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# TM 5-237

## DEPARTMENT OF THE ARMY TECHNICAL MANUAL

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# SURVEYING COMPUTER'S MANUAL

HEADQUARTERS, DEPARTMENT OF THE ARMY
OCTOBER 1964

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TECHNICAL MANUAL
No. 5-237

HEADQUARTERS
DEPARTMENT OF THE ARMY
WASHINGTON, D.C., *30 October 1964*

# SURVEYING COMPUTER'S MANUAL


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# CHAPTER 1
# INTRODUCTION

## 1. Purpose

This manual is published to serve as a reference and guide for the survey computer in accomplishing geodetic and topographic survey computations; to standardize the methods and procedures for completing these computations; to standardize tabulation procedures for numerical figures and results for use in records or for dissemination; and to familiarize survey computers with the accepted methods of performing survey computations.

## 2. Scope

This manual contains descriptive material, references, sample solutions, and sample tabulations for all types of computations that are *not* usually completed in the field notebooks and which may be encountered in military topographic surveys. The instruction for completion of each of the computations contains both descriptive material and a detailed solution. Unless otherwise stated, all grid coordinates will be Universal Transverse Mercator Grid coordinates.

## 3. References

TM 5–236 contains many of the tables used in performing the computations included in this manual. Basic topographic surveying methods are discussed in TM 5–441. Other references are included in appendix I.

## 4. Accuracy

*a.* The solutions found within this manual are designed to meet any foreseeable need of the military survey computer. Most of the included computations, unless otherwise stated, will meet the requirements for first order. Computations designed for first order accuracy may be adapted to lower order surveys by relaxing some of the refinements.

*b.* Users of this manual are encouraged to submit recommended changes or comments to improve the manual. Comments should be keyed to the specific page, paragraph, and line of the text in which change is recommended. Reasons should be provided for each comment to insure understanding and complete evaluation. Comments should be forwarded directly to the Commandant, U.S. Army Engineer School, Fort Belvoir, Va., 22060.

# CHAPTER 2

# ASTRONOMIC TABLES

## 5. Introduction

The astronomic determination of direction, latitude, longitude, and time depends upon the apparent movement of stellar bodies. Astronomic tables, in general, list the directions and positions of planets and stars, their rates of apparent movement, and certain numerical quantities necessary to convert their apparent motions to more usable information for any given instant. The most commonly used astronomic tables are the American Ephemeris and Nautical Almanac (AE&NA), Apparent Places of the Fundamental Stars (APFS), and the General Catalogue of 33342 Stars by Benjamin Boss. The first two sets of tables are published annually while the latter has been published for the epoch of 1950.

## 6. The American Ephemeris and Nautical Almanac (AE&NA)

The American Ephemeris and Nautical Almanac is published annually by the U.S. Naval Observatory and is distributed by the Army as a Technical Manual, TM 5-236-XX, with the third number indicating the year for which the data is applicable. The volume has undergone two important changes during recent years. One is the deletion of the ten-day stars while the other was the introduction of Ephemeris time which is a more precise time. Corrections, necessary to convert Universal time to Ephemeris time, are included in the Ephemeris.

*a. Tables Included in the AE&NA.*

(1) $\Delta T$, reduction from Universal time to Ephemeris time.
(2) Universal and Sidereal Time for $0^h$ UT.
(3) Sun (year)—for $0^h$ Ephemeris Time.
(4) Besselian and Independent Day Numbers.
(5) Mean places of stars.
(6) Table II—for finding the latitude by the observed altitude of Polaris and azimuth of Polaris at all hour angles.
(7) Table VIII—Sidereal time to mean solar time.
(8) Table IX—Mean solar time to sidereal time.
(9) Table X—Conversion of hours, minutes and seconds to decimals of a day.
(10) Table XI—Conversion of time to arc.
(11) Table XII—Conversion of arc to time.
(12) Table XIII—Interpolation constants.
(13) Table XIV—Second-Difference corrections.

*b. Use of Ephemeris Time.* Starting with the AE&NA for 1960, the tabular argument in the fundamental ephemerides of the sun, moon, and the planets is Ephemeris time. Ephemeris time is the uniform measure of time as defined by the laws of dynamics and determined in principle from the *orbital motions of the planets;* specifically *the orbital motions of the earth* as represented by Newcomb's "Tables of the Sun". Universal time is defined by the *rotational motion of the earth,* and is determined from the apparent diurnal motions which reflect this rotation. Because of variations in the rate of rotation, Universal time is not rigorously uniform.

*c. Universal and Sidereal Times.* Beginning with the 1960 Ephemeris the Sidereal time at $0^h$ Universal Time and the Universal Time at $0^h$ Sidereal time, which formerly were included in the Ephemeris of the Sun, are tabulated both for the mean equinox of the date, and for true equinox, with the short period terms of nutation included.

*d. Tables of the Sun.*

(1) *The date column.* The dates found in this column represent the instant at $0^h$ Ephemeris Time on the date indicated.
(2) *The apparent right ascension column.* The apparent right ascension (RA) is given in units of time for each day. The light-faced type, to the right and between the lines, gives the tabular differences in

seconds. Linear interpolation is made by multiplying these terms by the fraction of a day elapsed since $0^h$, and adding the result to the right ascension given for $0^h$ of the date. Note that the sun always increases in right ascension and the tabular differences are positive.

(3) *The apparent declination column.* The apparent declination ($\delta$) is given in the third column. Interpolated values are determined in the case of right ascensions as in (2) above. However, both the declination and tabular differences may be either positive or negative. Care must be exercised in preserving the proper sign. Both the apparent right ascension and declination of the sun contain the effects of long-period terms of nutation and aberration, and define the apparent place (as the observer sees it) of the *true* sun.

(4) *The horizontal parallax column.* Parallax is the displacement in the position of a heavenly body due to observations being made from the surface of the earth instead of at its center. Horizontal parallax is the angle at the center of the sun subtended by the earth's equatorial radius. For solar observations, the true parallax is equal to the product of the horizontal parallax and the cosine of the altitude, or the sine of the zenith distance.

(5) *The semidiameter column.* The amounts shown in this column are correct for horizontal semidiameter but, due to the effects of refractions, they will vary slightly for vertical semidiameter.

(6) *The equation of time column.* The sixth column gives the equation of time stated as apparent minus mean. Tabular differences are also furnished. Note that both the equation and the difference may have either algebraic sign.

*e. Table VIII. Sidereal into Mean Solar Time.* Table VIII of the Ephemeris gives the quantities that must be subtracted from a given time interval expressed in sidereal units in order to obtain the same interval expressed in mean-time units. Or it represents the amount a mean-time clock would lose as compared to a sidereal clock over a given sidereal interval. The main table is given at intervals of 1 sidereal minute for the entire day, the hour being given at the head of the column and the minutes down the left-hand side. The corrections for seconds are given at the extreme right. Interpolation is made as in table IX (*f* below). The sum of the quantities from the main and seconds table is then subtracted from the sidereal interval to obtain the corresponding mean-time interval.

*f. Table IX. Mean Solar into Sidereal Time.* This table of the Ephemeris is more frequently used than table VIII. It gives the quantities that must be added to a given interval expressed in mean-time units in order to obtain the same interval in sidereal units. It is the amount a sidereal clock gains with respect to a mean-time clock over a given mean-time interval. Table IX has the same form as table VIII. In order to convert a mean-time interval to a sidereal interval, first enter the main table in the column for the hour, and on the line corresponding to the last full mean-time minute. Next, the table to the right is entered using the seconds of mean time over the last full minute. Any interpolation for desired fractions of a second is made mentally. The final quantity from the seconds table is then added to the quantity taken from the main table. This gives the correction to be added to the mean-time interval.

*Example:* Find the sidereal-time interval corresponding to a mean-time interval of:

| | | | |
|---|---|---|---|
| | $17^h$ | $14^m$ | $12^s.7$ |
| From main table, correction for $17^h\ 14^m$ = | | $2^m$ | $49^s.860$ |
| From second table, correction for $12^s.7$ = | | | $0^s.035$ |
| Sidereal interval = | $17^h$ | $17^m$ | $02^s.595$ |

*g. Mean Places of Stars.* Mean places of 1078 stars are given in this table for the instant of the beginning of the Besselian year. This date occurs when the sun's mean longitude is 280° and falls very close to the beginning of the calendar year. The civil date is given in decimal days at the top of the page. The mean place does not coincide with the apparent place for the same date, but constitutes a base for the application of corrections in order to find the apparent place at any given date. The formulas for this reduction mean to apparent place, are given in the section of the American Ephemeris devoted to the use of the tables, under Stars. This reduction is seldom required of the computer. The mean-place tables are useful in preparing observing lists, and for

any purpose where a close value of the star's position is not required.

*h. Latitude From the Altitude of Polaris.* This table affords a means of determining the latitude when the altitude ($h$) of Polaris and the local sidereal time (LST) are known.

(1) Three corrections; $a_0$, $a_1$ and $a_2$ are extracted from table II. The arguments are the local sidereal time, the approximate latitude, and the month of observation respectively.

(2) The observed altitude is corrected for refraction. The latitude is determined by taking the algebraic sum of the corrected altitude and the three corrections from table II.

*i. Azimuth of Polaris.* Table II is also used to determine the azimuth of Polaris. The procedure is as follows:

(1) Extract the values for $b_0$, $b_1$ and $b_2$ from table II, using the local sidereal time (LST) of observation, the latitude and the month of observation respectively, as the arguments. These corrections are added algebraically.

(2) Multiply the quantity $(b_0+b_1+b_2)$ by the secant of the latitude to obtain the azimuth of Polaris, as referenced to the Pole.

*j. Besselian and Independent Day Numbers.* These day numbers, used for reducing mean to apparent place, are coefficients of the effects on the stars position caused by the processes of precession, nutation and aberration. They are computed by trigonometric means from the mean coordinates of the star and are dependent only on the Greenwich Ephemeric Time Date. Factors for proper motion are abstracted directly from the star catalogue. Either the Besselian or the Independent system may be used in the reductions although the Besselian system is more convenient for mass production.

*k. Table XII. Conversion of Arc to Time.* This table is often useful in avoiding division by 15. In successive columns, the time equivalents of degrees, minutes, seconds, and decimal seconds of arc are given.

*Example:* Convert 32°44′42″.15 to units of time.

| | | | |
|---|---|---|---|
| 32° | $=2^h$ | $08^m$ | |
| 44′ | = | $2^m$ | $56^s$ |
| 42″ | = | | $2^s.8$ |
| 0″.15 | = | | $0^s.01$ |
| Sum | $=2^h$ | $10^m$ | $58^s.81$ |

*l. Table XI. Conversion of Time to Arc.* This table is the inverse of table XII. The first part is a table in arguments of hours and minutes, from which is taken the arc equivalent of the even minute. The right hand section gives the arc equivalents of seconds and 100ths of seconds of time.

*Example:* Convert $2^h\ 10^m\ 58^s.81$ to arc units.

| | | | |
|---|---|---|---|
| $2^h\ 10^m$ | $=32°$ | 30′ | |
| $58^s$ | = | 14′ | 30″ |
| $0^s.81$ | = | | 12″.15 |
| Sum | $=32°$ | 44′ | 42″.15 |

*m. Table X. Conversion of Hours, Minutes, and Seconds to Decimals of a Day.* This table is useful in finding the decimal day equivalent to the UT for the purpose of entering the tables for the apparent places of stars. The equivalent of the hours and minutes is taken from the main table. That of the remaining seconds is given in the right hand column. This latter refinement is rarely necessary.

*n. Table XIII. Besselian Interpolation Constants.* The use of this table is explained in paragraph 9*d*.

## 7. The Apparent Places of the Fundamental Stars (APFS)

*a. Introduction.* The tables in the Apparent Places of the Fundamental Stars are the result of international cooperation which has reduced the duplication of certain parts of the ephemerides published by contributing nations. The data contained in this publication is limited to tables of the stars position and certain auxiliary tables.

*b. The Main Table.* The main table in the APFS lists the apparent positions at upper transit of 1483 stars, between 81° north and 81° south declination, at 10-day intervals throughout the year. The column headings contain the catalog number, name, magnitude and type of spectrum of each star. Right ascensions are listed in units of time to thousandths of seconds. Declinations are in units of arc to the hundredths of seconds. In both cases, the seconds values are followed by the tabular differences. At the foot of each page are found the mean place, secant, and tangent of $\delta$, and factors for computing short period terms of nutations for each star. The dates when two transits of the Greenwich meridian occur, during the same mean-time day, are also given.

*c. The Table of Circumpolar Stars.* Immediately following the main table is the table of Circumpolar Stars, which lists the apparent positions of 52 circumpolar stars for every upper star transit at Greenwich. The date refers to the civil day. Interpolation is made from the Greenwich Hour Angle of the star at the time of observation. The one-day interval, between tabulations, permits the inclusions of short period terms of nutation within the tabulated values. Right ascensions are given to two decimal places only, this being in the order of the uncertainty of circumpolar star positions. Otherwise the tables are similar to the 10-day star tables.

*d. The $FK_4$ System.* The "Apparent Places of Fundamental Stars" for 1964, and subsequent years, contains the 1535 stars in the Fourth Fundamental Catalogue ($FK_4$). This volume provides the mean and apparent places of 10-day and Circumpolar stars together with tables for their reduction.

*e. Table I.* Table I in the APFS furnishes factors for computing the short-period terms for the 10-day stars. The equations for accomplishing this are found at the foot of the pages of the table. The other necessary coefficients are tabulated under each star in the apparent-place table.

*f. Table II.* The sidereal time of $0^h$, Universal Time and the long- and short-period terms of the Equation of the Equinoxes are given. The apparent sidereal time is the sum of the mean sidereal time plus period terms.

*g. Table III.* Table III provides the conversion factors, mean solar to sidereal time and is identical to table IX of the AE&NA.

*h. Table IV.* Table IV is used for converting intervals of sidereal time to mean solar time and is identical to table VIII of the AE&NA.

*i. Table V.* Table V is used for reducing hours, minutes, and seconds to decimals of days and is similar to table X of the AE&NA with the exception that table X is a six-place table while table V is a five-place table.

*j. Table VI.* Table VI lists second difference corrections for use with linearly interpolated values. This table is somewhat different from the table of Besselian coefficients. The quantities given are for the term B'' $(\Delta_0''+\Delta_1'')$, the symbols being explained in paragraph 9*d*. The arguments are the fractional part $n$, and the double second difference $(\Delta_0''+\Delta_1'')$. The latter factor is at a tabular interval of 5 units of the last place of the fraction. The correction is always of opposite sign to $(\Delta_0''+\Delta_1'')$.

*k. Table VII.* Table VII is used for correcting the time of transit for the effect of diurnal aberration. The correction is rarely needed.

## 8. General Catalogue of 33342 Stars By Benjamin Boss

This catalogue consists of five volumes, the first being used for instructions and appendixes and the other four for mean places of stars. This catalogue is principally used by the geodesist in the determination of latitude. Mean to Apparent Place reduction is accomplished by means of the Besselian or Independent Day numbers from the AE&NA. Statistical and historical information contained in these tables is explained in the first volume.

## 9. Interpolation

*a. Introduction.* As the computer will have an almost continuous need for interpolation in the use of various tables, this paragraph will review linear and double interpolation.

*b. Linear Interpolation.* Geodetic surveys and astronomy will rarely have need for more than simple linear interpolation. This assumes that the function varies as a constant ratio, that is, as a straight line, between tabular values. Most functions are actually curves when plotted on the coordinate axis. Hence, a linear interpolation is subject to some error. The amount of the error depends on the sharpness of the curve and the spacing of the tabular values. All good tables are so arranged that the errors are nearly always negligible.

(1) When the interpolated value is taken as the proportional part of the difference between successive values given in the table, it is said to be interpolation on the chord. The tables of the American Ephemeris which give these tabular differences in smaller type are so arranged that the interpolation is along the chord.

(2) Another form of linear interpolation is said to be on the tangent. In this case, the small type figure is the rate of change, or slope of the tangent at the value of the function as given in the table. This form is more accurate than interpolation along the chord, provided not more than a half-interval is taken. These forms of

linear interpolation are shown in figure 1. The function is represented in the figure by the curve PQ, at which point its $y$ values are given in the table. It is desired to interpolate for the $y$ value when $x=0.8$. Point A represents the true value on the curve. Point B represents the value found by interpolation on the chord, the error being B—A. Interpolation along the tangent from the nearest tabular value Q gives a result at C. The error C—A is less than B—A. Point D is found by interpolating along the tangent from P, and the error is greater than that obtained on the chord

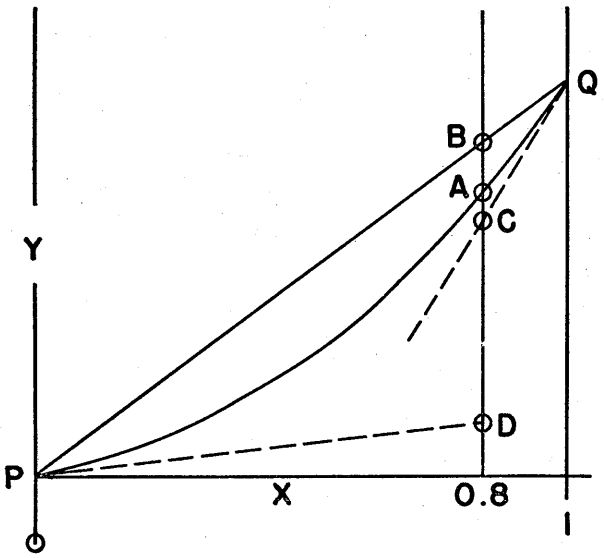



*Figure 1. Forms of linear interpolation.*

*c. Double Interpolation.* Double interpolation becomes necessary when a function is subject to two variables instead of one. The requirements of most double interpolations are met by a series of three chord interpolations; and the use of chord interpolations is recommended for this purpose. For example, the parallax of the sun varies during different dates of the year, and varies with the altitude of the various observations. The process of interpolation is as follows:

(1) *Example:* Determine the parallax for June 10, 1964 when the observed altitude is 33°31′19″.

(2) Parallax—from table VIII (app. III)

| *Date* | *Altitude* | *Parallax* | *Date* | *Altitude* | *Parallax* |
|---|---|---|---|---|---|
| June 1 | 36° | 7″.02 | July 1 | 36° | 7″.01 |
| June 1 | 33° | 7″.28 | July 1 | 33° | 7″.26 |

(3) Step 1—Interpolate for parallax on June 10 at 36° altitude. Use nearest tabulated value. $p=07''.02-1/3(07''.02-07''.01)=07''.017$

(4) Step 2—Interpolate for parallax on June 10 at 33° altitude. $p=07''.28-1/3(07''.28-07''.26)=7''.273$

(5) Step 3—Interpolate for parallax at altitude of 33°31′19″. $p=07.273-1/6(07''.273-07''.017)=07''.230=07''.23$

*d. Besselian Interpolation.* This method is required only in the highest class of computations. Interpolation to second and higher differences brings successively closer approximations of the true value on the curve. The tabular differences taken from the table are known as the first differences. The successive differences between the first differences are called second differences, and so on. The customary designation of these differences, to second differences only, appears below.

| *(Function) Tabular value* | *1st diff.* | *2d diff* |
|---|---|---|
| $F_{-1}$ | | |
| | $\Delta'_{-1/2}$ | |
| $F_0$ | | $\Delta''_0$ |
| | $\Delta'_{1/2}$ | |
| $F_1$ | | $\Delta''_1$ |
| | $\Delta'_{1\frac{1}{2}}$ | |
| $F_2$ | | |

The desired value $F_n$ lies at a fraction $n$, between $F_0$ and $F_1$. Bessel's formula is generally preferred. It is written as:

$$F_n=F_0+n\Delta'_{1/2}+B''(\Delta''_0+\Delta''_1)+[B''\,'\Delta''_{1/2}\,']+\ldots.$$

The quantity in the brackets represents the third difference, given for the sake of completeness. Only the first three terms are needed for second differences. The terms B″, B″ ′, and so on, are known as the Besselian interpolation constants. B″ and B″ ′ are given in table XV of the American Ephemeris for values of from 0 to 1. Their values are:

$$B''=\frac{n(n-1)}{2(2!)} \qquad B''\,'=\frac{n(n-1)(n-\frac{1}{2})}{3!}$$

and may be so derived in the absence of the tables. The terms $\Delta'_{1/2}$, $\Delta''_0$, and $\Delta''_1$, are taken from a tabulation as shown above.

*Example:* Find, with respect to second differences, the apparent declination of the sun at UT June 1.67, 1963.

| *Date* | *Decl.* | *1st diff.* | *2d diff.* |
|---|---|---|---|
| May 31 | $(F_{-1})+21°46'44''.6$ | | |
| | | $+522.2(=\Delta'_{-1/2})$ | |
| June 1 | $(F_0)+21°55'26''.8$ | | $-22.8(=\Delta_0'')$ |
| | | $+499.4(=\Delta'_{1/2})$ | |
| June 2 | $(F_1)+22°03'46''.2$ | | $-23.2(=\Delta_1'')$ |
| | | $+476.2(=\Delta'_{1\frac{1}{2}})$ | |
| June 3 | $(F_2)+22°11'42''.4$ | | |

The fraction $n=0.67$. Hence, from table XV, B'' is $-0.0553$. B''' if required would be $-0.0063$. Applying the formula:

$$F_n=+21°55'26.8+0.67(+499.4)$$
$$-0.0553(-46.0)=+22°01'03''.9.$$

This interpolation may be simplified with little loss of accuracy by taking the term, $B''(\Delta_0''+\Delta_1'')$ from table VI of the Apparent Places of Fundamental Stars. For the above example, $n=0.67$ and $(\Delta_0''+\Delta_1'')=46.0$. The correction taken from table VI is $+25$ in units of the last decimal place. This, added to $n\Delta'_{1/2}$ which is equal to $334''.6$, gives 337.1 as the interpolated difference. The final value is $+22°01'03''.9$, as by the first solution. Care should be taken that the double second difference and the figure taken from the table be in the proper decimal place.

# CHAPTER 3

# ASTRONOMIC OBSERVATION COMPUTATIONS

## Section I. CONVERSION OF TIME

### 10. Kinds of Time

The geodetic computer will be concerned with three kinds of time in astronomical and solar computations. They are apparent sidereal, mean solar, and apparent solar.

*a. Apparent Sidereal.* Apparent sidereal time is generally used in astronomical computations. Various expressions of sidereal time will confront the computer and the most common are listed below.

(1) Greenwich Sidereal Time (GST) is apparent sidereal time at the zero meridian of longitude near Greenwich, England. Greenwich Sidereal Time is zero hours at the instant of upper transit of the Greenwich meridian (0°λ) by the apparent motion of the vernal equinox.

(2) Local Sidereal Time is sidereal time at the local meridian, e.g., the meridian of a survey station, and is zero hours at the instant of upper transit of the local meridian by the apparent motion of the vernal equinox.

(3) Mean Sidereal Time is not used in astronomical computations.

*b. Mean Solar.* The mean solar day is measured by the fictitious mean sun between successive meridian passages. The solar year is identical in length to the sidereal year, but due to the apparent movement of the sun, it contains 1 day less. The mean solar day is therefore about 3 minutes and 56 seconds longer than the sidereal day. Mean solar time is that used in everyday life.

(1) Local Mean Time (LMT) is mean solar time at the local meridian and is the hour angle of the mean sun measured westward from the local meridian. Local Mean Time is 1200 hours at the instant of upper transit by the mean sun across the local meridian.

(2) Standard time is mean solar time at an adopted central meridian for a 15° wide time zone. In the United States, for example, the central meridians of the time zones are 75°, 90°, 105°, and 120° West of Greenwich corresponding to Eastern (EST), Central (CST), Mountain (MST), and Pacific (PST) time zones respectively. For any particular time zone the standard time is 1200 hours at the instant of upper transit by the mean sun across the central meridian of the zone.

(3) Daylight Saving Time is standard time plus one hour adopted for the general convenience of the public during the months of the year having the longest period of daylight hours, i.e., in the United States from April to October.

(4) Universal Time (UT) is mean solar time at the 0° meridian, and corresponds essentially to Greenwich civil time (GCT). There are three categories of Universal Time called UT0, UT1 and UT2.

(*a*) UT0 is mean solar time determined astronomically by individual observatories and referenced to the Greenwich meridian by application of difference in longitude. UT0 is not corrected for polar motion.

(*b*) UT1 is obtained by applying the correction for polar motion to the uncorrected Universal Time (UT0) by the observatory. The correction to the UT2 signal to obtain UT1 and UT0 is published in Time Service

publications of the major observatories. UT1 is equal to UT2 minus S, where S is the extrapolated seasonal variation in speed of rotation of the earth.

(*c*) UT2 is Universal Time (UT0) corrected for polar motion and for extrapolated seasonal variation in speed of rotation of the earth. Time service bulletins of the major observatories publish the correction to be applied to the time signal in order to obtain UT2.

*c. Apparent Solar.* Apparent solar time is kept by the actual sun. An apparent solar day is the interval between two successive meridian passages of the sun, and varies in length by about 30 minutes during the year, due to irregular apparent motion of the sun. Apparent time is necessary in computing some observations on the sun.

*d. Ephemeris.* Ephemeris Time (ET) is the independent variable in the gravitational theories of the Sun, Moon, and planets. If it is desired to convert Ephemeris Time to Universal Time, the following relationship may be used: UT=ET−ΔT. ΔT is the amount ET is ahead of UT and its value is published in the American Ephemeris and Nautical Almanac.

## 11. Conversion of Time

It is frequently required to convert one kind of time to another. This is done by the following processes:

*a.* To find the sidereal time of a given mean solar time (fig. 2), use the tables in part I of the American Ephemeris or table II in the Apparent Places of Fundamental Stars and DA Form 1900 (Conversion of Mean Time to Sidereal Time). These tables give the apparent sidereal time corresponding to $0^h$, Universal Time, for each day of the year. This is the mean solar time of the beginning of the day (midnight) at the Greenwich meridian.

(1) Find the Universal Time (UT, also called GCT) and date by adding algebraically the longitude in hours, minutes, and

PROJECT: 2 - 21 - 32

**CONVERSION OF MEAN TIME TO SIDEREAL TIME** (TM 5-237)

LOCATION: MARYLAND

ORGANIZATION: A & S, INC.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DATE | 9 July 1963 | | | | | | | | | | | |
| LONGITUDE W | ° 77 | ′ 04 | ″ 20.628 | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ |
| | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS |
| 1. Recorded {~~Local mean time~~ / Local std. time} | 21 | 07 | 52.093 | | | | | | | | | |
| 2. ~~(Watch)~~ (Chronometer) correction (F−, S+) | − 0 | 00 | 03.100 | | | | | | | | | |
| 3. Corrected {~~Local mean time~~ / Local std. time} | 21 | 07 | 48.993 | | | | | | | | | |
| 4. Longitude or time zone difference (W+, E−) | + 5 | 00 | 00.000 | | | | | | | | | |
| 5. Universal time (UT) (3+4) 10 July 1963 | 2 | 07 | 48.993 | | | | | | | | | |
| 6. Sidereal time of $0^h$ UT for Greenwich date | 19 | 08 | 47.571 | | | | | | | | | |
| 7. Correction for sidereal gain | + 0 | 00 | 20.997 | | | | | | | | | |
| 8. Greenwich sidereal time (GST) (5+6+7) | 21 | 16 | 57.561 | | | | | | | | | |
| 9. Longitude (hr., min., sec.) (W+, E−) | + 5 | 08 | 17.375 | | | | | | | | | |
| 10. Local sidereal time (LST) (8−9) | 16 | 08 | 40.186 | | | | | | | | | |

COMPUTED BY: W.R.Green - AMS — DATE: 13 July 1963

CHECKED BY: G.T.Tennis - AMS — DATE: 14 July 1963

DA FORM 1 FEB 57 1900

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420537

*Figure 2. Conversion of Mean Time to Sidereal Time (DA Form 1900).*

seconds of the place from Greenwich to the given mean time. If the given time is a standard time, add the number of hours corresponding to the time zone of the place. Longitudes and times west of Greenwich are positive, and east of Greenwich are negative. If the sum is more than 24 hours, subtract 24 hours and add 1 day to the date.

(2) Enter the table for the Greenwich date and find the sidereal time of $0^h$ UT from the table in the American Ephemeris, or in table II of the Apparent Places of Fundamental Stars. This term is also known as RAMS+$12^h$ (right ascension of mean sun). The sun's right ascension is measured from the upper meridian, while the beginning of the day is referred to the lower meridian. Hence, it is necessary to add 12 hours to the sun's RA.

(3) Since the sidereal units are shorter than mean time units, the sidereal time will constantly gain with respect to mean time, and a correction for this must be applied to the interval between $0^h$ UT and the UT of the observation. This is found in table IX, American Ephemeris, or table III, Apparent Places of Fundamental Stars. The tabular differences are minutes of mean time. An auxiliary listing in the right hand column of either table gives the correction for additional seconds in the mean time interval.

(4) Add the UT found in (1) above, the sidereal time of $0^h$ from (2) above, and the total correction from (3) above. This gives the Greenwich sidereal time (GST) of the given mean time.

(5) Subtract the longitude of the place from the GST to obtain the local sidereal time (LST) of the given mean time.

*b.* To find the local mean time (LMT) of a given sidereal time (fig. 3), use tables as noted and DA Form 1901 (Conversion of Sidereal Time To Mean Time).

(1) Add the longitude of the place to the local sidereal time to obtain the GST.

(2) Subtract from the GST the sidereal time of $0^h$ UT for the date to obtain the sidereal interval since $0^h$ UT.

(3) Subtract the correction, sidereal to mean solar time for this interval from table VIII, American Ephemeris, or table IV, Apparent Places of Fundamental Stars. This gives the UT.

(4) Subtract the longitude of the place from the UT to obtain the local mean time, or subtract the time zone correction to obtain local standard time.

*c.* To find the apparent solar time of a given mean time (fig. 4), use tables in part I of the American Ephemeris, or any other solar ephemeris, and DA Form 1902 (Conversion of Mean Time to Apparent Time).

(1) Add the longitude to the given local mean time to obtain UT.

(2) Take from the table the equation of time for the date. This value applies to $0^h$ UT. Note proper algebraic sign.

(3) Multiply the daily change of the equation of time by the fraction of a day elapsed since $0^h$ UT.

(4) Add algebraically the amounts from (2) and (3) above. The sum is the equation of time for the given time.

(5) Add algebraically the equation of time to the local mean time to obtain the local apparent time.

*d.* To find the local mean time of a given local apparent time (fig. 5), use the tables. The tables are made for mean time units, and since apparent time is given, a first approximation must be made for obtaining the equation of time.

(1) Find the Greenwich apparent time (GAT) by means of the longitude as above. This will seldom differ from the UT by more than 0.01 day.

(2) Subtract the equation of time for $0^h$ UT corrected for the elapsed interval in apparent time for the GAT. This gives a close approximation of the UT.

(3) Recompute the equation of time for the elapsed interval of mean time.

(4) Subtract this value from the LAT to obtain the LMT.

(5) The equation of time is the same at any given instant for all points in the world.

| PROJECT | 2-21-32 | | | | | | | | | | | CONVERSION OF SIDEREAL TIME TO MEAN TIME (TM 5-237) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LOCATION | MARYLAND | | | | | | ORGANIZATION | A & S, INC. | | | | | | |
| LOCAL DATE | 9 July 1963 | | | | | | | | | | | | | |
| LONGITUDE W | 77° | 04′ | 20.628″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | | |
| | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS | | |
| 1. Recorded local sidereal time (LST) | 17 | 08 | 46.896 | | | | | | | | | | | |
| 2. Watch correction (F−, S+) | − 0 | 00 | 03.602 | | | | | | | | | | | |
| 3. Corrected LST | 17 | 08 | 43.294 | | | | | | | | | | | |
| 4. Longitude (W+, E−) | + 5 | 08 | 17.375 | | | | | | | | | | | |
| 5. Greenwich sidereal time (GST) (3+4) | 22 | 17 | 00.669 | | | | | | | | | | | |
| 6. GST of 0h UT for Greenwich date* | | 08 | 47.571 | 10 | July | 1963 | | | | | | | | |
| | 19 | 04 | 51.014 | 9 | July | 1963 | | | | | | | | |
| 7. Sidereal interval since 0h UT (5−6) | | 08 | 13.098 | | | | | | | | | | | |
| | 03 | 12 | 09.655 | | | | | | | | | | | |
| 8. Correction for mean time lag | | | 30.834 | | | | | | | | | | | |
| | 0 | 00 | 31.481 | | | | | | | | | | | |
| 9. Universal time (UT) (7−8) | | 07 | 42.264 | | | | | | | | | | | |
| | 03 | 11 | 38.174 | | | | | | | | | | | |
| 10. ~~(Longitude)~~ or (time zone) difference | + 5 | 00 | 00.000 | | | | | | | | | | | |
| 11. ~~Local mean time (LMT)~~ Local std. time (L STD T) (9−10) | | 07 | 42.264 | | | | | | | | | | | |
| | 22 | 11 | 38.174 | | | | | | | | | | | |

*If Greenwich date is doubtful use local date for trial computation. At step (11) determine correct Greenwich date and if necessary rework computation from step (6) to end.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| J. A. Andrews - AMS | 13 July 1963 | G. T. Tennis - AMS | 14 July 1963 |

DA FORM 1 FEB 57 1901

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420985

*Figure 3. Conversion of Sidereal Time to Mean Time (DA Form 1901).*

| PROJECT 2-21-32 | CONVERSION OF MEAN TIME TO APPARENT TIME (TM 5-237) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LOCATION OHIO | ORGANIZATION A & S, INC. | | | | | | | | | | | |
| DATE | 18 MAY 1963 | | | | | | | | | | | |
| LONGITUDE W | ° 83 | ' 48 | " 24 | ° | ' | " | ° | ' | " | ° | ' | " |
| | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS | HOURS | MIN. | SECONDS |
| 1. Recorded local mean time (LMT) | 11 | 56 | 17.8 | | | | | | | | | |
| 2. Watch correction (F−, S+) | −0 | 00 | 03.1 | | | | | | | | | |
| 3. Corrected local mean time (LMT) | 11 | 56 | 14.7 | | | | | | | | | |
| 4. Longitude (TIME) | +5 | 35 | 13.6 | | | | | | | | | |
| 5. Universal time (UT) (3+4) | 17 | 31 | 28.3 | | | | | | | | | |
| 6. Equation of time for 0h Greenwich date | 0 | 03 | 41.0 | | | | | | | | | |
| 7. Variation of equation of time for Greenwich date | −0 | 00 | 02.0 | | | | | | | | | |
| 8. Fraction of day elapsed (5) ÷ 24 | | 0.730 day | | | | | | | | | | |
| 9. Correction to equation of time (7×8) | −0 | 00 | 01.5 | | | | | | | | | |
| 10. Corrected equation of time (6+9) | 0 | 03 | 39.5 | | | | | | | | | |
| 11. Local apparent time (LAT) (3+10) | 11 | 59 | 54.2 | | | | | | | | | |
| COMPUTED BY J. Green - AMS | DATE 20 MAY 1963 | | | CHECKED BY G.T. Tennis - AMS | | | DATE 21 MAY 1963 | | | | | |

DA FORM 1 FEB 57 1902

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420810

*Figure 4. Conversion of Mean Time to Apparent Time (DA Form 1902).*

CONVERSION OF APPARENT TIME TO MEAN TIME

Local Date 18 May 63

| | |
|---|---|
| Local Apparent Time (LAT) | $12^h00^m00^s$ |
| Longitude of Station, degrees | 83 48 24 W |
| Longitude of Station, hours + | $5^h35^m13^s.6$ |

| | |
|---|---|
| Recorded LAT | $12^h00^m00^s$ |
| Watch Correction | 00 |
| Corrected LAT | 12 00 00 |
| Longitude Difference + | 5 35 13.6 |
| Greenwich Apparent Time (GAT) | 17 35 13.6 |
| Equation of Time, 18 May (from Table) + | 3 41.0 |
| Correction for Interval= -2.00 (17.6/24) - | 0 01.5 |
| Corrected Equation of Time + | 3 39.5 |
| Approximate Universal Time | 17 31 34.1 |
| Correction for Interval= -2.00(17.53/24) - | 0 01.5 |
| Final Corrected Equation of Time + | 3 39.5 |

| | |
|---|---|
| Corrected LAT | 12 00 00.0 |
| Equation of Time + | 3 39.5 |
| Local Mean Time (LMT = LAT-Eq. of Time) | 11 56 20.5 |

*Figure 5. Conversion of apparent time to mean time.*

## Section II. COMPUTATION OF AZIMUTH

### 12. Method of Computation

*a.* The observation of astronomic azimuth consists of observing the angle between a mark on the earth's surface and a star or the sun. The computation consists of calculating the azimuth of the celestial body at the time of observation, then subtracting the measured angle from this value to obtain the azimuth of the mark.

*b.* The calculation of the azimuth of the star or sun involves the solution of the spherical triangle whose vertices are the pole, the observer's zenith, and the body observed. This triangle is known as the astronomic triangle or the PZS triangle (fig. 6). Since the body is apparently moving, the time of the observation must be known except in some special cases.

*c.* If the angles of a spherical triangle are designated A, B, and C, and the sides opposite them as $a$, $b$, and $c$, just as is customary in plane trigonometry, a fundamental formula can be derived for the solution of the triangle when any three of its elements are known.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

All other formulas for the solution of the spherical triangle may be derived from this fundamental equation.

*d.* In the astronomic triangle, the angle at the zenith, between the pole and the celestial body, is the azimuth of the body, hereafter designated A. The angle at the pole, between the zenith and the body, is the hour angle, designated $t$. The angle at the star or sun, usually denoted by $q$, is the parallactic angle. The parallactic angle will seldom be used in astronomic computations.

*e.* The side of the triangle opposite the azimuth angle, A, is the arc of the hour circle between

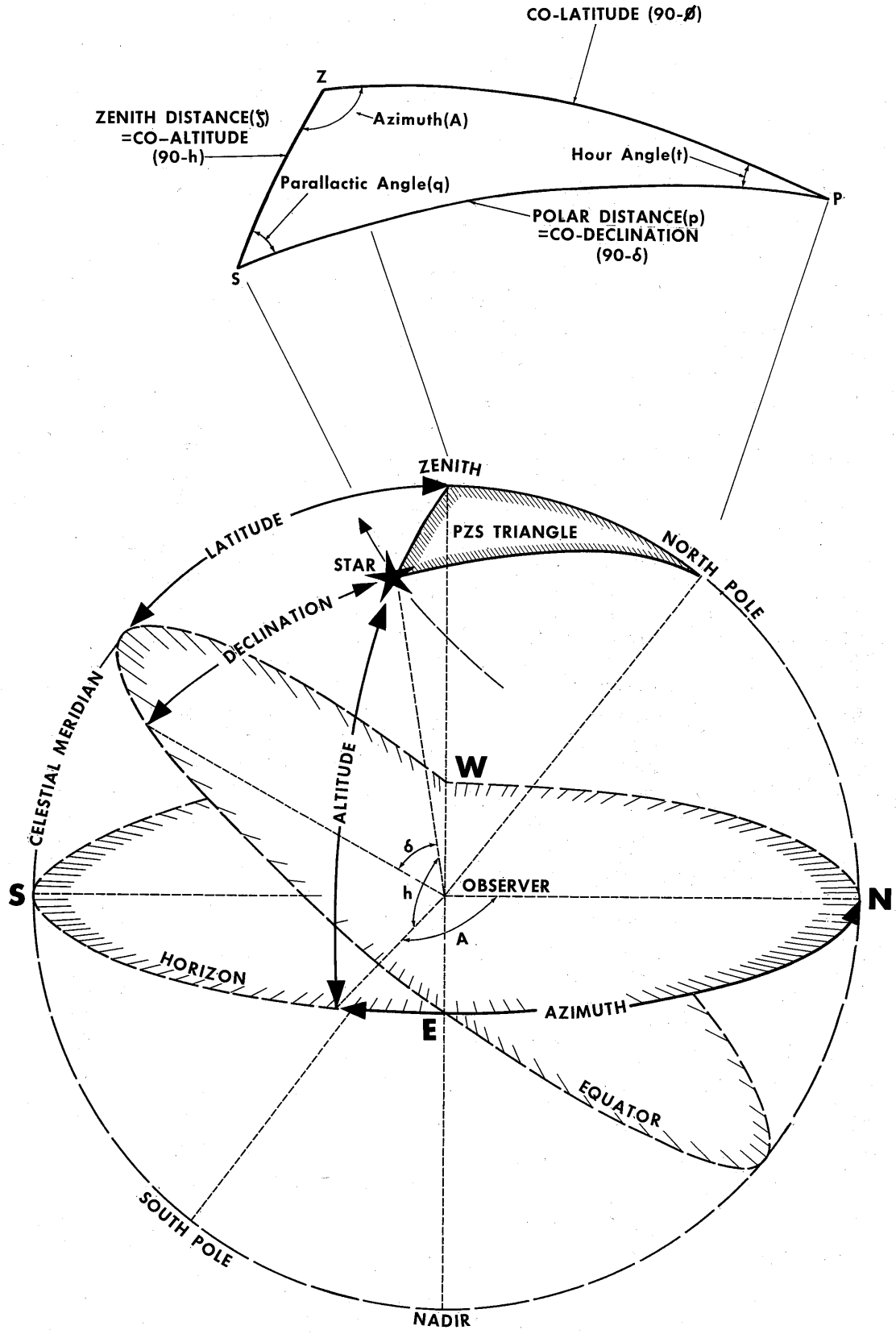



*Figure 6. PZS triangle.*

the pole and the star (or sun) and is known as the polar distance ($p$) or codeclination ($90°-\delta$). It is obtained by subtracting the star's declination from 90°. In most cases this subtraction need not be made, since the cofunction of the declination itself can be used instead. The side opposite the hour angle at the pole is the arc of the great circle between the zenith and the star. This is known as the star's zenith distance, designated by the Greek letter $\zeta$. The zenith distance is either observed directly, or its complement, the altitude ($h$), is subtracted from 90°. The cofunction of the altitude is frequently used in place of the required function of the zenith distance. The third side, lying opposite the parallactic angle, is the arc of the observer's meridian between the pole and the zenith. It is obtained by subtracting the observer's latitude from 90°, and is sometimes known as the colatitude ($90°-\phi$). In nearly all practical formulas, the cofunction of the latitude is used.

