater than 2 kilometers, it is easier to use the *graphic table of the errors of the bearings* (Pl. III), which gives on the scale of $\frac{1}{5000}$ the displacement of the bearing to the different distances from 2 to 18 kilometers. In the absence of this graphic table, the values of the p's for $dU^c=10$ mils are calculated by means of the formula

$$(p)\quad p^r\ 10\ \text{mils}=40\ \text{KD},$$

which gives p in millimeters on the scale of $\frac{1}{5000}$ in terms of D in millimeters of the construction sheet on the scale of $\frac{1}{20000}$, and in which the coefficient K has as value (Sec. 3):

$$\frac{3.1416}{3200}=\frac{1}{1000}\,(1-0.0183),$$

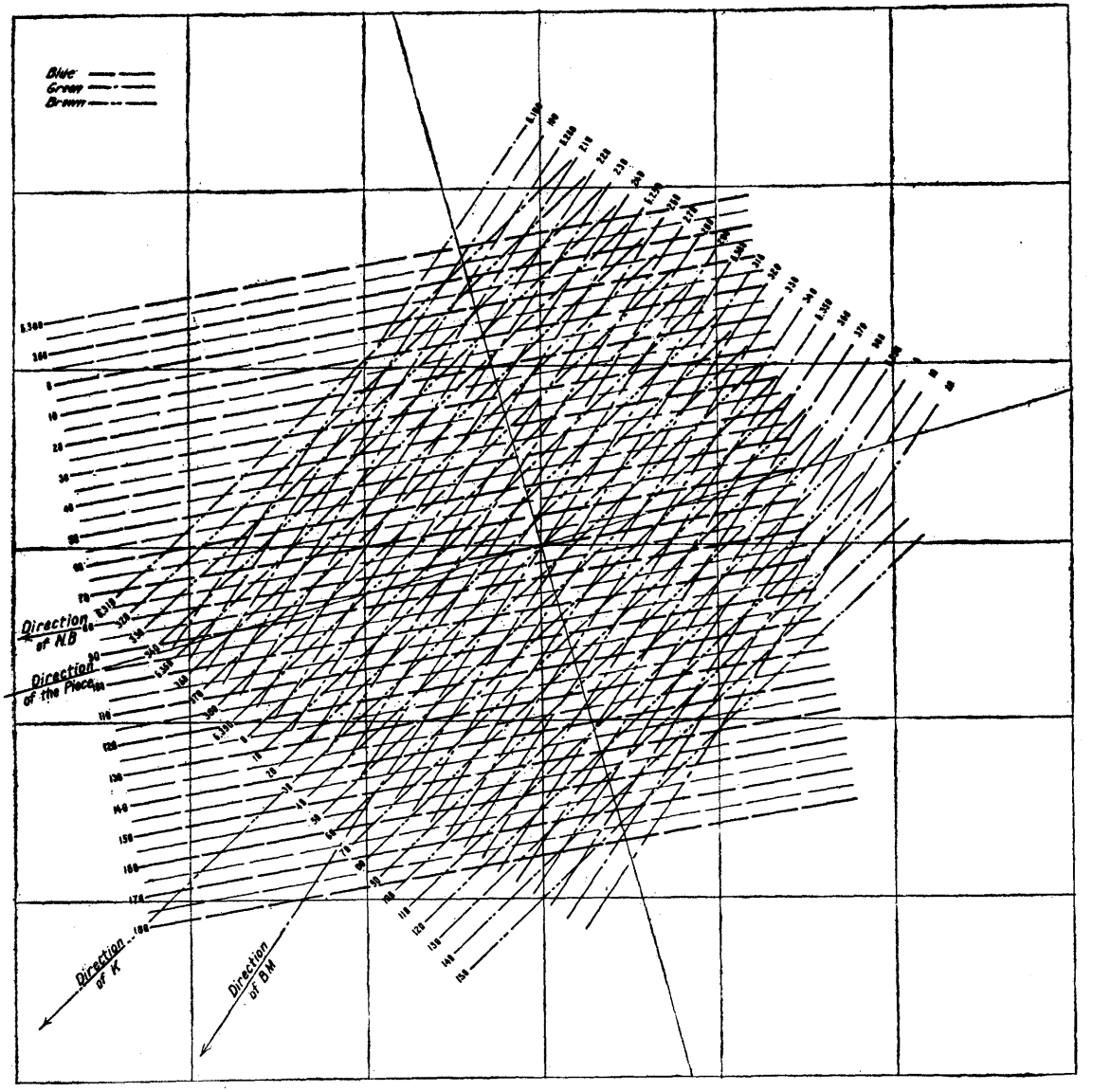
wherefore log K=4.99200. These lengths of p are then multiplied for 10 mils by 1, 2, 3, ... to have the distances of the successive divisions in multiples of 10 mils on the straight line XO, which gives the direction of the geometric locus. The divisions on the left of this line with respect to the observation station are indicated by the − sign, those on the right by the + sign.

If the lack of parallelism of the bearings becomes noticeable at the extremities of the graduation, it can be remedied easily by substituting for the single scale corresponding to the target two scales placed at a distance of 10 centimeters, for instance, from each side of the origin.

Instead of taking as origin of the graduation of each scale its point of intersection I with the corresponding straight line XO, it is preferable, in case the target is well determined and fixed, to trace the divisions which correspond to round numbers of mils of the angle α so as to avoid having to reckon the difference dU^c for each geometric locus to be constructed. The value of α for the point I being U^c-U_R, the adjustment to be applied to the divisions toward the right is the product of the value of p for 10 mils multiplied by the fraction of 10 mils obtained by subtracting U^c-U_R from the value of the next highest multiple of 10 mils above the actual value of α. Example II at the end of the pamphlet gives the disposition to adopt for all these calculations.

Target : Left gun of the battery 86-52

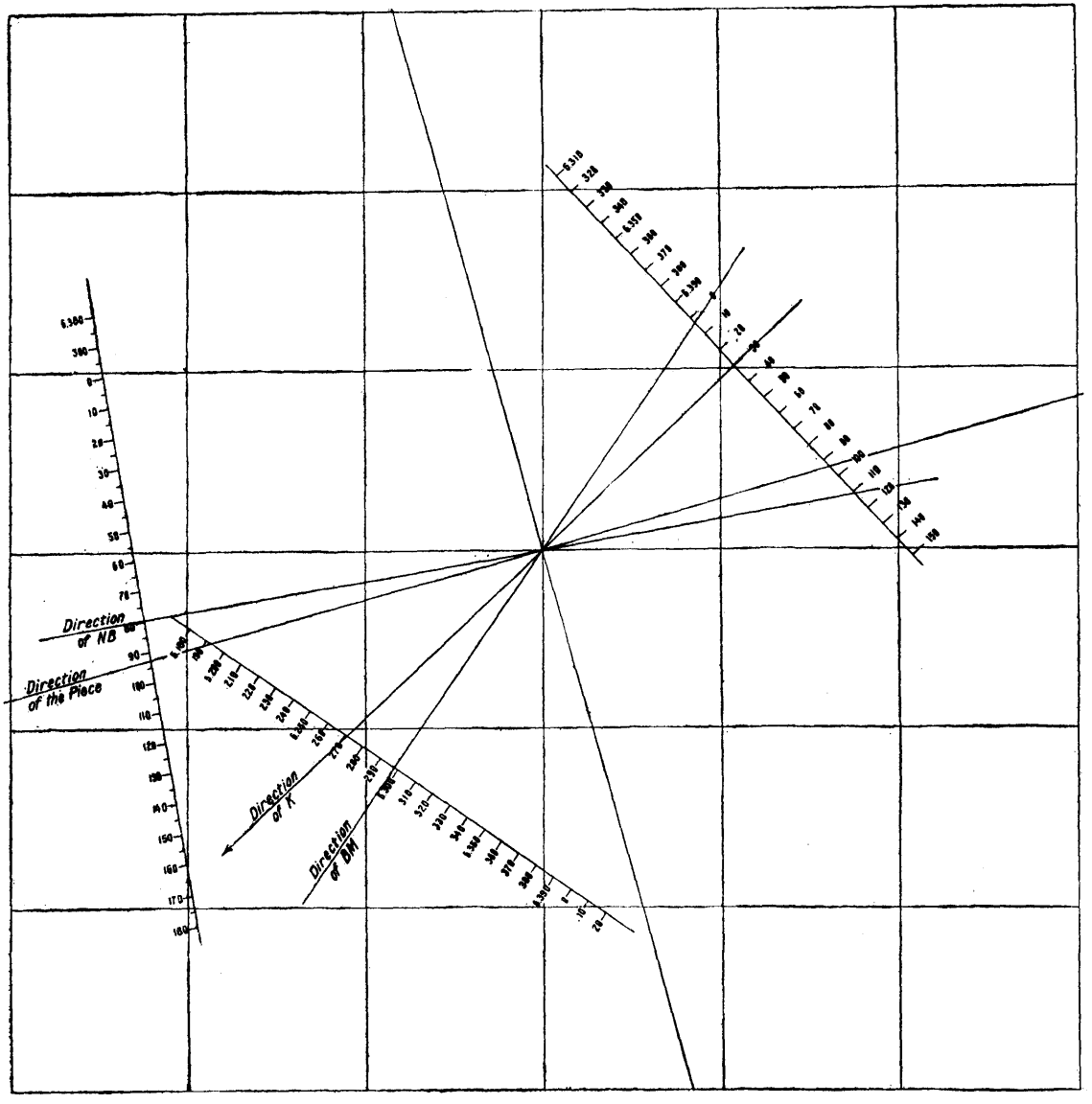
Plate I



Scale of $\frac{1}{5.000}$

Target : Left gun of the battery 86-52

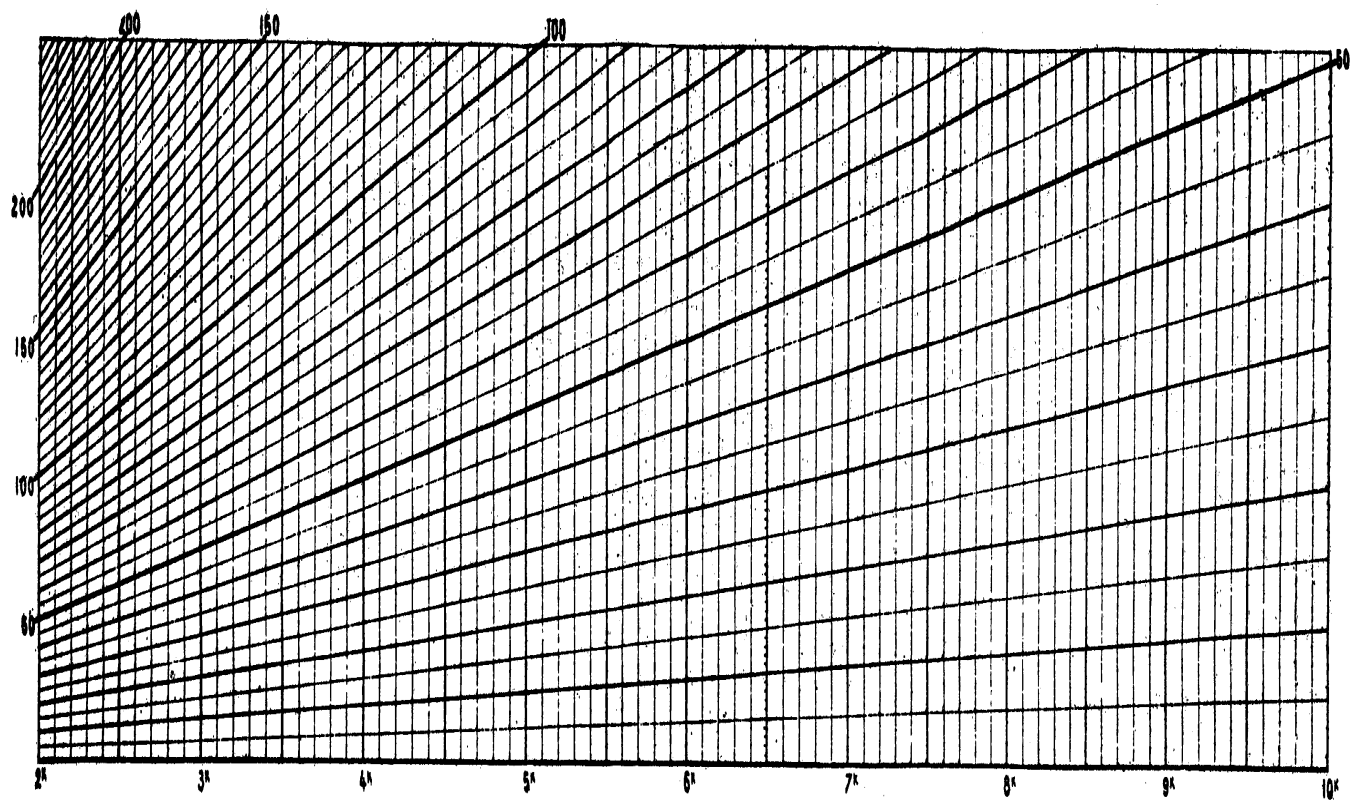
Plate II

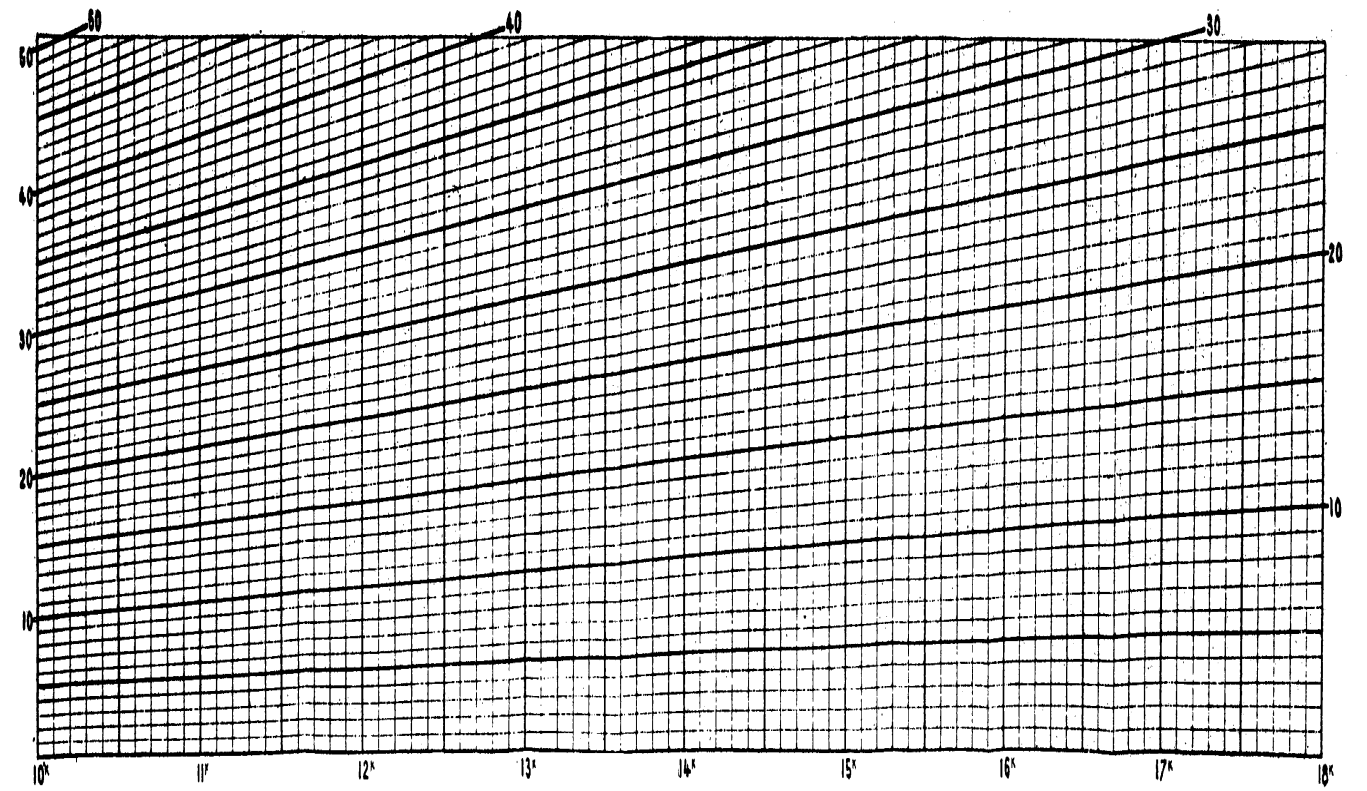


Scale of $\frac{1}{5.000}$.