*f.* The hour angle ($t$) is obtained from the recorded time of the observation and the right ascension of the body observed. In the case of a star, the local sidereal time (LST) is required. This may be found by observing the altitude, or the time of transit across the meridian of known stars; or from radio time signals, provided the longitude of the station is well known. The hour angle ($t$) equals the LST minus the right ascension. The hour angle is measured westward from the upper meridian from $0^h$ to $24^h$. For convenience, the $t$ angle is limited to the first 2 quadrants ($0^h$ to $12^h$) on the computing forms, and is considered as measured both west and east from the upper meridian. The latter direction is considered negative. Should the hour angle, found by subtracting the right ascension from the LST, fall between $12^h$ and $24^h$, it is subtracted from $24^h$ to obtain the negative $t$ angle.

*g.* The right ascensions and declinations of the stars are given in the Apparent Places of Fundamental Stars. These are given at intervals of Universal Time (UT). Hence, the local observed time must be converted to UT before taking out the right ascensions and declinations, as explained in paragraph 9. The right ascension and declination of the sun are given in the American Ephemeris and many other publications.

*h.* Since only three parts of the astronomic triangle are required in order to compute the azimuth, different combinations may be observed in the field. Thus, we may be given (1) the latitude and declination, and observe the altitude; (2) the latitude and declination, and observe (indirectly) the hour angle; (3) the declination, and observe the altitude and hour angle. There are also special cases, such as observations at elongation or culmination, when the star's position is found by trial without knowing the time.

*i.* The computer frequently must apply some corrections to observed values. Since the star (or sun) is generally observed at considerable altitude, an error is introduced in projecting its direction downward to the horizon whenever the axis of the telescope is not truly horizontal. For the inclinations involved, the correction, $c$, is—

$$c''=i''\tan h$$

in which $c$ and $i$ are in seconds of arc; $i$ is the inclination of the telescope axis as determined by the readings of the plate level or a striding level; and $h$ is the altitude of the star. This correction is not required in third-order computations.

(1) *Computing the inclination correction.* In order to compute the inclination, $i$, the sensitivity value of the level bubble in seconds of arc per division, and the displacement of the bubble in divisions from its level position must be known. The scale should be read at both the left and right hand ends of the bubble on both the direct and reversed pointings on the star.

(*a*) If the scale reads continuously from one end of the tube to the other, the record appears as—

| | *Left* | *Right* |
|---|---|---|
| Direct | 07. 5 | 16. 8 |
| Reversed | 17. 7 | 08. 5 |
| | 10. 2 | 08. 3 |
| | +01. 9 | |

The final figure +01.9 is the inclination factor, and is actually four times the mean displacement of the bubble for the two pointings. It is found as follows: The smaller value is subtracted from the larger in each of the columns, indicating readings taken at the left and right ends of the bubble. The difference in the right hand column is then subtracted algebraically from that in the left. That is, if left is greater than right, the inclination is positive; if right is greater than left,

it is negative. In case a striding level is used, it should be reversed on the axis during each pointing, D and R. A record identical to the above, and computed in the same manner will be obtained for each of the pointings. The final inclination factor is then the algebraic mean of the inclinations of the two pointings.

(*b*) Occasionally, a record will be made by reading the scale outward in both directions from its middle, as—

| | *Left* | *Right* |
|---|---|---|
| Direct | 05. 0 | 04. 3 |
| Reversed | 05. 2 | 04. 0 |
| | 10. 2 | 08. 3 |
| | +01. 9 | |

In this case, the columns are added; then right is subtracted from left, as before. When observing on Polaris, some observers may mark the columns west and east instead of left and right. The inclination factor must then be multiplied by the level factor, $\frac{d}{4}\tan h$, in which $d$ is the value of a division of the level scale in seconds of arc, and $h$ is the altitude of the object observed. The value $d$ for the instrument used must be furnished the computer. The final figure,

$$i\times\frac{d}{4}\tan h$$

is the correction which must be added algebraically to the circle reading taken on the star (or sun).

(2) *Correction for refraction.* An inclined ray of light is subject to bending in passing through the earth's atmosphere, as a result of which all observed objects appear too high. This bending of the ray is known as refraction, and varies in amount with the angle the light ray makes with the vertical, the temperature of the air, the barometric pressure, and to a lesser degree, the relative humidity. The humidity can be disregarded in all work unless specifically required. Table V, appendix III is used in finding the mean astronomic refraction and the corrections to be applied to the mean for varying temperatures and pressures. To use this table, proceed as follows:

(*a*) Enter the table at the apparent zenith distance of the object, and by interpolation, find the mean refraction. This value applies to a standard temperature of 10° C. (50° F.) and a barometric pressure of 760 millimeters (29.9 inches) of mercury.

(*b*) From the table of corrections for temperatures other than 50° F., determine the multiplier ($C_T$) of the mean refraction for the observed temperature.

(*c*) From the table of corrections for barometric pressures other than 29.9 inches, determine the multiplier ($C_B$) of the mean refraction for the observed barometric pressure.

(*d*) Find the refraction correction, $r$, by multiplying together the mean refraction and the two factors.

$$r=r_m\times C_B\times C_T$$

(*e*) The computer should judge whether the class of observation requires the corrections for nonstandard atmosphere.

## 13. Observation on a Close Circumpolar Star at Elongation

*a.* The stars ordinarily used for observation on a close circumpolar star at elongation are Polaris and 51 H.Cephei in the northern hemisphere, and $\sigma$ Octantis in the southern hemisphere. The reduction formula is:

$$\sin A_E=\frac{\cos\delta}{\cos\phi}$$

or

$$\sin A_E=\sin p\sec\phi$$

in which $p$ is the polar distance ($90-\delta$) of the star. The procedure (fig. 7) is as follows:

(1) Obtain $\delta$ for the date from the Apparent Places of Fundamental Stars.

(2) Solve the formula.

(3) Apply correction for diurnal aberration, if warranted by precision desired.

$$\text{Diurnal aberration}=0.32\frac{\cos A\cos\phi}{\cos h},$$

plus in the northern, and minus in the southern hemisphere.

(4) Subtract the observed angle, mark to

star, correcting reading of circle on the star for inclination.

(5) The azimuth of the star is measured east or west from the meridian, according to whether eastern or western elongation was observed. The field notes should state which, or at least record a time from which it can be determined.

(6) The above formula is exact and may be applied to any star at elongation (method 1, fig. 7).

*b.* The approximate formula for close circumpolar stars only is:

$$A_E''=p'' \sec \phi.$$

This formula is adequate for computing most azimuths observed by this method. $A_E$ and $p$ are stated in seconds of arc. $\delta$ is obtained as above and subtracted from 90° to obtain $p$ (method 2, fig. 7).

*c.* In table II, AE&NA, the azimuth of Polaris is obtained by determining the product of the quantity $(b_0+b_1+b_2)$ and the secant of the latitude. The factor "$b_0$" varies with the local sidereal time, the factor "$b_1$" varies with the latitude and the factor "$b_2$" varies with the month of the year. All values must be interpolated as accurately as possible (method 3, fig. 7).

*d.* In observations on a close circumpolar star near its point of elongation, it is possible to obtain one direct and one reversed pointing so near to the point of elongation that the observations may be computed as if made at the instant of elongation. In most cases, additional pointings are needed, particularly if a repeating theodolite is being used, since the required pointings cannot be obtained within the time limit. Pointings at some distance from elongation may be easily reduced if accurate time is available. The formulas given in *a* above are used for the computations. The time of each pointing on the star should be recorded. The formula for reduction to the instant of elongation is:

$$A_c''=405{,}000 \sin 1'' \tan A_E(T-T_E)^2$$

in which $A_E$ is the azimuth of elongation, T is the observed time, and $T_E$ the time of elongation, $T-T_E$ being expressed in minutes of sidereal time.

(1) The correction or reduction to elongation for Polaris can be obtained from table VI in appendix III. Along the left margin are the minutes of sidereal time and along the top are the azimuths of Polaris from north. In most cases, double interpolation is required to extract the desired value.

(2) The correction will always reduce, numerically, the angle between the position of elongation and the meridian. The mean correction for the star is applied to the azimuth of elongation before subtracting the angle, mark to star.

*e.* For the determination of the Local Hour Angle at Elongation, the formulas are—

$$\cos t_e=\frac{\tan \phi}{\tan \delta}=\tan \phi \cot \delta.$$

The body is on the meridian at the instant when the local sidereal time and the right ascension of the body are equal. The body is at western elongation at this instant plus the time interval represented by $t_e$. Eastern elongation takes place at culmination minus the time interval represented by $t_e$ or plus the quantity $(24^h-t_e)$. *Example:* If the observer's latitude is 38°39′33.″8 and the declination of Polaris is 89°05′15.″2, cos $t_e$ is equal to the product of tan 38°39′33.″8 × cot 89°05′15.″2 = 0.79998800(0.01592651) = 0.01274102. $t_e=89°16'12''=5^h57^m04.8^s$. Polaris will be on the meridian when its right ascension and the local sidereal time are equal, or at $1^h08^m58^s.9$ local sidereal time. Western elongation comes $5^h57^m04.8^s$ later and eastern elongation $5^h57^m04.8^s$ earlier. The conversion from sidereal to civil, or standard, time is explained in paragraph 11*b*.

## 14. Observations on a Close Circumpolar Star at Any Hour Angle

*a.* The advantages of this method are that the star may be observed at any time it is visible and an unlimited number of observations may be taken. The two common methods of determining the azimuth are known as the direction method and the hour angle method. Both methods use the same basic formulas which are as follows:

$$\tan A=-\frac{\sin t}{\cos \phi \tan \delta-\sin \phi \cos t}$$

$$\cot A=\frac{\sin \phi \cos t-\cos \phi \tan \delta}{\sin t}$$

$$\tan A=-\cot \delta \sec \phi \sin t \left(\frac{1}{1-a}\right)$$

## COMPUTATION OF AZIMUTH USING A CIRCUMPOLAR STAR AT ELONGATION

Station: Tap  $\phi = 38^\circ 39' 33.''8$ N  Date: 9 July 1963
$\lambda = 78^\circ 44' 37.''3$ W  Elongation: E
Position of Polaris: RA: $01^h 57^m 55.^s5$  Time: $01^h 08^m 58.^s9$ L std. T
$\delta : +89^\circ 05' 15.''2$

Method ①: Solution by formula $\sin A_E = \frac{\cos \delta}{\cos \phi}$

| | | |
|---|---|---|
| Cos δ | (89° 05′ 15.″2) | = 0.01592449 |
| Cos φ | (38° 39′ 33.″8) | = 0.78087338 |
| Sin $A_E$ | | = 0.02039318 |
| $A_E$ | | = +01.° 10′ 06.″7 |
| Mark to Star | | 158 32 16.1 |
| Astronomic Azimuth | | 202° 37′ 50.″6 |

Method ②: Solution by formula $A_E = \rho'' \sec \phi$

δ (89° 05′ 15.″2)
$\rho'' = 90^\circ - (89^\circ 05' 15.''2) = 0^\circ 54' 44.''8 = 3284.''8$
Sec φ (38° 39′ 33.″8) = 1.2806 1735
$A_E = 1.2806\ 1735\ (3284.''8) = 4206.''6 = +01^\circ 10' 06.''6$
Mark to Star  158° 32′ 16.″1
Astronomic Azimuth  202° 37′ 50.″5

Method ③: Solution by Table II, American Ephemeris

LST = $19^h 59^m 42^s$  ($20^h 00^m$)
φ = 38° 39′ 33.″8  (38° 40′)
Date = 9 July 1963
$b_o$ = +54.′5  Sec φ = 1.2806 1735
$b_1$ = 0.0
$b_2$ = + 0.3
Total = +54.′8
$A_E = 1.2806\ 1735\ (54.'8) = 70.1778 = +01^\circ 10' 10.''7$
Mark to Star  158° 32′ 16.″1
Astronomic Azimuth  202° 37′ 54.″6

*Figure 7. Computation of azimuth using a circumpolar star at elongation.*

Where: A=Azimuth of star, as reckoned from the observer's meridian in a clockwise direction

$a$=cot $\delta$ tan $\phi$ cos $t$

$t$=The local hour angle, reckoned westward from upper culmination.

Tables for $\log\left(\frac{1}{1-a}\right)$ are found in TM 5-236.

*b.* The direction method is named for the type of theodolite used in the observations; this method is the one most commonly used for high order astronomic azimuths. Computation can be made with natural functions, or logarithmic functions (fig. 8).

(1) Correct the mean recorded times of each position or set for the chronometer error. When a sidereal chronometer is used, this will give the local sidereal time (LST), the chronometer correction being obtained from observations taken at the station, or from radio time signals and the station longitude. If a mean-time chronometer or watch is used, it is customary to obtain the correction to local standard time. This is then converted to UT, thence to GST, and by applying the longitude, to LST.

(2) Obtain the right ascension and declination of the star from one of the ephemerides for the date and UT of observation, using the mean epoch of a series of observations which should not extend beyond a period of 4 hours.

(3) Subtract the star's right ascension from LST to obtain the hour angle of the star ($t$) and convert to units of arc.

(4) Solve the formula for A, the azimuth of the star.

(5) Determine the correction for curvature (table VII) when applicable and apply to A to give the correct azimuth of star. This correction numerically decreases the value of A.

(6) Determine the level correction and correct the circle readings on the star. If the altitude is not observed, it may be computed in the case of Polaris from Table II, AE&NA, or for any star from the formulas:

$$\sin h=\sin \phi \sin \delta+\cos \phi \cos \delta \cos t$$

or

$$\cos h=\frac{\cos \delta \sin t}{-\sin A}=\frac{\cos \delta \sin t}{-\tan A \cos A}$$

Computation of $h$ to the nearest minute of arc is sufficient.

(7) Subtract the corrected reading on the star from the circle reading on the mark.

(8) Add algebraically the corrected azimuth of the star from North and 180° to (7) to obtain the azimuth of the mark from South.

(9) Abstract the results of all positions or sets, apply the rules for rejection, and take the mean of the acceptable observations. Record this information on DA Form 1962 (fig. 9).

(10) Determine the probable error of observation.

(11) Apply correction for diurnal aberration.

(12) Apply correction for elevation of mark by formula:

$$C=+0.000109\ h \cos^2 \phi \sin 2A$$

where $h$=elevation of mark.

An accurate sea level reduction chart may be used.

(13) When the $x$ and $y$ of the instantaneous north pole are known for a given date, the correction to be applied to the astronomic azimuth to reference the azimuth to the mean pole is computed by the formula: $\Delta\alpha=(x \sin \lambda-y \cos \lambda) \sec \phi$ wherein West longitude is considered positive.

*c.* The hour angle method normally is used for lower order azimuths when the time of observation is less accurately determined. Solution can be made using natural function or logarithmic functions (fig. 10).

## 15. Observations on East-West Stars

*a. Basic Considerations.* When high order astronomic azimuths are needed and close circumpolar stars cannot be seen, East-West stars which reach elongation at approximately 15° altitude may be observed between approximately 7½° and 22½° altitude. At latitudes of less than 2°, stars crossing the prime vertical at approximately 30° altitude may be observed. The declination of the observed stars should be four to five times the observer's latitude for the elongating stars and approximately one-half the latitude for stars crossing the prime vertical. For lower order azimuths, the altitudes of east-

| PROJECT | LOCATION | AZIMUTH BY DIRECTION METHOD (TM 5-237) | | |
|---|---|---|---|---|
| | MARYLAND | | | |
| ORGANIZATION | MARK | LATITUDE ($\phi$) | LONGITUDE ($\lambda$) | STATION |
| USAMS | MAP | 38° 57′ 02.67″ | 5h 08m 28.86s | NP (AMS, 1958) |

| CHRON. NR. | INST. (NR.) | LEVEL VALUE (d) | ECC.* (INST.) ~~(SIGNAL)~~ | OBSERVER | G. CIVIL DAY |
|---|---|---|---|---|---|
| 12460 | T3 53010 | 6.462 | 29.953 m | F.E.W. | APR. 11. 131 |

| Date 19 63, position | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Chronometer reading | 10 07 48.3 | 10 14 35.0 | 10 33 56.4 | 10 40 16.7 |
| Chronometer correction | - 08 29.5 | - 08 29.6 | - 08 29.7 | - 08 29.7 |
| Sidereal time | 9 59 18.8 | 10 06 05.4 | 10 25 26.7 | 10 31 47.0 |
| RA($\alpha$) of POLARIS (star) | 1 56 45.0 | 1 56 45.0 | 1 56 45.0 | 1 56 45.0 |
| HA(t) of star (time) | 8 02 33.8 | 8 09 20.4 | 8 28 41.7 | 8 35 02.0 |
| t of star (arc) | 120 38 27.0 | 122 20 06.0 | 127 10 25.5 | 128 45 30.0 |
| Decl. ($\delta$) of star | 89 05 32.59 | 89 05 32.59 | 89 05 32.59 | 89 05 32.59 |
| Constants for star | Sin $\phi$ .628 65203 | Cos $\phi$ .777 68672 | Tan $\delta$ 63.122643 | Cos $\phi$ Tan $\delta$ 49.089 64119 |
| Sin t | + .860 37903 | + .844 93526 | + .796 80684 | + .779 79344 |
| Cos t | - .509 65472 | - .534 86859 | - .604 23412 | - .626 03690 |
| Sin $\phi$ cos t | - .320 39547 | - .336 24622 | - .379 85300 | - .393 55937 |
| Cos $\phi$ tan $\delta$ − sin $\phi$ cos t | 49.410 03666 | 49.425 88741 | 49.469 49419 | 49.483 20056 |
| $-\text{Tan } A = \frac{\sin t}{\cos\phi \tan\delta - \sin\phi \cos t}$ | .017 41304 | .017 09499 | .016 10703 | .015 75875 |
| A (Az. of star from N.)† | - 0 59 51.3 | - 0 58 45.8 | - 0 55 22.0 | - 0 54 10.2 |
| Diff. in time between D. & R. | 1 m. 35 s. | 1 m. 35 s. | 2 m. 02 s. | 1 m. 18 s. |
| Curvature correction | — | — | — | — |
| Altitude of star (h) | 38° 28′ 59″ | 38° 27′ 44″ | 38° 23″ 53′ | 38° 22′ 45″ |
| $\frac{d}{4}$ tan h (level factor) | 1.284 | 1.283 | 1.280 | 1.279 |
| Inclination | + 1.2 | + 2.3 | + 1.5 | + 3.3 |
| Level correction | + 01.5 | + 03.0 | + 01.9 | + 04.2 |
| Circle reading on star | 170 35 09.8 | 181 36 47.1 | 203 41 12.4 | 215 40 39.2 |
| Corr. reading on star | 170 35 11.3 | 181 36 50.1 | 203 41 14.3 | 215 40 43.4 |
| Circle reading on Mark | 00 00 11.3 | 11 00 44.6 | 33 01 47.0 | 44 59 59.2 |
| Diff. (Mark minus star) | 189 25 00.0 | 189 23 54.5 | 189 20 32.7 | 189 19 15.8 |
| Corr. Az. of star, from N.† | - 59 51.3 | - 58 45.8 | - 55 22.0 | - 54 10.2 |
| | 180° 00′ 00″.0 | 180° 00′ 00″.0 | 180° 00′ 00″.0 | 180° 00′ 00″.0 |
| Azimuth of MAP (clockwise from south) | 8° 25′ 08.7″ | 8° 25′ 08.7″ | 8° 25′ 10.7″ | 8° 25′ 05.6″ |

To the mean result from the above computation must be applied corrections for diurnal aberration, elevation of mark, and eccentricity (if any) of station and mark. Carry times and angles to tenths of seconds only.
*Give volume and page of record for eccentricity, if any. † Minus, if west of north.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| L.E. Smith - AMS | 2 OCT. '63 | O.R. Niska - AMS | 11 DEC. '63 |

DA FORM 1 FEB 57 1903

GPO 908848

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420809

① Natural functions, DA Form 1903

*Figure 8. Computation of azimuth using a circumpolar star at any hour angle (direction method).*

| PROJECT | LOCATION | AZIMUTH BY DIRECTION METHOD (LOGARITHMIC) (TM 5-237) | | |
|---|---|---|---|---|
| ORGANIZATION | MARK | LATITUDE ($\phi$) | LONGITUDE ($\lambda$) | STATION |
| USAMS | MAP | 38° 57′ 02.67″ | 5h 08m 28.86s | NP (AMS, 1958) |
| CHRON. NR. | INST. (NR.) | LEVEL VALUE (d) | ECC.* (INST.) (SIGNAL) | OBSERVER | G. CIVIL DAY |
| 12460 | T3 53010 | 6.462 | 29.953 m | F. E. W. | APR. 11.131 |

| Date 1963, position | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Chronometer reading | 10 07 48.3 | 10 14 35.0 | 10 33 56.4 | 10 40 16.7 |
| Chronometer correction | − 08 29.5 | − 08 29.6 | − 08 29.7 | − 08 29.7 |
| Sidereal time | 9 59 18.8 | 10 06 05.4 | 10 25 26.7 | 10 31 47.0 |
| RA($\alpha$) of POLARIS (star) | 1 56 45.0 | 1 56 45.0 | 1 56 45.0 | 1 56 45.0 |
| HA(t) of star (time) | 8 02 33.8 | 8 09 20.4 | 8 28 41.7 | 8 35 02.0 |
| t of star (arc) | 120 38 27.0 | 122 20 06.0 | 127 10 25.5 | 128 45 30.0 |
| Decl. ($\delta$) of star | 89 05 32.59 | 89 05 32.59 | 89 05 32.59 | 89 05 32.59 |
| Log cot $\delta$ | 8.199 815 | 8.199 815 | 8.199 815 | 8.199 815 |
| Log tan $\phi$ | 9.907 606 | 9.907 606 | 9.907 606 | 9.907 606 |
| Log cos t | 9.707 276N | 9.728 247N | 9.781 205N | 9.796 600N |
| Log a (to 5 places) | 7.814 70 N | 7.835 67 N | 7.888 63 N | 7.904 02 N |
| Log cot $\delta$ | 8.199 815 | 8.199 815 | 8.199 815 | 8.199 815 |
| Log sec $\phi$ | 0.109 195 | 0.109 195 | 0.109 195 | 0.109 195 |
| Log sin t | 9.934 690 | 9.926 823 | 9.901 353 | 9.891 980 |
| Log $\frac{1}{1-a}$ | 9.997 175 | 9.997 036 | 9.996 653 | 9.996 532 |
| Log (−tan A)(to 6 places) | 8.240 875 | 8.232 869 | 8.207 016 | 8.197 522 |
| A (Az. of star from N.)† | −0 59 51.3 | −0 58 45.8 | −0 55 22.0 | −0 54 10.2 |
| Diff. in time between D. & R. | 1m 35s | 1m 35s | 2m 02s | 1m 18s |
| Curvature correction | − | − | − | − |
| Altitude of star (h) | 38° 28′ 59″ | 38° 27′ 44″ | 38° 23′ 53″ | 38° 22′ 45″ |
| $\frac{d}{4}$ tan h (level factor) | 1.284 | 1.283 | 1.280 | 1.279 |
| Inclination | +1.2 | +2.3 | +1.5 | +3.3 |
| Level correction | +01.5 | +03.0 | +01.9 | +04.2 |
| Circle reading on star | 170 35 09.8 | 181 36 47.1 | 203 41 12.4 | 215 40 39.2 |
| Corr. reading on star | 170 35 11.3 | 181 36 50.1 | 203 41 14.3 | 215 40 43.4 |
| Circle reading on Mark | 00 00 11.3 | 11 00 44.6 | 33 01 47.0 | 44 59 59.2 |
| Diff. (Mark minus star) | 189 25 00.0 | 189 23 54.5 | 189 20 32.7 | 189 19 15.8 |
| Corr. Az. of star, from N.† | − 59 51.3 | − 58 45.8 | − 55 22.0 | − 54 10.2 |
| | 180° 00′ 00″.0 | 180° 00′ 00″.0 | 180° 00′ 00″.0 | 180° 00′ 00″.0 |
| Azimuth of MAP (clockwise from south) | 8° 25′ 08.7″ | 8° 25′ 08.7″ | 8° 25′ 10.7″ | 8° 25′ 05.6″ |

To the mean result from the above computation must be applied corrections for diurnal aberration, elevation of mark, and eccentricity (if any) of station and mark. Carry times and angles to tenths of seconds only.
*Give volume and page of record for eccentricity, if any. † Minus, if west of north.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| L. E. Smith - AMS | 2 OCT. 63 | O. R. Niska - AMS | 12 DEC. '63 |

DA FORM 1 FEB 57 1904

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420843

② Logarithmic functions, DA Form 1904

*Figure 8*—Continued.

## AZIMUTH SUMMARY

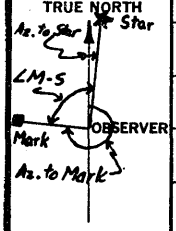
| PROJECT | | | | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|---|---|---|
| LOCATION, Maryland | | | | | ORGANIZATION USAMS | |
| STATION NP(AMS,1958) | | | | | | |
| | APR 11.131 | | MAY 24.081 | | | |
| | 8° 25′ | V | 8° 25′ | V | | |
| 1 | 08.″7 | -2.3 | 08.″4 | -2.0 | | |
| 2 | 08.7 | -2.3 | 06.6 | -0.2 | | |
| 3 | 10.7 | -4.3 | 06.1 | +0.3 | | |
| 4 | 05.6 | +0.8 | 05.2 | +1.2 | | |
| 5 | 03.9 | +2.5 | 07.5 | -1.1 | | |
| 6 | 06.8 | -0.4 | 05.5 | +0.9 | $\Sigma V^2 =$ | 79.73 |
| 7 | 07.0 | -0.6 | 04.3 | +2.1 | | |
| 8 | 07.0 | -0.6 | 08.3 | -1.9 | .6745 $\sqrt{\frac{79.73}{32(31)}}$ | $= \pm 0.''19$ |
| 9 | 03.6 | +2.8 | 06.4 | 0 | | |
| 10 | 05.7 | +0.7 | 05.6 | +0.8 | .6745 $\sqrt{\frac{79.73}{32-1}}$ | $= \pm 1.''08$ |
| 11 | 06.9 | -0.5 | 07.1 | -0.7 | | |
| 12 | 05.7 | +0.7 | 07.8 | -1.4 | | |
| 13 | 06.2 | +0.2 | 05.5 | +0.9 | | |
| 14 | 03.1 | +3.3 | 06.8 | -0.4 | | |
| 15 | 05.6 | +0.8 | 06.8 | -0.4 | | |
| 16 | 07.5 | -1.1 | 05.3 | +1.1 | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| MEAN OBSERVED AZIMUTH | | | | 8° | 25′ 06.″43 | ± 0.″19 |
| DIURNAL ABERRATION | | | | | + 00.32 | |
| ELEVATION OF MARK (120 FT.) | | | | | 00.00 | |
| ECCENTRICITIES | | | | | 03 02.93 | |
| FINAL ASTRONOMICAL AZIMUTH | | | | 8° | 28′ 09.″68 | ± 0.″19 |
| | | | | | | |
| 32 OBSERVATIONS | | | | | | |
| 0 REJECTIONS | | | | | | |
| PROBABLE ERROR OF A SINGLE OBSERVATION = | | | | | | ± 1.″08 |
| | | | | | | |
| | | | | | | |
| | | | | | | |

| TABULATED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| L.E. Smith - AMS | 2 OCT. 63 | O.R. Nistka - AMS | 12 DEC. '63 |

DA FORM 1 FEB 57 1962 GPO 908847 U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 9. Summary of azimuth observations.*

**DIAGRAM**




| | | |
|---|---|---|
| PROJECT: Test | LOCATION: Missouri | **AZIMUTH BY HOUR ANGLE METHOD** (TM 5-237) |
| ORGANIZATION: AMS | LATITUDE (φ): 37° 38' 47.8" N | LONGITUDE (λ): 91° 44' 18.0" W |
| STATION: LENOX | MARK: LENOX Azimuth Mark | INSTRUMENT (Number and type): |
| STANDARD TIME (Meridian): 90° | CELESTIAL BODY(S): Polaris | WATCH ~~FAST (−)~~ SLOW (+): 7 sec. |
| WATCH COMPARED (Time): | DATE: 5 Nov. 53 | OBSERVER: N. C. S. |
| WEATHER: Cool & Clear | | |

| | SET NR 1 | | | | | | SET NR 2 | | | | | | SET NR 3 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TIME | | | HOR. ANGLE | | | TIME | | | HOR. ANGLE | | | TIME | | | HOR. ANGLE | | |
| | HRS. | MIN. | SEC. | ° | ′ | ″ | HRS. | MIN. | SEC. | ° | ′ | ″ | HRS. | MIN. | SEC. | ° | ′ | ″ |
| Mean | 17 | 10 | 28.0 | 79 | 41 | 11.0 | 17 | 13 | 00.0 | 79 | 41 | 11.0 | 17 | 16 | 52.5 | 79 | 41 | 03.5 |

**SUN OBSERVATION**

[For star observation, use factors enclosed in brackets]

| | | | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. |
| 1 | Mean time of observation | | 17 | 10 | 28.0 | 17 | 13 | 00.0 | 17 | 16 | 52.5 |
| 2 | Watch correction | ± | + | | 7.0 | + | | 7.0 | + | | 7.0 |
| 3 | Time Zone Correction (TZC) | ± | +6 | | | +6 | | | +6 | | |
| 4 | UT of observation (1+2+3) | | 23 | 10 | 35.0 | 23 | 13 | 07.0 | 23 | 16 | 59.5 |
| 5 | $0^h$ Greenwich EQT [or Sid. T.] | ± | + 2 | 55 | 42.7 | 2 | 55 | 42.7 | 2 | 55 | 42.7 |
| 6 | UT × var. EQT per hour [or table III Eph.] | ± | + | 3 | 48.4 | | 3 | 48.9 | | 3 | 49.5 |
| 7 | (5+6) correct EQT [or G. Sid. T. (4+5+6)] | ± | 2 | 10 | 06.1 | 2 | 12 | 38.6 | 2 | 16 | 31.7 |
| 8 | (4+7) GAT [or RA] | | 1 | 52 | 33.0 | 1 | 52 | 33.0 | 1 | 52 | 33.0 |
| 9 | GHA in time (GAT−12h) [or (7−8)] | | 0 | 17 | 33.1 | 0 | 20 | 05.6 | 0 | 23 | 58.7 |
| 10 | GHA in arc | | 4 | 23 | 16.5 | 5 | 01 | 24.0 | 5 | 59 | 40.5 |
| 11 | Longitude, West (−), ~~East (+)~~ | ± | -91 | 44 | 18.0 | -91 | 44 | 18.0 | -91 | 44 | 18.0 |
| 12 | LHA (10+11)=t (or 360°−LHA=−t) | | -87 | 21 | 01.5 | -86 | 42 | 54.0 | -85 | 44 | 37.5 |

| | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ |
| t ± | 87 | 21 | 01.5 | 86 | 42 | 54.0 | 85 | 44 | 37.5 |
| Lat. (φ) ± | 37 | 38 | 47.8 | 37 | 38 | 47.8 | 37 | 38 | 47.8 |
| Dec. (δ) ± | 89 | 03 | 04.8 | 89 | 03 | 04.8 | 89 | 03 | 04.8 |
| Sin t ± | -.99893094 | | | -.99835685 | | | -.99724210 | | |
| Cos t ± | +.04622747 | | | +.05730266 | | | +.07421728 | | |
| Sin φ ± | +.61078950 | | | +.61078950 | | | +.61078950 | | |
| Cos φ ± | +.79179302 | | | +.79179302 | | | +.79179302 | | |
| Tan δ ± | +60.39059 | | | +60.39059 | | | +60.39059 | | |
| −Tan A ± | -.02090312 | | | -.02089406 | | | -.02087524 | | |
| A (E ~~W~~) | 1 | 11 | 51.0 | 1 | 11 | 49.1 | 1 | 11 | 45.2 |
| Azimuth of S | 1 | 11 | 51.0 | 1 | 11 | 49.1 | 1 | 11 | 45.2 |
| ∠, Mark to S − | 79 | 41 | 11.0 | 79 | 41 | 11.0 | 79 | 41 | 03.5 |
| Tr. Az. to Mark | 281 | 30 | 40.0 | 281 | 30 | 38.1 | 281 | 30 | 41.7 |

| | | | |
|---|---|---|---|
| Mean true azimuth to Mark | 281 | 30 | 39.9 |
| Grid correction ± | | | |
| Grid azimuth to Mark | | | |
| Magnetic azimuth to Mark | | | |
| Magnetic declination E(−), W(+) | | | |

$$-\text{Tan A} = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}$$

If LHA is greater than 180°, subtract from 360° and reverse sign.

Obtain δ from Ephemeris.

Check signs and quadrants by use of sketch.

| | | | |
|---|---|---|---|
| COMPUTED BY: R.A. Smith | DATE: 1 Feb 56 | CHECKED BY: W. C. Aumen | DATE: 1 Feb 56 |

DA FORM 1 FEB 57 1905

① Natural functions, DA Form 1905

*Figure 10. Computation of azimuth using a circumpolar star at any hour angle (angle method).*

**AZIMUTH BY HOUR ANGLE METHOD (Logarithmic)** (TM 5-237)

DIAGRAM — TRUE NORTH; Star, Az. to Star, L M-S, Mark, Az. to Mark, OBSERVER

| | | | |
|---|---|---|---|
| PROJECT: Test | LOCATION: Missouri | | |
| ORGANIZATION: AMS | LATITUDE: 37°38'47.8" N | LONGITUDE: 91°44'18.0" W | STATION: LENOX |
| MARK: LENOX Azimuth Mark | INSTRUMENT (Number and type): | | STANDARD TIME (Meridian): 90° |
| CELESTIAL BODY(S): Polaris | WATCH ~~FAST (−)~~ SLOW (+): 7 sec. | | WATCH COMPARED (Time): |
| DATE: 5 Nov. 53 | OBSERVER: N. C. S. | | WEATHER: Cool & Clear |

| | SET NR 1 | | | | | | SET NR 2 | | | | | | SET NR 3 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TIME | | | HOR. ANGLE | | | TIME | | | HOR. ANGLE | | | TIME | | | HOR. ANGLE | | |
| | HRS. | MIN. | SEC. | ° | ′ | ″ | HRS. | MIN. | SEC. | ° | ′ | ″ | HRS. | MIN. | SEC. | ° | ′ | ″ |
| Mean | 17 | 10 | 28.0 | 79 | 41 | 11.0 | 17 | 13 | 00.0 | 79 | 41 | 11.0 | 17 | 16 | 52.5 | 79 | 41 | 03.5 |

**SUN OBSERVATION**

[For star observation, use factors enclosed in brackets]

| | | | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. |
| 1 | Mean time of observation | | 17 | 10 | 28.0 | 17 | 13 | 00.0 | 17 | 16 | 52.5 |
| 2 | Watch correction | ± | + | | 07.0 | | | 07.0 | | | 07.0 |
| 3 | Time Zone Correction (TZC) | ± | +6 | | | 6 | | | 6 | | |
| 4 | UT of observation (1+2+3) | | 23 | 10 | 35.0 | 23 | 13 | 07.0 | 23 | 16 | 59.5 |
| 5 | $O^h$ Greenwich EQT [or Sid. T.] | ± | 2 | 55 | 42.7 | 2 | 55 | 42.7 | 2 | 55 | 42.7 |
| 6 | UT × var. per hour [or table III Eph.] | ± | | 3 | 48.4 | | 3 | 48.9 | | 3 | 49.5 |
| 7 | (5+6) correct EQT [or G. Sid. T. (4+5+6)] | ± | 2 | 10 | 06.1 | 2 | 12 | 38.6 | 2 | 16 | 31.7 |
| 8 | (4+7) GAT [or RA] | | 1 | 52 | 33.0 | 1 | 52 | 33.0 | 1 | 52 | 33.0 |
| 9 | GHA in time (GAT−12h) [or (7−8)] | | 0 | 17 | 33.1 | 0 | 20 | 05.6 | 0 | 23 | 58.7 |
| 10 | GHA in arc | | 4 | 23 | 16.5 | 5 | 01 | 24.0 | 5 | 59 | 40.5 |
| 11 | Longitude, West (−), ~~East (+)~~ | ± | −91 | 44 | 18.0 | 91 | 44 | 18.0 | 91 | 44 | 18.0 |
| 12 | LHA (10+11)=t (or 360°−LHA=−t) | | −87 | 21 | 01.5 | −86 | 42 | 54.0 | −85 | 44 | 37.5 |

δ = 89°03'04.8"

| | SET NR 1 | SET NR 2 | SET NR 3 |
|---|---|---|---|
| Log sin φ + | 9.78589156 | | |
| Log cos t + | 8.66490015 | 8.75817478 | 8.87050500 |
| (Sum) log A + | 8.45079171 | 8.54406634 | 8.65639656 |
| A | 0.02823525 | 0.03499986 | 0.04533113 |
| Log cos φ + | 9.89861167 | | |
| Log tan δ + | 1.78096930 | | |
| (Sum) log B + | 1.67958097 | | |
| B | 47.81685038 | | |
| B − A | 47.78861513 | 47.78185052 | 47.77151925 |
| Log sin t − | 9.99953546 | 9.99928580 | 9.99880060 |
| Log (B − A) + | 1.67932445 | 1.67926296 | 1.67916905 |
| Log(− tan A)(diff.) | 8.32021101 | 8.32002284 | 8.31963155 |
| A (E ~~or W~~) | 1° 11' 50.9" | 1° 11' 49.1" | 1° 11' 45.2" |
| Azimuth of S | 1 11 50.9 | 1 11 49.1 | 1 11 45.2 |
| ∠, Mark to S − | 79 41 11.0 | 79 41 11.0 | 79 41 03.5 |
| Tr. Az. to Mark | 281 30 39.9 | 281 30 38.1 | 281 30 41.7 |

| | | | |
|---|---|---|---|
| Mean true azimuth to Mark | | 281 | 30 | 39.9 |
| Grid correction | ± | | | |
| Grid azimuth to Mark | | | | |
| Magnetic azimuth to Mark | | | | |
| Magnetic declination E(−), W(+) | | | | |

$$-\text{Tan } A = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}$$

If LHA is greater than 180°, subtract from 360° and reverse sign.

Obtain δ from Ephemeris.

Check signs and quadrants by use of sketch.

COMPUTED BY: R.A. Smith — DATE: 6 Feb 55

CHECKED BY: W. C. Aumen — DATE: 6 Feb. 55

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② Logarithmic functions, DA Form 1906

*Figure 10*—Continued.

west stars can be observed at any latitude by the equal altitude method; i.e., a pair consisting of an east star and a west star should be observed at the same altitude. The stars should be near the prime vertical, or when the latitude is near 30°, the declination of the stars should be approximately twice the latitude. For any type of observation on east-west stars, the accuracy of the observers latitude is critical.

*b. The Hour Angle Method.* Either the Direction or the Hour Angle Method may be used as outlined in paragraph 14. Because of the rapid movement of the stars as compared to circumpolar stars, recording of time is more critical and chronographic recording of time is recommended. Since the stars' azimuth will be between 45° and 90° from the meridian, DA Forms 1903 and 1905 should be modified to a cotangent format (fig. 11) such as:

$$-\cot A=\frac{\cos\phi\tan\delta}{\sin t}-\frac{\sin\phi}{\tan t}$$

(1) Usually, it is easier to compute the stars' azimuth for each direct and each reverse pointing on a star and then determine an azimuth for each such pair. This eliminates the necessity for computing the curvature correction. If the mean time of the direct and reverse pointings is used, the curvature correction is computed by the following formula:

$$\text{Curvature correction}=\frac{\sin A\cos\phi\sec^2 h\,(\cos h\sin\delta-2\cos A\cos\phi)\,(t)^2\sin 1''}{8}$$

where $t$ is the time interval between the direct and reverse pointings expressed in seconds of arc.

(2) The diurnal aberration correction is applied to the azimuth of the mark reckoned clockwise from the observer's meridian. It is positive if the star is observed north of the observer's latitude and negative if observed south of the observer's latitude.

*c. The Altitude Method.* The formulas are:

$$\cos A=\frac{\sin\delta-\sin h\sin\phi}{\cos h\cos\phi}$$

$$\cos\frac{1}{2}A=\sqrt{\frac{\cos s\cos(s-p)}{\cos\phi\cos h}}$$

$$\tan\frac{1}{2}A=\sqrt{\frac{\sin(s-\phi)\sin(s-h)}{\cos s\cos(s-p)}}$$

in which $s=\frac{1}{2}(\phi+h+p)$, $p$ being the polar distance of the star.

(1) The second and third formulas are preferred for logarithmic computations. DA Form 1907 (Azimuth by Altitude Method) is used for the computations by the first formula (fig. 12), and DA Form 1908 (Azimuth by Altitude Method, Logarithmic) is used for the computation by the second formula (fig. 12).

(2) The procedure is as follows:

(*a*) Arrange the observations by positions in the case of the direction method, and by sets if by repetition. Find the mean horizontal circle readings and the corresponding mean vertical angles for each position or set.

(*b*) Correct the mean observed vertical angles for refraction.

(*c*) Obtain $\delta$ for the date from an ephemeris and subtract from 90° to obtain $p$ when the formula requires it.