ε

$y = \rho$ on the scale $\frac{1}{5000}$





Service Géographique de l'Armée

16057°. (To face page 26.) No. 3

On the construction sheet on the scale of $\frac{1}{5000}$ thus prepared, the loci of X′ are constructed very quickly; it is only necessary to draw parallels to their directions through the division points a_1, a_2, a_3, a_4 of their scales. The operation may be still further simplified by tracing in advance the bearings passing through the points of division of the scales, so as to be able to trace those of X′ by interpolation at sight. But it is then indispensable for clearness to use a particular color or type of line for each system of lines. Plates I and II give specimens of construction sheets thus prepared with or without a system of parallels.

15. *Completion of the preparation of the construction sheets on the scale $\frac{1}{5000}$ at the moment of the adjustment.*—When the gun or battery P, which is to direct its fire on the target X, is designated, the direction XP (fig. 6) on the construction sheet on the scale of $\frac{1}{20000}$ where the location of P has been marked in advance according to its coordinates is taken and transferred from X (fig. 7) to XP on the sheet on the scale $\frac{1}{5000}$. It is likewise possible to trace, on a perpendicular XQ to XP drawn through X, or through any other point, a graduation in mils or in decigrads so as to give to the battery the error in direction in mils or in decigrads, as preferred, and thus to avoid the transformation of the linear deviation X′ξ, in one or other of these angular unities. It appeared from the preceding section that, on the scale of $\frac{1}{5000}$, a division of 10 mils on this graduation has as length in millimeters

$$40 \times K \times \overline{XP},$$

$\overline{XP}$ being in millimeters, on the scale of $\frac{1}{20000}$, the distance from the gun to the target, and K being always the factor of transformation to pass from the value of an angle in mils to its value in radians.

The length of the division of 10 decigrads equals in millimeters

$$40 \times K' \times \overline{XP},$$

$\overline{XP}$ being expressed in the unity as above, and K′ designating the factor of transformation to pass from the angle reckoned in decigrads to its value in radians:

$$K' = \frac{3.1416}{2000} = 1.5708 \times 10^{-3}$$
$$(\log K' = \bar{3}.19612)$$

For greater clearness, it is well to use a special color to trace the line XP, its perpendicular XQ and the scale of the latter, even on the construction sheet in black. (See the example II at end for the arrangement of the calculation.)

16. *Adjustment on a target visible from the observation stations during the adjustment.*—This second case is that of a target either constantly visible, or intermittently visible, as a gun to be fired on the flashes of which the stations can see.

In this case, it is no longer necessary to assume that the position of the target is known in advance and fixed during the firing, since the section can determine it before and during the adjustment. The section always starts by locating the target. This first operation is indispensable in the case of an enemy gun to be fired on which must either be identified or determined, according as it is known or not, so that it can be signaled to the Artillery.

It is in every case very useful to furnish the goniometric reading which, in each station, corresponds to the sighting of the target, which from now on is taken as common reference point for the adjustment, instead of the ordinary reference points. It is useless to try to secure great precision in this determination; *an approximate position quickly obtained* (with wires, for instance), *provided it admits verification*, is sufficient, not only to identify a known target, but to enable the Artillery, if a new target is involved, to fire an initial percussion shell into its vicinity.

The adjustment is then carried out as in the preceding case, that is, by constructing geometric loci of each of the bursts so as to have their position with reference to the target. Only, instead of comparing the calculated reference angle U^c with the observed reference angle $U_R+\alpha$ to obtain the angular deviation dU^c of the burst and of the target, the goniometric reading of the last sighting of the target (supposedly made exactly) is subtracted from that of the burst. It goes without saying that the graduations of the scales of the p's ought, in this case, to be connected no longer with the reference points of the observatory stations as origins, but with the directions of the loci passing through the target.