(*d*) Compute by the applicable formula.

(*e*) Subtract angle, mark to star.

(*f*) Determine the mean of the azimuths determined by an east star and by a west star. When more than one east and one west star are observed, the final azimuth is the mean of the average value obtained from all east stars, and from all west stars.

(*g*) Apply diurnal aberration, etc., if necessary for the required accuracy.

## 16. Observations on the Sun

*a. Altitude Method.* The altitude method (fig. 13) is preferred when the time of the observation is not precisely known. It is subject to considerable error on account of inaccuracies in the altitude, declination, or latitude. Hence, these should be taken from the record book and tables with care. The altitude method is not suitable for observations on the sun when it is near the meridian.

| PROJECT | LOCATION | AZIMUTH BY DIRECTION METHOD (TM 5-237) | | |
|---|---|---|---|---|
| | SOLOMON | | | |
| ORGANIZATION | MARK | LATITUDE ($\phi$) " | LONGITUDE ($\lambda$) | STATION |
| USAMS | TIP | S 05° 15' 22.36 | $-10^h 18^m 16.^s 6$ | RAIL |
| CHRON. NR. | INST. (NR.) | LEVEL VALUE (d) | ECC.* (INST.) (SIGNAL) | OBSERVER |
| 2E 10001 | T-3 26593 | 6.311 | | J. DOE |

| G. CIVIL DAY |
|---|
| 6.404 JAN. 63 |

| | | | | |
|---|---|---|---|---|
| Date 19 63, position | 3 (D) | 3 (R) | | |
| Chronometer reading | 02 48 21.8 | 02 49 00.5 | | |
| Chronometer correction | + 1.1 | + 1.1 | | |
| Sidereal time | 02 48 22.9 | 02 49 01.6 | | |
| RA($\alpha$) of 308 (star) | 08 05 58.8 | 08 05 58.8 | | |
| HA(t) of star (time) | 18 42 24.1 | 18 43 02.8 | | |
| t of star (arc) | 280 36 01.5 | 280 45 42.0 | | |
| Decl. ($\delta$) of star | -24 11 48.53 | -24 11 48.53 | | |
| Constants for star | Sin $\phi$ -.091 60957 | Cos $\phi$ +.995 79500 | Tan $\delta$ -.449 35098 | Cos $\phi$ Tan $\delta$ -.447 46146 |
| Sin t | -.982 93402 | -.982 41240 | | |
| ~~Cos t~~ TAN t / Cos t | - 5.34 32378 / +.183 95850 | - 5.26 13053 / +.186 72408 | | |
| ~~Sin $\phi$ cos t~~ (1) Cos $\phi$ TAN $\delta$ / SIN t | +.455 23041 | +.455 47212 | | |
| ~~Cos $\phi$ tan $\delta$ − sin $\phi$ cos t~~ (2) SIN $\phi$ / TAN t | +.017 14495 | +.017 41195 | | |
| ~~Tan A = (1) − (2) / ...~~ Cot A = (1) − (2) sin t | +.438 08546 | +.438 06017 | | |
| A (Az. of star from S) † | - 66 20 32.9 | - 66 20 37.3 | | |
| Diff. in time between D. & R. | m. s. | m. s. | m. s. | m. s. |
| Curvature correction | | | | |
| Altitude of star (h) | 11° 48' 29.73" | 11° 57' 19.21" | ° ' " | ° ' " |
| $\frac{d}{4}$ tan h (level factor) | .330 | .334 | | |
| Inclination | + 0.5 | | | |
| Level correction | + 0.2 | | | |
| MEAN Circle reading on star | 197 27 28.4 | | | |
| Corr. reading on star | 197 27 28.6 | | | |
| CORRECTED Circle reading on Mark | 22 01 20.9 | | | |
| Diff. (Mark minus star) | 184 33 52.3 | | | |
| MEAN ~~Corr~~. Az. of star, from S † | -66 20 35.1 | | | |
| | ~~180° 00' 00".0~~ | ~~180° 00' 00".0~~ | 180° 00' 00".0 | 180° 00' 00".0 |
| Azimuth of TIP (clockwise from south) | 118 13 17.2 | | | |

To the mean result from the above computation must be applied corrections for diurnal aberration, elevation of mark, and eccentricity (if any) of station and mark. Carry times and angles to tenths of seconds only.
* Give volume and page of record for eccentricity, if any. † Minus, if west of north.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| G. T. Tennis - AMS | MAY '63 | J. A. Andrews - AMS | MAY '63 |

DA FORM 1 FEB 57 1903

GPO 908848

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420809

*Figure 11. Computation of azimuth using east-west stars.*

**AZIMUTH BY ALTITUDE METHOD** (TM 5-237)

DIAGRAM: TRUE NORTH; Star; Az. to Star; Az. to Mk; OBSERVER; LM-5; Mark

| | | | |
|---|---|---|---|
| PROJECT: 1722-24 | LOCATION: England | | |
| ORGANIZATION: A&S Inc. | LATITUDE: 51°23'30"N | LONGITUDE: 0°02'09"W | STATION: ⊙9 |
| MARK: Lamp | INSTRUMENT (Number and type): | | STANDARD TIME (Meridian): |
| CELESTIAL BODY(S): γ Cygni | WATCH FAST (−) SLOW (+): | | WATCH COMPARED (Time): |
| DATE: 7 June 55 | OBSERVER: E.T.Ennett | | WEATHER: Clear & Warm |

| | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Hor. Angle | Vert. Angle | | Hor. Angle | Vert. Angle | | Hor. Angle | Vert. Angle | |
| Mean | 303° 31' 36" | 40° 48' 05" | | | | | | | |
| Parallax | | | | | | | | | |
| Mean refraction | | − 1' 07" | | | | | | | |
| h (sum) | | 40° 46' 58" | | | | | | | |
| | Hrs. | Min. | Sec. | Hrs. | Min. | Sec. | Hrs. | Min. | Sec. |
| Mean time | 21 | 16 | | | | | | | |
| Watch correction | | — | | | | | | | |
| TZC | — | | | | | | | | |
| Universal time (UT) | 21 | 16 | | | | | | | |
| δ at 0h UT ± | ° | ' | " | ° | ' | " | ° | ' | " |
| UT × δ var. per hr. ± | | | | | | | | | |
| δ ± + | 40 | 06 | 41 | | | | | | |
| h | 40 | 46 | 58 | | | | | | |
| φ | 51 | 23 | 30 | | | | | | |
| Sin δ ± + | 0.64427566 | | | Set 2 and Set 3 | | | | | |
| Sin h + + | 0.65319303 | | | are computed | | | | | |
| Sin φ + + | 0.78142972 | | | by method used | | | | | |
| Cos h ± + | 0.75719143 | | | in Set 1. | | | | | |
| Cos φ ± + | 0.62399326 | | | | | | | | |
| Cos A ± + | 0.28329357 | | | | | | | | |
| A (E ~~or W~~) | 73° | 32' | 35" | ° | ' | " | ° | ' | " |
| Azimuth of S | 73 | 32 | 35 | | | | | | |
| Angle, Mark to S − | 303 | 31 | 36 | | | | | | |
| True Az. to Mark | 130 | 00 | 59 | | | | | | |
| Mean true azimuth to Mark | ° | ' | " | | | | | | |
| Grid correction | | | | | | | | | |
| Grid azimuth | | | | | | | | | |
| Mag. azimuth to Mark | | | | | | | | | |
| Mag. declination | | | | | | | | | |

$$\text{Cos A} = \frac{\sin\delta - \sin h \sin\phi}{\cos h \cos\phi}$$

Computation = Three sets are computed separately for check, refraction and parallax corrections are obtained from TM 5-236. Apply watch correction to observed mean time. TZC = time zone correction to universal time.

δ = declination, (+) if north, (−) if south. h = altitude. φ = latitude, (+) if north, (−) if south.
A = Astronomic azimuth east or west of north. If cos A is (−), A is between 90° and 180°.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.A.Smith | 10 Aug. 55 | W. C. Asmen | 11 Aug. 55 |

DA FORM 1 FEB 57 1907

① Natural function solution, DA Form 1907

*Figure 12. Computation of azimuth using the altitude method (star).*

**AZIMUTH BY ALTITUDE METHOD** (Logarithmic) (*TM 5-237*)

DIAGRAM: TRUE NORTH, Star, Az. to Star, Az. to Mk, OBSERVER, Mark

| | | | |
|---|---|---|---|
| PROJECT: 1722-24 | LOCATION: England | | |
| ORGANIZATION: A&S, Inc. | LATITUDE: 51° 23' 00" N | LONGITUDE: 0° 02' 09" W | STATION: 09 |
| MARK: Lamp | INSTRUMENT (*Number and type*) | | STANDARD TIME (*Meridian*) |
| CELESTIAL BODY(S): Y Cygni | WATCH FAST (−) SLOW (+) | | WATCH COMPARED (*Time*) |
| DATE: 7 June 55 | OBSERVER: E. T. Ennett | | WEATHER: Clear & Warm |

| | SET NR 1 | | | | | | SET NR 2 | | | | | | SET NR 3 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | HOR. ANGLE | | | VERT. ANGLE | | | HOR. ANGLE | | | VERT. ANGLE | | | HOR. ANGLE | | | VERT. ANGLE | | |
| | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ |
| Mean | 303 | 31 | 36 | 40 | 48 | 05 | | | | | | | | | | | | |
| Parallax | | | | | | | | | | | | | | | | | | |
| Mean refraction | | | | − | 1 | 07 | | | | | | | | | | | | |
| h (sum) | | | | 40 | 46 | 58 | | | | | | | | | | | | |

| | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. |
|---|---|---|---|---|---|---|---|---|---|
| Mean time | 21 | 16 | | | | | | | |
| Watch correction | | — | | | | | | | |
| TZC | — | | | | | | | | |
| Universal time (UT) | 21 | 16 | | | | | | | |

| | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|
| δ at 0h UT ± | | | | | | | | | |
| UT × δ var. per hr ± | | | | | | | | | |
| δ ± | + 40 | 06 | 41 | | | | | | |

| | | Logarithms | | | Logarithms | | | Logarithms |
|---|---|---|---|---|---|---|---|---|
| p | 49 53 19 | | p | | | p | | |
| φ | 51 23 30 | Sec 0.20482010 | φ | | Sec | φ | | Sec |
| h | 40 46 58 | Sec 0.12079431 | h | | Sec | h | | Sec |
| 2 | 142 03 47 | | 2 | | | 2 | | |
| s | 71 01 53.5 | Cos 9.51194726 | s | | Cos | s | | Cos |
| p | 49 53 19 | | p | | | p | | |
| s−p | 21 08 34.5 | Cos 9.96973435 | s−p | | Cos | s−p | | Cos |
| 1. (sum) | | 19.80729602 | 2. (sum) | | | 3. (sum) | | |
| Sum÷2=log cos A/2 | | 9.90364801 | Sum÷2=log cos A/2 | | | Sum÷2=log cos A/2 | | |

| | NR 1 | NR 2 | NR 3 | | |
|---|---|---|---|---|---|
| A/2 | 36 46 17.6 | | | Mean true azimuth to Mark | |
| A, (E or W) | 73 32 35 | | | Grid correction ± | |
| Azimuth of S | 73 32 35 | | | Grid azimuth to Mark | |
| Angle, Mark to S − | 303 31 36 | | | Magnetic azimuth to Mark | |
| True azimuth to Mark | 130 00 59 | | | Magnetic declination E(−), W(+) | |

Computations:—Three sets are computed separately for check. Refraction and parallax from TM 5-236.

TZC=Time zone correction to universal time.

$s = \frac{1}{2}(\phi + h + p)$

δ Correction = UT × variation per day ÷ 24
If δ is (+), p = 90° − δ; if δ is (−), p = 90° + δ
(s − p) = arithmetical difference, always positive.

Formula is $\cos \frac{1}{2} A = \sqrt{\frac{\cos s \cos (s-p)}{\cos \phi \cos h}}$

A = astronomic azimuth east or west of north.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| J. D. Vance | 16 Mar. 56 | R. A. Smith | 5 April 56 |

DA, FORM FEB 57 1908

② Logarithmic functions, DA Form 1908

*Figure 12*—Continued.

DIAGRAM — TRUE NORTH




**AZIMUTH BY ALTITUDE METHOD** (TM 5-237)

| PROJECT | LOCATION | | |
|---|---|---|---|
| W-8058 | Virginia | | |
| **ORGANIZATION** | **LATITUDE** | **LONGITUDE** | **STATION** |
| A & S, Inc. | 38° 39' 49"N. | 77° 09' 00"W. | BENVENUE |
| **MARK** | **INSTRUMENT** (Number and type) | | **STANDARD TIME** (Meridian) |
| GUNSTON | | | 75° |
| **CELESTIAL BODY(S)** | **WATCH FAST (−) SLOW (+)** | | **WATCH COMPARED** (Time) |
| Sun | — | | — |
| **DATE** | **OBSERVER** | | **WEATHER** |
| 17 Sept. 52 | T. F. Smith | | Clear & Warm |

| | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Hor. Angle | Vert. Angle | | Hor. Angle | Vert. Angle | | Hor. Angle | Vert. Angle | |
| Mean | 07° 19' 24" | 49° 16' 24" | | 12° 46' 06" | 50° 40' 07" | | 18° 30' 58" | 51° 48' 19" | |
| Parallax | | + | 06 | | + | 06 | | + | 05 |
| Mean refraction | | − | 50 | | − | 48 | | − | 46 |
| h (sum) | | 49° | 15' 40" | | 50° | 39' 25" | | 51° | 47' 38" |
| | Hrs. | Min. | Sec. | Hrs. | Min. | Sec. | Hrs. | Min. | Sec. |
| Mean time | 10 | 44 | | 10 | 59 | | 11 | 14 | |
| Watch correction | | | | | | | | | |
| TZC | +5 | | | +5 | | | +5 | | |
| Universal time (UT) | 15 | 44 | | 15 | 59 | | 16 | 14 | |
| δ at 0h UT ± | 2° | 22' | 11" | 2° | 22' | 11" | 2° | 22' | 11" |
| UT × δ var. per hr. ± | − | 15 | 13 | − | 15 | 27 | − | 15 | 42 |
| δ ± + | 02 | 06 | 58 | 02 | 06 | 44 | 02 | 06 | 29 |
| h + | 49 | 15 | 40 | 50 | 39 | 25 | 51 | 47 | 38 |
| φ + | 38 | 39 | 49 | 38 | 39 | 49 | 38 | 39 | 49 |
| Sin δ ± + | 0.03692471 | | | 0.03685688 | | | 0.03678421 | | |
| Sin h + + | 0.75769156 | | | 0.77336403 | | | 0.78579093 | | |
| Sin φ + + | 0.62474687 | | | 0.62474687 | | | 0.62474687 | | |
| Cos h ± + | 0.65261283 | | | 0.63396220 | | | 0.61849221 | | |
| Cos φ ± + | 0.78082735 | | | 0.78082735 | | | 0.78082735 | | |
| Cos A ± − | 0.85647490 | | | 0.90158855 | | | 0.94036592 | | |
| A (E ~~or W~~) | 148° | 55' | 23" | 154° | 22' | 04" | 160° | 06' | 47" |
| Azimuth of S | 148 | 55 | 23 | 154 | 22 | 04 | 160 | 06 | 47 |
| Angle, Mark to S − | 07 | 19 | 24 | 12 | 46 | 06 | 18 | 30 | 58 |
| True Az. to Mark | 141 | 35 | 59 | 141 | 35 | 58 | 141 | 35 | 49 |
| Mean true azimuth to Mark | 141° | 35' | 55" | | | | | | |
| Grid correction | | | | | | | | | |
| Grid azimuth | | | | | | | | | |
| Mag. azimuth to Mark | | | | | | | | | |
| Mag. declination | | | | | | | | | |

$$\text{Cos A} = \frac{\sin \delta - \sin h \sin \phi}{\cos h \cos \phi}$$

Computation = Three sets are computed separately for check, refraction and parallax corrections are obtained from TM 5-236. Apply watch correction to observed mean time. TZC = time zone correction to universal time.

δ = declination, (+) if north, (−) if south. h = altitude. φ = latitude, (+) if north, (−) if south.
A = Astronomic azimuth east or west of north. If cos A is (−), A is between 90° and 180°.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R A Smith | 12 Nov 54 | W. C. Cumen | 13 Nov 54 |

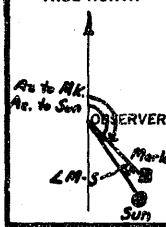
DA FORM 1 FEB 57 1907

① Natural functions, DA Form 1907

*Figure 13. Computation of azimuth using the altitude method (sun).*

**AZIMUTH BY ALTITUDE METHOD** (Logarithmic) (TM 5-237)

DIAGRAM — TRUE NORTH




| | | | |
|---|---|---|---|
| PROJECT: W-8058 | LOCATION: Virginia | | |
| ORGANIZATION: A & S | LATITUDE: 38°39'49"N | LONGITUDE: 77°09'00"W | STATION: BENVENUE |
| MARK: GUNSTON | INSTRUMENT (Number and type): | | STANDARD TIME (Meridian): 75° |
| CELESTIAL BODY(S): Sun | WATCH FAST (−) SLOW (+): — | | WATCH COMPARED (Time): |
| DATE: 17 Sept 52 | OBSERVER: T. F. Smith | | WEATHER: Clear & Warm |

| | SET NR 1 | | | | | | SET NR 2 | | | | | | SET NR 3 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | HOR. ANGLE | | | VERT. ANGLE | | | HOR. ANGLE | | | VERT. ANGLE | | | HOR. ANGLE | | | VERT. ANGLE | | |
| Mean | 07° | 19' | 24" | 49° | 16' | 24" | 12° | 46' | 06" | 50° | 40' | 07" | 18° | 30' | 58" | 51° | 48' | 19" |
| Parallax | | | | | + | 06 | | | | | + | 06 | | | | | + | 05 |
| Mean refraction | | | | | − | 50 | | | | | − | 48 | | | | | − | 46 |
| h (sum) | | | | 49 | 15 | 40 | | | | 50 | 39 | 25 | | | | 51 | 47 | 38 |

| | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. |
| Mean time | 10 | 44 | | 10 | 59 | | 11 | 14 | |
| Watch correction | | | | | | | | | |
| TZC | +5 | | | +5 | | | +5 | | |
| Universal time (UT) | 15 | 44 | | 15 | 59 | | 16 | 14 | |
| δ at 0h UT ± | 2° | 22 | 11" | 2° | 22' | 11" | 2° | 22' | 11" |
| UT × δ var. per hr ± | − | 15 | 13 | − | 15 | 27 | − | 15 | 42 |
| δ ± | + 02 | 06 | 58 | + 02 | 06 | 44 | + 02 | 06 | 29 |

| | SET NR 1 | | | | SET NR 2 | | | | SET NR 3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p | 87 | 53 | 02 | Logarithms | 87 | 53 | 16 | Logarithms | 87 | 53 | 31 | Logarithms |
| φ | 38 | 39 | 49 | Sec 0.10744499 | 38 | 39 | 49 | Sec 0.10744499 | 38 | 39 | 49 | Sec 0.10744499 |
| h | 49 | 15 | 40 | Sec 0.18534439 | 50 | 39 | 25 | Sec 0.19793663 | 51 | 47 | 38 | Sec 0.20866577 |
| 2 ⌊ | 175 | 48 | 31 | | 177 | 12 | 30 | | 178 | 20 | 58 | |
| s | 87 | 54 | 15.5 | Cos 8.56310849 | 88 | 36 | 15 | Cos 8.38666797 | 89 | 10 | 29 | Cos 8.15846250 |
| p | 87 | 53 | 02 | | 87 | 53 | 16 | | 87 | 53 | 31 | |
| s−p | 0 | 01 | 14 | Cos 9.99999997 | 0 | 42 | 59 | Cos 9.99996605 | 1 | 16 | 58 | Cos 9.99989115 |
| 1. (sum) | | | | 18.85589784 | 2. m) | | | 18.69201564 | 3. (sum) | | | 18.47446441 |
| Sum÷2=log cos A/2 | | | | 9.42794892 | Sum÷2=log cos A/2 | | | 9.34600182 | Sum÷2=log cos A/2 | | | 9.23723220 |

| | NR 1 | | | NR 2 | | | NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| A/2 | 74° | 27' | 41.5" | 77° | 11' | 02" | 80° | 03' | 23.6" |
| A, (E or W) | 148° | 55' | 23" | 154° | 22' | 04" | 160° | 06' | 47" |
| Azimuth of S | 148° | 55' | 23" | 154° | 22' | 04" | 160° | 06' | 47" |
| Angle, Mark to S − | 07° | 19' | 24" | 12° | 46' | 06" | 18° | 30' | 58" |
| True azimuth to Mark | 141° | 35' | 59" | 141° | 35' | 58" | 141° | 35' | 49" |

| | | | |
|---|---|---|---|
| Mean true azimuth to Mark | 141° | 35' | 55" |
| Grid correction ± | | | |
| Grid azimuth to Mark | | | |
| Magnetic azimuth to Mark | | | |
| Magnetic declination E(−), W(+) | | | |

Computations:—Three sets are computed separately for check. Refraction and parallax from TM 5-236.

TZC = Time zone correction to universal time.

$s = \frac{1}{2}(\phi + h + p)$

δ Correction = UT × variation per day ÷ 24

If δ is (+), $p = 90° - \delta$; if δ is (−), $p = 90° + \delta$

(s − p) = arithmetical difference, always positive.

Formula is $\cos \frac{1}{2} A = \sqrt{\frac{\cos s \cos (s-p)}{\cos \phi \cos h}}$

A = astronomic azimuth east or west of north.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| J. D. Vance | 16 Mar. 56 | R. Asmith | 5 April 56 |

DA FORM 1 FEB 57 1908

② Logarithmic functions, DA Form 1908

*Figure 13*—Continued.

(1) The formulas for logarithmic computations are:

$$\cos\frac{1}{2}A=\sqrt{\frac{\cos s\cos(s-p)}{\cos\phi\cos h}}$$

$$\tan\frac{1}{2}A=\sqrt{\frac{\sin(s-h)\sin(s-\phi)}{\cos s\cos(s-p)}}$$

in which

$h$=observed altitude corrected for refraction

$\phi$=latitude of station

$p$=polar distance of sun $(90°-\delta)$

$s=\frac{1}{2}(h+\phi+p)$.

This first formula is simpler, while the second is slightly more precise.

(2) The formula for machine computation is:

$$\cos A=\frac{\sin\delta-\sin h\sin\phi}{\cos h\cos\phi}$$

(3) The procedure of computation is as follows:

(*a*) Compute each position by the direction method, or each set by the repetition method, separately. The observation may consist of a series of positions (D&R observations) or a single pointing on the mark followed by a series of direct pointings on the sun, and an equal number of reversed pointings on the sun, followed by a final pointing on the mark.

(*b*) For each position or set, determine the mean of the recorded times and the horizontal and the vertical circle readings. For each position or set, the angle from the mark to the star is the difference between the mean circle reading on the mark and the mean reading on the star regardless of the number of repetitions.

(*c*) Each mean vertical angle must be corrected for parallax and refraction. Tables of parallax (table VIII) and refraction (table V) are included in appendix III. For lower order observations the parallax may be neglected since it is never greater than 9″.

(*d*) Convert the local time of observation to Ephemeris time as explained in paragraph 10*d*.

(*e*) Extract, from the AE&NA, the declination of the sun for the Ephemeris time of the observation.

(*f*) The value of $p$ is determined by subtracting the declination of the sun from 90°. If the sun is south of the equator the declination is considered to be negative. The quantity $(s-p)$ is always considered positive.

(*g*) Solve for the value of "A" by use of the formula.

(*h*) Subtract the horizontal angle, mark to sun, from the computed azimuth of the sun to determine the azimuth from station to the mark. The final azimuth is the mean of all the azimuth values.

(*i*) Either DA Form 1907 or 1908 may be used for this computation.

*b. Hour Angle Method.* The hour angle method (fig. 14) is preferred when the time of the observation is accurately known. It is not greatly affected by errors in the latitude and declination, and should always be used when necessary to observe the sun near the meridian.

(1) The formulas are:

$$\tan\frac{1}{2}(A-q)=\frac{\sin\frac{1}{2}(\phi-\delta)}{\cos\frac{1}{2}(\phi+\delta)}\cot\frac{1}{2}t$$

$$\tan\frac{1}{2}(A+q)=\frac{\cos\frac{1}{2}(\phi-\delta)}{\sin\frac{1}{2}(\phi+\delta)}\cot\frac{1}{2}t$$

or

$$\tan A=-\frac{\sin t}{\cos\phi\tan\delta-\sin\phi\cos t}.$$

When A exceeds 45° from the meridian, the last may be stated as follows:

$$-\cot A=\frac{\cos\phi\tan\delta}{\sin t}-\frac{\sin\phi}{\tan t}.$$

The algebraic signs of the functions may be maintained, or the following rules followed in the case of the first formula:

(*a*) $t$ is taken less than $12^h$ and positive. If over $12^h$, subtract from $24^h$.

(*b*) If $t$ is less than $12^h$, the azimuth is to the west of the meridian. If greater than $12^h$, it is to the east.

(*c*) $\frac{1}{2}(A-q)$ and $\frac{1}{2}(A+q)$ are taken out as less than 90°.

(*d*) When $(\phi-\delta)$ is positive, add numerically the values of $\frac{1}{2}(A-q)$ and $\frac{1}{2}(A+q)$ to obtain the azimuth angle between

**AZIMUTH BY HOUR ANGLE METHOD** (TM 5-237)

DIAGRAM: TRUE NORTH; Az. to Mk; Az. to Sun; OBSERVER; Sun; Mark; ∠ Mk-S

| | | | |
|---|---|---|---|
| PROJECT: 400-23 | LOCATION: Virginia | | |
| ORGANIZATION: A&S | LATITUDE (φ): 38°42'36"N | LONGITUDE (λ): 77°08'46"W | STATION: 0.54 |
| MARK: 02 | INSTRUMENT (Number and type): | | STANDARD TIME (Meridian): 75° |
| CELESTIAL BODY(S): Sun | WATCH ~~FAST (-)~~ SLOW (+): 24.1 sec | | WATCH COMPARED (Time): |
| DATE: 8 Dec. 52 | OBSERVER: C.J. Brooks | | WEATHER: Cold |

| | SET NR 1 | | | | | | SET NR 2 | | | | | | SET NR 3 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TIME | | | HOR. ANGLE | | | TIME | | | HOR. ANGLE | | | TIME | | | HOR. ANGLE | | |
| | HRS. | MIN. | SEC. | ° | ′ | ″ | HRS. | MIN. | SEC. | ° | ′ | ″ | HRS. | MIN. | SEC. | ° | ′ | ″ |
| Mean | 15 | 42 | 22.6 | 83 | 19 | 55 | 15 | 51 | 34.3 | 84 | 56 | 30 | 16 | 01 | 13.2 | 86 | 35 | 42 |

**SUN OBSERVATION**

[For star observation, use factors enclosed in brackets]

| | | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. |
| 1 | Mean time of observation | 15 | 42 | 22.6 | 15 | 51 | 34.3 | 16 | 01 | 13.2 |
| 2 | Watch correction ± | | + | 24.1 | | + | 24.1 | | + | 24.1 |
| 3 | Time Zone Correction (TZC) ± | +5 | | | +5 | | | +5 | | |
| 4 | UT of observation (1+2+3) | 20 | 42 | 46.7 | 20 | 51 | 58.4 | 21 | 01 | 37.3 |
| 5 | $0^h$ Greenwich EQT ~~[or Sid. T.]~~ ± | + | 8 | 14.2 | + | 8 | 14.2 | + | 8 | 14.2 |
| 6 | UT × var. EQT per hour ~~[or table III Epin.]~~ ± | | − | 22.8 | | − | 23.0 | | − | 23.2 |
| 7 | (5+6) correct EQT ~~[or Gr. Sid. T. (4+5+6)]~~ ± | + | 7 | 51.4 | + | 7 | 51.2 | + | 7 | 51.0 |
| 8 | (4+7) GAT ~~[or R.A.]~~ | 20 | 50 | 38.1 | 20 | 59 | 49.6 | 21 | 09 | 28.3 |
| 9 | GHA in time (GAT−12h) ~~[or (7−8)]~~ | 8 | 50 | 38.1 | 8 | 59 | 49.6 | 9 | 09 | 28.3 |
| 10 | GHA in arc | 132 | 39 | 31.5 | 134 | 57 | 24.0 | 137 | 22 | 04.5 |
| 11 | Longitude, West (−), ~~East (+)~~ ± | 77 | 08 | 46.0 | 77 | 08 | 46.0 | 77 | 08 | 46.0 |
| 12 | LHA (10+11)=t ~~(or 360° − LHA = t)~~ | 55 | 30 | 45.5 | 57 | 48 | 38.0 | 60 | 13 | 18.5 |

| | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ |
| t ± | 55 | 30 | 46 | 57 | 48 | 38 | 60 | 13 | 18 |
| Lat. (φ) ± | 38 | 42 | 36 | | | | | | |
| Dec. (δ) ± | -22 | 47 | 05 | -22 | 47 | 07 | -22 | 47 | 10 |
| Sin t ± | .82425248 | | | .84629132 | | | .86795332 | | |
| Cos t ± | .56622243 | | | .53272037 | | | .49664578 | | |
| Sin φ ± | .62537886 | | | | | | | | |
| Cos φ ± | .78032127 | | | | | | | | |
| Tan δ ± | .42004756 | | | .42005897 | | | .42007608 | | |
| −Tan A ± | -1.20880188 | | | 1.28044947 | | | 1.35960567 | | |
| A ~~(East~~ W) | 129 | 35 | 59 | 127 | 59 | 20 | 126 | 20 | 05 |
| Azimuth of S | 230 | 24 | 01 | 232 | 00 | 40 | 233 | 39 | 55 |
| ∠, Mark to S − | 83 | 19 | 55 | 84 | 56 | 30 | 86 | 35 | 42 |
| Tr. Az. to Mark | 147 | 04 | 06 | 147 | 04 | 10 | 147 | 04 | 13 |

| | | | |
|---|---|---|---|
| Mean true azimuth to Mark | 147 | 04 | 10 |
| Grid correction ± | | | |
| Grid azimuth to Mark | | | |
| Magnetic azimuth to Mark | | | |
| Magnetic declination E(−), W(+) | | | |

$$-\text{Tan A} = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}$$

If LHA is greater than 180°, subtract from 360° and reverse sign.

Obtain δ from Ephemeris.

Check signs and quadrants by use of sketch.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.A. Smith | 9 Feb 55 | W. C. Aumen | 9 Feb 55 |

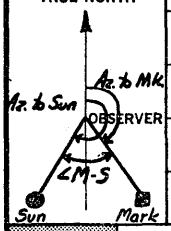
DA FORM 1 FEB 57 1905

① Natural functions, DA Form 1905

*Figure 14. Computation of azimuth using the sun at any hour angle*

**AZIMUTH BY HOUR ANGLE METHOD (Logarithmic)** (TM 5-237)

DIAGRAM




| | | | |
|---|---|---|---|
| PROJECT: 400-23 | LOCATION: Virginia | | |
| ORGANIZATION: A&S | LATITUDE: 38°42'36"N | LONGITUDE: 77°08'46"W | STATION: 054 |
| MARK: 02 | INSTRUMENT (Number and type): | | STANDARD TIME (Meridian): 75° |
| CELESTIAL BODY(S): Sun | WATCH ~~FAST (-)~~ SLOW (+): 24.1 sec. | | WATCH COMPARED (Time): |
| DATE: 8 Dec. 52 | OBSERVER: C. J. Brooks | | WEATHER: Cold |

| | SET NR 1 | | | | | | SET NR 2 | | | | | | SET NR 3 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TIME | | | HOR. ANGLE | | | TIME | | | HOR. ANGLE | | | TIME | | | HOR. ANGLE | | |
| | HRS. | MIN. | SEC. | ° | ′ | ″ | HRS. | MIN. | SEC. | ° | ′ | ″ | HRS. | MIN. | SEC. | ° | ′ | ″ |
| Mean | 15 | 42 | 22.6 | 83 | 19 | 55 | 15 | 51 | 34.3 | 84 | 56 | 30 | 16 | 01 | 13.2 | 86 | 35 | 42 |

**SUN OBSERVATION**

[For star observation, use factors enclosed in brackets]

δ
1. -22°47'05"
2. -22 47 07
3. -22 47 10

| | | | SET NR 1 | | | SET NR 2 | | | SET NR 3 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. | HRS. | MIN. | SEC. |
| 1 | Mean time of observation | | 15 | 42 | 22.6 | 15 | 51 | 34.3 | 16 | 01 | 13.2 |
| 2 | Watch correction | ± | + | | 24.1 | + | | 24.1 | + | | 24.1 |
| 3 | Time Zone Correction (TZC) | ± | +5 | | | +5 | | | +5 | | |
| 4 | UT of observation (1+2+3) | | 20 | 42 | 46.7 | 20 | 51 | 58.4 | 21 | 01 | 37.3 |
| 5 | $0^h$ Greenwich EQT ~~[or Sid. T.]~~ | ± | + | 8 | 14.2 | + | 8 | 14.2 | + | 8 | 14.2 |
| 6 | UT × var. per hour ~~[or table III Eph.]~~ | ± | − | | 22.8 | − | | 23.0 | − | | 23.2 |
| 7 | (5+6) correct EQT ~~[or G. Sid. T. (4+3+6)]~~ | ± | + | 7 | 51.4 | + | 7 | 51.2 | + | 7 | 51.0 |
| 8 | (4+7) GAT ~~[or Rat]~~ | | 20 | 50 | 38.1 | 20 | 59 | 49.6 | 21 | 09 | 28.3 |
| 9 | GHA in time (GAT−12h) ~~[or (7−8)]~~ | | 8 | 50 | 38.1 | 8 | 59 | 49.6 | 9 | 09 | 28.3 |
| 10 | GHA in arc | | 132 | 39 | 31.5 | 134 | 57 | 24.0 | 137 | 22 | 04.5 |
| 11 | Longitude, West (−), ~~East (+)~~ | ± | 77 | 08 | 46.0 | 77 | 08 | 46.0 | 77 | 08 | 46.0 |
| 12 | LHA (10+11) = t (~~or 360° − LHA = t~~) | | 55 | 30 | 45.5 | 57 | 48 | 38.0 | 60 | 13 | 18.5 |

| | SET NR 1 | SET NR 2 | SET NR 3 |
|---|---|---|---|
| Log sin φ | + 9.79614319 | | |
| Log cos t | + 9.75298707 | 9.72649931 | 9.69604675 |
| (Sum) log A | + 9.54913026 | 9.52264250 | 9.49218994 |
| A | + 0.35410353 | 0.33315206 | 0.31059177 |
| Log cos φ | + 9.89227345 | | |
| Log tan δ | − 9.62329847 | 9.62331026 | 9.62332795 |
| (Sum) log B | − 9.51557192 | 9.51558371 | 9.51560140 |
| B | − 0.32777205 | 0.32778095 | 0.32779430 |
| B − A | − 0.68187558 | 0.66093301 | 0.63838607 |
| Log sin t | + 9.91606027 | 9.92751989 | 9.93849637 |
| Log (B − A) | − 9.83370513 | 9.82015745 | 9.80508340 |
| Log(− tan A)(diff.) | 0.08235514 | 0.10736244 | 0.13341297 |
| A ~~(E or~~ W) | 129°35'59" | 127°59'20" | 126°20'05" |
| Azimuth of S | 230 24 01 | 232 00 40 | 233 39 55 |
| ∠, Mark to S − | 83 19 55 | 84 56 30 | 86 35 42 |
| Tr. Az. to Mark | 147 04 06 | 147 04 10 | 147 04 13 |

| | | |
|---|---|---|
| Mean true azimuth to Mark | 147 04 10 | |
| Grid correction ± | | |
| Grid azimuth to Mark | | |
| Magnetic azimuth to Mark | | |
| Magnetic declination E(−), W(+) | | |

$$-\text{Tan } A = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}$$

If LHA is greater than 180°, subtract from 360° and reverse sign.

Obtain δ from Ephemeris.

Check signs and quadrants by use of sketch.

COMPUTED BY: R.A. Smith — DATE: 9 Feb 55

CHECKED BY: W. C. Aumen — DATE: 9 Feb 55

DA FORM 1 FEB 57 1906

② Log functions, DA Form 1906

*Figure 14*—Continued.

the meridian and the sun. If negative, subtract numerically. $q$ is the parallactic angle, which cancels in the solution.

(2) The second is the standard azimuth formula.

(3) The procedure is as follows:

(*a*) Compute each position or set separately, using the means of the times of pointings with the corresponding means of the horizontal circle readings.

(*b*) Convert the local standard time of the observation to GAT.

(*c*) Subtract $12^h$ from GAT to obtain GHA, and convert GHA to units of arc.

(*d*) Subtract the longitude of the station from GHA to obtain $t$, the local hour angle of the sun. West longitudes are plus, east longitudes minus. If using rules to disregard signs, subtract $t$ from 360° when it is over 180°.

(*e*) Take declination of sun from an ephemeris for date and UT of observation.

(*f*) Apply formula.

(*g*) Subtract angle, mark to sun.

(4) All pointings on the sun refer to its center. The method of pointing on the sun should always be recorded by the field party. The computer should inspect this record and apply any corrections for semidiameter or other corrections that may be required. Ordinarily, opposite tangents will be observed so that a mean of the readings will refer to the center.

## Section III. DETERMINATION OF LATITUDE

### 17. Relation of Latitude to Zenith Distance

Nearly all observations for latitude, other than those using the astrolabe or zenith camera, consist of measuring the altitude of a celestial body when it is on or near the meridian. The latitude of a place may be defined as the altitude of the pole or the declination of the zenith at the place. Either can be obtained from the meridian altitude or zenith distance of a body of known declination. The equation is:

$$\phi = \zeta + \delta$$

where $\zeta$ is the meridian zenith distance and $\delta$ is the body's declination. $\zeta$ is positive when the body is toward the equator from the zenith, negative when toward the pole. $\phi$ and $\delta$ are positive north of the equator, and negative south of it.

### 18. Latitude by the Altitude of a Circumpolar Star at Culmination

*a. Formula.* Latitude, by this method, is detemined by use of the following formula:

$$\phi = h \pm p$$

Where: $h$=the corrected altitude of the star.

$p$=the polar distance $(90° - \delta)$. $p$ is negative if the star is between the zenith and the pole but is positive if beyond the pole.

*b. Procedure of Computation.* The computation is as follows:

(1) Apply the correction for refraction to the observed altitude.

(2) Extract, from the Apparent Places of the Fundamental Stars, the declination of the star for the date and time of the observation.

(3) If the star was observed at upper culmination, subtract the value of $p$ from the corrected altitude $(90° - \delta)$. If the star was observed at lower culmination, the value of $p$ must be added to the corrected altitude (fig. 15).

(4) If the position of the star, during observation, was *not* recorded, the following may help in determining the position:

(*a*) If the local sidereal time and the right ascension of the star are equal, the position is upper culmination.

(*b*) If the local sidereal time is equal to RA+$12^h$, the position is lower culmination.

### 19. Latitude From the Zenith Distance of a Close Circumpolar Star at Any Hour Angle

*a. The Formula.* Latitude, by this method, is determined by use of the following formula:

$$\phi = h - p\cos t + \tfrac{1}{2}p^2 \sin^2 t \cot \zeta \sin 1'' - \tfrac{1}{3}p^3 \cos t \sin^2 t \sin^2 1'' + \tfrac{1}{8}p^4 \sin^4 t \cot^3 \zeta \sin^3 1''.$$

*Latitude by Altitude of Polaris at Culmination*

| | | | |
|---|---|---|---|
| *Local Date* 18 May 54 | *Eastern Standard Time of observ.* 10:40PM | | |
| *Polaris observed at lower culmination.* | *Observed altitude of star* | 42° | 17′ 35″ |
| *Temperature* 45°F | *Barometric Pressure* | 30.4 inches | |
| *1. Mean refraction, Table V, Appendix III (use zenith distance of star as argument)* | | 0° | 01′ 04″ |
| *2. Correction to refraction for Temperature (same Table as 1)* | | + | 01 |
| *3. Correction to refraction for Pressure, (same Table as 1.)* | | + | 01 |
| *4. Corrected refraction (1+2+3)* | | 0° | 01 06 |
| *5. Observed altitude of star* | | 42 | 17 35 |
| *6. Corrected altitude of star (5-4)* | | 42 | 16 29 |
| *7. Declination of star (for date)* | | 89 | 02 58 |
| *8. Polar Distance, p, of star (90°-Declination)* | | +0 | 57 02 |
| *9. Latitude (6+8)* | | 43° | 13′ 31″ |

*Figure 15. Computation of latitude by altitude of a circumpolar star at culmination.*

Where: $h$=corrected altitude
$p$=polar distance (in seconds of arc)

*b. Procedure of Computation.* The computation (fig. 16) is as follows:

(1) Abstract, from the field records, the zenith distance and the sidereal time of observation. Correct the time for chronometer error. Apply the refraction, level, and collimation corrections to the observed zenith distance.

(2) Abstract, from the Apparent Places of the Fundamental Stars, the declination and the right ascension of the star at the time of observation. Determine the local hour angle ($t$) by use of the formula $t$=LST−RA of star and the polar distance by use of the formula $p=90°-\delta$. Reduce the polar distance to seconds of arc.

(3) From the appropriate function tables, obtain sin $t$, cos $t$, sin 1″ and cot $\zeta$.

(4) Apply the formula given above, the terms of which are the star's elevation above or below the pole. DA Form 2839, Latitude From Zenith Distance of Polaris, is designed for a logarithmic solution of this method, and can be used for other close circumpolar stars.

## 20. Latitude From the Meridian Zenith Distance of Any Star

*a. The Formula.* Latitude is determined from the meridian zenith distance of any star by use of the formula:

$$\phi=\delta+\zeta.$$

Where: $\delta$=declination of star; positive if north of equator, negative if south.

$\zeta$=meridian zenith distance of star corrected for refraction. $\zeta$ is positive when the star is on the opposite side of the zenith from the pole; negative when the star is between the zenith and the pole.

*b. Procedure of Computation.* The computation (fig. 17) is accomplished as follows:

(1) Determine the refraction correction and apply it to the observed zenith distance to determine the corrected zenith distance.

(2) Extract, from the Apparent Places of the Fundamental Stars, the declination of the star for the date and time of observation.

(3) Should there be any confusion in algebraic signs, a diagram together with a roughly known value of the latitude will indicate the proper procedure. This confusion might occur in the case of a subpolar star observed at a very high latitude.

(4) Substitute known values in the formula and solve for the latitude.

## 21. Latitude by Use of Table II, AE&NA

*a. The Formula.* The following formula is used in determining latitude by use of table II, AE&NA:

$$\phi=h+(a_0+a_1+a_2).$$

Where: $h$=observed altitude, corrected for refraction.

$a_0$, $a_1$, $a_2$=factors in table II, AE&NA.

*b. Procedure of Computation.* The computation (fig. 18) is as follows:

(1) Extract, from the field records, the mean observed altitude and the date and time of observation. Reduce the time to local sidereal time if necessary.

(2) Determine the correction for refraction and apply it to the observed altitude to determine the corrected altitude.

(3) Interpolate in table II, AE&NA, for the values of $a_0$, $a_1$, and $a_2$.

(4) Substitute known values in the formula and solve for the latitude.

| PROJECT | LATITUDE FROM ZENITH DISTANCE OF POLARIS (TM 5-237) | | |
|---|---|---|---|
| OBSERVER R. S. | UNIVERSAL DATE 12.18 July '63 | STATION NORTH | |
| ASST. OBSERVER F. E. W. | RECORDER R. A. G. | TEMP. 36 °F | BARO. 29.80 " |
| INST. WILD T-4 No. 37446 | LEVEL VALUE " d/2 = 0.977 | REFRACTION FACTOR 1.0239 | POSITION NO. 1 |

| | h m s | | ° ' " |
|---|---|---|---|
| CHRON. TIME | 17 32 17.9 | $\zeta_o$ | 28 22 16.95 |
| CHRON. CORR. | 0.0 | LEVEL CORR. | + 00.98 |
| LOCAL SID. TIME | 17 32 17.9 | REFRACTION CORR | + 32.01 |
| R.A. | − 01 57 59.1 | COLLIMATION | + 11.70 |
| HOUR ANGLE (t) | 15 34 18.8 | $\zeta$ | 28 23 01.64 |
| t | 233° 34' 42.0" | | |
| | | | 90° 00' 00 00" |
| LOG p | 3.516 4957 | $\delta$ | − 89 05 15.30 |
| LOG COS t | 9.773 5840 - 10 | p | ° 54' 44.70 " |
| LOG I | 3.290 0797 | p | 3284.70 " |
| | | | |
| LOG 0.5 | 9.698 9700 - 10 | LOG 1/3 | 9.522 88 - 10 |
| 2 LOG p | 7.032 9914 | 3 LOG p | 10.549 49 |
| 2 LOG SIN t | 9.811 2348 - 10 | LOG COS t | 9.773 58 - 10 |
| LOG COT $\zeta$ | 0.2673390 | 2 LOG SIN t | 9.811 23 - 10 |
| LOG SIN 1" | 4.685 5749 - 10 | 2 LOG SIN 1" | 9.371 15 - 20 |
| LOG II | 1.496 1101 | LOG III | 9.028 33 - 10 |
| | | | |
| I | + 1950.20 " | LOG 1/8 | 9.096 91 - 10 |
| II | + 31.34 " | 4 LOG p | 14.065 98 |
| III | + 00.11 " | 4 LOG SIN t | 9.622 47 |
| IV | + 00.00 " | 3 LOG COT $\zeta$ | 0.802 02 |
| SUM | + 1981.65 " | 3 LOG SIN 1" | 4.056 72 - 20 |
| SUM | ° 33' 01.65" | LOG IV | 7.644 10 - 10 |
| h (90° - $\zeta$) | 61 36 58.36 | | |
| $\phi_A$ | 62 10 00.01 | | |
| | | | |

$\phi_A$ = h - I + II - III + IV

**SIGNS:** I and III are PLUS in the first and fourth quadrants.
II and IV are always PLUS.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R. Salvermoser - AMS | July '63 | T. E. Wilson - AMS | July '63 |

DA FORM 2839, 1 OCT 64

*Figure 16. Computation of latitude by altitude of a circumpolar star at any hour angle, DA Form 2839.*

*Latitude from a Meridian Zenith Distance*

*Station: Hoff*

*Celestial Body: α Aurigae (Capella)*

*Date: 20 Feb 55*

*Longitude: 75°30'W*

*Time Zone Meridian: 75°W*

| | | | |
|---|---|---|---|
| *Local Standard Time* | $18^h$ | $15^m$ | $10^s$ |
| *Time Zone Correction* | +5 | | |
| *Universal Time* | $23^h$ | $15^m$ | $10^s$ |
| *Altitude* | 44° | 16′ | 23″ |
| *Refraction* | − | 0 | 59 |
| *h* | 44 | 15 | 24 |
| *Zenith Distance (ζ)* | −45° | 44′ | 36″ |
| *Declination (δ)* | 45 | 57 | 27 |
| *Latitude (φ)* | 0° | 12′ | 51″ |

*Figure 17. Computation of latitude from the meridian zenith distance of a star.*

## 22. Latitude by Circummeridian Altitudes of a Star

This is a more accurate method for observing latitude by means of the instruments ordinarily available. It is an extension of the method of meridian distance and the accuracy is increased by taking a greater number of pointings. A series of altitudes or zenith distances is observed on a star at recorded times during a period of a few minutes before and after transit. From the hour angle of the star at the time of each observation, the observed altitudes are reduced to the equivalent meridian altitude. The observing period should not exceed from 10 minutes before to 10 minutes after transit. The procedure is as follows:

*a.* Reduce each observed altitude (or zenith distance) to the meridian (fig. 19).

(1) Compute the LST of the pointing by correcting the recorded time for chronometer error. The numerical difference between the LST and the star's right ascension is the hour angle ($t$).

(2) Let $A=\frac{\cos\phi\cos\delta}{\sin\zeta}$, in which $\phi$ is a closely scaled map latitude, or is a trial value computed from an altitude taken near the meridian. $\zeta$ should be the value nearest the meridian. Also, let $m=\frac{2\sin^2\left(\frac{t}{2}\right)}{\sin 1''}$ in which $t$ is in seconds of arc (table IX).

(3) Then $h_m=h+Am$ or $\zeta_m=\zeta-Am$; $h$ or $\zeta$ being the observed values, corrected for refraction, for the pointing being reduced.

*b.* After all observations are reduced, take the mean of all consistent values.

*c.* Apply formula: $\phi=\zeta+\delta$.

*d.* If for any reason it is necessary to reduce observations taken more than 10 minutes from the meridian, the formula,

$$\sin h_m=\sin h+\cos\phi\cos\zeta\ 2\sin^2\left(\frac{t}{2}\right)$$

should be used.

*e.* DA Form 2840, Field Computation of Latitude-Circummeridian Zenith Distances, is designed for a logarithmic solution of this method.

## 23. Precise Latitude by Circummeridian Altitudes of Stars

*a.* Circummeridian altitudes can be used for the determination of precise latitude at latitudes

*COMPUTATION OF LATITUDE BY ALTITUDE OF POLARIS (TABLE II, AE & NA)*

*DATE:* 11 APRIL 1963

*LOCAL SIDEREAL TIME:* $09^h\ 59^m\ 19^s$

*ALTITUDE OF POLARIS:* 38° 28′ 59″

| | | | |
|---|---|---|---|
| ALTITUDE (h) | = | | 38° 28′ 59.0″ |
| TABLE II AE & NA | | | |
| $a_0$ | = | + | 28 11.9 |
| $a_1$ | = | − | 06.9 |
| $a_2$ | = | | 00.0 |
| LATITUDE OF STATION | | | 38° 57′ 04.0″ |

*Figure 18. Computation of latitude by altitude of Polaris (table II, AE&NA).*

## FIELD COMPUTATION OF LATITUDE - CIRCUMMERIDIAN ZENITH DISTANCES
(TM 5-237)

| | | | |
|---|---|---|---|
| PROJECT: GREENLAND ASTRO | | STATION: ISUNGUA ECC. | |
| LOCATION: DISKO IS. | ORGANIZATION: USAMS | DATE: 21 AUG. 61 PM | ELEVATION: |
| INSTRUMENT: WILD T-4 NO. 37446 | CHRONOMETER: 2E 12001 | TEMP.: -0.1 °C | BAROM.: 26.81 Hg. |
| OBSERVER: R. SALVERMOSER | ASST. OBSERVER: J.M. LEWIS | RECORDER: G.E. O'KEEFE | |
| APPROX. POSITION: $\phi$: N 69° 46' 55.5" | $\lambda$: + 03h 28m 10.5s | ECCENTRICITY: 0 | |
| DIRECTION: FROM: / TO: | | STAR NO.: 17 | ~~(North)~~ (South) |

| POSITION NO. | 11 | 12 | 13 | 14 |
|---|---|---|---|---|
| CHRON. READING | 00 35 06.7 | 00 35 42.2 | 00 36 11.5 | 00 36 43.7 |
| CHRON. CORRECTION | + 01.9 | + 01.9 | + 01.9 | + 01.9 |
| SIDEREAL TIME | 00 35 08.6 | 00 35 44.1 | 00 36 13.4 | 00 36 45.6 |
| R. A. OF STAR | 00 34 51.0 | 00 34 51.0 | 00 34 51.0 | 00 34 51.0 |
| HOUR ANGLE (t) | 17.6 | 53.1 | 01 22.4 | 01 54.6 |
| M | 0.17 | 1.54 | 3.71 | 7.16 |
| LOG. COS $\phi_A$ | 9.538 5632 | | | |
| LOG. COS $\delta$ | 9.772 5056 | | | |
| − LOG. SIN $\zeta_1$ | 9.442 9473 | | | |
| SUM LOG. A | 9.868 1215 | | | |
| A | 0.73811 | | | |
| ZENITH DISTANCE $\zeta$ | 16° 05' 42.0" | 16° 05' 42.9" | 16° 05' 44.0" | 16° 05' 46.9" |
| REFRACTION | + 15.5" | + 15.5" | + 15.5" | + 15.5" |
| CORRECTED $\zeta$ | 16° 05' 57.5" | 16° 05' 58.4" | 16° 05' 59.5" | 16° 06' 02.4" |
| A • M | − 00.1" | − 01.1" | − 02.7" | − 05.3" |
| $\zeta_1$ | 16° 05' 57.4" | 16° 05' 57.3" | 16° 05' 56.8" | 16° 05' 57.1" |
| DECLINATION $\delta$ | 53° 40' 59.2" | 53° 40' 59.2" | 53° 40' 59.2" | 53° 40' 59.2" |
| LATITUDE $\phi_A$ | 69° 46' 56.6" | 69° 46' 56.5" | 69° 46' 56.0" | 69° 46' 56.3" |

REMARKS

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R. Salvermoser - AMS | 13 July '63 | F. E. Wilson - AMS | 13 July '63 |

DA FORM 2840, 1 OCT 64

*Figure 19. Computation of latitude from circummeridian altitudes of a star, DA Form 2840.*

between 60° and 90°, where other precise methods are impractical. Observations should be made with a broken telescope theodolite.

(1) This method requires a number of zenith distances observations at recorded times of selected fundamental catalogue (APFS) stars, during the period extending from 5 minutes before to 5 minutes after transit. The stars are observed in pairs one north and one south of the observer with the zenith distance of each being less than 30° to minimize irregular refraction conditions. For the pair, the difference in their zenith distances should not exceed 3° and preferably should be less than 1°. Ten to twelve pairs are required and the time difference between observing the stars of any one pair should not exceed 1 hour. If a close circumpolar star, at any hour angle, is used as the north star of a pair, see paragraph 19 for computation.

(2) When a pair of stars are close to the zenith (i.e. their azimuth factors (A) are less than 0.15 as determined from the formula, $A=\sin\zeta \sec\delta$), they will move fast with respect to azimuth and the instrument should be clamped in the meridian. The star is bisected in rapid succession with the horizontal crosshair, noting in the recording where the star crosses the vertical hair. A meridian correction can be computed and applied to the hour angle before reducing the zenith distance to the meridian if the telescope was not properly alined.

*b*. The procedures followed in the computation are:

(1) Abstract the field data from the field books (figs. 20 and 21).

(*a*) Chronometer times and date of observed zenith distances.

(*b*) Observed zenith distance corrected for the bubble correction. This inclination correction is computed by the formula zenith distance=observed zenith distance$+\frac{d}{2}$ (L–R) where $d$ is the vertical circle level value in seconds of arc and (L–R) is the difference between the left and right level readings.

(*c*) The radio time signals for computing the chronometer correction.

(*d*) The temperature and barometric pressure readings.

(2) When the computations are to be completed using normal observations.

(*a*) The rigorous equation for the reduction of the observed circummeridian zenith distances of a star to the meridian is:

$$\zeta_1=\zeta-\left(\frac{\cos\phi\cos\delta}{\sin\zeta_1}\right)\left(\frac{2\sin^2\frac{1}{2}t}{\sin 1''}\right)+\left(\frac{\cos\phi\cos\delta}{\sin\zeta_1}\right)^2\left(\frac{2\cot\zeta_1\sin^4\frac{1}{2}t}{\sin 1''}\right)$$

Substituting:

$$m=\frac{2\sin^2\frac{1}{2}t}{\sin 1''} \qquad n=\frac{2\sin^4\frac{1}{2}t}{\sin 1''}$$

$$A=\frac{\cos\phi\cos\delta}{\sin\zeta_1} \qquad B=A^2\cot\zeta_1$$

We have: $\zeta_1=\zeta-Am+Bn$, and
$\zeta_1=\zeta+Am+Bn$, for subpolar stars.

In the above equation:

$\zeta=$ Observed zenith distance corrected for inclination, refraction, and for index error of the vertical circle (collimation and zenith point error).

$\zeta_1=$ Zenith distance of star on the meridian.

$\phi=$ astronomic latitude of the station.

$t=$ hour angle of the star at the instant of observation.

$\delta=$ declination of the star at transit.

In the above equation, the third term was neglected. This third term is:

$$+4/3\,\frac{(1+3\cot^2\zeta_1)}{\sin 1''}\cdot A^3\sin^6\frac{1}{2}t$$

(*b*) After finding the mean zenith distance of the two stars of the pair, the latitude for the pair can be computed as follows.

$\frac{\phi_s+\phi_n}{2}=\phi$ the latitude for the pair, where

$\phi_s=\delta+\zeta_1$, for stars south of the zenith, and

$\phi_n=\delta-\zeta_1$, for stars north of the zenith.

STATION ISUNGUA Ecc. DATE 21AUG 1961 P.M.

INSTRUMENTMAN R. Salvermoser INSTRUMENT T-4 # 37446

RECORDER G.E. O'Keefe WEATHER PARTLY CLOUDY, HAZY

ASST. Obs. J.M. LEWIS

| POS. REP. | OBJECT OBSERVED | CHRON TIME H | M | S | STOP WATCH | TEL D/R | CIRCLE ° | ' | MICRO./VERN. 1ST/A(') | 2D/B(') | MEAN ' | MEAN D/R' | DIR/ANG ' | LEVEL L | R | L-R | REMARKS | LEVELS W | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | STAR # 17 | 00 | 29 | 22.9 | | | 16 | 06 | 23.6 | 23.5 | 23.55 | | | 4.0 | 4.2 | -.2' | | | |
| 2 | SOUTH | | 29 | 59.8 | | | | 06 | 14.5 | 14.4 | 14.45 | | | 4.0 | 4.1 | -.1' | | | |
| 3 | CIRCLE LEFT | | 30 | 34.0 | | | | 06 | 06.4 | 06.3 | 06.35 | | | 4.0 | 4.1 | -.1' | | | |
| 4 | | | 31 | 09.6 | | | | 06 | 00.2 | 00.2 | 00.20 | | | 4.1 | 4.1 | 0' | | | |
| 5 | | | 31 | 46.5 | | | | 05 | 54.6 | 54.5 | 54.55 | | | 4.1 | 4.1 | 0' | | | |
| 6 | | | 32 | 21.8 | | | | 05 | 49.5 | 49.7 | 49.60 | | | 4.1 | 4.1 | 0' | | | |
| 7 | | | 32 | 51.6 | | | | 05 | 46.1 | 46.1 | 46.10 | | | 4.1 | 4.1 | 0' | | | |
| 8 | | | 33 | 19.1 | | | | 05 | 43.6 | 43.4 | 43.50 | | | 4.1 | 4.1 | 0' | | | |
| 9 | | | 33 | 51.1 | | | | 05 | 42.0 | 42.0 | 42.00 | | | 4.1 | 4.1 | 0' | | | |
| 10 | | | 34 | 22.8 | | | | 05 | 42.0 | 42.1 | 42.05 | | | 4.2 | 4.0 | +.2' | | | |
| 11 | | | 35 | 06.7 | | | | 05 | 42.0 | 41.8 | 41.90 | | | 4.1 | 4.0 | +.1' | | | |
| 12 | | | 35 | 42.2 | | | | 05 | 42.9 | 42.7 | 42.80 | | | 4.1 | 4.0 | +.1' | | | |
| 13 | | | 36 | 11.5 | | | | 05 | 43.8 | 44.0 | 43.90 | | | 4.1 | 4.0 | +.1' | | | |
| 14 | | | 36 | 43.7 | | | | 05 | 46.8 | 46.8 | 46.80 | | | 4.1 | 4.0 | +.1' | | | |
| 15 | | | 37 | 13.5 | | | | 05 | 49.4 | 49.4 | 49.40 | | | 4.0 | 4.1 | -.1' | | | |
| 16 | | | 37 | 46.9 | | | | 05 | 54.0 | 53.9 | 53.95 | | | 4.0 | 4.1 | -.1' | | | |
| 17 | | | 38 | 23.0 | | | | 05 | 59.8 | 59.7 | 59.75 | | | 4.0 | 4.1 | -.1' | | | |
| 18 | | | 39 | 03.0 | | | | 06 | 07.1 | 07.1 | 07.10 | | | 4.0 | 4.1 | -.1' | | | |
| 19 | | | 39 | 36.9 | | | | 06 | 14.8 | 14.9 | 14.85 | | | 4.1 | 4.1 | 0' | | | |
| 20 | | 00 | 40 | 12.8 | | | 16 | 06 | 23.2 | 23.2 | 23.20 | | | 4.1 | 4.1 | 0' | | | |
| | | | | | | | | | | | | | | | | | BAROMETER: 26.81" | | |
| | | | | | | | | | | | | | | | | | TEMP: -0.1°C | | |

*Figure 20. Field observations—Star 17.*

STATION ISUNGUA ECC. DATE 21 AUG 1961 P.M

INSTRUMENTMAN R. SALVERMOSER INSTRUMENT T-4 # 374

RECORDER G. E. O'KEEFE WEATHER PARTLY CLOUDY, HAZY

ASST. OBS: J. M. LEWIS

| POS. / REP. | OBJECT OBSERVED | CHRON TIME H | M | S | STOP WATCH | TEL D / R | CIRCLE ° | ' | MICRO. / VERN. 1ST / A(') | 2D / B(') | MEAN ° | MEAN D / R° | DIR / ANG ' | REMARKS L | R | L−R' | LEVELS W | E |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | STAR # Na | 00 | 57 | 30.4 | | | 16 | 15 | 45.7 | 45.8 | 45.75 | | | 4.0 | 4.0 | .0 | | |
| 2 | NORTH | | 58 | 10.9 | | | | | 44.1 | 44.1 | 44.10 | | | 4.2 | 3.9 | +.3 | | |
| 3 | CIRCLE LEFT | | 58 | 51.6 | | | | | 42.3 | 42.5 | 42.40 | | | 4.2 | 3.9 | +.3 | | |
| 4 | | | 59 | 33.7 | | | | | 40.9 | 41.0 | 40.95 | | | 4.4 | 3.8 | +.6 | | |
| 5 | | 01 | 00 | 14.9 | | | | | 41.0 | 41.0 | 41.00 | | | 4.4 | 3.8 | +.6 | | |
| 6 | | | 00 | 59.4 | | | | | 38.9 | 39.1 | 39.00 | | | 4.7 | 3.7 | +1.0 | | |
| 7 | | | 01 | 48.2 | | | | | 39.4 | 39.3 | 39.35 | | | 4.8 | 3.8 | +1.0 | | |
| 8 | | | 02 | 26.3 | | | | | 38.9 | 38.8 | 38.85 | | | 4.8 | 3.7 | +1.1 | | |
| 9 | | | 03 | 23.4 | | | | | 38.8 | 38.8 | 38.80 | | | 4.9 | 3.6 | +1.3 | | |
| 10 | | | 04 | 02.4 | | | | | 38.6 | 38.7 | 38.65 | | | 4.9 | 3.5 | +1.4 | | |
| 11 | | | 04 | 44.0 | | | | | 38.5 | 38.5 | 38.50 | | | 5.0 | 3.4 | +1.6 | | |
| 12 | | | 05 | 23.0 | | | | | 39.3 | 39.4 | 39.35 | | | 4.8 | 3.7 | +1.1 | | |
| 13 | | | 06 | 01.3 | | | | | 39.3 | 39.3 | 39.30 | | | 4.8 | 3.7 | +1.1 | Relevelled | |
| 14 | | | 07 | 09.8 | | | | | 42.8 | 42.8 | 42.80 | | | 3.8 | 4.7 | −0.9 | | |
| 15 | | | 07 | 52.4 | | | | | 44.5 | 44.5 | 44.50 | | | 3.9 | 4.6 | −0.7 | | |
| 16 | | | 08 | 27.8 | | | | | 45.0 | 45.0 | 45.00 | | | 3.9 | 4.6 | −0.7 | | |
| 17 | | | 09 | 09.7 | | | | | 46.0 | 45.8 | 45.90 | | | 4.0 | 4.3 | −0.3 | | |
| 18 | | 01 | 09 | 54.1 | | | 16 | 15 | 48.0 | 48.0 | 48.00 | | | 4.1 | 4.3 | −0.2 | | |

BARO: 26.81" Hg

TEMP: +0.2°C



*Figure 21. Field observations—Star Na.*

(*c*) Southern latitude and declinations are to be considered negative, the sign of the cosine being plus (+), and the sign of the tangent, cotangent, and sine being minus (−).

(*d*) The factors $m$ and $n$ are functions of the hour angle $t$. Values of $m$ may be found in table IX, appendix III. Values of $n$ are tabulated in table X, appendix III.

(*e*) Constants A and B are computed for each star. Since closely approximated values of $\phi$ and $\zeta_1$ are required, it will be necessary to recompute the meridian reductions if the values used vary by more than $5''$ from the true value.

(*f*) The approximate values for $\phi$ and $\zeta_1$ may be determined as follows:

Select the observation nearest to the transit (minimum zenith distance) for the north and south star of a pair. Correct these two zenith distances for refraction and level error. Compute the latitude for the north and the south star. The difference between the two results is equal to the double collimation error (index error) of the vertical circle. The mean of the two results is equal to the approximate latitude. If the computed latitude for the north star is higher than the one for the south star, then half of the double collimation error will be added to the zenith distance of the north star and will be subtracted from the zenith distance of the south star to obtain the approximate corrected meridian zenith distances.

(*g*) Having found the arithmetical means of the zenith distances of the two stars comprising one pair, we have the latitude:

$\phi_n = \delta - \zeta_1$, for the north star, and

$\phi_s = \delta + \zeta_1$, for the south star

Then the latitude derived from the pair is:

$$\phi = \frac{\phi_n + \phi_s}{2}$$

(*h*) The arithmetic mean of the latitude results of all acceptable pairs determines the observed astronomic latitude of the station.

(*i*) No further adjustment is necessary if the hour angles are distributed symmetrically before and after transit; if the observer has bisected the star always at the center of the cross wires; and if the recorded time in reality corresponds with the exact local sidereal time of the bisections. An approximate method of adjustment is employed in order to determine the error in $t$; to correct all individual meridian distances; and to obtain a more accurate final zenith distance of the star.

(3) The computations of observations, when the instrument is clamped in the meridian, are as follows·

(*a*) The reduction of the observations to the meridian are made by the following formula:

$$m' = \frac{\sin^2 \frac{1}{2}t}{\sin 1''} \times \sin 2\delta, \text{ where}$$

$t$ = the true hour angle of the star at observation.

$m'$ = the correction to reduce the measured zenith distance to what it would have been if observed at transit.

Table IX may be utilized for this computation. Using $t$ as the argument, the $m$ values from the tables must be multiplied by $\frac{\sin 2\delta}{2}$, which is a constant for all observations of the star.

(*b*) Since the horizontal center wire is seldom exactly perpendicular to the observer's meridian, it becomes necessary to determine the inclination of the cross wire in order to reduce all observations as if observed along a perfectly adjusted cross wire, that is, in a plane perpendicular to the observer's. The adjustment of the reduced zenith distances due to inclination of the cross wire is shown in figure 22.

(1) Explanation of the terms:

PROJECT: HYPOTHETICAL

TABULATION OF GEODETIC DATA (TM 5-237)

LOCATION: $\phi = 70^\circ N$ $\delta = +54^\circ 59' 40''$

ORGANIZATION:

| STATION t | s | $ZD_o$ | $V_o$ | [tt] | [t $V_o$] | $\Delta ZD$ | ZD | V |
|---|---|---|---|---|---|---|---|---|
| $05^m 00^s$ | -300 | 35.″05 | -14.53 | | | -15.00 | 20.″05 | -.02 |
| 04 12 | -252 | 32.60 | -12.08 | | | -12.60 | 20.00 | +.03 |
| 03 25 | -205 | 30.28 | -9.76 | | | -10.25 | 20.03 | .00 |
| 02 10 | -130 | 26.90 | -6.38 | | | -6.50 | 20.40 | -.37 |
| 01 30 | -90 | 24.30 | -3.78 | | | -4.50 | 19.80 | +.23 |
| 00 10 | -10 | 20.09 | +0.43 | | | -0.50 | 19.59 | +.44 |
| 00 53 | +53 | 17.65 | +2.87 | | | +2.65 | 20.30 | -.27 |
| 01 47 | +107 | 14.70 | +5.82 | | | +5.35 | 20.05 | -.02 |
| 02 51 | +171 | 11.41 | +9.11 | | | +8.55 | 19.96 | +.07 |
| 04 04 | +244 | 07.92 | +12.60 | | | +12.20 | 20.12 | -.09 |
| 05 03 | +303 | 04.85 | +15.67 | 415,473 | 20,724.33 | +15.15 | 20.00 | +.03 |
| | -109 | $20.52^3$ | -.03 | + | + | | $20.02^7$ | +.03 |

$\Delta ZD$ | a | $\eta$

+11 | -109 | -0.03

+9.90909 | +0.00237

+415,473.0000 | -20,724.3300

-1,080.0908 | - 0.2976

+414,392.9092 | -20,724.6276

$a$ = +0.05001202

$\Delta ZD$ = +.00237

-.49557 (-a) (+9.909)

= -.493

$ZD_o$ 20.523

ZD = 20.″030

TABULATED BY: RS | DATE: 1/62 | CHECKED BY: RaS | DATE: 7/62

DA FORM 1 FEB 57 1962

GPO 921961

U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 22. Adjustment of zenith distance for inclination of cross wire.*

$t$=the hour angle of the star

$ZD_0$=the observed zenith distance corrected for curvature and refraction.

$v$=the residual from the mean

$\Delta ZD$=correction applied to $ZD_0$

$a$=inclination of the cross wire, or $\Delta ZD$ per unit of $t$.

The normal equations are:

$n(\Delta ZD)-[t]a+[v]=0$

$-[t](\Delta ZD)+[tt]a-[tv]=0$

(2) After solving for $a$ and $\Delta ZD$ the individual $\Delta ZD$'s may be computed as follows:

$at=\Delta ZD$; then the final ZD is

$ZD=ZD_0+\Delta ZD$

(c) There remains another source of error of the observed zenith distance if the line of sight (collimation axis of the telescope) does not lie in the plane of the meridian. The equation for the correction is:

$\Delta\zeta_1=-Am$, where

$$A=\frac{\cos\phi\cos\delta}{\sin\zeta_1}, \text{ and } m=\frac{2\sin^2\frac{1}{2}t}{\sin 1''},$$

$\Delta\zeta_1$=the correction to be applied to the reduced mean zenith distance of the star, and

$t$=the meridian error of the instrument expressed as an hour angle (local sidereal time at transit minus right ascension of the star). The correction is applied so that it will numerically decrease the zenith distance.

*c.* Following is a completed example of the field notes and computations for one set of stars using this method. This pair of stars was observed at station Test on 21 August 1961, P.M. date. The longitude of the station is $03^h\ 28^m\ 10.0^s$ W. The chronometer correction for both stars is $+1.9^s$. The constant for the vertical circle level is 2.845″ per div. The stars observed are APFS stars 17 and *Na*

(1) The Greenwich Civil Day and the short period terms are computed for each star as follows:

| | *Star 17* | | | *Star Na* | | |
|---|---|---|---|---|---|---|
| Local sidereal Time | $00^h$ | $34^m$ | $51^s$ | $01^h$ | $03^m$ | $25^s$ |
| Longitude | 03 | 28 | 10 | 03 | 28 | 10 |
| Sid. Time at $0^h$ | −22 | 00 | 14 | −22 | 00 | 14 |
| Universal Time | 06 | 02 | 47 | 06 | 31 | 21 |
| | (22 Aug 61) | | | (22 Aug 61) | | |
| Greenw. Civil Day | 362.7833$^m$ | | | 391.35 | | |
| | 1444 | | | 1444 | | |
| Greenw. Civil Day | 22.251 Aug 61 | | | 22.271 Aug 61 | | |
| | $d\psi=-0.042$ | | | | | |
| | $d\epsilon=0.077$ | | | not applicable | | |

(2) The right Ascension and Declination for each star are then completed.

| *Star 17* | *Star Na* |
|---|---|
| Longitude 0. 145 day west of Greenwich | |
| $n$ 0. 8145 | $n$ 0. 145 |
| B″ $\frac{0.8145\ (-0.1855)}{4}=-0.0378$ | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| RA for 14 Aug UC | $00^h$ | $34^m$ | $50^s.745$ | RA for 22 Aug UC | $01^h$ | $03^m$ | $25.32^s$ |
| 0. 8145 (+0. 294) | + | | .2395 | 0. 145 (0. 24) | | | +0. 03 |
| −0. 0378 (−0. 099) | + | | .0036 | | | | |
| (0. 067) (−0. 042)+(−0. 090) (0. 077) | + | | .0048 | | | | |
| Right ascension | 00 | 34 | 50. 99 | | 01 | 03 | 25. 35 |

*Note.* The FK4–FK3 corrections are to be applied for 1962 and 1963 only.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Declination 14 Aug UC | 53° | 40′ | 56. 85″ | Declination 22 Aug UC | 86° | 02′ | 49. 47″ |
| (0. 8145) (20. 95) | + | | 2. 403 | (0. 145) (0. 33) | | | +0. 048 |
| −0. 0378 (0. 29) | − | | .011 | | | | |
| (0. 39) (−0. 042)+(0. 15) (−0. 077) | − | | .028 | | | | |
| Declination | 53° | 40′ | 59. 21″ | | 86° | 02′ | 49. 52″ |

*Note.* The FK4–FK3 corrections are to be applied for 1962 and 1963 only.

(3) Approximate Meridian Zenith Distances and preliminary Latitude are then determined as follows:

| | *Star 17 (South star)* | | *Star Na (North star)* |
|---|---|---|---|
| Observed Zenith Distance (position 11) | 16°05′42″ | (pos. 9) | 16°15′40″ |
| Refraction | + 16″ | | + 16″ |
| Corrected Zenith Distance | +16°05′58″ | | −16°15′56″ |
| Declination | +53°40′59″ | | +86°02′50′ |
| Latitude ($\phi$) | +69°46′57″ | | +69°46′54″ |
| Mean Latitude (preliminary) | 69°46′55″5 | | |

Zenith Distance ($\zeta_1$) of star 17: 16°05′58″−1.5″=16°05′56.5″

Zenith Distance ($\zeta_1$) of star Na: 16°15′56″−1.5″=16°15″54.5″

(4) For the computation of the Corrected Local Sidereal Time, refraction, and corrected observed zenith distance (figs. 23 and 24), Roman numerals were assigned to the columns for clarity in the explanation.

(*a*) *Column I.* Number of position.

(*b*) *Column II.* The corrected local sidereal time is equal to the observed (chronometer) time plus or minus the chronometer correction.

(*c*) *Column III.* The observed zenith distances as taken from the field records.

(*d*) *Column IV.* Level correction.

$$=\left(\frac{L-R}{2}\right)(2.845'').$$

(*e*) *Column V.* The refraction correction is always plus, increasing the zenith distances numerically. The formula for the correction is: $(r_m)$ $(C_B)$ $(C_T)$ (table V).

(*f*) *Column VI.* The corrected observed zenith distance is equal to the observed zenith distance+level correction+refraction correction.

(5) In the computation of the final zenith distances (figs. 25 and 26), the Roman numerals are continued.

(*a*) *Column VII.* The hour angle $t$ is equal to the corrected local sidereal time of the observation minus the right ascension of the star (II−R.A.).

(*b*) *Column VIII.* Factor $m$ to be found in table IX.

(*c*) *Column IX.* Factor $n$, to be found in table X.

(*d*) *Column X.* $Am$, where $A=\frac{\cos\phi\cos\delta}{\sin\zeta_1}$

(*e*) *Column XI.* $Bn$, where $B=A^2\cot\zeta_1$.

(*f*) *Column XII.* $\zeta_1$, the meridian zenith distance (VI+X+XI).

(6) The adjustment of the reduced zenith distance is based on the assumption that there is a constant error in the corrected hour angles at the instant of each bisection. There are a number of sources producing this error, some of which are listed below:

(*a*) The star is not observed at the intersection of the cross wires.

(*b*) Delay in time from the instant of bisection to the instant of reading the chronometer.

(*c*) Personal error of the recorder in reading the chronometer.

(*d*) Horizontal collimation error of the instrument.

(7) The adjustment of the zenith distances (values listed in column XII of the sample computation) is carried out by solving two simultaneous equations:

Equation I: $n$ (ZD)+$[t]$ $a-[\zeta_1]=0$

Equation II: $n$ (ZD)+$[t]$ $a-[\zeta_1]=0$

(*a*) In equation I, $n$ denotes the number of observations before transit; ZD is the final adjusted mean zenith distance of the star reduced to the meridian; $[t]$ is the algebraic sum of the hour angles before transit (in seconds of time); $a$ is the correction for the zenith distance per second of the hour angle $t$; and $[\zeta_1]$ is the sum of the zenith distances before transit.

(*b*) In equation II, the explanations for equation I holds, except that "after transit" should be substituted for "before transit."

(*c*) After the unknown $a$ is computed, column XIII may be computed by the formula: XIII=(*a*) (*t*), in which $t$ is to be taken from Column VII (in seconds of time). Apply Column XIII to Column XII to obtain the adjusted zenith distances.

PROJECT: GREENLAND ASTRO

TABULATION OF GEODETIC DATA (TM 5-237)

LOCATION: Sta. ISUNGUA Ecc.

ORGANIZATION: USAMS, Obs. R.S.

STATION: STAR #17(South)

G.C.D. = 22.251 AUG. 1961

BARO. = 26.81" TEMP. = -0.1°C

| | | | | |
|---|---|---|---|---|
| δ | 53° 40′ | 59.″21 | cos ø | 0.345 5917 |
| RA | 00h 34m | 50.s99 | cos δ | 0.592 2507 |
| $S_1$ | 16° 05′ | 56.″5 | SIN $S_1$ | 0.277 2984 |
| ø | 69° 46′ | 55.″5 | COT $S_1$ | 3.464 8020 |

Chron. Corr.: +1.s9

C = $C_B$ $C_T$ = 0.928

| I | II | III | IV | V | VI |
|---|---|---|---|---|---|
| Pos. | Corr. Loc. Sid. T. | Obs. Z.D. | Level | Refr. | Corr. Obs. Z.D. |
| | h m s | ° ′ ″ | | | ° ′ ″ |
| 1 | 00 29 24.8 | 16 06 23.55 | -.28 | + 15.53 | 16 06 38.80 |
| 2 | 30 01.7 | 14.45 | -.14 | + 15.52 | 29.83 |
| 3 | 30 35.9 | 06.35 | -.14 | + 15.52 | 21.73 |
| 4 | 31 11.5 | 00.20 | .00 | + 15.52 | 15.72 |
| 5 | 31 48.4 | 16 05 54.55 | .00 | + 15.52 | 10.07 |
| 6 | 32 23.7 | 49.60 | .00 | + 15.52 | 05.12 |
| 7 | 32 53.5 | 46.10 | .00 | + 15.52 | 01.62 |
| 8 | 33 21.0 | 43.50 | .00 | + 15.51 | 16 05 59.01 |
| 9 | 33 53.0 | 42.00 | .00 | + 15.51 | 57.51 |
| 10 | 34 24.7 | 42.05 | +.28 | + 15.51 | 57.84 |
| 11 | 35 08.6 | 41.90 | +.14 | + 15.51 | 57.55 |
| 12 | 35 44.1 | 42.80 | +.14 | + 15.51 | 58.45 |
| 13 | 36 13.4 | 43.90 | +.14 | + 15.51 | 59.55 |
| 14 | 36 45.6 | 46.80 | +.14 | +.15.52 | 16 06 02.46 |
| 15 | 37 15.4 | 49.40 | -.14 | + 15.52 | 04.78 |
| 16 | 37 48.8 | 53.95 | -.14 | + 15.52 | 09.33 |
| 17 | 38 24.9 | 59.75 | -.14 | + 15.52 | 15.13 |
| 18 | 39 04.9 | 16 06 07.10 | -.14 | + 15.52 | 22.48 |
| 19 | 39 38.8 | 14.85 | .00 | + 15.52 | 30.37 |
| 20 | 00 40 14.7 | 23.20 | .00 | + 15.53 | 38.73 |

TABULATED BY: R.S. DATE: 1/62

CHECKED BY: RaS DATE: 7/62

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*Figure 23. Correction to observed zenith distance—Star 17.*

PROJECT: GREENLAND ASTRO

TABULATION OF GEODETIC DATA (TM 5-237)

LOCATION: Sta. ISUNGUA Ecc.

ORGANIZATION: USAMS, Obs. R. S.