The advantages of this method of proceeding are numerous. In the first place, the errors in range and in direction do not depend on the error which may exist in the position of the target, at least within the limits in which this error remains slight. They are also independent of the errors of position of the observatories, since the common reference point is near the bursts (remark at the end of Sec. 4). From this point of view, the method is especially appropriate for adjustments to be carried out in open warfare, when there is generally no time to determine the exact positions of the stations. Finally, if a mobile or shifting target (gun mounted on rails, neighboring gun continuing the fire of the first) is to be fired on, the rather

frequent observation of the flashes of its rounds gives an opportunity of following the displacements or changes, modifying the adjustment accordingly, and consequently avoiding the waste of ammunition that would be involved in the continuation of the fire with the elements from the initial adjustment. It is scarcely necessary to add that, if the change of position of the target became so important that the directions of the loci of the gun P and of the scales were no longer sufficiently exact, it would be necessary to replace the construction sheet on the scale $\frac{1}{5000}$ by another relating to the last observed position of the target.

EXAMPLE I.

Calculation of U_{R1}, reference angle of the observatory O_1.

$x_{R1}=45702.5$ $\quad$ $y_{R1}=87382.3$

Formulæ: $\tan U_{R1}=\dfrac{x_{R1}-x_{o1}}{y_{R1}-y_{o1}}$ $\quad$ $x_{o1}=34226.0$ $\quad$ $y_{o1}=84482.0$

$x_{R1}-x_{o1}=+11476.5$ $\quad$ $y_{R1}-y_{o1}=+2900.3$

$\log (x_{R1}-x_{o1})=4.05981$

$\log (y_{R1}-y_{o1})=3.46244$

$\log \tan U_{R1}=\ \ .059737$

$U_{R1}=1347.9$ mils.

Calculation of the intersection of the division d=800 mils of the graduation relative to the observation station O_1, with the line x=47000:

Formulæ: $y=y_{o1}+(47{,}000-x_{o1})\cot (U_{R1}+800 \text{ mils})$. $\quad$ log.

47,000 $\quad$ $U_{R1}=1{,}347.9$ mils $\quad$ 4.1063

$x_{o1}=34{,}226$ $\quad$ 800 $\quad$ $\overline{1}.7756-$

$(47{,}000-x_{o1})=12{,}774$ $\quad$ $U_{R1}+800=2{,}147.9$ mils $\quad$ $3.8819-$

2^e term $=-7{,}619$

$y_o=\ 84{,}482$

$y\ =\ 76{,}863$

EXAMPLE II.

Calculation of the values of a division of 10 mils of each scale.

Formula: $p\ p_2$ 10 mils=40 KD. log K=$\bar{4}$.9920.

Observation stations.	O_1	O_2	O_3	Products of p by 1, 2, 3.....10			
				Factor.	O_1	O_2	O_3
					mm.	*mm.*	*mm.*
D in mm. scale of $\frac{1}{20000}$	220.4	154.1	149.8	1	8.7	4.8	5.9
log D.	2.3432	2.1878	2.1755	2	17.3	9.6	11.8
log K.	$\bar{4}$.9920	$\bar{4}$.9920	$\bar{4}$.9920	3	26.0	14.4	17.6
log 40.	1.6021	1.6021	1.6021	4	34.6	19.2	23.5
				5	43.3	24.0	29.4
log p.	0.9373	0.6819	0.7696	6	52.0	28.9	35.3
				7	60.6	33.7	41.2
				8	69.3	38.5	47.0
$p\ p_2$ 10 mils.	8.66 mm.	4.81 mm.	5.88 mm.	9	77.9	43.3	52.9
				10	86.6	48.1	58.8

Calculation of the adjustments to be applied toward the right to the divisions of the scales when the reference points are taken as origins of the graduations.

Formulae: Adjustment=[(Next highest multiple of 10 above the real value of α)$-(U^c-U_R)$]

Observation stations.	O_1	O_2	O_3
U^c	1,427.5	830.7	7,012.5
U_R	1,347.9	801.0	714.1
U^c-U_R	79.6	29.7	6,298.4
Next highest multiple of 10	80	30	6,300
$\alpha-(U^c-U_R)$	.4 mil.	.3 mil.	1.6 mil.
Adjustment.	.3 mm.	.1 mm.	.9 mm.

OCTOBER, 1916.