STATION: STAR Na (North)

G.C.D. = 22.271 AUG. 1961

$\delta = 86^\circ\ 02'\ 49.''52$ | $RA = 01^h\ 03^m\ 25.^s35$ | $\phi_1 = 16^\circ\ 15'\ 54.''5$ | $\phi = 69^\circ\ 46'\ 55.''5$

$\cos \phi = 0.345\ 5917$ | $\cos \delta = 0.068\ 9366$ | $\sin \phi_1 = 0.280\ 0827$ | $\cot \phi_1 = 3.427\ 4733$

BARO. = 26.''81 Hg. | TEMP. = +0.°2 C

Chron. Corr. = +1.$^s$9

$C = C_B C_T$ = 0.927

| I | II | III | IV | V | VI |
|---|---|---|---|---|---|
| Pos. | Corr. Loc. Sid. T. | Obs. Z.D. | Level | Refr. | Corr. Obs. Z.D. |
| | h m s | ° ′ ″ | ″ | ″ | ° ′ ″ |
| 1 | 00 57 32.3 | 16 15 45.75 | 0.00 | +15.68 | 16 16 01.43 |
| 2 | 58 12.8 | 44.10 | +0.43 | +15.68 | 00.21 |
| 3 | 58 53.5 | 42.40 | +0.43 | +15.68 | 15 58.51 |
| 4 | 59 35.6 | 40.95 | +0.85 | +15.68 | 57.48 |
| 5 | 01 00 16.8 | 41.00 | +0.85 | +15.68 | 57.53 |
| 6 | 01 01.3 | 39.00 | +1.42 | +15.68 | 56.10 |
| 7 | 01 50.1 | 39.35 | +1.42 | +15.68 | 56.45 |
| 8 | 02 28.2 | 38.85 | +1.56 | +15.68 | 56.09 |
| 9 | 03 25.3 | 38.80 | +1.85 | +15.68 | 56.33 |
| 10 | 04 04.3 | 38.65 | +1.99 | +15.68 | 56.32 |
| 11 | 04 45.9 | 38.50 | +2.28 | +15.68 | 56.46 |
| 12 | 05 24.9 | 39.35 | +1.56 | +15.68 | 56.59 |
| 13 | 06 03.2 | 39.30 | +1.56 | +15.68 | 56.54 |
| 14 | 07 11.7 | 42.80 | -1.28 | +15.68 | 57.20 |
| 15 | 07 54.3 | 44.50 | -1.00 | +15.68 | 59.18 |
| 16 | 08 29.7 | 45.00 | -1.00 | +15.68 | 59.68 |
| 17 | 09 11.6 | 45.90 | -0.43 | +15.68 | 16 16 01.15 |
| 18 | 01 09 56.0 | 48.00 | -0.28 | +15.68 | 03.40 |

TABULATED BY: RS | DATE: 1/62 | CHECKED BY: RaS | DATE: 7/62

DA FORM 1 FEB 57 1962

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*Figure 24. Correction to observed zenith distance—Star Na.*

PROJECT: GREENLAND ASTRO — TABULATION OF GEODETIC DATA (TM 5-237)

~~LOCATION~~: Sta. ISUNGUA Ecc. — ORGANIZATION: USAMS

~~STATION~~: STAR #17 — $A = \frac{\cos\phi \cos\delta}{\sin\xi_1} = 0.73811$ — $B = A^2 \cot\xi_1 = 1.88765$

| Pos. | t | m | n | Am | Bn | $\xi_1$ | at | Adj. $\xi_1$ | V |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | 16° 05′ | | | |
| | VII | VIII | IX | X | XI | XII | XIII | XIV | XV |
| 1 | −5m 26.s2 | 58.03 | .0082 | −42.84 | +.02 | 55.″98 | +.58 | 56.56 | .00 |
| 2 | 4 49.3 | 45.65 | .0051 | −33.69 | +.01 | 56.15 | +.51 | 56.66 | −.10 |
| 3 | 4 15.1 | 35.49 | .0031 | −26.20 | +.01 | 55.54 | +.45 | 55.99 | +.57 |
| 4 | 3 39.5 | 26.28 | .0017 | −19.40 | .00 | 56.32 | +.39 | 56.71 | −.15 |
| 5 | 3 02.6 | 18.19 | − | −13.43 | − | 56.64 | +.32 | 56.96 | −.40 |
| 6 | 2 27.3 | 11.84 | | −8.74 | | 56.38 | +.26 | 56.64 | −.08 |
| 7 | 1 57.5 | 7.53 | | −5.56 | | 56.06 | +.21 | 56.27 | +.29 |
| 8 | 1 30.0 | 4.42 | | −3.26 | | 55.75 | +.16 | 55.91 | +.65 |
| 9 | 0 58.0 | 1.83 | | −1.35 | | 56.16 | +.10 | 56.26 | +.30 |
| 10 | 0 26.3 | 0.38 | | −0.28 | | 57.56 | +.05 | 57.61 | −1.05 |
| 11 | +0 17.6 | 0.17 | | −0.13 | | 57.42 | −.03 | 57.39 | −.83 |
| 12 | 0 53.1 | 1.54 | | −1.14 | | 57.31 | −.09 | 57.22 | −.66 |
| 13 | 1 22.4 | 3.71 | | −2.74 | | 56.81 | −.15 | 56.66 | −.10 |
| 14 | 1 54.6 | 7.16 | | −5.28 | | 57.18 | −.20 | 56.98 | −.42 |
| 15 | 2 24.4 | 11.37 | | −8.39 | | 56.39 | −.26 | 56.13 | +.43 |
| 16 | 2 57.8 | 17.24 | | −12.73 | | 56.60 | −.32 | 56.28 | +.28 |
| 17 | 3 33.9 | 24.96 | | −18.42 | − | 56.71 | −.38 | 56.33 | +.23 |
| 18 | 4 13.9 | 35.16 | .0030 | −25.95 | +.01 | 56.54 | −.45 | 56.09 | +.47 |
| 19 | 4 47.8 | 45.18 | .0050 | −33.35 | +.01 | 57.03 | −.51 | 56.52 | +.04 |
| 20 | 5 23.7 | 57.14 | .0079 | −42.18 | +.01 | 56.56 | −.58 | 55.98 | +.58 |
| | | | | | | $56.55^4$ | | $56.55^8$ | +.05 |
| | $\frac{a}{A}$ = +.00241 | 10ZD | −1711.8 | a −562.54 | = 0 | | | | |
| | $\Delta t$ = +2.s2 | 10ZD | +1669.2 | a −568.55 | = 0 | | | | |
| | | | −3381.0 | a + 6.01 | = 0 | | | | |
| | $PE_{sgl.}$ = ±.″32 | | | a | = −0.001,777,6 | | | | |
| | PE = ±.″07 | | | | | | | | |
| | Rej. L. = ±1.″06 | 10ZD | = 568.55 | −2.967 | = 565.583 | | ZD = $56.55^8$ | | |
| | | | | | Corr. | for error | in t | + .002 | |

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*Figure 25. Computation of final zenith distance—Star 17.*

PROJECT: GREENLAND ASTRO — TABULATION OF GEODETIC DATA (TM 5-237)

LOCATION: Sta. ISUNGUA Ecc. — ORGANIZATION: USAMS

STATION: STAR #Na

$A = \frac{\cos\phi \cos\delta}{\sin\xi_1} = 0.08506$ $\quad B = A^2 \cot\xi_1 = 0.02480$

| Pos. | t | m | n | Am | Bn | $\xi_1$ | at | Adj. $\xi_1$ | v |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | 16° 15′ | | | |
| | VII | VIII | IX | X | XI | XII | XIII | XIV | XV |
| 1 | −5m 53.s1 | 68.00 | | − 5.78 | | 55.″65 | +.09 | 55.74 | −.05 |
| 2 | 5 12.6 | 53.29 | | − 4.53 | | 55.68 | +.08 | 55.76 | −.07 |
| 3 | 4 31.9 | 40.32 | | − 3.43 | | 55.08 | +.07 | 55.15 | +.54 |
| 4 | 3 49.8 | 28.80 | | − 2.45 | | 55.03 | +.06 | 55.09 | +.60 |
| 5 | 3 08.6 | 19.40 | | − 1.65 | | 55.88 | +.05 | 55.93 | −.24 |
| 6 | 2 24.1 | 11.33 | | − 0.96 | | 55.14 | +.04 | 55.18 | +.51 |
| 7 | 1 35.3 | 4.95 | | − 0.42 | | 56.03 | +.02 | 56.05 | −.36 |
| 8 | 0 57.2 | 1.78 | | − 0.15 | | 55.94 | +.01 | 55.95 | −.26 |
| 9 | 0 00.1 | 0.00 | | 0.00 | | 56.33 | .00 | 56.33 | −.64 |
| 10 | +0 38.9 | 0.83 | | − 0.07 | | 56.25 | −.01 | 56.24 | −.55 |
| 11 | 1 20.5 | 3.54 | | − 0.30 | | 56.16 | −.02 | 56.14 | −.45 |
| 12 | 1 59.5 | 7.78 | | − 0.66 | | 55.93 | −.03 | 55.90 | −.21 |
| 13 | 2 37.8 | 13.58 | | − 1.16 | | 55.38 | −.04 | 55.34 | +.35 |
| 14 | 3 46.3 | 27.93 | | − 2.38 | | 54.82 | −.06 | 54.76 | +.93 |
| 15 | 4 28.9 | 39.43 | | − 3.35 | | 55.83 | −.07 | 55.76 | −.07 |
| 16 | 5 04.3 | 50.50 | | − 4.30 | | 55.38 | −.08 | 55.30 | +.39 |
| 17 | 5 46.2 | 65.37 | | − 5.56 | | 55.59 | −.09 | 55.50 | +.19 |
| 18 | 6 30.6 | 83.21 | | − 7.08 | | 56.32 | −.10 | 56.22 | −.53 |
| | | | | | | 55.69° | | 55.68″6 | +.08 |

NEGLIGIBLE

CHECK

9ZD − 1652.7a − 500.76 = 0

9ZD + 1933.0a − 501.66 = 0

− 3585.7a + 0.90 = 0

$\frac{a}{A}$ = .00295 $\quad$ a = $\quad$ −0.000,251,0

Δt = +2.s7 $\quad$ 9ZD = + 500.76 $\quad$ ZD = 55.″68″6

PE sgl. = ±.″31 $\quad$ + .415 $\quad$ +.00° *

PE = ±.″07 $\quad$ + 501.175

Rej. L. = ±1.″01

* Corr. for error in "t"

TABULATED BY: RS — DATE: 1/62

CHECKED BY: RaS BENDIX COMPUTER — DATE: 7/62

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*Figure 26. Computation of final zenith distance—Star Na.*

(*d*) ZD, the mean of the final adjusted zenith distances, may be computed by substituting $a$ in one of the two equations.

(*e*) In the event that observations before and after transit are not balanced in number of observations, it is advisable to employ a least squares adjustment of the reduced zenith distances (values in column XII of the sample computations). Figure 27 shows the least squares adjustment of star 17. The two normal equations are:

$$n(\Delta\zeta_1)-[t]a+[v]=0$$
$$-[t](\Delta\zeta_1)+[tt]a-[tv]=0$$

(8) In the above adjustment, the zenith distances will be reduced to a meridian which is, by the amount of *dt*, off the astronomical meridian. Normally, not a large error in $t$ is to be expected if all precautions in the determination of the local sidereal time of the bisections were taken. The correction to be applied to the mean of the adjusted meridian zenith distance of the star is:

$\text{Correction}=\text{A}\cdot\frac{\sin^2\frac{1}{2}dt}{\sin 1''}\text{A}m$, where $dt$ is the average error in the hour angles.

(*a*) The average error in the hour angles (*dt*) may be found in the following manner:

Differentiating the formula $\zeta_1-\zeta=-\text{A}m+\text{B}n$, and neglecting the second term, we can say that

$$\zeta_1-\zeta=\frac{\sin^2\frac{1}{2}t}{\sin 1''},$$

$$d\ (\zeta_1-\zeta)=\frac{\text{A}}{\sin 1''}\cdot\sin t''\cdot dt(\text{radians})=$$

$$\text{A}\cdot\sin t''\cdot dt''$$

(*b*) Assuming that *dt* (error in hour angles) and $t$ (hour angle) are both in error by 1 second of time, then:

$d\ (\zeta_1-\zeta)=\text{A}\ (0.000,072,722)\ (15)=0.00109''\text{A}$, which means that the change in the zenith distance due to an error of 1 second in $t$ is equal to $0.00109''\ (\text{A})\cdot(dt)$.

This relation holds true since the sine and arc of $t$ are linear, when $t$ is small.

(*c*) From this, it follows that the average error in $t$ may be computed by the formula: $dt^s=\frac{a}{0.00109(\text{A})}$. If $a$ is plus, then the chronometer time of the bisections is less than the correct local sidereal time, and the final adjusted mean zenith distance will be lower (numerically) than the zenith distances from column XII of the sample computation. If $a$ is minus, the opposite would apply.

(9) The latitude from one pair of star (fig. 28) is equal to $\frac{\phi_s+\phi_n}{2}$, where $\phi_s=\delta+\zeta_1$, for the star south of the zenith and $\phi_n=\delta-\zeta_1$, for the star north of the zenith. The mean of the summation of the results of all pairs will give the latitude of the station. Eccentric reduction, reduction to sea level, and the correction for the variation of the pole must be applied (fig. 29).

(10) The probable errors (PE) are computed as follows:

(*a*) PE of a single pointing on a star:

$$e_o=\pm 0.6745\sqrt{\frac{[vv]}{(n-1)}}$$

(*b*) PE of the arithmetic mean of the reduced zenith distances:

$$r_0=\pm\frac{e_0}{\sqrt{n}}$$

(*c*) PE of a single latitude pair:

$$e=\pm 0.6745\sqrt{\frac{[vv]}{(n-1)}}$$

(*d*) PE of the arithmetic mean of all pairs (latitude result);

$$r=\frac{e}{\sqrt{n}}$$

In the above equations,
$v$=the residual, and $n$=number of observations.

(*e*) It should be understood that the probable errors thus derived represent the probable errors of the observations only, and do not include constant

| PROJECT: GREENLAND ASTRO | | | | | | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|---|---|---|---|---|
| LOCATION: Sta. ISUNGUA Ecc. | | | | | | | ORGANIZATION: USAMS | |
| STATION: STAR # 17 | | | | | | | | |
| n | t | $\zeta_1$ | $V_1$ | [tt] | [tV] | at | Adj. $\zeta_1$ | $V_2$ |
| | | 16° 05' | | | | | 16° 05' | |
| 1 | - 326.2 | 55.98 | + 0.57 | | | + 0.45 | 56.43 | + 0.13 |
| 2 | - 289.3 | 56.15 | + 0.40 | | | + 0.40 | 56.55 | + 0.01 |
| 3 | - 255.1 | 55.54 | + 1.01 | | | + 0.35 | 55.89 | + 0.67 |
| 4 | - 219.5 | 56.32 | + 0.23 | | | + 0.30 | 56.62 | - 0.06 |
| 5 | - 182.6 | 56.64 | - 0.09 | | | + 0.25 | 56.89 | - 0.33 |
| 6 | - 147.3 | 56.38 | + 0.17 | | | + 0.20 | 56.58 | - 0.02 |
| 7 | - 117.5 | 56.06 | + 0.49 | | | + 0.16 | 56.22 | + 0.34 |
| 8 | - 90.0 | 55.75 | + 0.80 | | | + 0.12 | 55.87 | + 0.69 |
| 9 | - 58.0 | 56.16 | + 0.39 | | | + 0.08 | 56.24 | + 0.32 |
| 10 | - 26.3 | 57.56 | - 1.01 | | | + 0.04 | 57.60 | - 1.04 |
| 11 | + 17.6 | 57.42 | - 0.87 | 757 704.62 | - 1052.171 | - 0.02 | 57.40 | - 0.84 |
| 12 | + 53.1 | 57.31 | - 0.76 | | | - 0.07 | 57.24 | - 0.68 |
| 13 | + 82.4 | 56.81 | - 0.26 | | | - 0.11 | 56.70 | - 0.14 |
| 14 | + 114.6 | 57.18 | - 0.63 | | | - 0.16 | 57.02 | - 0.46 |
| 15 | + 144.4 | 56.39 | + 0.16 | | | - 0.20 | 56.19 | + 0.37 |
| 16 | + 177.8 | 56.60 | - 0.05 | | | - 0.25 | 56.35 | + 0.21 |
| 17 | + 213.9 | 56.71 | - 0.16 | | | - 0.30 | 56.41 | + 0.15 |
| 18 | + 253.9 | 56.54 | + 0.01 | | | - 0.35 | 56.19 | + 0.37 |
| 19 | + 287.8 | 57.03 | - 0.48 | | | - 0.40 | 56.63 | - 0.07 |
| 20 | + 323.7 | 56.56 | - 0.01 | | | - 0.45 | 56.11 | + 0.45 |
| | - 42.6 | 56.55 | - 0.09 | | | + 0.04 | 56.56 | + 0.07 |
| | $\Delta\zeta_1$ | = + .01 | | | | [$V_2$] | /n = .00 | |
| | | 56.56 | | CHECK | | | 56.56 | |
| | | $\Delta\zeta_1$ | | a | | | n | |
| | | + 20.000 | | + 42.6 | | | - 0.090 | |
| | | | | - 2.13 | | | + 0.0045 | |
| PE = ± 0.08 | | | + | 757 704.62 | | | + 1052.1710 | |
| | | | - | 90.738 | | | + .1917 | |
| | | | + | 757 613.882 | | | + 1052.3627 | |
| | | | | a = | | | - .001 | |
| | | | | $\Delta\zeta_1$ = | | | + .007 | |
| TABULATED BY: RS | | DATE: 1/62 | | CHECKED BY: RAS | | | DATE: 7/62 | |

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*Figure 27. Adjustment of zenith distance—star 17.*

PROJECT: GREENLAND ASTRO

TABULATION OF GEODETIC DATA (TM 5-237)

LOCATION: Sta. ISUNGUA Ecc.

ORGANIZATION: USAMS

STATION

| STAR | ZENITH DIST. (1) | DECLINATION (2) | LATITUDE (1 + 2) | |
|---|---|---|---|---|
| Na (N) | -16° 15′ 55.″69 | +86° 02′ 49.″52 | 69° 46′ 53.″83 | |
| 17 (S) | +16 05 56.56 | +53 40 59.21 | 55.77 | |
| | | $\phi_m = \frac{\phi_N + \phi_S}{2}$ = | 69° 46′ 54.″80 | ±0.″05 |
| $PE_{PAIR}$ = | $\pm\sqrt{\frac{.07^2 + .07^2}{4}}$ | = ±0.″05 | | |

TABULATED BY: RS DATE: 7/62

CHECKED BY: RAS DATE: 7/62

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*Figure 28. Observed latitude—single pair of stars (17 & Na).*

## SUMMARY OF LATITUDE RESULTS

| PROJECT: GREENLAND ASTRO | | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|---|
| LOCATION: Sta. ISUNGUA Ecc. | | | ORGANIZATION: USAMS | |
| STATION | | | | |
| PAIR NO. | $\phi$ | $V_1$ | $V_2$ | [VV] |
| 1 | 69° 46′ 54.″80 | + 0.″13 | + 0.″05 | |
| 2 | 55.60 | (- 0.67) R | — | |
| 3 | 54.70 | + 0.23 | + 0.15 | |
| 4 | 54.87 | + 0.06 | - 0.02 | |
| 5 | 55.02 | - 0.09 | - 0.17 | |
| 6 | 54.76 | + 0.17 | + 0.09 | |
| 7 | 54.67 | + 0.26 | + 0.18 | |
| 8 | 55.13 | - 0.20 | - 0.28 | |
| 9 | 54.83 | + 0.10 | + 0.02 | |
| Mn. or Sum | (54.″93) | - 0.01 | | |
| | 54.85 | | + 0.02 | (0.6769) |
| | | | | 0.1736 |
| $e_1$ | $= \pm .6745\sqrt{\frac{0.6769}{8}} = \pm 0.″20$ | | Rej. Limit = ±.20 | x 2.84 = ±0.″57 |
| $e_2$ | $= \pm .6745\sqrt{\frac{0.1736}{7}} = \pm 0.″11$ | | Rej. Limit = ±.11 | x 2.76 = ±0.30 |
| $r$ | $= \frac{\pm 0.11}{\sqrt{8}} = \pm 0.″04$ | | | |
| MEAN OBSERVED ASTRONOMIC LATITUDE : | | | 69° | 46′ 54.″85 |
| | | | | ± 0.″04 |
| TABULATED BY: RS | DATE: 1/62 | CHECKED BY: RAS | | DATE: 7/62 |

DA, FORM 1 FEB 57 1962 GPO 921961 U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 29. Summary of latitude results—circummeridian altitude method.*

errors of the instrument and the errors introduced by inconsistent refraction conditions.

(11) Chauvenet's rejection rule should be applied for the rejection of single pointings on the star (Column XIV of the sample computations), and for the rejection of latitude pairs. The rejection factors are found in table XI.

## 24. Latitude By the Horrebow-Talcott Method

*a.* The Horrebow-Talcott method of first-order latitude determination is used in most of the world today for latitudes up to approximately 60°. This method utilizes pairs of stars on or near the meridian and of approximately equal zenith distances observed north and south of the observer. Excellent results are obtained because the method depends entirely on differential measurements which can be accurately determined.

*b.* The basic formula used in computing latitude from observations made with a Wild T-4 broken telescope theodolite is:

$$\phi=\frac{1}{2}(\delta+\delta')+\frac{1}{2}R(M_W-M_E)$$
$$+\frac{1}{8}\left(\frac{d+d_1}{2}\right)[(n+n_1+s+s_1)_W-(n+n_1+s+s_1)_E]$$
$$+\frac{1}{2}(r-r')+\frac{1}{2}(m+m').$$

Where: $\delta$ & $\delta'$ are the apparent declination of the stars of a pair.

$M_W$ & $M_E$ are the micrometer readings with ocular west and east respectively.

$n$, $n_1$, $s$, $s_1$ are the readings of the north and south ends of the two levels.

$d$ & $d_1$ are the values of one division of each of the levels.

$r-r'$ is the difference in refraction of the two stars.

$m+m'$ is the sum of the meridian distance corrections.

R is the value of one turn of the latitude micrometer.

*Note.* If the Wild T-4 is not used, the micrometer and level sections of the formula (i.e. $m$, $n$, $s$, and $d$) must conform to the instrument used.

*c.* The first step in the computation of the apparent places is to obtain the mean place of each star for the epoch of observation from a standard star catalogue. It should be noted that the apparent places of stars for observations made subsequent to July 1st must be computed for the epoch of the following calendar year, since the Besselian and Independent Day Numbers in the AE&NA are tabulated on that basis. The best available star catalogue is the "General Catalogue of 33342 Stars for the Epoch 1950," by Benjamin Boss. In the Horrebow-Talcott method the right ascensions need be accurate only to the nearest second.

(1) This accuracy will usually be attained by adding algebraically to the catalogue right ascension the product of the annual variation and the number of years between the epoch of observation and the epoch of the catalogue. If the epoch of observation is earlier than the epoch of the catalogue, then the difference in years must be considered negative.

(2) If we denote by $\alpha_o$, the mean right ascension for the beginning of the year nearest the time of observation ($t_o$) and by $\alpha_m$ the catalogue right ascension, then the complete formula for use with the Boss General Catalogue, Epoch 1950, is:

$$\alpha_o=\alpha_m+(t_o-1950)\text{ An. Var.}$$
$$+\frac{1}{2}(t_o-1950)^2\left(\frac{1}{100}\right)\text{ Sec. Var.}$$
$$+\left(\frac{t_o-1950}{100}\right)^3 3^d t.$$

In the case of circumpolar stars, it may be necessary to use more than two terms of the above formula.

(3) If $\zeta_m$ represents the catalogue declination and $\zeta_o$ the mean declination for the beginning of the year ($t_o$), then the complete formula for mean declination is:

$$\delta_o=\delta_m+(t_o-1950)\text{ An. Var.}$$
$$+\frac{1}{2}(t_o-1950)^2\left(\frac{1}{100}\right)\text{ Sec. Var.}$$
$$+\left(\frac{t_o-1950}{100}\right)^3 3^d t.$$

All the terms of the above formula may be needed to determine the mean declinations with the accuracy required by the latitude. Usually the mean declinations are computed accurately to the nearest hundredth of a second.

(4) The above formulas for reducing the mean right ascension and declination of a star

from the epoch of the catalogue to the epoch of observation contain all the elements necessary for the reduction. These elements consist of first- and second-order terms of the precession and proper motion. The terms of the formulas are arranged according to ascending powers of the time interval from the epoch of the catalogue to the epoch of observation in the manner indicated in these formulas.

*d.* After the mean places of the stars for the epoch of observation have been obtained, the apparent declination of each star is computed by means of Besselian and Independent Day Numbers. Using the trigonometrical expressions for Bessel's star constants (*Formulas for the Reduction of Stars*, AE&NA, any year) the formula for apparent declination customarily expressed in terms of Besselian Star Constants and Besselian Day Numbers has been transformed into:

$$\delta=\delta_o+\tau\mu'+\iota\cos\delta_o+\mathrm{X}\sin\alpha_o+\mathrm{Y}\cos\alpha_o+\Delta\delta_\delta+\Delta\delta_\alpha.$$

Where: $\mathrm{X}=-(\mathrm{B}+\mathrm{C}\sin\delta_o)$

$\mathrm{Y}=\mathrm{D}\sin\delta_o+\mathrm{A}$

(1) DA Form 2865 (Reduction, Mean to Apparent Declination) has been arranged for this computation (fig. 30).

(2) $\alpha_o$ and $\delta_o$ are computed for the "appropriate year" and entered on the form along with their natural sines and cosines, each in its designated column. Four place tables provide the required accuracy of the trigonometric functions. Be sure to use the correct signs of these functions.

(3) The proper motion in declination, $\mu'$, is taken from the star catalogue (G.C.). (If the change in proper motion per 100 years, namely, 100 $\Delta\mu'$, is large enough it must be taken into account in taking the value of $\mu'$ from the catalogue.)

(4) Usually when the period of the observations does not exceed 4 hours, the values of A, B, C, D, $\tau$, and $\iota$ can be obtained with sufficient accuracy by using the mean Universal time, commonly abbreviated UT, of the observations. If the mean right ascensions of the set of stars are spaced with approximate regularity, the mean of the first and last right ascensions will provide a sufficiently accurate mean local sidereal time of the observations. Sometimes when there is a pronounced break in the regularity, the set should be divided and the mean epoch computed for each part.

(5) To the mean local sidereal time, add the longitude if west of Greenwich or subtract if east, thus obtaining the corresponding Greenwich sidereal time. From this Greenwich sidereal time or as abbreviated, GST, subtract the Greenwich sidereal time of the nearest preceding $0^h$ UT. Divide this interval expressed in minutes by 1444 and the result will be the fractional part of a civil day from $0^h$, UT chosen.

(6) After quantities A, B, C, D, $\tau$, and $\iota$ have been determined, compute the X and Y values for all the stars. When these have been entered in the proper places on the form, all the necessary information for computing the apparent declination of a star is on the same horizontal line with the exception of $\tau$ and $\iota$ which are at the bottom of the form. By machine, all the steps of the computation from mean to apparent declination may be carried through in one continuous operation. The signs of the products must be carefully adhered to.

(7) The corrections $\Delta\delta_\delta$ and $\Delta\delta_\alpha$ must be applied to the apparent declination. These corrections reduce the values obtained from the system of the General Catalogue to the system of the FK4. The requisite tables may be requested from the Americas Division, Department of Geodesy, Army Map Service, Washington, D.C. Distribution of the tables will be made as soon as they are available.

*e.* The information which should be entered by the observing party on DA Form 2842 (Latitude Computation) (fig. 31) consists of the following:

(1) At the top of the form, the names of the station, chief of party and observer, the date of observation, the kind and number of the instrument used, the number of the chronometer, and the elevation of the station if available.

(2) In the designated columns for each star, the star catalogue number; the position of the star with respect to the zenith N or S; the position, E or W, of the ocular during the observation on the

PROJECT | STATION: MAP ASTRO 2 | LOCATION: MARYLAND, USA | DATE: 2 MAY 1963 PM | REDUCTION, MEAN TO APPARENT DECLINATION (TM 5-237)

| CATALOG STAR NO. | Mag. | MEAN RIGHT ASCENSION $\alpha_0$ | MEAN DECLINATION $\delta_0$ | $\sin\delta_0$ | $\cos\delta_0$ | $\mu'$ | X | $\sin\alpha_0$ | Y | $\cos\alpha_0$ | * $\Delta\delta_\delta$ | * $\Delta\delta_\alpha$ | APPARENT DECLINATION $\delta$ | PE $\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 19467 | 4.1 | 14h 23m 56s | 52° 01' 15.78" | .7882 | .6154 | -.404 | +7.372 | -.5875 | -10.820 | -.8092 | +.19 | +.15 | 52° 01' 16.69" | |
| 19528 | 7.0 | 27 07 | 26 01 05.84 | .4387 | .8987 | -.024 | +2.504 | -.5987 | -6.020 | -.8009 | .19 | .15 | 26 01 04.07 | |
| 19687 | 6.5 | 34 26 | 23 24 37.30 | .3973 | .9177 | +.015 | +1.927 | -.6240 | -5.452 | -.7814 | .20 | .16 | 23 24 35.18 | |
| 19742 | 5.5 | 37 05 | 54 10 57.09 | .8109 | .5852 | -.024 | +7.688 | -.6330 | -11.132 | -.7742 | .16 | .16 | 54 10 57.62 | |
| 19800 | 6.4 | 40 13 | 21 16 52.25 | .3629 | .9318 | -.061 | +1.448 | -.6435 | -4.979 | -.7654 | .21 | .17 | 21 16 49.86 | |
| 19907 | 6.6 | 45 20 | 56 46 14.14 | .8365 | .5480 | -.005 | +8.045 | -.6604 | -11.483 | -.7509 | .14 | .16 | 56 46 14.44 | |
| 20041 | 7.0 | 51 50 | 20 02 19.97 | .3427 | .9395 | -.070 | +1.166 | -.6815 | -4.702 | -.7318 | .22 | .18 | 20 02 17.32 | |
| 20151 | 7.1 | 56 22 | 57 47 49.57 | .8462 | .5329 | -.025 | +8.180 | -.6958 | -11.617 | -.7182 | .14 | .17 | 57 47 49.30 | |
| 20308 | 5.6 | 15 04 12 | 48 17 35.07 | .7466 | .6653 | +.022 | +6.792 | -.7199 | -10.249 | -.6940 | .22 | .18 | 48 17 33.68 | |
| 20421 | 7.1 | 09 20 | 29 22 06.39 | .4904 | .8715 | -.032 | +3.224 | -.7353 | -6.730 | -.6777 | .19 | .19 | 29 22 03.68 | |
| 20489 | 6.2 | 12 35 | 31 55 30.17 | .5288 | .8487 | -.027 | +3.759 | -.7448 | -7.258 | -.6672 | .20 | .18 | 31 55 27.46 | |
| 20586 | 6.8 | 17 00 | 45 45 13.80 | .7163 | .6977 | +.004 | +6.370 | -.7576 | -9.833 | -.6528 | .21 | .18 | 45 45 11.57 | |
| 20696 | 5.1 | 21 40 | 30 25 13.23 | .5063 | .8623 | -.195 | +3.445 | -.7707 | -6.949 | -.6372 | .19 | .18 | 30 25 10.10 | |
| 20741 | 6.9 | 23 56 | 47 11 25.18 | .7336 | .6796 | -.038 | +6.611 | -.7770 | -10.070 | -.6295 | .21 | .18 | 47 11 22.65 | |
| 21032 | 5.4 | 36 30 | 40 28 22.34 | .6491 | .7607 | +.053 | +5.434 | -.8103 | -8.910 | -.5860 | .18 | .19 | 40 28 18.95 | |
| 21086 | 7.1 | 39 03 | 37 38 06.61 | .6106 | .7919 | -.035 | +4.898 | -.8168 | -8.381 | -.5770 | .20 | .19 | 37 38 03.04 | |
| 21246 | 5.1 | 46 06 | 62 42 48.21 | .8887 | .4584 | -.061 | +8.772 | -.8341 | -12.200 | -.5516 | .11 | .09 | 62 42 45.03 | |
| 21321 | 7.3 | 50 18 | 15 20 48.17 | .2647 | .9643 | -.142 | +0.080 | -.8441 | -3.630 | -.5362 | .26 | .20 | 15 20 44.64 | |
| 21534 | 4.9 | 59 58 | 29 57 12.27 | .4993 | .8664 | -.015 | +3.348 | -.8660 | -6.852 | -.5001 | .19 | .20 | 29 57 07.95 | |
| 21684 | 6.6 | 16 05 53 | 47 36 09.58 | .7385 | .6743 | - | +6.680 | -.8786 | -10.138 | -.4776 | .22 | .19 | 47 36 04.89 | |
| 21761 | 6.5 | 09 12 | 43 54 56.81 | .6936 | .7204 | -.309 | +6.054 | -.8854 | -9.521 | -.4648 | .20 | .19 | 43 54 51.81 | |
| 21863 | 5.4 | 13 17 | 33 57 03.88 | .5585 | .8295 | -.086 | +4.172 | -.8935 | -7.665 | -.4490 | .21 | .19 | 33 56 58.95 | |
| 21937 | 6.6 | 16 52 | 25 59 08.09 | .4381 | .8989 | -.004 | +2.495 | -.9004 | -6.012 | -.4350 | .19 | .19 | 25 59 03.41 | |
| 22003 | 6.7 | 19 28 | 52 07 33.65 | .7894 | .6139 | -.014 | +7.389 | -.9053 | -10.837 | -.4247 | .19 | .19 | 52 07 28.23 | |
| 22094 | 6.9 | 23 29 | 55 25 06.19 | .8233 | .5676 | -.358 | +7.861 | -.9126 | -11.302 | -.4088 | .15 | .19 | 55 25 00.43 | |
| 22216 | 6.0 | 29 38 | 22 16 25.14 | .3790 | .9254 | -.004 | +1.672 | -.9233 | -5.200 | -.3842 | .21 | .18 | 22 16 20.39 | |
| 22342 | 6.6 | 35 16 | 14 32 55.60 | .2512 | .9679 | - | -0.108 | -.9324 | -3.445 | -.3614 | .26 | .17 | 14 32 51.53 | |
| 22382 | 6.4 | 36 32 | 63 08 48.55 | .8922 | .4517 | -.091 | +8.820 | -.9344 | -12.248 | -.3562 | .11 | .14 | 63 08 42.16 | |

| | h | m | h | m |
|---|---|---|---|---|
| MEAN EPOCH OBSERVATION | 16 | 22.0 | | |
| CORRECTION FOR LONGITUDE | 5 | 08.5 | | |
| GREENWICH SIDEREAL TIME | 21 | 30.5 | | |
| SIDEREAL TIME $0^h$ G C T. | 14 | 40.7 | | |
| SIDEREAL INTERVAL | 6 | 49.8 | | |
| GREENWICH CIVIL DAY | MAY | 3.284 | | |

BESSELIAN AND INDEPENDENT DAY NUMBERS

| | |
|---|---|
| A | + 0.005 |
| B | + 3.607 |
| C | - 13.929 |
| D | - 13.734 |
| i | - 6.041 |
| τ | + 0.3346 |

The mean epoch should be for a period not exceeding 4 hours. Use all sines and cosines to 4 decimal places; compute X and Y to 3 decimal places. *Reduction from Boss General Catalogue System to FK System to 2 decimal places.

$\delta = \delta_0 + \eta\mu' + i\cos\delta_0 + X\sin\alpha_0 + Y\cos\alpha_0 + \Delta\delta_\delta + \Delta\delta_\alpha$

**Formulas:** $X = -(B + C\sin\delta_0)$

$Y = A + D\sin\delta_0$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| L. E. Smith - AMS | July '63 | O. R. Niska - AMS | DEC. '63 |

DA FORM 2865, 1 OCT 64

*Figure 30. Reduction, mean to apparent declination (DA Form 2865).*

**LATITUDE COMPUTATION** (TM 5-237)

| | | | |
|---|---|---|---|
| STATION: MAP ASTRO 2 | LOCATION: MARYLAND | PROJECT: | INSTRUMENT (Type and No.): WILD T-4 NO. 56095 |
| PAGE NO.: 1 | NO. OF PAGES: 5 | | |
| STATION ELEVATION: 75 M | OBSERVER: H. N. CADDESS | PARTY CHIEF: R. SALVERMOSER | DATE: 2 May 1963 PM |
| MICRO HALF-TURN VALUE: 1/2 R = 76.5 | LEVEL VALUE: d + $d_1$ / 16 = .1168 | CHRONOMETER: 12460 | |

| PAIR | BOSS STAR NO. | N OR S | E OR W | MICROMETER READING (t d) | MICROMETER W - E (t d) | LEVEL N (d) | LEVEL S (d) | LEVEL W - E (d) | CHRONOMETER TIME OF OBSERVATION (h m s) | MERIDIAN DISTANCE (s) | DECLINATION (° ' ") | HALF SUM OF DECLINATIONS (° ' ") | CORRECTIONS MICRO (' ") | CORRECTIONS LEVEL (") | CORRECTIONS REF (") | CORRECTIONS MERID (") | LATITUDE (° ' ") | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 19467 | N | E | 11 45.9 | | 30.1 | 53.7 | | 14 24 11.4 | -16 | 52 01 16.69 | | | | | .03 | 38 56 | TIPPED LATE |
| 1 | | | | | -3.248 | 128.6 | 151.0 | +6.6 | | | | 39 01 10.38 | -04 08.47 | +0.77 | -.08 | | 62.63 | |
| | 19528 | S | W | 8 21.1 | | 55.6 | 32.0 | | 14 27 07.2 | -1 | 26 01 04.07 | | | | | | | TEMP. MEAN = 38 °F |
| | | | | | | 152.4 | 130.0 | | | | | | | | | | | BAR. MEAN = 30.20 in. |
| | 19687 | S | W | 12 93.9 | | 49.8 | 26.1 | | 14 34 27.4 | -2 | 23 24 35.18 | | | | | | | $(1.024)^{CT}(1.010)^{CB}$ = 1.034 |
| 2 | | | | | +7.239 | 147.0 | 124.3 | +1.0 | | | | 38 47 46.40 | +09 13.78 | +0.12 | +.17 | | 60.47 | |
| | 19742 | N | E | 5 70.0 | | 25.8 | 49.4 | | 14 37 07.4 | -3 | 54 10 57.62 | | | | | | | |
| | | | | | | 124.2 | 146.8 | | | | | | | | | | | |
| | 19800 | S | E | 11 83.0 | | 32.5 | 56.1 | | 14 40 14.4 | -2 | 21 16 49.86 | | | | | | | |
| 3 | | | | | -3.547 | 130.0 | 152.5 | +4.3 | | | | 39 01 32.15 | -04 31.35 | +0.50 | -.08 | | 61.23 | |
| | 19907 | N | W | 8 28.3 | | 57.3 | 33.6 | | 14 45 27.5 | -8 | 56 46 14.44 | | | | | .01 | | TIPPED LATE |
| | | | | | | 153.5 | 131.0 | | | | | | | | | | | |
| | 20041 | S | E | 8 87.6 | | 25.2 | 49.0 | | 14 51 51.3 | -2 | 20 02 17.32 | | | | | | | |
| 4 | | | | | +1.542 | 124.1 | 146.8 | -0.4 | | | | 38 55 03.31 | +01 57.96 | -0.05 | +.04 | | 61.26 | |
| | 20151 | N | W | 10 41.8 | | 48.7 | 25.0 | | 14 56 23.3 | -2 | 57 47 49.30 | | | | | | | |
| | | | | | | 146.9 | 124.1 | | | | | | | | | | | |
| | 20308 | N | W | 10 62.2 | | 53.2 | 29.7 | | 15 04 13.2 | -2 | 48 17 33.68 | | | | | | | |
| 5 | | | | | +5.601 | 149.6 | 126.9 | +33.3 | | | | 38 49 48.68 | +07 08.48 | +3.89 | +.12 | | 61.33 | |
| | 20421 | S | E | 5 02.1 | | 20.7 | 44.5 | | 15 09 57.0 | -37.5 | 29 22 03.68 | | | | | .16 | | TIPPED LATE |
| | | | | | | 119.0 | 141.9 | | | | | | | | | | | |
| | 20489 | S | E | 9 40.4 | | 21.5 | 45.1 | | 15 13 05.0 | -30 | 31 55 27.46 | | | | | .11 | | TIPPED LATE |
| 6 | | | | | +5.247 | 120.6 | 143.0 | -3.3 | | | | 38 50 19.52 | +06 41.40 | -0.39 | +.12 | | 60.76 | |
| | 20586 | N | W | 14 65.1 | | 43.9 | 20.0 | | 15 17 01.5 | -2 | 45 45 11.57 | | | | | | | |
| | | | | | | 143.0 | 120.0 | | | | | | | | | | | |

COMPUTED BY: L. E. Smith - AMS | DATE: July '63 | CHECKED BY: O. R. Niska - AMS | DATE: DEC. '63

DA FORM 2842, 1 OCT 64

*Figure 31. Latitude Computation (DA Form 2842).*

star; the micrometer reading in turns of the screw and in divisions and tenths of division of the micrometer head; the respective readings of the north and south ends of the two latitude levels; and the chronometer time of observation.

*f.* In the office, the computer will complete the computation.

(1) If the observing party has not furnished the approximate elevation of the station, the value of one half turn of the micrometer, and the level value, then these data should be entered at the top of the page.

(2) The apparent declinations computed on DA Form 2865 (Reduction, Mean to Apparent Declination) should be entered in the column designated "Declination", and the half-sum of these declinations, for each pair, computed and entered in the next column.

(3) The algebraic difference of the micrometer readings for each pair (in the sense ocular west minus ocular east is positive) is then placed in the "Diff. Z.D." column, usually in decimal form. This difference is then converted to seconds of arc by multiplying by the value of one half turn of the micrometer and the result placed in the micrometer correction column.

(4) Next, the algebraic difference of the sum of the level readings for the star with ocular west minus the sum of the level readings with ocular east is set down in the designated column. This difference multiplied by the level value, i.e., by $\left(\frac{d+d_1}{16}\right)$, constitutes the level correction.

(5) The approximate meridian distance is computed by the formula $\alpha_o-(t+\Delta t)$, where $\alpha_o$ is the mean right ascension, $t$ the chronometer time of observation and $\Delta t$ the correction to the chronometer time obtained from a radio time signal. This distance is entered in the proper column on DA Form 2842 (Latitude Computation). If, for any reason, the observer has not observed the star on the meridian, it should be noted under "Remarks", giving an estimation of the time of observation before or after transit.

(6) If a star is observed off the meridian while the line of collimation of the telescope remains in the meridian, the measured zenith distance is in error on account of the curvature of the apparent path of the star. Let $m$ be the correction to reduce the measured zenith distance to what it would have been if the star had been observed on the meridian.

Then,

$$m=\frac{\sin^2 \frac{1}{2}\tau}{\sin 1''} \sin 2\delta$$

in which $\tau$ is the hour-angle of the star. The signs are such that the correction to the latitude $\left(\frac{m}{2}\right)$ is always plus for the stars of positive declination and minus for star of negative declination (south of the equator), regardless of whether the star is to the northward or to the southward of the zenith. $\frac{m}{2}$ or $\frac{m'}{2}$ is then always applied as a correction to the latitude with the sign of the right-hand member of the above equation. For a subpolar, $180°-\delta$ must be substituted for $\delta$, making the correction negative for a northern subpolar, and positive for a southern subpolar. Table XII gives the corrections to the latitude computed from the above formula. If both stars of a pair are observed off the meridian, two such corrections must be applied to the computed latitude.

(7) Although the difference in refraction of a pair of stars used in the Horrebow-Talcott method is small, it must be applied as a correction to the latitude. The refraction for each star of a pair is very nearly proportional to the tangent of the zenith distance, so that the differential refraction will be very nearly proportional to the square of the secant of the mean zenith distance. In addition, the differential refraction depends upon the pressure and temperature of the atmosphere at the time of the observation. For a mean state of the atmosphere (pressure 29.9 inches or 76 cm. and temperature 50° F. or 10° C.), the correction to be applied to the latitude for differential refraction will be given by the formula:

$$\frac{r-r'}{2}=\frac{57''.9}{2} \sin (\zeta-\zeta') \sec^2 \zeta$$

where $r$ and $\zeta$ are the refraction and zenith distance, respectively, of the star observed with ocular East, the primed letters referring to the star observed with ocular West. Differential refraction will, therefore, have the same sign as half the difference of the zenith distances as measured by the micrometer. The two zenith distances of a pair of stars used in the Horrebow-Talcott method are so nearly equal that either may be used to determine the $\sec^2 \zeta$ in the formula.

(*a*) Table XIII has been computed by means of the above formula with half the difference of the zenith distances as measured by the micrometer for one argument and the mean zenith distance for the other.

(*b*) In as much as the refraction obtained from the above table is only valid for the assumed mean state of the atmosphere, it will be necessary to apply to this differential refraction, factors obtained from the regular refraction (table V) in order to reduce it to the differential refraction for the pressure and temperature at the time of observation.

(*c*) When the micrometer, level, refraction, and meridian-distance corrections have been combined algebraically with the mean of the declinations of a pair of stars, the latitude as determined from observations on that pair of stars is obtained.

(8) The correction to the mean latitude and the correction to the value of one-half turn of the micrometer are computed by the method of least squares. Separate computation of each night's observation is not necessary unless there has been a distinct change in the value of one-half turn of the micrometer. Such a change should have been recorded in the field records.

(*a*) DA Form 2843 (Astronomic Latitude Summary) should be used for the adjustment (fig. 32). The data in the first four columns are obtained directly from DA Form 2842, Latitude Computation, the micrometer differences being taken to the nearest tenth.

(*b*) Before proceeding further with the adjustment, it is necessary to find out which, if any, of the results are to be rejected. An absolute rejection limit of 3″ from the mean of all the latitudes in column 4, each considered to have unit weight, is first used. Then a mean of the remaining latitudes is taken, and the probable error of a single observation, $e_p$, is computed. Any latitude with a residual, $\Delta\phi$, equal to or greater than $3\frac{1}{2}\, e_p$ is automatically rejected. In addition, other values may be rejected if the residual is excessive when compared with all others. Before final rejection of any value, the records should be reviewed to determine whether a star has been misidentified, level values follow pattern, or the turns of the micrometer may be a full turn in error. These are the most common causes of error. Notes by the observer may indicate doubtful observations. Another criterion for rejection is table XI. For a small number of observations, its use may increase the number of rejections.

(*c*) After all rejections have been made, the accepted observations remain to be adjusted in order to determine from the observations themselves the most probable value of a turn of the micrometer and the most probable latitude of the station. The information at the foot of the third column is now entered. Then a mean $\phi_m$ is taken of the unrejected latitudes in column 4 and the difference $\Delta\phi = \phi_m - \phi$ entered in the next column and summed algebraically at the foot of the column.

(*d*) If $p$ is the number of accepted latitudes, $c$ the amount in seconds by which the mean latitude deviates from the probable latitude, and $r$ the amount by which the preliminary value of one half-turn of the micrometer is to be corrected, then there will be $p$ equations of the form.

$$c - Mr + \Delta\phi = v.$$

Inserting the condition that $[v^2]$ or $[vv]$ must be a minimum, the normal equations to be solved for $c$ and $r$ are

PROJECT

**ASTRONOMIC LATITUDE SUMMARY**
*(TM 5-237)*

| OBSERVER | DATE | STATION |
|---|---|---|
| H.N. CADDESS | 2 MAY 1963 PM | MAP ASTRO 2 |

| CHIEF OF PARTY | INSTRUMENT | LOCATION |
|---|---|---|
| R. SALVERMOSER | WILD T-4 NO. 56095 | MARYLAND |

| STAR NUMBER BOSS | STAR NUMBER GEN. CATALOG | Mic. Diff. M | $\phi$ 38° 57′ | $\Delta\phi$ | $\frac{}{\Delta\phi}^2$ | Mr | ADJUSTED $\phi$ | $\Delta\phi$ | $\frac{}{\Delta\phi}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 19467 | 19528 | -3.2 | 02.63 | -1.45 | | -.13 | 02.50 | -1.30 | |
| 19687 | 19742 | +7.2 | 00.47 | +0.71 | | +.30 | 00.77 | +0.43 | |
| 19800 | 19907 | -3.5 | 01.23 | -0.05 | | -.14 | 01.09 | +0.11 | |
| 20041 | 20151 | +1.5 | 01.26 | -0.08 | | +.06 | 01.32 | -0.12 | |
| 20308 | 20421 | +5.6 | 01.33 | -0.15 | | +.23 | 01.56 | -0.36 | |
| 20489 | 20586 | +5.2 | 00.76 | +0.42 | | +.21 | 00.97 | +0.23 | |
| 20696 | 20744 | +6.9 | 01.54 | -0.36 | | +.28 | 01.82 | -0.62 | |
| 21032 | 21086 | -4.8 | 00.67 | +0.51 | | -.20 | 00.47 | +0.73 | |
| 21246 | 21321 | -3.7 | 00.68 | +0.50 | | -.15 | 00.53 | +0.67 | |
| 21534 | 21684 | +8.2 | 00.40 | +0.78 | | +.34 | 00.74 | +0.46 | |
| 21761 | 21863 | +0.8 | 00.32 | +0.86 | | +.03 | 00.35 | +0.85 | |
| 21937 | 22003 | -4.9 | 02.23 | -1.05 | | -.20 | 02.03 | -0.83 | |
| 22094 | 22216 | +4.9 | R | | | | | | |
| 22342 | 22382 | +4.9 | 01.13 | +0.05 | | +.20 | 01.33 | -0.13 | |
| 22646 | 22785 | -0.1 | 02.21 | -1.03 | | .00 | 02.21 | -1.01 | |
| 22866 | 22980 | -3.9 | R | | | | | | |
| 23132 | 23225 | +3.5 | 01.75 | -0.57 | | +.14 | 01.89 | -0.69 | |
| 23433 | 23574 | -3.3 | 01.64 | -0.46 | | -.14 | 01.50 | -0.30 | |
| 23770 | 23919 | -0.2 | 01.29 | -0.11 | | -.01 | 01.28 | -0.08 | |
| 24003 | 24155 | +3.1 | 00.76 | +0.42 | | +.13 | 00.89 | +0.31 | |
| 24279 | 24433 | +0.5 | 00.99 | +0.19 | | +.02 | 01.01 | +0.19 | |
| 24538 | 24613 | -2.7 | 00.86 | +0.32 | | -.11 | 00.75 | +0.45 | |
| 24699 | 24816 | +2.9 | 01.41 | -0.23 | | +.12 | 01.53 | -0.33 | |
| 24920 | 25040 | -4.8 | 00.86 | +0.32 | | -.20 | 00.66 | +0.54 | |
| 26290 | 26358 | -2.3 | 00.49 | +0.69 | | -.09 | 00.40 | +0.80 | |
| 26475 | 26542 | -1.2 | R | | | | | | |
| 26632 | 26749 | -1.9 | 00.47 | +0.71 | | -.08 | 00.39 | +0.81 | |
| 26996 | 27047 | -6.9 | 02.21 | -1.03 | | -.28 | 01.93 | -0.73 | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| + SUM | | 50.3 | 12.12 | 6.48 | | | 14.15 | 6.54 | |
| - SUM | | 42.3 | 17.47 | 6.57 | | | 15.71 | 6.65 | |
| ALGEBRAIC SUM | | +8.0 | 29.59 | -.09 | 10.0279 | | 29.86 | +.08 | 9.2468 |
| MEAN | | +0.32 | 01.18 | | | | 01.20 | | |

| | UNADJUSTED | ADJUSTED |
|---|---|---|
| MICROMETER ONE-HALF TURN VALUE | 76.5 | 76.54126 |
| (1) MEAN $\phi$ 12 PAIRS WITH PLUS MICRO. DIFF. | 01.01 | 01.18 |
| (2) MEAN $\phi$ 13 PAIRS WITH MINUS MICRO. DIFF. | 01.34 | 01.21 |
| DIFFERENCE (2) − (1) | +.33 | +.03 |

NORMAL EQUATIONS

PC − [M]r + [(Δφ)] = 0 — − [M]c + [MM]r − [M(Δφ)] = 0

25c − 8.0r + (−.09)Δφ = 0 — −8.0c + 463.52r − 18.992 Δφ = 0

c = +.02 | r = +.04126 | $e_p$ = ± .428 | $e_\phi$ = ± .086 | $e_r$ = ± .01992

| | |
|---|---|
| MEAN OBSERVED LATITUDE | N 38° 57′ 01.20″ |
| REDUCTION TO SEA LEVEL (Sta. Elev. 75 M) | S 00.01 |
| REDUCTION FOR ECCENTRICITY | N 02.45 |
| ASTRONOMIC LATITUDE OF GEODETIC STATION | N 38 57 03.64 ± 0.09 |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| L.E. Smith - AMS | July '63 | O.R. Nieska - AMS | DEC. '63 |

DA FORM 2843, 1 OCT 64

*Figure 32. Astronomic Latitude Summary (DA Form 2843).*

$$pc-[M]r+[\Delta\phi]=0$$

$$-[M]c+[MM]r-[M\Delta\phi]=0$$

where [ ] is the standard symbol indicating summation in operations with least squares. In the solution by the Doolittle method (fig. 33), two columns are added to obtain the proper coefficients necessary to determine the probable errors of the micrometer and the latitude. The equations take the form:

$$pc-[M]r+[\Delta\phi]+1.0+0=0$$

$$-[M]c+[MM]r-[M\Delta\phi]+0+1.0=0$$

(*e*) Each micrometer difference, M, of the accepted latitudes is now multiplied by the value of $r$ and the results placed in the proper column. Then to each preliminary latitude there is added algebraically the corresponding M$r$ to produce the corrected latitude which is entered in the designated column of DA Form 2843 (Astronomic Latitude Summary). A mean of these latitudes is now taken and entered at the foot of the column. In the next column the difference, mean latitude minus individual latitude, is set down for each pair of stars. The sum of the squares of these new $\Delta\phi$'s is entered in the next column. The mean of the corrected latitude is the mean observed astronomical latitude of the latitude station, uncorrected, however, for elevation of station above sea level or variation of the pole.

(*f*) The probable errors have been computed by means of the following formulas with data used in the adjustment.

$$e_p=\pm\sqrt{\frac{0.455[\Delta\phi^2]}{p-2}}$$

$$e_\phi=\pm\sqrt{\frac{0.455[\Delta\phi^2]}{(p-2)\left(p-\frac{[M]^2}{[MM]}\right)}}=\pm e_p\sqrt{\frac{1}{p}-\frac{[MM]}{[M]^2}}$$

$$e_r=\pm\sqrt{\frac{0.455[\Delta\phi^2]}{(p-2)\left([MM]-\frac{[M]^2}{p}\right)}}$$

$$=\pm e_p\sqrt{\frac{1}{[MM]}-\frac{p}{[M]^2}}$$

(9) The correction to the latitude to reduce it to sea level is given by the following formula:

$$\Delta\phi=-0''.000171\ h\sin 2\phi$$

where $\Delta\phi$ is the correction in seconds of arc, $h$ the elevation of the station in meters, and $\phi$ is the latitude. This correction can be obtained directly from table XIV.

(10) When the $x$ and $y$ of the instantaneous north pole are known for a given date, the reduction to be applied to an astronomical latitude observed in west longitude ($\lambda$) to reduce it to the mean pole is as follows:

$$\Delta\phi=-(x\cos\lambda+y\sin\lambda),$$

$x$ and $y$ are in seconds of arc.

(11) If the observations have been made from an eccentric station, the reduction to the geodetic station is computed by cosine of the azimuth x distance in meters divided by difference per second of arc in meters. The sign of the correction should be checked on an oriented sketch of the eccentricity.

## 25. Latitude By Meridian Zenith Distance of the Sun

This is a rough method used for convenience in observing and it will not yield precise results. The computations are similar to those in paragraph 20, except for the necessary determination of the sun's coordinates.

*a.* The field observations are usually made by following the sun in altitude near noon and accepting the highest obtained altitude as the meridian altitude. This ignores the slight difference in the meridian and maximum altitudes due to the changing declination. In finding the maximum altitude by trial, it is seldom possible to secure a reversed pointing. Hence, the observed altitude must be corrected for index error of the vertical circle, refraction, semidiameter, and parallax.

*b.* A more accurate method consists in knowing the meridian from a previous azimuth observation, or in computing the exact time of the sun's transit from a known watch time and the station longitude. The vertical circle is then read a few seconds before transit, the telescope reversed and the other limb observed. The index and semidiameter corrections are thus eliminated.

| PROJECT | | TABULATION OF GEODETIC DATA (TM 5-237) | | |
|---|---|---|---|---|
| LOCATION, MARYLAND | | ORGANIZATION USAMS | | |
| STATION MAP ASTRO 2 | SOLUTION OF LATITUDE ADJUSTMENT | | | |
| C | r | $\overline{\Delta\phi}$ | R | S |
| 25.00000 | -8.00000 | -0.09000 | +1.00000 | 0.00000 |
| c = | +0.32000 | +0.00360 | -0.04000 | 0.00000 |
| | +463.52000 | -18.99200 | 0.00000 | +1.00000 |
| | - 2.56000 | - 0.02880 | +0.32000 | 0.00000 |
| | +460.96000 | -19.02080 | +0.32000 | +1.00000 |
| | r = | + 0.04126 | -0.00069 | -0.00217 |
| r = +.04126 | | | | |
| c = +.01680 | | | | |
| | | + 10.02790 | | |
| | | - 0.00032 | -0.04000 | |
| | | - 0.78480 | - 0.00022 | -0.00217 |
| | | 9.24278 | .04022 | .00217 |
| $e_p$ = | $\sqrt{\frac{(9.24278)(0.45495)}{23}}$ = | | ± 0.42758 | |
| $e_\phi$ = | $e_p\sqrt{.04022}$ = | | ± 0.08575. | |
| $e_r$ = | $e_p\sqrt{.00217}$ = | | ± 0.01992 | |
| TABULATED BY L. E. Smith - AMS | DATE July '63 | CHECKED BY O. R. Nieka - AMS | DATE DEC. '63 | |

DA FORM 1 FEB 57 1962 GPO 908847 U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 33. Solution of latitude adjustment.*

*c.* The computations for *a* above, are—

(1) Correct the observed altitude (or zenith distance) for index error (if known), refraction, semidiameter from the American Ephemeris or equivalent, and parallax from table VIII. Parallax may be neglected in rough work and when the index error is unknown.

(2) Find the sun's declination as follows:

(*a*) Accepting the observation as having been made on the meridian, the local apparent time is $12^h$.

(*b*) Add the longitude to obtain Greenwich apparent time (GAT).

(*c*) Subtract the equation of time from GAT to obtain UT.

(*d*) Correct the apparent declination for the date for the elapsed UT from $0^h$.

(*e*) In case the local standard time of the observation is recorded, the UT is found at once by adding the time zone difference.

(3) Apply the formula:

$$\phi=\delta+\zeta$$

or $$\phi=\delta+(90°-h)$$

## 26. Latitude By Circummeridian Zenith Distances of the Sun

This is an extension of the previous method for greater accuracy. It is similar to the method in paragraph 22. The observations are made starting about 10 minutes before local apparent noon and continued for about the same interval after noon. Pointings are made in direct and reversed positions alternately upon the different limbs. The computation procedure is as follows:

*a.* Take the means of each pair of D and R pointings and the means of their vertical readings.

*b.* Determine the hour angles ($t$) of the mean time of pointings on each pair. These are the difference between the observed time and the time of transit.

*c.* Scale the approximate latitude from a map, or compute a trial latitude using the highest observed altitude.

*d.* Apply the formula, finding the corrections $Am$ for each value of $t$, and find the equivalent meridian altitude or zenith distance by the equation:

$$h_m=h+Am \text{ or } \zeta_m=\zeta-Am$$

*e.* Mean all the consistent values of $h_m$ (or $\zeta_m$), and apply corrections for refraction and parallax.

*f.* Obtain the sun's meridian declination by finding the UT of transit and using tables of the ephemeris.

*g.* Apply the following formula:

$$\phi=\delta+\zeta_m$$

$$\phi=\delta+(90°-h_m)$$

# Section IV. DETERMINATION OF LONGITUDE

## 27. Basic Method

The longitude ($\lambda$) of a place is the arc of the equator between the meridian of the place and the primary meridian of Greenwich. Since there is a direct relationship between longitude and time, determination of the time at the place with respect to the time at the meridian of Greenwich will establish the longitude of the place. Present day radio time signals broadcast by WWV, WWVH, GBR, JJY, and several other major observatories have been synchronized and provide an excellent means of obtaining time at the meridian of Greenwich. Time at the place is determined by observations on various stars using several different methods and procedures.

## 28. Determination of Longitude By Star Transits

The most direct method of determining longitude is by observing the instant of transit of known stars over the observer's meridian. At that instant, the observer's hour angle is $0^h$ and the local sidereal time is equal to the right ascension of the star. This method is applicable to any class of observation but is seldom used except for first or second order work since the preparatory work of placing some types of instruments in the meridian (para. 29) will provide a longitude having the required accuracy for lower order work.

*a.* The instruments used in this method are usually large meridian transits or universal type theodolites with very sensitive levels and impersonal type, automatic recording eyepiece micrometers.

*b.* The following formulas and identities are applicable:

$$\Delta\lambda-Aa+(\alpha+\Delta\alpha-t-\Delta t)=v$$

$$A=\sin\zeta\sec\delta=\sin\phi-\tan\delta\cos\phi$$

$$B=\cos\zeta\sec\delta=\cos\phi+\tan\delta\sin\phi$$

$$C=\sec\delta$$

$$k=0^s.0213\cos\phi\sec\delta$$

$$l=\tfrac{R}{200}(m+s)\,C$$

$b = i(d)/15\ (n)$

The symbols in the above equations are:

$\Delta\lambda$ = correction to an assumed longitude
A = azimuth factor of the star
$a$ = azimuth of the line of collimation (amount by which the instrument is off the meridian assuming the collimation error of the instrument to be negligible)
$\alpha$ = right ascension
$\Delta\alpha$ = the short period terms of right ascension
$t$ = mean time of transit corrected for levels, diurnal aberration, width of contact strips and lost motion
$\Delta t$ = chronograph correction
$v$ = residual of a star in the solution of a star set (usually six stars)
$\zeta$ = zenith distance
$\delta$ = declination
$\phi$ = astronomic latitude of observer
B = level factor
C = collimation factor
k = diurnal aberration. The sign is minus for stars observed at upper culmination and plus for subpolar stars (i.e., observed at lower culmination)
$l$ = correction for width of contact strips and lost motion
R = equatorial value of one turn of the micrometer in seconds of time
$m$ = lost motion in terms of divisions of one turn of the micrometer
$s$ = average width of contact strips in terms of divisions of one turn of the micrometer
$b$ = inclination error in seconds of time
$i$ = mean level value in seconds of arc per division of bubble
$d$ = difference of bubble readings (refer to instrument manual to determine sign)
$n$ = number of level bubble readings

*c.* The following data should be furnished by the field party:

(1) DA Form 2844 (Longitude Record) containing the following information (fig. 34):
  (*a*) date and headings
  (*b*) star names and/or numbers
  (*c*) level records
  (*d*) time of radio time signal comparisons
  (*e*) remarks as applicable
(2) Chronograph sheets and/or tapes which contain the record of the star trackings and radio time signals
(3) Instrument and level constants including information as to when and how determined
(4) All data abstracted on the proper forms including the scalings from the chronograph sheets and/or tapes
(5) Field computations

*d.* Scaling of the Favog Chronograph record is fully covered in TM 5–6675–210–15. In scaling other types of chronograph records a suitable glass scaler or a variable scale may be utilized. Figure 35 is an example, using a glass scaler.

(1) In scaling radio time signals on DA Form 2845 (Radio Time Signals) (fig. 35②), 20 breaks should be adequate if good reception was obtained. The mean epoch of the radio time signals is reduced to the nearest second and the mean chronograph time is corrected to that epoch.
(2) In scaling the star transits (fig. 36), at least 10 matching pairs of breaks are required. If the residuals of the least square solution for the star set appear erratic, it may be desirable to scale all recorded matching pairs of breaks, make the obvious rejections, reject others on the basis of pattern, and then obtain a new mean value for use in the computations. With an experienced observer, it is seldom necessary to scale additional pairs except under conditions where it was difficult to track the star.

*e.* DA Form 2847 (Comparison of Chronometer and Radio Signals) is used for radio time signal comparison computations. The procedure is as follows (fig. 37):

(1) Fill in all headings. The latitude and longitude should be the closest approximation which is available.
(2) Enter the year of observation and the meridian of the local time which is being recorded.
(3) For each column, enter the local date, recorded local standard time, the chronograph time of signal, transmitting station, and frequency on which received.
(4) To the local standard time, add if west (and subtract if east) the meridian of the local standard time expressed in time and fill in the appropriate date and Universal Time (UT).
(5) From the American Ephemeris and Nautical Almanac published for the year of

| PROJECT | LONGITUDE RECORD (Original Transit Level Readings) (TM 5-237) | | | |
|---|---|---|---|---|
| LOCATION: MARYLAND | STATION: MAP ASTRO 2 | | | |
| ORGANIZATION: USAMS | CHIEF OF PARTY: R. SALVERMOSER | CHRONOMETER: 12474 | DATE (Local): 18 JUNE '63 PM | |
| OBSERVER: R. SALVERMOSER | INSTRUMENT (Type and No.): WILD T-4 No. 56095 | | RECORDER: H. N. CADDESS | |

| SET NO. 2 | | | SET NO. 3 | | |
|---|---|---|---|---|---|
| STARS (Or Signal*) | LEVELS W. | LEVELS E. | STARS (Or Signal*) | LEVELS W. | LEVELS E. |
| 531 (E) A -.36 | 72.0 | 34.1 | 572 (E) A +.20 | 70.2 | 30.8 |
| α 14h 23m 56s | 33.3 | 71.1 | α 15 26 15 | 33.0 | 72.6 |
| ΣA -.36 | 38.7 | 37.0 | ΣA +.20 | 37.2 | 41.8 |
| 534 (W) A +.16 | 33.1 | 71.0 | 578 (W) A +.23 | 33.0 | 72.5 |
| 14 30 14 | 72.2 | 34.3 | 15 33 04 | 69.4 | 29.8 |
| ΣA -.20 | 39.1 | 36.7 | ΣA +.43 | 36.4 | 42.7 |
| 540 (E) A -.16 | 72.0 | 33.8 | 1412 (E) A -.20 | 69.3 | 29.8 |
| 14 37 28 | 31.9 | 70.1 | 15 37 03 | 33.8 | 73.3 |
| ΣA -.36 | 40.1 | 36.3 | Picked up late ΣA +.23 | 35.5 | 43.5 |
| 1386 (W) A +.02 | 31.9 | 70.3 | 583 (W) A +.41 | 33.2 | 73.0 |
| 14 47 39 | 71.1 | 32.5 | 15 44 25 | 71.1 | 31.3 |
| Dim thru haze ΣA -.34 | 39.2 | 37.8 | Picked up late ΣA +.64 | 37.9 | 41.7 |
| 551 (E) A +.42 | 71.0 | 32.1 | 1416 (E) A -.08 | 70.3 | 30.8 |
| 14 54 24 | 32.0 | 71.0 | 15 51 21 | 33.6 | 73.1 |
| Picked up late ΣA +.08 | 39.0 | 38.9 | ΣA +.56 | 36.7 | 42.3 |
| 555 (W) A -.04 | 32.1 | 71.1 | 595 (W) A -.48 | 33.8 | 73.4 |
| 15 00 44 | 71.3 | 32.2 | 15 56 54 | 69.1 | 29.3 |
| ΣA +.04 | 39.2 | 38.9 | Picked up late ΣA +.08 | 35.3 | 44.1 |
| 1395 (E) A -.24 | 71.3 | 32.0 | | | |
| 15 04 10 | 32.6 | 71.8 | | | |
| Very dim thru haze ΣA -.20 | 38.7 | 39.8 | | | |
| 563 (W) A +.10 | 32.0 | 71.3 | | | |
| 15 13 58 | 73.0 | 33.7 | | | |
| ΣA -.10 | 41.0 | 37.6 | | | |

REMARKS

WWV 15 MC

SET 2 (1) 1418 (2) 1434 (3) 1440 (4) 1445 (5) 1516

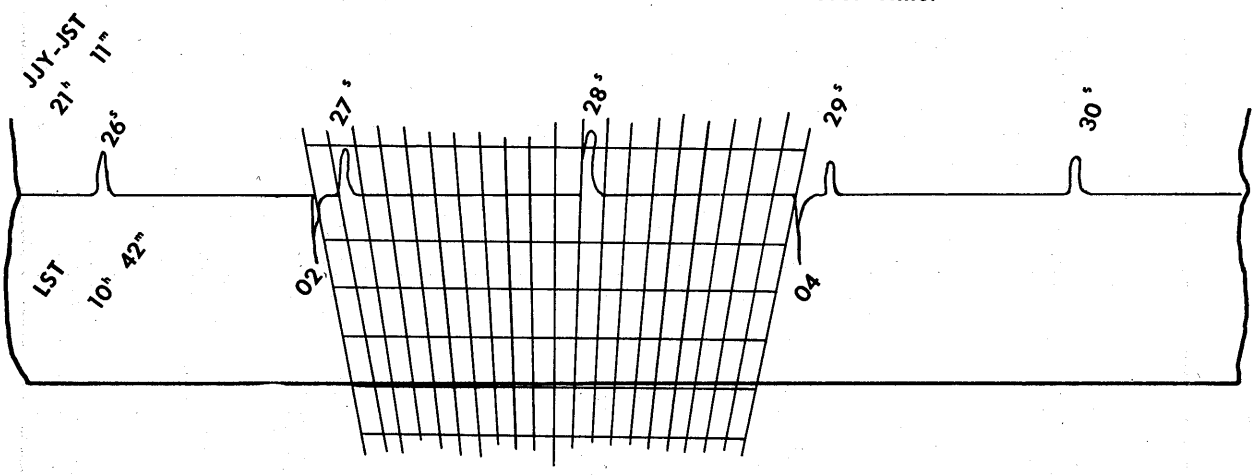
SET 3 (1) 1523 (2) 1541 (3) 1559

*In proper sequence with stars to show time of reception, include following data for each time signal received: Time (Local, standard, etc.); radio-station identification; frequency.

DA FORM 2844, 1 OCT 64

*Figure 34. Longitude Record, DA Form 2844.*

**NOTE:** As shown here, it is not uncommon to scale the chronometer time of the ending of the JJY signal rather than the resumption, 0.02 is then added to the scaled chronometer time.




① Graphic sample

*Figure 35. Time signal scalings.*

the observations, obtain both the sidereal time of $0^h$UT (apparent sidereal time, HA of first point of Aries) for the UT date and the change in nutation. "Change in Nutation" is the proportional part of the UT day multiplied by the tabular difference of the Equation of the Equinoxes for the date. The sign of the tabular difference is determined by reversing the sign of the equation of the Equinox at $0^h$ and adding algebraically to the equation of the Equinox at $24^h$.

(6) From table IX in the Ephemeris (AE&NA) determine the correction mean solar to sidereal time for the UT or multiply the UT expressed in minutes by 0.1642746 which is the rate of change per minute.

(7) The transmission time between the transmitter and the astronomic station may be determined by first using the formula; Cos D=sin $\phi_1$ sin $\phi_2$+cos $\phi_1$ cos $\phi_2$ cos $\Delta\lambda$ where $\phi_1$ and $\phi_2$ are the latitudes of the respective stations, $\Delta\lambda$ is the difference in longitude, and D is expressed in degrees of arc and decimals thereof. Then the correction for transmission time becomes $\Delta T=0.000401$ D, the constant being based upon a speed of 278,000 km/second for short wave reception.

(8) The "correction to signal" is obtained from "Time Service Bulletins" published periodically by the Observatories monitoring the time signal. If the monitoring observatory is not close to the transmitting station, it will be necessary to correct for the time of transmission between the two points to obtain the correction to signal at the transmitter. The UT0 corrections are currently being used. If a common pole is adopted in the future, it may be desirable to convert to UT1. The correction used should be identified so that future conversions may be made if warranted.

(9) To obtain the Greenwich sidereal time, add (3) through (8) above.

(10) From (9) above, subtract the approximate longitude to obtain the local sidereal time (LST).

(11) The chronometer correction is the difference between the LST and the chronometer time of signal. The correction is positive if the chronometer is slow and negative if fast.

(12) The rate per minute of the chronometer is determined by dividing the difference between two chronometer corrections in seconds by the difference in their chronometer times in minutes. The rate is positive if the chronometer is losing and negative if gaining.

*f.* Computation of factors A, B, C, *k* and *l* are

| PROJECT | | | | | RADIO TIME SIGNALS (TM 5-237) | | | |
|---|---|---|---|---|---|---|---|---|
| OBSERVER | R. SALVERMOSER | DATE | 18 JUNE 1963 PM | STATION | MAP ASTRO 2 | | | |
| RECORDER | H. CADDESS | TIME ZONE OF SIGNAL | R | FREQ OF SIGNAL | WWV, 15 MCS | | | |

| Sending Time of Signal 20 h. | | Local Chronometer Time of Signal 14 h. | | Correction to Reduce to Mean Epoch | Local Chronometer Time of Mean Epoch 14 h. 12 m. | Sending Time of Signal 21 h. | | Local Chronometer Time of Signal 14 h. | | Correction to Reduce to Mean Epoch | Local Chronometer Time of Mean Epoch 14 h. 42 m. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| m. | s. | m. | s. | s. | s. | m. | s. | m. | s. | s. | s. |
| 34 | 22 | 12 | 08.62 | .049 | 26.669 | 04 | 19 | 42 | 10.54 | .057 | 31.597 |
| | 24 | | 10.64 | .044 | .684 | | 21 | | 12.55 | .052 | .602 |
| | 26 | | 12.63 | .038 | .668 | | 23 | | 14.56 | .047 | .607 |
| | 28 | | 14.64 | .033 | .673 | | 25 | | 16.56 | .041 | .601 |
| | 30 | | 16.65 | .027 | .677 | | 27 | | 18.56 | .036 | .596 |
| | 31 | | 17.65 | .025 | .675 | | 29 | | 20.57 | .030 | .600 |
| | 33 | | 19.65 | .019 | .669 | | 31 | | 22.57 | .025 | .595 |
| | 35 | | 21.65 | .014 | .664 | | 33 | | 24.58 | .019 | .599 |
| | 37 | | 23.66 | .008 | .668 | | 35 | | 26.59 | .014 | .604 |
| | 39 | | 25.66 | .003 | .663 | | 37 | | 28.59 | .008 | .598 |
| | 41 | | 27.67 | -.003 | .667 | | 40 | | 31.58 | .000 | .580 |
| | 43 | | 29.67 | -.008 | .662 | | 42 | | 33.59 | -.005 | .585 |
| | 45 | | 31.68 | -.014 | .666 | | 44 | | 35.59 | -.011 | .579 |
| | 47 | | 33.68 | -.019 | .661 | | 48 | | 39.61 | -.022. | .588 |
| | 49 | | 35.69 | -.025 | .665 | | 50 | | 41.61 | -.027 | .583 |
| | 51 | | 37.69 | -.030 | .660 | | 52 | | 43.61 | -.033 | .577 |
| | 53 | | 39.69 | -.036 | .654 | | 54 | | 45.62 | -.038 | .582 |
| | 55 | | 41.70 | -.041 | .659 | | 56 | | 47.63 | -.044. | .586 |
| | 57 | | 43.71 | -.047 | .663 | | 58 | | 49.64 | -.049 | .591 |
| | 58 | | 44.72 | -.049 | .671 | | 60 | | 51.64 | -.055 | .585 |
| | | | | | MN .666[9] | | | | | | MN .591[75] |

| | | | |
|---|---|---|---|
| MEAN EPOCH | 20 34 40.2 | MEAN EPOCH | 21 04 39.2 |
| MEAN SCALED READINGS | .667[5] | MEAN SCALED READINGS | .589[5] |
| CORR. MN TO SID ( 0.2 ) | -.000[5] | CORR. MN TO SID ( 0.8 ) | .002[2] |
| ADOPTED MEAN EPOCH | 20 34 40 | ADOPTED MEAN EPOCH | 21 04 40 |
| CORR CHRON TIME | 14 12 26.667 | CORR CHRON TIME | 14 42 31.591[7] |
| COMPUTED BY L.E. Smith - AMS | DATE JULY '63 | CHECKED BY O.R. Niska - AMS | DATE DEC. '63 |

DA FORM 2845, 1 OCT 64

② DA Form 2845 (Radio Time Signals)

*Figure 35*—Continued.

| STATION | PROJECT | ORGANIZATION | SET NO. | STAR TRANSIT SCALINGS (TM 5-237) |
|---|---|---|---|---|
| MAP ASTRO 2 | TEST | US AMS | 2 | |
| LOCATION | OBSERVER | INSTRUMENT | LOCAL DATE | * No individual sum shall exceed the mean by more than ± 0.2 |
| MARYLAND, USA | R. SALVERMOSER | WILD T-4 56095 | 18 JUN. 63 PM | |

| STAR 531 | | | | STAR 534 | | | | STAR 540 | | | | STAR 1386 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| TR | IN 14h 22m | | SUMS | TR | IN 14h 29m | | SUMS | TR | IN 14h 36m | | SUMS | TR | IN 14h 46m | | SUMS |
| | s | s | s | | s | s | s | | s | s | s | | s | s | s |
| 1 | 17.5 | 37.7 | 55.2 | 4 | 22.5 | 68.7 | 91.2 | 2 | 18.5 | 39.6 | 58.1 | 1 | 39.1 | 43.2 | 82.3 |
| 2 | 19.4 | 36.0 | 55.4 | 5 | 23.4 | 67.5 | 90.9 R | 3 | 20.1 | 38.0 | 58.1 | 2 | 40.4 | 41.9 | 82.3 |
| 3 | 21.0 | 34.2 | 55.2 | 6 | 24.8 | 66.4 | 91.2 | 4 | 21.4 | 36.7 | 58.1 | 3 | 41.7 | 40.6 | 82.3 |
| 5 | 24.5 | 30.9 | 55.4 | 7 | 25.9 | 65.1 | 91.0 | 5 | 23.1 | 35.1 | 58.2 | 4 | 43.0 | 39.2 | 82.2 |
| 6 | 26.0 | 29.3 | 55.3 | 8 | 27.2 | 63.9 | 91.1 | 6 | 24.3 | 33.7 | 58.0 | 5 | 44.3 | 38.0 | 82.3 |
| 7 | 27.7 | 27.5 | 55.2 | 9 | 28.4 | 62.8 | 91.2 | 7 | 25.8 | 32.3 | 58.1 | 6 | 45.6 | 36.7 | 82.3 |
| 8 | 29.2 | 26.1 | 55.3 | C | 29.6 | 61.5 | 91.1 | 8 | 27.3 | 30.8 | 58.1 | 7 | 46.8 | 35.5 | 82.3 |
| 9 | 31.2 | 24.4 | 55.6 | 1 | 30.8 | 60.4 | 91.2 | 9 | 28.8 | 29.5 | 58.3 | 8 | 48.2 | 34.1 | 82.3 |
| D | 32.6 | 22.6 | 55.2 | 2 | 31.8 | 59.2 | 91.0 | C | 30.1 | 28.0 | 58.1 | 9 | 49.4 | 32.8 | 82.2 |
| 2 | 36.3 | 19.3 | 55.6 | 3 | 33.2 | 57.9 | 91.1 | 1 | 31.6 | 26.6 | 58.2 | C | 50.8 | 31.6 | 82.4 |
| 3 | 37.7 | 17.5 | 55.2 | 4 | 34.3 | 56.7 | 91.0 | 2 | 33.1 | 25.1 | 58.2 | 1 | 52.0 | 30.2 | 82.2 |
| 4 | 39.3 | 16.0 | 55.3 | 5 | 35.6 | 55.7 | 91.3 R | 3 | 34.5 | 23.8 | 58.3 | 2 | 53.4 | 28.9 | 82.3 |
| 5 | 41.0 | 14.4 | 55.4 | 6 | 36.8 | 54.4 | 91.2 | 4 | 35.9 | 22.4 | 58.3 | 3 | 54.8 | 27.6 | 82.4 |
| 6 | 42.9 | 12.7 | 55.6 | 7 | 37.9 | 53.1 | 91.0 | 5 | 37.2 | 20.8 | 58.0 | 4 | 55.9 | 26.3 | 82.2 |
| 7 | 44.4 | 11.2 | 55.6 | 8 | 39.1 | 52.1 | 91.2 | 6 | 38.9 | 19.4 | 58.3 | 5 | 57.2 | 25.0 | 82.2 |
| 8 | 46.1 | 09.4 | 55.5 | 9 | 40.3 | 50.9 | 91.2 | 7 | 40.3 | 17.9 | 58.2 | 6 | 58.6 | 23.7 | 82.3 |
| 9 | 47.7 | 07.7 | 55.4 | B | 41.5 | 49.6 | 91.1 | 8 | 41.6 | 16.5 | 58.1 | 7 | 59.8 | 22.4 | 82.2 |
| C | 49.4 | 06.2 | 55.6 | 1 | 42.5 | 48.3 | 90.8 R | 9 | 43.2 | 15.1 | 58.3 | 8 | 61.1 | 21.2 | 82.3 |
| 1 | 51.1 | 04.6 | 55.7 R | | | | | B | 44.5 | 13.8 | 58.3 | 9 | 62.4 | 19.9 | 82.3 |
| 2 | 52.8 | 02.8 | 55.6 | | | | | 1 | 46.0 | 12.3 | 58.3 | B | 63.7 | 18.6 | 82.3 |
| OUT | 14h 25m | | *MEAN 55.40 | OUT | 14h 30m | | *MEAN 91.12 | OUT | 14h 38m | | *MEAN 58.18 | OUT | 14h 48m | | *MEAN 82.28 |
| TRANSIT TIME | 14h 23m 57.700 | | | TRANSIT TIME | 14h 30m 15.560 | | | TRANSIT TIME | 14h 37m 29.090 | | | TRANSIT TIME | 14h 47m 41.140 | | |
| Scaled By R. Salvermoser - AMS | | | | DATE JUNE 1963 | | | | Checked By L. E. Smith - AMS | | | | DATE July 1963 | | | |

DA FORM 2846, 1 OCT 64

*Figure 36. Star Transit Scalings (DA Form 2846).*

made directly on DA Form 2848 (Astronomic Longitude Data) (fig. 38).

(1) Extract tan $\delta$ and sec $\delta$ from the fundamental catalogue at the bottom of the page for each star. If greater accuracy is required, the $\delta$ should be determined to the nearest second and tan $\delta$ to five decimal places.

(2) Apply appropriate formulas as listed in *b* above, and as listed on the form.

*g.* The level correction to the time of transit is computed as follows:

(1) On DA Form 2844 (Longitude Record) (fig. 34), subtract the west bubble readings, the difference is considered positive; then subtract the east bubble readings with the difference considered negative. The inclination is the algebraic sum of these two results, which is entered on DA Form 2848 (fig. 38).

(2) Multiply the result of (1) above, by $i/60$, where $i$ is the mean level value in seconds per division of the bubble. This result is the inclination error in seconds of time ($b$).

(3) The total level correction in seconds of time is the product B$b$ which is entered on the appropriate line.

*h.* The uncorrected transit time is the mean of the 10 or more matching pairs of breaks from $d(2)$ above.

*i.* The mean corrected $t$ then becomes the sum of the scaled transit time $+(l)+(k)+(\mathrm{B}b)$ which is entered on the $t$ line.

*j.* In computing $\alpha$ and $\Delta\alpha$, it is necessary to proceed as follows (fig. 39):

(1) Determine the mean epoch of each star set (usually six stars). The mean of $t$ for the first and last stars of the set will be sufficient if the star transits are evenly spaced.

(2) If the chronometer correction is greater than 10 minutes, it will be necessary to

| PROJECT | COMPARISON OF CHRONOMETER AND RADIO SIGNALS (TM 5-237) | | | |
|---|---|---|---|---|
| LOCATION: MARYLAND | CHRONOMETER NO. ~~Mean~~ Sidereal: 12474 | | STATION: MAP ASTRO 2 | |
| ORGANIZATION: USAMS | LATITUDE | LONGITUDE Time 5h 08m 29.s000 \| Arc | | |
| LOCAL DATE: JUNE 1963 | 18 JUNE | 18 JUNE | 18 JUNE | 18 JUNE |
| STANDARD TIME OF SIGNAL 75° W MERIDIAN | 20 H 34 M 40 S | 21 H 04 M 40 S | 21 H 39 M 33 S | 22 H 25 M 30 S |
| CHRONOMETER TIME OF SIGNAL | 14 12 26.667 | 14 42 31.592 | 15 17 30.316 | 16 03 34.858 |
| TRANSMITTING STATION | WWV | WWV | WWV | WWV |
| FREQUENCY OF SIGNAL | 15 MC | 15 MC | 15 MC | 15 MC |
| UNIVERSAL TIME (U. T.) OF SIGNAL — DATE JUN. | 19 | 19 | 19 | 19 |
| UNIVERSAL TIME (U. T.) OF SIGNAL — TIME | 1 H 34 M 40.s000 | 2 H 04 M 40.s000 | 2 H 39 M 33.s000 | 3 H 25 M 30.s000 |
| SIDEREAL TIME OF 0h U. T. | 17 45 59.845 | 17 45 59.845 | 17 45 59.845 | 17 45 59.845 |
| CHANGE IN NUTATION | .000 | + .001 | + .001 | + .001 |
| CORRECTION - MEAN SOLAR TO SIDEREAL TIME | 15.551 | 20.480 | 26.210 | 33.758 |
| TRANSMISSION TIME | 0 | 0 | 0 | 0 |
| CORRECTION TO SIGNAL * | - .101 | - .101 | - .101 | - .101 |
| G.S.T. OF SIGNAL | 19 20 55.295 | 19 51 00.225 | 20 25 58.955 | 21 12 03.503 |
| LONGITUDE OF STATION | 5 08 29. | 5 08 29. | 5 08 29. | 5 08 29. |
| LOCAL SIDEREAL TIME | 14 12 26.295 | 14 42 31.225 | 15 17 29.955 | 16 03 34.503 |
| CHRONOMETER TIME OF SIGNAL | 14 12 26.667 | 14 42 31.592 | 15 17 30.316 | 16 03 34.858 |
| CHRONOMETER CORRECTION | - .372 | -.367 | -.361 | -.355 |
| Δ CORRECTION | s | +0.s005 | +0.s006 | +0.s006 |
| Δ CHRONOMETER | M | 30.M082 | 34.M979 | 46.M076 |
| RATE PER MINUTE | | +.000 166 21 | +.000 171 53 | +.000 130 22 |

REMARKS

* Preliminary correction $UT_0$ signal.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| L.E. Smith - AMS | July '63 | O.R. Niska - AMS | July '63 |

DA FORM 2847, 1 OCT 64

*Figure 37. Comparison of Chronometer and Radio Signals (DA Form 2847).*

**ASTRONOMIC LONGITUDE DATA** (TM 5-237)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| PROJECT: TEST | OBSERVER: R. SALVERMOSER | | CHRONOMETER: 12474 | SET NO: 2 | | | | |
| LOCATION: MARYLAND | RECORDER: H. CADDESS | | INSTRUMENT Type and No.: WILD T-4 56095 | | STATION: MAP ASTRO 2 | | | |
| ORGANIZATION: USAMS | LEVEL VALUE (dt): d/60 = 0.s01868 | | LOCAL DATE: 18 JUN. 63 PM | GREENWICH DATE: 19.091 JUN. 63 | $\eta$ = .521 | $d\Psi$ = -.207 | $d\epsilon$ = -.010 | |
| LATITUDE ($\phi$): N 38° 57' 01" | SINE $\phi$: +.62865 | COSINE $\phi$: +.77769 | LONGITUDE($\lambda$): 5h 08m 29s | | ½ $\frac{R}{100}$ (m + s) = .085³ | | PAGE 2 OF PAGES 6 | |
| STAR | 531 | 534 | 540 | 1386 | 551 | 555 | 1395 | 563 |
| DECLINATION ($\delta$) | +52° 01' 28" | +30 32 06 | +44 33 59 | +37 57 55 | +14 35 45 | +40 32 18 | +48 17 46 | +33 27 14 |
| TAN $\delta$ | +1.281 | +0.590 | +0.985 | +0.780 | +0.260 | +0.855 | +1.122 | +0.661 |
| C = sec $\delta$ | 1.625 | 1.161 | 1.404 | 1.268 | 1.033 | 1.316 | 1.503 | 1.199 |
| INCLINATION | +1.7 | +2.4 | +3.8 | +1.4 | +0.1 | +0.3 | -1.1 | +3.4 |
| b (Incl. x dt) | +.03176 | +.04483 | +.07098 | +.02615 | +.00187 | +.00560 | -.02055 | +.06351 |
| B | 1.583 | 1.149 | 1.397 | 1.268 | 0.941 | 1.315 | 1.483 | 1.193 |
| Scaled Transit Time | 14h 23m 57.700s | 14h 30m 15.560s | 14h 37m 29.090s | 14h 47m 41.140s | 14h 54m 30.065s | 15h 00m 34.810s | 15h 04m 14.020s | 15h 14m 02.300s |
| $l$ (+) | .139 | .099 | .120 | .108 | .088 | .112 | .128 | .102 |
| $\kappa$ (-)* | -.027 | -.019 | -.023 | -.021 | -.017 | -.022 | -.025 | -.020 |
| Bb | +.050 | +.052 | +.099 | +.033 | +.002 | +.007 | -.030 | +.076 |
| t 14h | 23 57.862 | 30 15.692 | 37 29.286 | 47 41.260 | 54 30.138 | 00 34.907 | 04 14.093 | 14 02.458 |
| $\Delta$ t | -.370 | -.369 | -.368 | -.366 | -.365 | -.364 | -.363 | -.362 |
| $\alpha$ 14h | 23 57.542 | 30 15.315 | 37 28.945 | 47 40.877 | 54 29.682 | 00 34.525 | 04 13.808 | 14 02.063 |
| $\Delta\alpha$ | -.009 | -.011 | -.010 | -.010 | -.012 | -.010 | -.009 | -.010 |
| $\alpha+\Delta\alpha-(t+\Delta t)$ | +.041 | -.019 | +.017 | -.027 | -.103 | -.028 | +.069 | -.043 |
| A | -.368 | +.170 | -.137 | +.022 | +.426 | -.036 | -.244 | +.115 |

A = sin $\zeta$ sec $\delta$ = sin $\phi$ − tan $\delta$ cos $\phi$
B = cos $\zeta$ sec $\delta$ = cos $\phi$ + tan $\delta$ sin $\phi$
C = sec $\delta$
b = (ΔW − ΔE) d/60

$l$ = ½ $\frac{R}{100}$ (m + s) sec $\delta$
$l$ IS ALWAYS POSITIVE.
$\kappa$ = − 0.s0213 cos $\phi$ sec $\delta$
*FOR STARS OBSERVED AT LOWER CULMINATION $\kappa$ IS POSITIVE.

$\Delta\alpha$ = $d\alpha(\psi)\cdot d\psi + d\alpha(\epsilon)\cdot d\epsilon$
B = -.062

| COMPUTED BY | DATE |
|---|---|
| L. E. Smith - AMS | July 63 |
| CHECKED BY | DATE |
| O. R. Nieka - AMS | DEC. 63 |

DA FORM 2848, 1 OCT 64

*Figure 38. Astronomic Longitude Data (DA Form 2848).*

apply the chronometer correction to the mean epoch.

(3) Add albebraically the λ (+ if west, − if east) to the Mean Epoch.

(4) Determine the time interval in sidereal units by subtracting the value of the sidereal time of $0^h$UT for the nearest preceding date.

(5) Convert the time intervals to minutes and divide by 1444 to obtain the decimal part of a day. This is the Greenwich civil date or the UT date.

(6) Use the UT date to interpolate for $d\psi$ and $d\epsilon$ in table 1 of the Fundamental Catalogue.

(7) Interpolate for $\alpha$ in the Fundamental Catalogue including the second difference interpolation as explained in Chapter 2. Since the interpolation factor $\eta$ is equal to the λ expressed as a decimal of a day for one day stars, then for 10 day stars it is equal to $\frac{1}{10}\lambda$ plus 1/10 the number of days between the date of tabulation and the date of observation. As a check, subtract the UT date which is listed to one decimal for the date of tabulation from the UT date of observation computed in (5) above and divide by 10. The first two decimals of this approximate $\eta$ should agree with the precise $\eta$.

(8) Compute $\Delta\alpha$ by formula $[d\alpha(\psi)\times d\psi + d\alpha(\epsilon)\times d\epsilon]$ and enter on form.

(9) If $\alpha$ and $\Delta\alpha$ are not in the $FK_4$ system and corrections to that system are available, it will be necessary to apply these corrections to either $\alpha$ or $\Delta\alpha$.

*k.* The chronometer correction may be determined from the following formula:

$$\Delta T = C_o + (t - t_o)r$$

Where $C_o$ is the chronometer correction of the nearest preceding chronometer/radio comparison, $t$ is the chronometer time of the star observation,

| PROJECT | | TABULATION OF GEODETIC DATA (TM 5-237) | | |
|---|---|---|---|---|
| LOCATION, MARYLAND | | ORGANIZATION USAMS | | |
| STATION MAP ASTRO 2 | INTERPOLATION FACTORS | | | |
| | SET 1 | SET 2 | SET 3 | SET 4 |
| $t + \Delta t$ (1ST) | 12 43 25 | 14 23 58 | 15 26 20 | 16 18 40 |
| $t + \Delta t$ (LAST) | 13 15 54 | 15 14 02 | 15 56 57 | 17 25 47 |
| MEAN | 12 59 40 | 14 49 00 | 15 41 38 | 16 52 14 |
| APPROX. $\lambda$ | 5 08 29 | 5 08 29 | 5 08 29 | 5 08 29 |
| G.S.T. | 18 08 09 | 19 57 29 | 20 50 07 | 22 00 43 |
| S.T. of $0^h$ U.T. | 16 07 26 | 17 46 00 | 17 46 00 | 17 46 00 |
| SID. INTERVAL | 2 00 43 | 2 11 29 | 3 04 07 | 4 14 43 |
| G.C.D. | MAY 25.084 | JUNE 19.091 | JUNE 19.128 | JUNE 19.176 |
| $d\psi$ $d\varepsilon$ | +.007 -.082 | -.207 -.010 | -.205 -.011 | -.203 -.013 |
| $B'$ $\eta$ | -.005 .021 | -.062 .521 | -.062 .521 | -.062 .521 |
| | | | | |
| | | | | |
| | | | | |
| G.C.D. | 25.1 | 19.1 | | |
| TABULATION DATE | 24.9 | 13.9 | | |
| APPROX. $\eta$ | .02 | .52 | | |
| ACTUAL $\eta$ | .021 | .521 | | |
| | | | | |
| | | | | |
| COMPUTATION | OF $\eta$ | $\lambda/24 = 0^d.214$ , $\therefore$ FOR | | 10 DAY |
| | | | STAR, $\eta$ = | .X21 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
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| | | | | |
| | | | | |
| | | | | |
| TABULATED BY L. E. Smith - AMS | DATE July '63 | CHECKED BY O. R. Niska - AMS | DATE DEC. '63 | |

DA FORM 1 FEB 57 1962 GPO 903847 U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 39. Computation of interpolation factors.*

$t_o$ is the chronometer time of the signal from which $C_o$ was obtained, and $r$ is the rate per minute by which the chronometer fails to keep sidereal time within the interval containing the star observation. The algebraic signs of $C_o$ and $r$ will determine the sign of the correction.

*l.* Compute $(\alpha+\Delta\alpha-t-\Delta t)$ for each star.

*m.* Abstract on DA Form 2849 (Least Squares Adjustment of Longitude) (fig. 40) A and $(\alpha+\Delta\alpha-t-\Delta t)$ for each star, set up and solve simultaneously the two normal equations:

$$n\Delta\lambda-[A]a+[(\alpha+\Delta\alpha-t-\Delta t)]=0$$
$$-[A]\Delta\lambda+[AA]a-[A(\alpha+\Delta\alpha-t-\Delta t)]=0$$

Then use $a$ to determine each stars residual by the following formula:

$$\Delta\lambda-Aa+(\alpha+\Delta\alpha-t-\Delta t)=v$$

*n.* In the above least squares solution of the star sets, it is not anticipated that good observations with present day equipment should result in any $v$ being greater than 0.080 second. If any $v$ is greater than this amount, an examination of that star should be made to determine whether any errors in scaling, level reading, or other factors may be the cause. If the value is not consistent with the remainder of the star set, it may be rejected. Any star with a $v$ of more than 0.2 second of time is always rejected.

*o.* The summary of results is computed on DA Form 1962 (fig. 41).

(1) In the "Summary of Results" observations by a competent observer using good equipment should be such that the difference between the mean observed $\Delta\lambda$ and any individual $\Delta\lambda$ should not exceed 0.04 second. Any star set having such a large residual should be reviewed to determine whether any errors have been made or whether one of the stars of the set might have an "A" factor which was distorting the $\Delta\lambda$ of the set. In a final examination, any star set having a large residual not balanced by one, equal to or only slightly smaller, with an opposite sign is to be rejected.

(2) The probable error of the longitude is computed from the standard probable error formula:

$$PE=0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}$$

where $v$=the deviation from the mean and $n$=the number of values

(3) The correction to the geodetic station, if applicable, is computed by the formula $C=S \sin A \cdot H$.

C=correction in seconds of arc.

S=distance in meters.

H=Reciprocal of the meters per second for longitude using the $\phi$ of the station as the argument in the extraction from the appropriate tables of meridianal arc from the Spheroid used.

A=azimuth to geodetic station.

An oriented sketch of the eccentricity will provide a means of verifying the sign of the reduction.

(4) When the $x$ and $y$ of the instantaneous north pole are known for a given date, the correction to be applied to an observed astronomic longitude to reference it to the mean pole is:

$$\Delta\lambda''(\text{arc})=(x \sin \lambda-y \cos \lambda) \tan \phi$$

West longitude is considered positive.

## 29. Longitude Using Time By Transits of Pairs of Stars Over a Great Circle Approximating the Meridian

*a.* This method (fig. 42) is suitable for use with a small theodolite or transit, and is the method used for placing an instrument accurately in the meridian. It consists of selecting two stars, one north and one south of the zenith, which will transit at a convenient time and interval, say 2 to 5 minutes apart. These stars must have high azimuth factors, that is, at least 0.75 and preferably higher. The azimuth factor (A) is computed by the formula:

$$A=\sin \zeta/\cos \delta$$

A value of the latitude is necessary, preferably within a few minutes. The instrument is carefully leveled and pointed as nearly in the meridian as possible by a ground azimuth, corrected pointing on Polaris, or even by magnetic compass. The zenith distance of the first star is computed by $\zeta=\phi-\delta$, and set on the vertical circle. When the star appears in the field, it is placed on the horizontal wire and the time recorded when it crosses the vertical wire. Without unclamping the horizon motion, the telescope is pointed to the zenith distance of the second star, whose transit is recorded in the same manner. If the telescope

**LEAST SQUARES ADJUSTMENT OF LONGITUDE** (TM 5-237)

SET NO. 2

| PROJECT | LOCATION | STATION |
|---|---|---|
| | MARYLAND | MAP ASTRO 2 |

| OBSERVER | INSTRUMENT (Type and No.) | CHRONOMETER | DATE |
|---|---|---|---|
| R. S. | WILD T-4 No. 56095 | 12474 | 19.091 JUNE '63 |

| STAR | A | $\alpha+\Delta\alpha-t-\Delta t$ | AA | $A(\alpha+\Delta\alpha-t-\Delta t)$ | $A_a$ | $\Delta\lambda$ | V |
|---|---|---|---|---|---|---|---|
| 531 | -.368 | +.041 | | | +.073 | +.032 | -.019 |
| 534 | +.170 | -.019 | | | -.034 | -.015 | +.028 |
| 540 | -.137 | +.017 | | | +.027 | +.010 | +.003 |
| 1386 | +.022 | -.027 | | | -.004 | +.023 | -.010 |
| 551 | +.426 | -.103 | | | -.084 | +.019 | -.006 |
| 555 | -.036 | -.028 | | | +.007 | +.035 | -.022 |
| 1395 | -.244 | +.069 | | | +.048 | -.021 | +.034 |
| 563 | +.115 | -.043 | | | -.023 | +.020 | -.007 |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | -.052 | -.093 | +.43911 | -.08589 | | +.013 | +.001 |

| | | |
|---|---|---|
| | +8.00000 | +.05200 -.09300 |
| $\Delta\lambda$ = | | -.00650 +.01162 |
| | | +.43911 +.08589 |
| | | -.00034 +.00060 |
| | | +.43877 +.08649 |
| | | $a$ = -.19712 |
| | | $\Delta\lambda$ = +.01290 |

NORMAL EQUATIONS

$n\Delta\lambda-[A]a+[(\alpha+\Delta\alpha-t-\Delta t)]=0$

$-[A]\Delta\lambda+[AA]a-[A(\alpha+\Delta\alpha-t-\Delta t)]=0$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| L. E. Smith - AMS | July '63 | O. R. Niska - AMS | July '63 |

DA FORM 2849, 1 OCT 64

*Figure 40. Least squares solution of star set.*

| PROJECT | | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|---|
| LOCATION MARYLAND | | | ORGANIZATION USAMS | |
| STATION MAP ASTRO 2 | ASTRONOMIC LONGITUDE SUMMARY | | | |
| | G. C. D. | a | Δλ | V |
| SET 1 | 25.084 MAY '63 | +0.186 | +0.024 | -0.012 |
| 2 | 19.091 JUNE '63 | -0.197 | +0.013 | -0.001 |
| 3 | 19.128 " | +0.140 | +0.027 | -0.015 |
| 4 | 19.176 " | -0.130 | -0.004 | +0.016 |
| 5 | 19.248 " | -0.222 | +0.018 | -0.006 |
| 6 | 19.285 " | -0.297 | -0.004 | +0.016 |
| | | | | $\Sigma V = -.002$ |
| | | MN. Δλ = | +0.012 | $\Sigma V^2 = .000918$ |
| | | | | $e_\lambda = \pm {}^s.0037$ |
| | | | | (or) = ± ".06 |
| ASSUMED LONGITUDE λ | | | + 05h 08m | 29s.000 |
| MEAN Δλ | | | + | 00.012 |
| MEAN OBSERVED LONGITUDE | (TIME) | | + 05 08 | 29.012 ± s.004 |
| | (ARC) | | W 77° 07' | 15".18 ± ".06 |
| ASTRO. LONGITUDE (MAP ASTRO 2) | | | W 77° 07' | 15".18 ± ".06 |
| REDUCTION TO GEODETIC STATION | | | E | 02".54 |
| ASTRO. LONGITUDE (NP(AMS, 1958)) | | | W 77° 07' | 12".64 ± .06 |

| TABULATED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| L. E. Smith - AMS | July '63 | O. R. Niska - AMS | July '63 |

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GPO 921961

U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 41. Summary of results.*

LONGITUDE BY TIME OF TRANSIT OF PAIRS OF STARS OVER A GREAT CIRCLE APPROXIMATING THE MERIDIAN

STATION: CASS  DATE: 15 FEB. '56  TIME: $20^h40^m$ E.S.T.  LATITUDE ($\phi$): $38°08'$

| STARS | RIGHT ASCENSION ($\alpha$) | $\delta$ | $\beta$ | A |
|---|---|---|---|---|
| 22 H CAML | $6^h\ 14^m\ 05^s.5$ | $+69°\ 20'$ | $-31\frac{1}{4}$ | -1.47 |
| $\beta$ CANIS MAJORIS | 6 18 39.5 | $-30\ 03$ | 68 | +1.07 |

OBSERVATION

| STARS | t | $(\alpha-t)$ |
|---|---|---|
| 22 H CAML | 6 13 47.1 | +18.4 |
| $\beta$ CANIS MAJORIS | 6 18 44.6 | -5.1 |

SOLUTION OF $\Delta T$

$\Delta T - 1.47a - 18.4 = 0$

$\Delta T + 1.07a + 05.1 = 0$

$1.07\Delta T - 1.51290 - 19.688 = 0$

$1.47\Delta T + 1.51290 + 7.497 = 0$

$2.54\Delta T \quad -12.191 = 0$

$\Delta T = 4^s.80$

| RADIO SIGNAL | |
|---|---|
| 15 FEB. 1956 | $21^h\ 00^m\ 00^s.0$ |
| TIME ZONE CORR. | + 5 |
| U.T., 16 FEB. | 2 00 00.0 |
| SID. T. AT $0^h$ U.T. | 9 39 53.6 |
| CORR. FOR SID. GAIN | + 0 19.7 |
| G. S. T. | 11 40 13.3 |
| CHRON. T. OF SIGNAL | $6^h\ 31^m\ 54^s.2$ |
| CORR. ($\Delta T$) | + 04.8 |
| L. S. T. OF SIGNAL | 6 31 59.0 |
| G. S. T. OF RADIO SIG. | $11^h\ 40^m\ 13^s.3$ |
| L. S. T. OF RADIO SIG. | - 6 31 59.0 |
| LONGITUDE (TIME) | $5^h08^m14^s.3$ |
| (ARC) | $77°03'34''.5$ |

*Figure 42. Computation of longitude using time by transits of pairs of stars over a great circle approximating the meridian.*

axis is not accurately leveled, it will be necessary to obtain bubble readings and apply corrections.

*b.* The computation procedure is as follows:

(1) Determine the right ascension and declination of the stars from the ephemeris. To accomplish this, the approximate local standard time must be converted to UT by the difference for the time zone.

(2) Determine the azimuth factors of the stars.

(3) Apply level corrections to observed time of transit. The correction to the time of transit for the level error is: $B \times i \times d/60$, where B, a factor of the star $=\cos\zeta/\cos\delta$, $d$ is the value of 1 division of the bubble tube in seconds of arc, and $i$ is the inclination factor as explained under computation of azimuth.

(4) Write an equation for each star:

$$\Delta T + Aa - (\alpha - t) = 0$$

Where T is the desired chronometer correction; A is the azimuth factor; $a$ is the angle between the meridian and the line of collimation in seconds of time; $\alpha$ is the star's right ascension; and $t$ is the recorded chronometer time. The values of A and $(\alpha - t)$ are known.

(5) Solve the two equations for $\Delta T$.

(6) $\Delta T + t =$ the required local sidereal time. There remains an error due to the collimation error of the telescope. This should be made a minimum by careful adjustment. The effect can be reduced by observing a second pair of stars with the telescope in the reversed position, and meaning the values of $\Delta T$.

(7) Obtain the chronometer time of a radio time signal by comparisons. Add the chronometer correction (mean $\Delta T$) to obtain the LST of signal.
(8) Compute GST of signal.
(9) Subtract LST from GST to obtain longitude.

## 30. Longitude Using Time By the Altitudes of Stars Near the Prime Vertical

*a.* This method (fig. 43) is most commonly used by surveyors. It is frequently used to determine the LST for azimuth observations in the absence of radio equipment or of an accurate longitude. An approximate value of the latitude is required.

*b.* The observations are taken on a star near the prime vertical and having an altitude of from 30° to 50°. A number of sets, each consisting of one pointing in the direct and one in the reversed positions, are ordinarily taken. The accuracy is greatly increased by observing pairs of stars at nearly the same altitudes and relationship to the prime vertical to the east and west. When this latter system is used, the reversal of the telescope between consecutive pointings can be dispensed with and all pointings can be made with the telescope in the same position. DA Form 1909, Longitude by the Altitude of Stars Near the Prime Vertical is used for the computation.

*c.* The computation procedure is as follows:
(1) Correct the mean altitude (or zenith distance) of each star, for refraction.
(2) Determine the mean time of observation corresponding to the above.
(3) Obtain the declination and right ascension of the star for the date and UT of observation from the ephemeris. An approximate local standard time converted to UT is sufficient for this purpose.
(4) Apply the following formula:

$$\sin^2 \frac{1}{2} t = \frac{\sin \frac{1}{2}[\zeta + (\phi - \delta)] \sin \frac{1}{2}[\zeta - (\phi - \delta)]}{\cos \phi \cos \delta}$$

where $t$ is the required hour angle, $\delta$ the declination, and $\phi$ the latitude. The value of $\frac{1}{2} t$ will be positive for a west star, negative for an east star.
(5) Add the hour angle $t$ (in time) to the right ascension to obtain the LST.
(6) Subtract the chronometer reading from LST to obtain the chronometer correction.
(7) After comparing the chronometer and the radio time signal, that is, obtaining the chronometer reading for the time signal, add to this reading the chronometer correction found in (6) above, to obtain the corrected chronometer time of the signal. This is the LST of the signal.
(8) Compute the GST of the signal by the rules for conversion of time.
(9) The longitude is the difference between the GST and the LST.

## 31. Longitude Using Time By the Altitude of the Sun

This is, in principle, the same as the method of paragraph 30. The differences lie only in the calculation of the sun's coordinates and time conversion. The computation procedure is as follows:

*a.* Extract the mean vertical circle readings and times of each pair of pointings, D and R.

*b.* Correct the vertical angles for refraction and parallax.

*c.* Obtain the sun's declination and right ascension for the date and time of observation.

*d.* Apply formula, preferably using DA Form 1909. In this case, $t$ is the local apparent time (LAT).

*e.* Convert LAT to local mean time (LMT) by subtracting the equation of time.

*f.* Subtract the chronometer time of observation from the LMT to obtain chronometer correction.

*g.* After comparing chronometer and time signal, add the chronometer correction to the chronometer time of the signal to obtain the LMT of the signal.

*h.* Subtract LMT from UT of signal to obtain longitude.

## LONGITUDE BY THE ALTITUDE OF STARS NEAR THE PRIME VERTICAL (TM 5-237)

| PROJECT | STATION | |
|---|---|---|
| 7-6.333 | Doyle | |
| **LOCATION** | **ORGANIZATION** | **DATE** |
| Ohio | A&S, Inc. | 3 Dec. 43 |
| **INSTRUMENT** (Type and number) | **CHRONOMETER** | **APPROXIMATE ANGLE BETWEEN STAR AND POLARIS** |
| 368 | #235 | 96° 05′ |
| **OBSERVER** | | **CHRONOMETER TIME OF ANGLE READING** |
| J. D. Vance | | 18h 56m |

### COMPUTATION OF TIME

| | STAR {East / ~~West~~} α Arietis | | STAR {East / ~~West~~} α Arietis | |
|---|---|---|---|---|
| | h. m. s. | ° ′ ″ | h. m. s. | ° ′ ″ |
| Chron. Reading, Zenith Dist. | 18 56 58.2 | 45 00 20.0 | 19 01 27.0 | 44 10 28.8 |
| Refraction | | + 56.8 | | + 55.1 |
| Corrected Z. D. = ζ | | 45 01 17 | | 44 11 24 |
| log cos φ , φ | 9.8779372 | 40 58 34 | | |
| log cos δ , δ | 9.9633858 | 23 11 53 | | |
| log cos φ + log cos δ = log D, φ − δ | 9.8413230 | 17 46 41 | 9.8413230 | 17 46 41 |
| log sin ½ [ζ + (φ − δ)], ½ [ζ + (φ − δ)] | 9.7168423 | 31 23 59 | 9.7116378 | 30 59 02.5 |
| log sin ½ [ζ − (φ − δ)], ½ [ζ − (φ − δ)] | 9.3720087 | 13 37 18 | 9.3587956 | 13 12 21.5 |
| Sum two log sines = log N | 9.0888510 | | 9.0704334 | |
| log N − log D = log sin² ½ t | 9.2475280 | | 9.2291104 | |
| log sin ½ t , ½ t (arc) | 9.6237640 | 335 08 02.3 | 9.6145552 | 335 41 23.5 |
| | h. m. s. | | h. m. s. | |
| t (time) , t (arc) | 20 41 04.3 | 310 16 04.6 | 20 45 31.1 | 311 22 47.0 |
| Right ascension of star | 2 04 00.7 | | 2 04 00.7 | |
| Sidereal time | 22 45 05.0 | | 22 49 31.8 | |
| Chronometer reading | 18 56 58.2 | | 19 01 27.0 | |
| Chronometer correction | +3 48 06.8 | | +3 48 04.8 | |

The chronometer correction is plus if the chronometer is slow, and minus if fast. Carry all angles to seconds only, all time to tenths of seconds, and all logarithms to seven decimal places.

### COMPUTATION OF LONGITUDE

| TIME OF RADIO SIGNAL | | TRANSMITTING STATION | |
|---|---|---|---|
| 18h 52m 47.9 E.Std.T. | | WWV | |
| Chronometer reading (Sid.T.) | 19h 25m 40.0s | Std. time 75° mer. | 18h 52m 47.9s |
| Chronometer correction | +3 48 05.8 | TZC | +5 |
| LST | 23 13 45.8 | UT | 23 52 47.9 |
| TZC = time zone correction | | Sid. T. at 0h UT | 4 43 47.804 |
| Longitude (λ) = GST − LST | | Corr. (table III) | + 3 55.372 |
| | | GST | 28 40 31.076 |
| | | LST | −23 13 45.800 |
| Longitude (λ) (arc) | 81° 41′ 19.″14 | Longitude (λ) | 5 26 45.276W |
| COMPUTED BY R.A. Smith | DATE 16 Jan. 54 | CHECKED BY W. C. Aumen | DATE 17 Jan. 54 |

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*Figure 43. Computation of Longitude By the Altitudes of Stars Near the Prime Vertical (DA Form 1909).*

# Section V. COMPUTATION OF LATITUDE AND LONGITUDE FROM OBSERVATIONS MADE WITH THE ASTROLABE

## 32. Basic Procedures

*a.* The astrolabe is an instrument used to obtain latitude and longitude by the observation of the times at which stars cross a circle of fixed altitude. These stars are distributed in the observing procedure, to each quadrant of azimuth, to minimize the effect of refraction.

*b.* The solution of the observation set requires an approximate or assumed geographic position of the observing station. This position should be near the true position in order that the intercepts in the graphic or analytic solution will be small.

*c.* Two procedures for reduction of an astrolabe observation set to a latitude and longitude are discussed. The graphic solution is the established method for computing third and fourth order astronomic position and the analytical solution is used for the reduction of first and second order observations using impersonal timing equipment. The number of significant figures used in the computation is dependent upon the order of the work.

*d.* The Field Record, DA Form 1910 (Observations, Astro-Fix) (fig. 44), contains the names of the stars, approximate azimuths of the stars observed, observed chronometer times, and chronometer corrections or stop watch readings. When an electronic method of timing is used (as in the example shown), the scaled results of the time comparisons and corrected chronometer times of star transits are recorded in column (d) of DA Form 1910. This form is used both for an observing program and for a record of the field observations. While sidereal time equipment is preferred, mean time equipment may be used. If more than one transit wire is used, it is customary for low orders of work to mean all transit readings on the several wires, but for higher orders of precision a single wire transit is required.

*e.* Computations for observations up to 70° latitude may be made by the so-called sine-cosine formulas using DA Form 1911, Altitude and Azimuth (Sin-Cos) (fig. 45 ①), or DA Form 1912, Altitude and Azimuth (Sin-Cos) (Logarithmic), (fig. 45 ②). Astrolabe observations at higher latitudes are of little value. The sine-cosine formulas are as follows:

$$\sin H_c = \sin \phi_a \sin \delta + \cos \phi_a \cos \delta \cos t$$

$$\sin Z = \frac{\cos \delta \sin t}{\cos H_c}$$

where $H_c$ refers to the computed altitude of the star distinct from the observed altitude; $\phi_a$ is the assumed latitude; and $t$ is the hour angle of the star in the shortest direction from the meridian.

*f.* The values of $H_c$ are computed for each star. Each value is then subtracted from an assumed approximate altitude of the fixed circle to obtain differences called intercepts, which are then adjusted to reduce the assumed position to the computed position. The azimuths of the stars are used in this adjustment.

## 33. Computation Procedure

*a.* From the recorded time, compute the UT for the purpose of finding the declinations and right ascensions of the stars. An average time may be taken by inspecting the record, or if the observations are at fairly well distributed intervals, the mean of the times of the first and last stars is sufficient.

*b.* Enter all the stars and their declinations and right ascensions on the form. Declinations should be to the nearest second of arc and Right Ascensions (RA) to the nearest $0^s.1$ of time, for third order work. If impersonal timing equipment is used, and higher orders of precisions are desired, the declination should be determined to the nearest 0.01 second of arc.

*c.* Correct the recorded time for the chronometer error found by comparison with radio time signal, and covert from GST to LST by subtracting the assumed longitude.

*d.* Compute the numerical difference between LST and RA of each star in such a manner as to make the difference always less than $12^h$. One method is to apply the equation, $t$=LST−RA, and if $t$ exceeds $12^h$, subtract $t$ from $24^h$. Disregard algebraic signs.

*e.* Solve equation for $H_c$. Complete the computation for all stars up to this point, and inspect the values of $H_c$ for uniformity. Reject any outstanding values after checking to eliminate errors.

| PROJECT | OBSERVATIONS, ASTRO-FIX (TM 5-237) |
|---|---|
| SPAN | |

| LOCATION | ORGANIZATION | DATE |
|---|---|---|
| RYUKYU IS. | USAMS | 22.431 NOV. '59 |

| WATCH FAST (−) SLOW (+) | ASSUMED LAT. ($L_A$) | ASSUMED LONG. ($\lambda_A$) | STATION |
|---|---|---|---|
| | 30° 27′ 03.2″ | 08h 40m 52.070s | ERABU |

| INSTRUMENT (Number and type) | OBSERVER | | RECORDER |
|---|---|---|---|
| ASTROLABE / AUTOMATIC TIMER | R.L. JONES | $\delta\psi$ = +0.10<br>$\delta\varepsilon$ = 0.00 | W. E. ADAMS |

DESCRIPTION OF STATION; REFERENCES, CROSS BEARINGS, SKETCHES, ETC.

For an astrolabe, put L. Sid. T. in column (e); for a transit, put vertical angle in column (e)

| NR | STAR (a) | WATCH TIME (b) H | M | S | STOPWATCH (c) M | S | CORRECTED TIME (d) H | M | S | LOCAL SID. T. (e) | | | AZIMUTH (α) (f) ° | ′ | REMARKS (g) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 752 | γ SAGITTAE | | | | | | 18 | 17 | 10.403 | 22 | 00 | 30.051 | 255 | 37 | |
| 1602 | β PISCIUM | | | | | | 18 | 21 | 52.215 | | 05 | 12.635 | 150 | 46 | |
| 768 | ε DELPHINI | | | | | | 18 | 27 | 00.692 | | 10 | 21.957 | 235 | 19 | |
| 892 | ι PISCIUM | THESE TWO COLUMNS | | | | | 18 | 44 | 19.615 | | 27 | 43.724 | 143 | 07 | |
| 33 | μ ANDROMEDAE | NOT APPLICABLE IF | | | | | 18 | 49 | 50.308 | | 33 | 15.323 | 65 | 17 | |
| 902 | ω PISCIUM | TIME SCALED FROM | | | | | 18 | 56 | 04.659 | | 39 | 30.699 | 138 | 37 | |
| 765 | γ CYGNI | CHRONOGRAPH | | | | | 18 | 57 | 37.064 | | 41 | 03.356 | 298 | 29 | |
| 42 | β ANDROMEDAE | RECORDS. | | | | | 19 | 02 | 18.254 | | 45 | 45.316 | 71 | 01 | |
| 17 | ξ CASSIOPEIAE | | | | | | 19 | 06 | 07.308 | | 49 | 34.997 | 31 | 44 | |
| 777 | α CYGNI (DENEB) | | | | | | 19 | 10 | 42.862 | | 54 | 11.305 | 308 | 43 | |
| 45 | υ PISCIUM | | | | | | 19 | 17 | 18.562 | 23 | 00 | 48.089 | 88 | 01 | |
| 780 | ε CYGNI | | | | | | 19 | 22 | 32.838 | | 06 | 03.226 | 285 | 51 | |
| 2 | β CASSIOPEIAE | | | | | | 19 | 28 | 17.500 | | 11 | 48.831 | 14 | 24 | |
| 804 | 1 PEGASI | | | | | | 19 | 40 | 45.769 | | 24 | 19.149 | 256 | 10 | |
| 52 | 51 ANDROMEDAE | | | | | | 19 | 45 | 23.119 | | 28 | 57.258 | 44 | 12 | |
| 64 | α TRIANGULI | | | | | | 19 | 48 | 22.104 | | 31 | 56.733 | 83 | 02 | |
| 50 | η PISCIUM | | | | | | 19 | 53 | 10.327 | | 36 | 45.745 | 114 | 23 | |
| 1559 | ν CYGNI | | | | | | 19 | 54 | 20.715 | | 37 | 56.327 | 287 | 43 | |
| 73 | γ ANDROMEDAE P* | | | | | | 19 | 59 | 28.227 | | 42 | 59.667 | 57 | 28 | |
| 66 | β ARIETIS | | | | | | 20 | 02 | 39.885 | | 46 | 16.863 | 101 | 39 | |
| 821 | $\pi^2$ CYGNI | | | | | | 20 | 06 | 12.269 | | 49 | 49.829 | 317 | 24 | |
| 74 | α ARIETIS | | | | | | 20 | 10 | 11.769 | | 53 | 49.984 | 95 | 30 | |
| 878 | γ PISCIUM | | | | | | 20 | 23 | 08.192 | 24 | 06 | 48.533 | 206 | 33 | |
| | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| D.D. H. | NOV. '59 | M. K. | NOV. '59 |

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*Figure 44. Observation schedule and field data.*

| PROJECT | ALTITUDE AND AZIMUTH (SIN-COS) (TM 5-237) | | |
|---|---|---|---|
| SPAN | | | |
| LOCATION: RYUKYU IS. | ORGANIZATION: USAMS | | DATE: 22.431 Nov. 59 |
| STATION: ERABU | ASSUMED LAT. ($L_A$): 30° 27′ 03.2″ | ASSUMED LONG. ($\lambda_A$): 08h 40m 52.070s | WATCH FAST (−) SLOW (+) |
| INSTRUMENT (Number and type): ASTROLABE/AUTOMATIC TIMER | OBSERVER: R. L. JONES | $\delta\psi$ = +0.10 | $\delta\varepsilon$ = 0.00 |

| Star | 1 752 | | | 2 1602 | | | 3 768 | | | 4 892 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Declination (δ) ± | +19° | 23′ | 13.″267 S | +03° | 36′ | 21.″079 S | +11° | 10′ | 12.″077 S | +05° | 24′ | 38.″074 S |
| Watch | | | | | | | | | | | | |
| Corr. slow+, fast− ± | | | | | | | | | | | | |
| UT −9h | 09 | 17 | 10.403 | 09 | 21 | 52.215 | 09 | 27 | 00.692 | 09 | 44 | 19.615 |
| G. Sid. T 22.43 40h UT | 04 | 00 | 56.049 | | | | | | | | | |
| Mean time interval to sid. time (corr.) | | 01 | 31.529 | | 01 | 32.301 | | 01 | 33.146 | | 01 | 35.990 |
| G. Sid. T. | 13 | 19 | 37.981 | 13 | 24 | 20.565 | 13 | 29 | 29.887 | 13 | 46 | 51.654 |
| Long. ($\lambda_A$) E+, W− ± | 08 | 40 | 52.070 | | | | | | | | | |
| Local Sid. T. | 22 | 00 | 30.051 | 22 | 05 | 12.635 | 22 | 10 | 21.957 | 22 | 27 | 43.724 |
| R. A. | 19 | 56 | 57.615 | 23 | 01 | 50.370 | 20 | 31 | 17.219 | 23 | 37 | 53.654 |
| M. A. | +02 | 03 | 32.436 | −00 | 56 | 37.735 | +01 | 39 | 04.740 | −01 | 10 | 09.930 |
| M. A. (arc) t | 30° | 53′ | 06.54″ | 345° | 50′ | 33.98″ | 24° | 46′ | 11.10″ | 342° | 27′ | 31.05″ |
| Sin $L_A$ | 0.50679963 | | | | | | | | | | | |
| Sin δ | 0.33194742 | | | 0.06289251 | | | 0.19372107 | | | 0.09429208 | | |
| A (product) | 0.16823083 | | | 0.03187390 | | | 0.09817777 | | | 0.04778719 | | |
| Cos $L_A$ | 0.86206388 | | | | | | | | | | | |
| Cos δ | 0.94329789 | | | 0.99802031 | | | 0.98105665 | | | 0.99554458 | | |
| Cos t | +0.85819798 | | | +0.96962821 | | | +0.90799881 | | | +0.95349955 | | |
| B (product) | 0.69787204 | | | 0.83422667 | | | 0.76792501 | | | 0.81831527 | | |
| A | 0.16823083 | | | 0.03187390 | | | 0.09817777 | | | 0.04778719 | | |
| Sin $H_C$* | 0.86610287 | | | 0.86610057 | | | 0.86610278 | | | 0.86610246 | | |
| $H_C$ | 60 | 00 | 31.9628 | 60 | 00 | 31.0124 | 60 | 00 | 31.9256 | 60 | 00 | 31.7934 |
| $H_O$ | 60 | 00 | 33.5800 | 60 | 00 | 33.5800 | 60 | 00 | 33.5800 | 60 | 00 | 33.5800 |
| Intercept ($H_O$>$H_C$= intercept "To") | | + | 01.6172 | | + | 02.5676 | | + | 01.6544 | | + | 01.7866 |
| Cos δ | 0.94329789 | | | 0.99802031 | | | 0.98105665 | | | 0.99554458 | | |
| Sin t | 0.51331884 | | | 0.24458361 | | | 0.41897275 | | | 0.30139443 | | |
| C (product) | 0.48421258 | | | 0.24409941 | | | 0.41103600 | | | 0.30005159 | | |
| Cos $H_C$ | 0.49986580 | | | 0.49986979 | | | 0.49986591 | | | 0.49986651 | | |
| Sin Z (C÷Cos $H_C$) | 0.96868516 | | | 0.48832599 | | | 0.82229252 | | | 0.60026344 | | |
| Z | 75° | 37′ | | 29° | 14′ | | 55° | 19′ | | 36° | 53′ | |
| Azimuth $Z_N$ | 255 | 37 | | 150 | 46 | | 235 | 19 | | 143 | 07 | |

*When L and δ have same sign: Sin $H_C$=A+B if M. A.<90° and A−B if M. A.>90°
When L and δ have opposite sign: Sin $H_C$=A−B if M. A.<90° and A+B if M. A.>90°

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| Haddox | JAN. '60 | Tennis | JAN. '60 |

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U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420550

① *Altitude and Azimuth (Sin-Cos) (DA Form 1911)*

*Figure 45. Computation of intercept and azimuth.*

| PROJECT | ALTITUDE AND AZIMUTH (SIN-COS) (LOGARITHMIC) (TM 5-237) | | |
|---|---|---|---|
| SPAN | | | |
| LOCATION RYUKYU IS. | ORGANIZATION USAMS | | DATE 22.431 Nov. '59 |
| STATION ERABU | ASSUMED LAT. ($L_A$) 30° 27′ 03.2 | ASSUMED LONG. ($\lambda_A$) 08h 40m 52.070 | WATCH FAST (−) SLOW (+) |
| INSTRUMENT (Number and type) ASTROLABE WITH AUTOMATIC TIMER | OBSERVER R. L. JONES | | $\delta\psi$ = +0.10 $\delta\varepsilon$ = 0.00 |

| Star | 1 752 | 2 1602 | 3 768 | 4 892 |
|---|---|---|---|---|
| Declination (δ) ± | +19 23 13.267 | +03 36 21.079 | +11 10 12.077 | +05 24 38.074 |
| Watch | | | | |
| Corr. slow+, fast− ± | | | | |
| UT − 9h | 09 17 10.403 | 09 21 52.215 | 09 27 00.692 | 09 44 19.615 |
| G. Sid. T 22.43 d 0h UT | 04 00 56.049 | | | |
| Mean time interval to sid. time (corr.) | 01 31.529 | 01 32.301 | 01 33.146 | 01 35.990 |
| G. Sid. T. | 13 19 37.981 | 13 24 20.565 | 13 29 29.887 | 13 46 51.654 |
| Long. ($\lambda_A$) E+, W− ± | 08 40 52.070 | | | |
| Local Sid. T. | 22 00 30.051 | 22 05 12.635 | 22 10 21.957 | 22 27 43.724 |
| R. A. | 19 56 57.615 | 23 01 50.370 | 20 31 17.217 | 23 37 53.654 |
| M. A. | +02 03 32.436 | −00 56 37.735 | +01 39 04.740 | −01 10 09.930 |
| M. A. (arc) t | 30 53 06.54 | 345 50 33.98 | 24 46 11.10 | 342 27 31.05 |
| Log sin $L_A$ | 9.70483629 | | | |
| Log sin δ | 9.52106930 | 8.79859894 | 9.28717684 | 8.97447522 |
| Log A (sum) | 9.22590559 | 8.50343523 | 8.99201313 | 8.67931151 |
| A | 0.16823083 | 0.03187390 | 0.09817777 | 0.04778719 |
| Log cos $L_A$ | 9.93553945 | | | |
| Log cos δ | 9.97464886 | 9.99913938 | 9.99169409 | 9.99806072 |
| Log cos t | 9.93358749 | 9.98660524 | 9.95808527 | 9.97932049 |
| Log B (sum) | 9.84377580 | 9.92128407 | 9.88531881 | 9.91292066 |
| B | 0.69787204 | 0.83422667 | 0.76792501 | 0.81831527 |
| Sin $H_C$* | 0.86610287 | 0.86610057 | 0.86610278 | 0.86610246 |
| Log sin $H_C$ | | | | |
| $H_C$ | 60 00 31.9628 | 60 00 31.0124 | 60 00 31.9256 | 60 00 31.7934 |
| $H_O$ | 60 00 33.5800 | 60 00 33.5800 | 60 00 33.5800 | 60 00 33.5800 |
| Intercept ($H_O > H_C$ = intercept "To") | + 01.6172 | + 02.5676 | + 01.6544 | + 01.7866 |
| Log cos δ | 9.97464886 | 9.99913938 | 9.99169409 | 9.99806072 |
| Log sin t | 9.71038720 | 9.38842736 | 9.62218578 | 9.47913522 |
| Log C (sum) | 9.68503606 | 9.38756674 | 9.61387987 | 9.47719594 |
| Log cos $H_C$ | 9.69885342 | 9.69885688 | 9.69885355 | 9.69885403 |
| Log sin Z: (diff.) | 9.98618264 | 9.68870986 | 9.91502632 | 9.77834191 |
| Z | 75° 37′ | 29° 14′ | 55° 19′ | 36° 53′ |
| Azimuth $Z_N$ | 255° 37′ | 150° 46′ | 235° 19′ | 143° 07′ |

*When L and δ have same sign: Sin $H_C$ = A + B if M. A. < 90° and A − B if M. A. > 90°
When L and δ have opposite sign: Sin $H_C$ = A − B if M. A. < 90° and A + B if M. A. > 90°

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| D. D. H | Nov. '59 | M. K. | Nov. '59 |

DA FORM 1 FEB 57 1912

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420844

② *Altitude and Azimuth (Sin-Cos) (Logarithmic) (DA Form 1912)*

*Figure 45*—Continued.

*f.* Select an arbitrary value for $H_o$ to an even minute or second of arc slightly greater than the highest retained value of $H_c$. The quantity $H_o - H_c$, the intercept, will then always be positive.

*g.* Solve formula for the azimuth, A, of each star using a single value of $H_c$.

*h.* On DA Forms 1911 and 1912, the first quadrant angle is represented by the letter Z. To determine the quadrant the azimuth angle falls in when measured from north ($Z_N$) one of the following methods can be used.

(1) There is no satisfactory way to show the quadrant of the azimuth angle by using signs and therefore they are omitted on the computation forms DA 1911 and DA 1912.
   (*a*) A North star with a negative meridian angle (MA) would be in the first quadrant.
   (*b*) A South star with a negative meridian angle would be in the second quadrant.
   (*c*) A South star with a positive meridian angle would be in the third quadrant.
   (*d*) A North star with a positive meridian angle would be in the fourth quadrant.
(2) When the assumed latitude and a star's declination are nearly the same, determination as to the star being a north or south star can be determined from DA Form 1910, Observation, Astro-fix, where the approximate azimuths of the observations have been computed.

## 34. Graphical Solution

DA Form 1913 may be used for plotting the intercepts derived above (fig. 46). This form does not require a protractor. The solution is as follows:

*a.* Plot the intercept of each star to a suitable scale from the central point of the plotting circle in the star's azimuth direction if the intercept is positive, or in the opposite direction if the intercept is negative. Small variations are displayed best by adding a constant to all intercepts before plotting. In the example given a constant of 50 seconds (50″) has been added to each intercept.

*b.* Draw lines through the points so laid off, at right angles to their azimuth lines. These are the line of position.

*c.* By trial, draw a circle as nearly tangent as possible to all the lines of position. A set of concentric circles drawn on a piece of transparent plastic will facilitate this operation (fig. 47). Due to errors in the observations, the circle can never be drawn truly tangent to all lines. The discrepancies should be equalized in amount and direction.

*d.* Mark the center of this circle. This represents the true position of the station.

*e.* Measure the seconds and fraction of a second by the scale of the graph in the direction parallel to the N–S axis of the plot between the assumed and true positions. This amount applied to the assumed latitude gives the observed latitude.

*f.* Measure similarly the E–W difference between the assumed and true positions. Divide this value in seconds of time by the cosine of the latitude, and apply the result to the assumed longitude to obtain the observed longitude.

*g.* The algebraic signs of the differences are apparent from the plot.

*h.* Should the assumed position be in error as much as 10 minutes of arc, the scale of the plot will be too small for the required accuracy. In this case, plot four stars in different quadrants to obtain a close approximation of the position. Recompute with this as the assumed position.

## 35. Analytic Solution

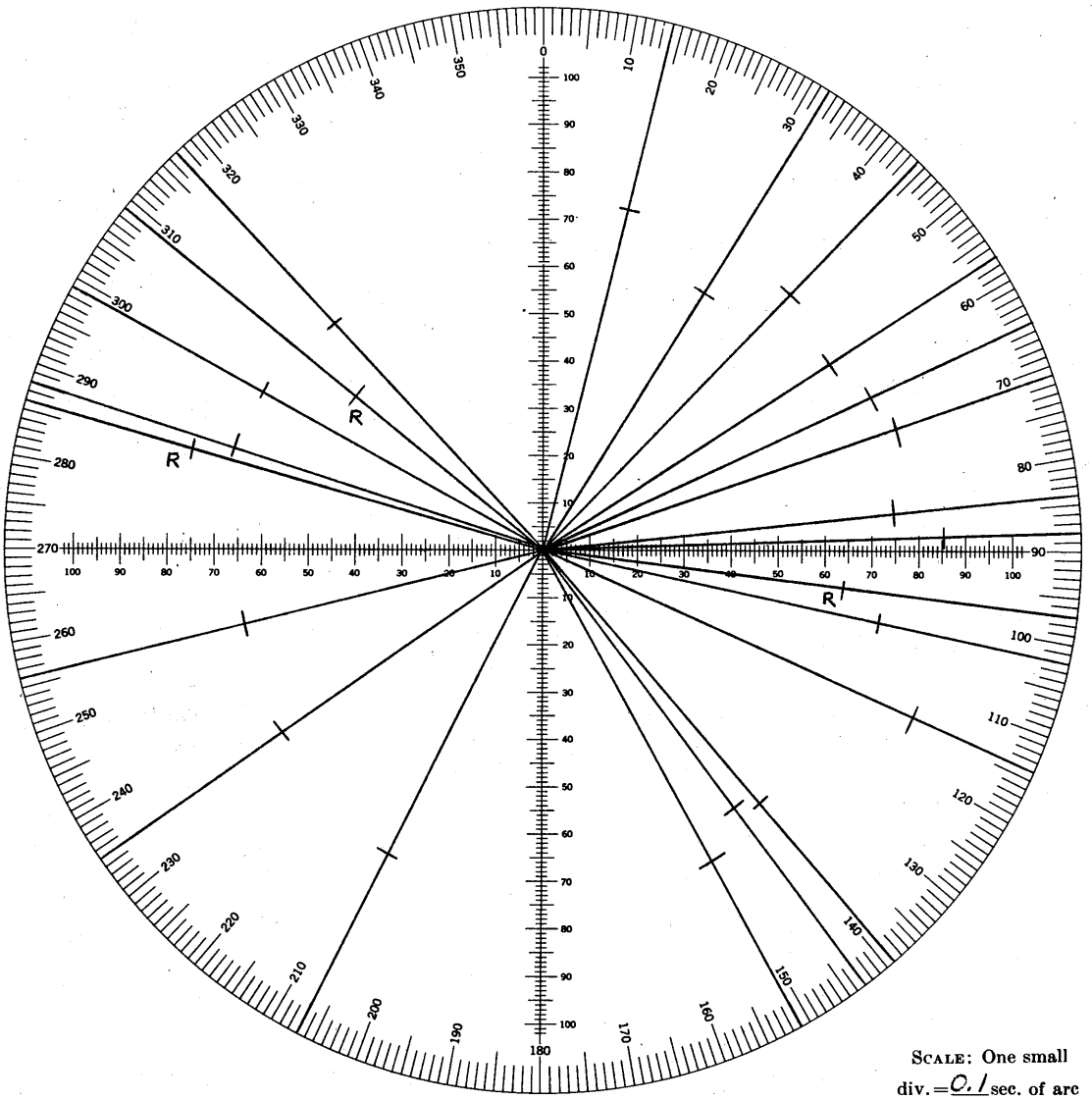
*a.* When the circummeridian intercepts and azimuths from North are known, the observation equation can be written in the following form:

$$v = \cos\phi_o(\Delta\lambda + \Delta T)\{\sin A_N\} + \Delta\phi\{\cos A_N\} + \{-1\}\Delta H - (H_o - H_c)$$

Where: $\{\}$, braced quantities indicate determined coefficients;
$\Delta\lambda$, is the change in longitude from assumed longitude;
$\Delta\phi$, is the change in latitude from assumed latitude;
$\Delta H$, is the adjusted intercept from adjusted position,
$\Delta T$, is the correction to the radio time signal;
$A_N$, is the azimuth of the star from North;
$(H_o - H_c)$, is the circummeridian intercept based on assumed position of station;
$v$, is the residual.

The international sign convention is used. This convention specifies that longitudes east of Greenwich, hour angles west of the meridian, and latitudes and declinations north of the equator are positive. Corrections are to be added al-

| PROJECT | ASTROLABE PLOTTING (TM 5-237) | |
|---|---|---|
| SPAN | | |
| LOCATION: RYUKYUS ISLANDS | ORGANIZATION: USAMS | DATE: 22.431 Nov. '59 |




SCALE: One small div. = 0.1 sec. of arc

| ASSUMED POSITION | | CORRECTIONS | | PLOTTED POSITION |
|---|---|---|---|---|
| LATITUDE ($\phi$) 30° 27' 03.2 N | $\Delta\phi$ | 0.2 S | | LATITUDE ($\phi$) 30° 27' 03.0 N |
| LONGITUDE ($\lambda$) 130° 13' 01.1 E | $\Delta\lambda$ | 0.6 E | | LONGITUDE ($\lambda$) 130° 13' 01.7 E |
| COMPUTED BY: Haddock | DATE: Jan. 63 | CHECKED BY: Tennis | | DATE: Jan 63 |

DA FORM 1 FEB 57 1913

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420713

*Figure 46. Graphical solution.*

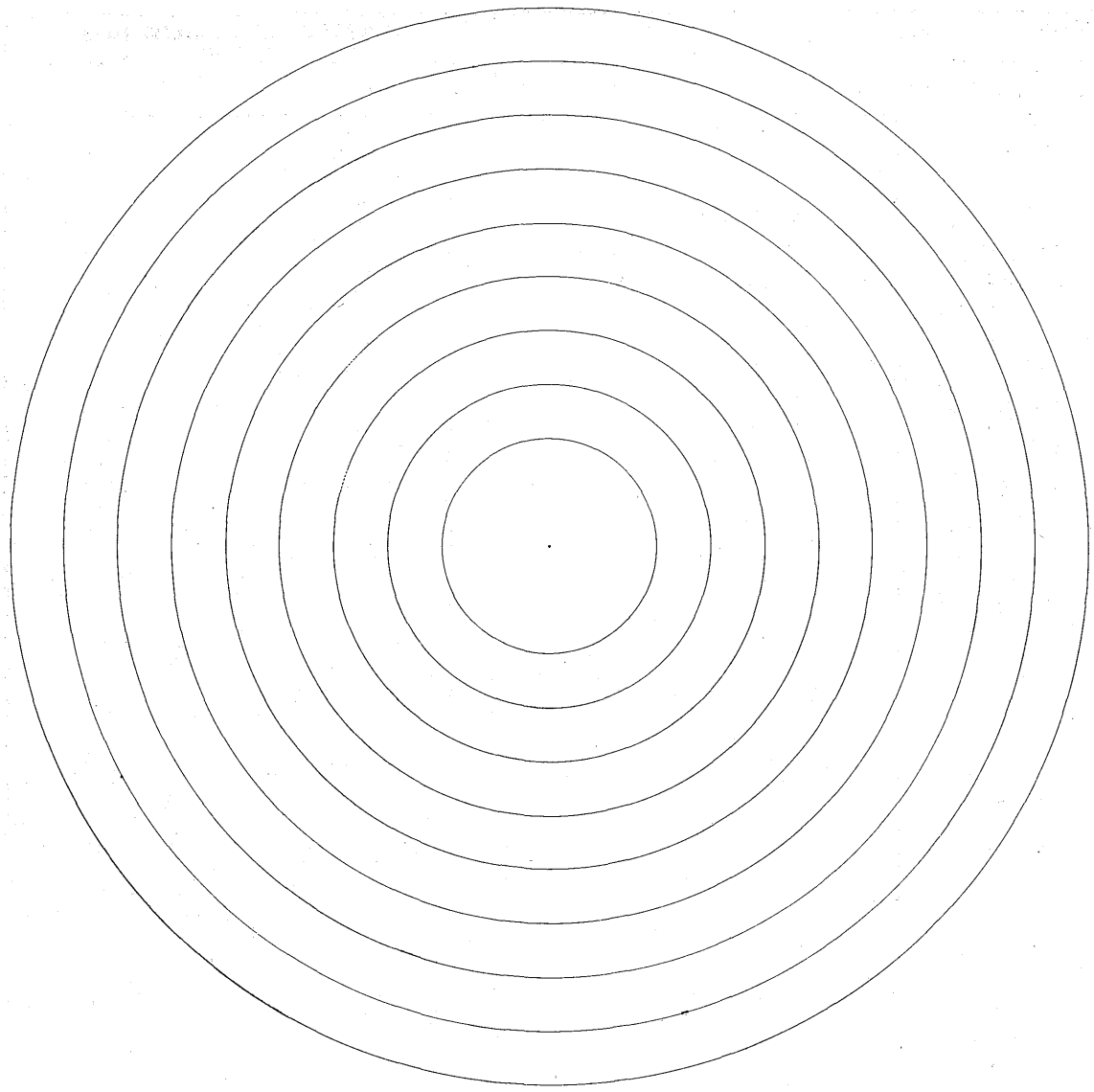


*Figure 47. Concentric circle overlay.*

gebraically. See sample formation of observation equations (fig. 48).

*b*. The normal equations are obtained by requiring that the sum of the squares of the residuals be a minimum. This requirement is obtained by finding the sum of the partial derivatives with respect to each variable in the squared observation equations. The symmetric normal equations are namely:

I. $[(\sin A_N)(\sin A_N)] \cos \phi_o(\Delta\lambda+\Delta T)$
$+[(\sin A_N)(\cos A_N)]\Delta\phi$
$+[(\sin A_N)(-1)]\Delta H-[(\sin A_N)(H_o-H_c)]$
$=0$

II. $[(\sin A_N)(\cos A_N)] \cos \phi_o(\Delta\lambda+\Delta T)$
$+[(\cos A_N)(\cos A_N)]\Delta\phi$
$+[(\cos A_N)(-1)]\Delta H-[(\cos A_N)(H_o-H_c)]$
$=0$

III. $[(-1)(\sin A_N)] \cos \phi_o(\Delta\lambda+\Delta T)$
$+[(-1)(\cos A_N)]\Delta\phi+\Delta H+[(H_o-H_c)]=0$

The brackets indicate a finite summation of the set. These equations are solved by the Doolittle Method. See sample formation of normal equations in figure 49.

*c*. Substitution of solved values back into the observation equations determines the residuals ($v$'s) for the star intercept observations. See figure 50 for sample forward solution and back solution and figure 51 for residuals and probable

PROJECT SPAN

**TABULATION OF GEODETIC DATA** (TM 5-237)

LOCATION RYUKYU IS.

ORGANIZATION USAMS

STATION ERABU

SET I

| n | SIN AN | COS AN | ΔH | $-(H_o - H_c)$ | $\Sigma a_{ij}$ |
|---|---|---|---|---|---|
| 1 | -0.9687 | -0.2483 | -1.0000 | -1.6172 | -3.8342 |
| 2 | +0.4883 | -0.8727 | | -2.5676 | -3.9520 |
| 3 | -0.8223 | -0.5691 | | -1.6544 | -4.0458 |
| 4 | Rejected | by Observer | | | |
| 5 | +0.6003 | -0.7998 | | -1.7866 | -2.9861 |
| 6 | +0.9079 | +0.4192 | | -2.6804 | -2.3533 |
| 7 | +0.6613 | -0.7502 | | -2.0852 | -3.1741 |
| 8 | -0.8788 | +0.4772 | | -1.7728 | -3.1744 |
| 9 | +0.9454 | +0.3260 | | -2.9518 | -2.6804 |
| 10 | +0.5249 | +0.8512 | | -1.4232 | -1.0471 |
| 11 | -0.7795 | +0.6264 | | -0.1357 | -1.2888 |
| 12 | +0.9993 | +0.0369 | | -3.5461 | -3.5099 |
| 13 | -0.9620 | +0.2732 | | -2.7266 | -4.4154 |
| 14 | +0.2465 | +0.9692 | | -2.4165 | -2.2008 |
| 15 | -0.9711 | -0.2388 | | -1.6696 | -3.8795 |
| 16 | +0.6966 | +0.7175 | | -2.5742 | -2.1601 |
| 17 | +0.9928 | +0.1201 | | -1.3712 | -1.2583 |
| 18 | +0.9110 | -0.4125 | | -3.6839 | -4.1854 |
| 19 | -0.9528 | +0.3036 | | -1.8941 | -3.5433 |
| 20 | +0.8427 | +0.5384 | | -2.2689 | -1.8878 |
| 21 | +0.9795 | -0.2016 | | -2.2625 | -2.4846 |
| 22 | -0.6767 | +0.7362 | | -1.4412 | -2.3817 |
| 23 | +0.9948 | -0.1017 | | -2.3882 | -2.4951 |
| 24 | -0.4470 | -0.8945 | | -2.1511 | -4.4926 |
| | | | | | -67.4307 |

TABULATED BY D.D. Haddox DATE JAN. 60 CHECKED BY G. Tennis DATE JAN. 60

DA FORM 1 FEB 57 1962

GPO 908847

U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 48. Formation of observation equations.*

| PROJECT: SPAN | | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|---|
| LOCATION: RYUKYU ISLAND | | | ORGANIZATION: USAMS | |
| STATION: ERABU | FORMATION OF NORMAL EQUATIONS | | | SET 1 |
| | | | | |
| | | | | |
| $\Sigma$ COL's 2×2 + | $\Sigma$ COL's 2×3 + | $\Sigma$ COL's 2×4 + | $\Sigma$ COL's 2×5 + | $\Sigma$ COL's 2×6 |
| $\Sigma$ SIN AN · SIN AN + | $\Sigma$ SIN AN · COS AN + | $\Sigma$ SIN AN · $\Delta H$ + | $\Sigma$ SIN AN · $[-(H_o - H_c)]$ | + $\Sigma$ SIN AN · ($\Sigma a_{ij}$) |
| = + (+15.4776) + | (−0.2103) + | (−3.3324) + | (−14.2536) + | (−2.3187) |
| | | | | |
| + | $\Sigma$ COL's 3×3 + | $\Sigma$ COL's 3×4 + | $\Sigma$ COL's 3×5 + | $\Sigma$ COL's 3×6 |
| + | $\Sigma$ COS AN · COS AN + | $\Sigma$ COS AN · $\Delta H$ + | $\Sigma$ COS AN · $[-(H_o - H_c)]$ | + $\Sigma$ COS AN · ($\Sigma a_{ij}$) |
| = + | (7.5232) + | (−1.3059) + | (−1.1969) + | (+4.8100) |
| | | | | |
| | + | $\Sigma$ COL's 4×4 + | $\Sigma$ COL's 4×5 + | $\Sigma$ COL's 4×6 |
| | + | $\Sigma \Delta H \cdot \Delta H$ + | $\Sigma \Delta H \cdot [-(H_o - H_c)]$ | + $\Sigma \Delta H \cdot (\Sigma a_{ij})$ |
| | = + | (+23.0000) + | (+49.0690) + | (+67.4307) |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| COEFFICIENTS TO NORMAL EQUATIONS | | | | |
| | | | | |
| I +15.4776 | −0.2103 − | 3.3324 − | 14.2536 − | 2.3187 |
| II | +7.5232 − | 1.3059 − | 1.1969 + | 4.8100 |
| III | + | 23.0000 + | 49.0690 + | 67.4307 |
| | | | | |
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| TABULATED BY: D. D. Haddox - AMS | DATE: JAN. '60 | CHECKED BY: G. T. Tennis - AMS | DATE: JAN. '60 | |

DA FORM 1 FEB 57 1962 GPO 908847 U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 49. Formation of normal equations.*

| PROJECT: SPAN | | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|---|
| LOCATION: RYUKYU ISLAND | | | ORGANIZATION: USAMS | SET 1 |
| STATION: ERABU | FORWARD SOLUTION AND BACK SOLUTION | | | |
| | | | | |
| I + 15.4776 | - 0.2103 | - 3.3324 | - 14.2536 | - 2.3187 |
| $\Delta L$ = | + 0.0136 | + 0.2153 | + 0.9209 | + 0.1498 |
| | | | | |
| II | + 7.5232 | - 1.3059 | - 1.1969 | + 4.8100 |
| | - 0.0029 | - 0.0453 | - 0.1938 | - 0.0315 |
| | + 7.5203 | - 1.3512 | - 1.3907 | + 4.7785 |
| | $\Delta\phi$ = | + 0.1797 | + 0.1849 | - 0.6354 |
| | | | | |
| III | | + 23.0000 | + 49.0690 | + 67.4307 |
| | | - 0.7175 | - 3.0688 | - 0.4992 |
| | | - 0.2428 | - 0.2499 | + 0.8587 |
| | | + 22.0397 | + 45.7503 | + 67.7900 |
| | | $\Delta H$ = | - 2.0758 | - 3.0758 |
| | | | | |
| $\Delta H$ = -2.07 | 58 | | | |
| $\Delta\phi$ = -2.075 | 8 (+0.1797) + 0. | 1849 = - 0.188 | 1 | |
| $\Delta L$ = -2.075 | 8 (+0.2153) - 0. | 1881 (+0.0136) | + 0.9209 = + 0. | 4714 |
| | | | | |
| $(\Delta\lambda + \Delta T)_{ARC}$ = | $\Delta L / \cos\phi_o$ | = + 0.″547 | $\lambda$ = | $\lambda_o + (\Delta\lambda + \Delta T) - \Delta T \pm D - \Delta t$ |
| $(\Delta\lambda + \Delta T)_{TIME}$ = | $\Delta L / (15 \cos\phi_o)$ | = + 0.$^s$036 | $\lambda_o$ = | 08$^h$ 40$^m$ 52.$^s$070 |
| | | | $(\Delta\lambda + \Delta T)_{TIME}$ = | + 0.036 ± 0.009 |
| | | CORRECTION TO SIGNAL | $-\Delta T$ : | -(- 0.001) |
| DIURNAL ABBERATION : D = 0.277 | | $\frac{\cos\phi_o}{15}$ (E+, W-) | = | + 0.016 |
| | TRANSMISSION TIME | $\Delta t$ = | | - 0.005 |
| | | | $\lambda_{TIME}$ = | 08$^h$ 40$^m$ 52.″116 ± 0.009 |
| | | | $\lambda_{ARC}$ = | 130° 13′ 01.″74 ± 0.13 |
| $\phi = \phi_o + \Delta\phi$ | $\pm \delta\phi_{(ECC)} - \delta\phi_{(ELEV)}$ | | | |
| $\phi_o$ = 30° | 27′ 03.″20 | | | |
| $\Delta\phi$ = | - 00.19 ± 0.17 | | | |
| $\delta\phi_{(ECC)}$ = | 00.00 | | | |
| $\delta\phi_{(ELEV)}$ = | - 00.09 | | ELEVATION OF ERABU = | 600.06 M |
| $\phi$ = 30° | 27′ 02.92 ± 0.17 | | | |
| TABULATED BY: D. D. Haddox - AMS | DATE: JAN. '60 | CHECKED BY: G. T. Tennis - AMS | DATE: JAN. '60 | |

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*Figure 50. Forward solution and back solution.*

PROJECT: SPAN

TABULATION OF GEODETIC DATA (TM 5-237)

LOCATION: RYUKYUS ISLAND

ORGANIZATION: USAMS

STATION: ERABU

SET 1

| NO. | RESIDUALS AND PROBABLE ERROR | | | | |
|---|---|---|---|---|---|
| | $\Delta L \cdot \sin A_N$ | $\Delta\phi \cdot \cos A_N$ | $-\Delta H$ | $(\overline{H_o - H_c})$ | $V_{(H_o-H_c)-(\overline{H_o-H_c})}$ |
| 1 | -0.4566 | +0.0467 | +2.0758 | 1.6659 | -0.0487 |
| 2 | +0.2302 | +0.1642 | | 2.4702 | +0.0974 |
| 3 | -0.3876 | +0.1070 | | 1.7952 | -0.1408 |
| 4 | | REJECTED | BY OBSERVER | | |
| 5 | +0.2830 | +0.1504 | | 2.5092 | -0.7226 |
| 6 | +0.4280 | -0.0789 | | 2.4249 | +0.2555 |
| 7 | +0.3117 | +0.1411 | | 2.5286 | -0.4434 |
| 8 | -0.4143 | -0.0898 | | 1.5717 | +0.2011 |
| 9 | +0.4457 | -0.0613 | | 2.4602 | +0.4916 |
| 10 | +0.2474 | -0.1601 | | 2.1631 | -0.7399 |
| 11 | -0.3675 | -0.1178 | | 1.5905 | -1.4548 |
| 12 | +0.4711 | -0.0069 | | 2.5400 | +1.0061 |
| 13 | -0.4535 | -0.0514 | | 1.5709 | +1.1557 |
| 14 | +0.1162 | -0.1823 | | 2.0097 | +0.4068 |
| 15 | -0.4578 | +0.0449 | | 1.6629 | +0.0067 |
| 16 | +0.3284 | -0.1350 | | 2.2692 | +0.3050 |
| 17 | +0.4680 | -0.0226 | | 2.5212 | -1.1500 |
| 18 | +0.4294 | +0.0776 | | 2.5828 | +1.1011 |
| 19 | -0.4491 | -0.0571 | | 1.5696 | +0.3245 |
| 20 | +0.3972 | -0.1013 | | 2.3717 | -0.1028 |
| 21 | +0.4617 | +0.0379 | | 2.5754 | -0.3129 |
| 22 | -0.3190 | -0.1385 | | 1.6183 | -0.1771 |
| 23 | +0.4689 | +0.0191 | | 2.5638 | -0.1756 |
| 24 | -0.2107 | +0.1683 | | 2.0334 | +0.1177 |
| | | | | $\Sigma V^2 =$ | 9.1929 |

$\rho = \pm\sqrt{\frac{\Sigma V^2}{N-3}} = \pm\sqrt{\frac{9.1929}{23-3}} = \pm 0.''6780$

$\sigma_{\lambda(ARC)} = \pm\sqrt{\frac{1}{\sin^2 A_N}} \cdot \frac{1}{\cos\phi_o} = \frac{0.6780}{\sqrt{75.4776}} \cdot \frac{1}{0.8621} = \pm 0.''1999$

$e_{\lambda(ARC)} = \pm 0.6745\,\sigma_{ARC} = \pm 0.''13$

$\sigma_{\lambda(TIME)} = \pm \frac{0.1999}{15} = \pm 0.^s013$

$e_{\lambda(TIME)} = \pm 0.6745\,\sigma_{\lambda TIME} = \pm 0.^s009$

$\sigma_\phi = \pm\sqrt{\frac{1}{\cos^2 A_N}} = \pm\frac{0.6780}{\sqrt{7.5232}} = \pm 0.2472$

$e_\phi = \pm 0.6745\,\sigma_\phi = \pm 0.''17$

$\sigma_H = \pm\sqrt{\rho^2 N} = \pm\frac{0.6780}{\sqrt{23}} = \pm 0.1414$

$e_H = \pm 0.6745\,\sigma_H = \pm 0.''095$

| TABULATED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| D. L. Haddox - AMS | JAN. '60 | G. T. Tennis - AMS | JAN. '60 |

DA FORM 1 FEB 57 1962

GPO 921961

U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 51. Residuals and probable error.*

error determinations. After the residuals are obtained, the probable errors are obtained with the following equations:

$$\rho = \pm\sqrt{\frac{[vv]}{n-3}}$$

$$\sigma_\lambda \text{ (arc)} = \pm\frac{\rho}{\sqrt{[\sin^2 A_N]}} \cdot \frac{1}{\cos \phi_o}$$

$$\sigma_\lambda \text{ (Time)} = \pm\frac{\sigma_\lambda \text{ (arc)}}{15}$$

$$\sigma_\phi = \pm\frac{\rho}{\sqrt{[\cos^2 A_N]}}$$

$$\sigma_H = \pm\frac{\rho}{\sqrt{n}}$$

$$e = \pm 0.6745\sigma$$

The statistical results are given in figure 51. Further rejection may be considered by using Chauvenet's Criteria, table XI in appendix III.

*d.* Considerable savings of computation time and effort will be realized by rejecting observations before the observation equations are formed. Plot azimuths and intercepts on DA Form 1913 as is done in paragraph 34. A transparent overlay is placed over the plot of azimuths and intercepts. The overlay is inscribed with a number of concentric circles whose diameters differ by two times the rejection limit. The rejection limit is generally five times the probable error that will be tolerated for the class of work. Thus, for first-order astronomic work, the probable error should be less than two tenths of a second ($0''.2$); and therefore, the rejection limit would be set at 1 second ($1''.0$) and the diameters of the concentric circles would differ by 2 seconds. The overlay is moved until a ring is found into which most plotted intercepts will fall. Those intercepts falling without the limiting rings are rejected. The formation of observation equations proceeds as in paragraph 34*a*.

*e.* The graphic and analytic determinations of astrolabe astronomic position are concluded with corrections for elevation, mean position of the pole, correction to signal, transmission time, and diurnal aberration.

(1) *Corrections to latitude.*

(*a*) When the observing station is at some elevation other than sea-level, a correction must be applied. This is a correction for the curvature of the vertical or, otherwise stated, for the fact that two level surfaces at different elevations are not parallel but converge slightly as the poles are approached. This correction can be determined by one of the following equations:

$$\Delta\phi_E = 0.000171\ h \sin 2\phi_o$$

$$\Delta\phi_E = 0.000052\ h' \sin 2\phi_o$$

in which $h$ is the altitude of the observing station in meters, $h'$ is the altitude of the observing station in feet, and $\phi_o$ is the assumed latitude of the station. The correction is always subtracted when the observing station is above sea level.

(*b*) Where the greatest accuracy in the astronomic latitude is required, as in first and second order astronomic observations, it is necessary to reduce the observed latitude to the mean position of the pole as is outlined in paragraph 24.

(2) *Corrections to longitude.* There are four corrections to the longitude, two caused by small errors in the time, one caused by a small error in the observed altitudes, and the fourth caused by the variation of the pole from its mean position.

(*a*) The transmission time correction is generally computed at the same time as the chronometer error is determined. This computation is outlined in paragraph 28*e*.

(*b*) The correction to signal is the correction to the time service clock as transmitted. The radio time service bulletins contain this correction generally for $0^h$UT and $1200^h$ UT for the date and year of observation. See example in figure 37 and explanation in paragraph 28*e*.

(*c*) Because of the rapid rotation of the earth, a star when observed is apparently displaced. The displacement is in the direction of rotation of the earth so a star toward the east will appear to be at a slightly lower altitude than the true altitude of the star. In the same way, a star toward the west will appear to be at a slightly higher altitude than the true altitude of the star. This effect is known as

diurnal aberration. The correction to the longitude of the observing station for this effect is found by the equation:

$$\Delta\lambda_D = 0''.2771 \cos \phi_0 \text{ (ARC)}$$

in which $\Delta\lambda_D$ is the diurnal correction to longitude and $\phi_o$ is the assumed latitude. The correction is added to the east longitude and subtracted from the west longitude.

(*d*) Where the greatest accuracy in the astronomic longitude is required, as in first- and second-order astronomic observations, it is necessary to reduce the observed longitude to the mean position of the pole as is outline in paragraph 28*o*.

*f*. The corrected astronomic positions are recorded on DA Form 2850, (Astronomic Results) (fig. 52). This report card is described in paragraph 36.

## Section VI. ASTRONOMIC RESULTS

### 36. Use of Form

The results of the computations of astronomic data should be summarily tabulated on DA Form 2850 (Astronomic Results). This form was designed to provide a final summary of the astronomic position, azimuth, and LaPlace azimuth computations for a station, and to further provide a convenient reference source for this information. An explanation of its preparation is outlined.

### 37. Tabulation of Results

*a*. All entries on the astronomic results form must be complete in all respects to avoid any ambiguity. This involves an accurate station designation, appropriate hemispheric reference in the latitude and longitude entries, reference pole for the azimuth entry, unit of measurement for the elevation entries, and careful identification of personnel, equipment, and dates. The form should not be considered complete until all entries have been verified by someone other than the person who made them.

*b*. The latitude, longitude, and azimuth entries are extracted from the respective computations. If cardinal directions are used, as on the example form in figure 53, there is no chance of error in the proper application of reduction data in the latitude and longitude entries. Care must be taken in indicating the proper sign of the corrections applied to the azimuth entries.

*c*. The present design of the astronomic results form does not include provision for certain corrections which are not habitually computed. Effects of polar motion on latitude, longitude, and azimuth are not included for this reason. Occasionally an astronomic azimuth is observed from an eccentric station and the necessary eccentric reduction may be entered as shown on the example.

*d*. The computation of the deflection components and the LaPlace correction for azimuth can only be made if the geodetic position of the station is known. It should be noted that the sign of the prime vertical deflection will be influenced by the consideration of East longitudes as positive, and west longitudes as negative values. Also, this consideration will affect the LaPlace azimuth and the note concerning the application of the correction must be carefully observed, i.e., $\alpha_G = \alpha_N -$ LaPlace Correction (subtract algebraically). The deflection in meridian is computed considering the sign of north latitudes as positive and south latitudes as negative.

*e*. A sketch of the geodetic connection should be prepared from data available from field records and computations. Reasonable care should be taken in depicting the relationship of the points involved. Sufficient data should be provided in order that eccentric reduction computations can be made.

## ASTRONOMIC RESULTS
*(TM 5-237)*

| | | | |
|---|---|---|---|
| PROJECT | SPAN | STATION | ERABU |
| | | LOCATION | RYUKYU ISLAND |

### LATITUDE

| OBSERVER AND ORGANIZATION | INSTRUMENT | CHRONOMETER | DATE OBSERVED |
|---|---|---|---|
| R.L. Jones - USAMS | Astrolabe/ Auto Timer | 496 | 22 Nov. 1959 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ELEVATION OF STATION | 600.06 m | MEAN OBSERVED LATITUDE | | ° | ′ | ″ | | ″ | |
| | | | N | 30 | 27 | 03.01 | ± | 0.17 |
| OBSERVATIONS ACCEPTED | 23 | SEA LEVEL REDUCTION | S | | | 00.09 | | |
| OBSERVATIONS REJECTED | 1 | ECCENTRIC REDUCTION | | | | 00.00 | | |
| PROBABLE ERROR (PAIR) | ± 0.17 | ASTRONOMIC LATITUDE | N | 30 | 27 | 02.92 | ± | 0.17 |

REMARKS

| COMPUTER | DATE | CHECKER | DATE |
|---|---|---|---|
| Haddox - AMS | Jan. 60 | Tennis - AMS | Jan. 60 |

### LONGITUDE

| OBSERVER AND ORGANIZATION | INSTRUMENT | CHRONOMETER | DATE OBSERVED |
|---|---|---|---|
| R.L. Jones - USAMS | Astrolabe/ Auto Timer | 496 | 22 Nov. 1959 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| STAR SETS ACCEPTED | 23 | MEAN OBSERVED TIME | | H | M | S | | S | |
| | | | + | 08 | 40 | 52.116 | ± | 0.009 | |
| STAR SETS REJECTED | 1 | MEAN OBSERVED ARC | | ° | ′ | ″ | | ″ | |
| | | | E | 130 | 13 | 01.74 | ± | 0.13 | |
| REMARKS | | ECCENTRIC REDUCTION | | | | 00.00 | | | |
| | | ASTRONOMIC LONGITUDE | E | 130 | 13 | 01.74 | ± | 0.13 | |

| COMPUTER | DATE | CHECKER | DATE |
|---|---|---|---|
| Haddox - AMS | Jan. 60 | Tennis - AMS | Jan. 60 |

### AZIMUTH

| OBSERVER AND ORGANIZATION | INSTRUMENT | CHRONOMETER | DATE OBSERVED |
|---|---|---|---|
| | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| MARK | | MEAN OBSERVED AZIMUTH | ° | ′ | ″ | ″ |
| | | | | | | ± |
| ELEVATION OF MARK | | DIURNAL ABERRATION | | | | |
| OBSERVATIONS ACCEPTED | | ELEVATION CORRECTION | | | | |
| OBSERVATIONS REJECTED | | ASTRONOMIC AZIMUTH (MEASURED FROM ) | | | | ± |

REMARKS

| COMPUTER | DATE | CHECKER | DATE |
|---|---|---|---|
| | | | |

| GEODETIC LATITUDE | GEODETIC LONGITUDE | SKETCH OF GEODETIC CONNECTION |
|---|---|---|
| DATUM | NOTE: NORTH LATITUDES AND EAST LONGITUDES POSITIVE | |
| DEFLECTION IN MERIDIAN $(\phi_A - \phi_G)$ | ″ | |
| DIFFERENCE IN LONGITUDE $(\lambda_A - \lambda_G)$ | | |
| PRIME VERTICAL DEFLECTION $(\lambda_A - \lambda_G) \cos \phi$ | | |
| LAPLACE CORRECTION $(\lambda_A - \lambda_G) \sin \phi_G$ | | |
| LAPLACE AZIMUTH $(\alpha_G)$ | ′ ″ | NOTE: $\alpha_G = \alpha_A$ - LAPLACE CORR (SUBTRACT ALGEBRAICALLY) |

DA FORM 2850, 1 OCT 64

*Figure 52. Astronomic results (astrolabe).*

## ASTRONOMIC RESULTS
*(TM 5-237)*

| | | | |
|---|---|---|---|
| PROJECT | | STATION | NP (AMS, 1958) |
| | | LOCATION | MARYLAND |

### LATITUDE

| OBSERVER AND ORGANIZATION | INSTRUMENT | CHRONOMETER | DATE OBSERVED |
|---|---|---|---|
| H.N.C. - AMS | Wild T-4 56095 | 12460 | 2 May 1963 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ELEVATION OF STATION | 75 m | MEAN OBSERVED LATITUDE | | ° | ′ | ″ | | ″ |
| | | | N | 38 | 57 | 01.20 | ± | 0.09 |
| OBSERVATIONS ACCEPTED | 25 pair | SEA LEVEL REDUCTION | S | | | 00.01 | | |
| OBSERVATIONS REJECTED | 3 pair | ECCENTRIC REDUCTION | N | | | 02.45 | | |
| PROBABLE ERROR (PAIR) | ± 0.″43 | ASTRONOMIC LATITUDE | N | 38 | 57 | 03.64 | ± | 0.09 |

REMARKS

Horrebow - Talcott method.

| COMPUTER | DATE | CHECKER | DATE |
|---|---|---|---|
| LES (AMS) | 23 July 1963 | ORN (AMS) | 11 Dec 1963 |

### LONGITUDE

| OBSERVER AND ORGANIZATION | INSTRUMENT | CHRONOMETER | DATE OBSERVED |
|---|---|---|---|
| R.S. - AMS | Wild T-4 56095 | 12474 | 24 May, 18 Jun 63 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| STAR SETS ACCEPTED | 6 | MEAN OBSERVED TIME | | H | M | S | | S |
| | | | | 05 | 08 | 29.012 | ± | 0.004 |
| STAR SETS REJECTED | 0 | MEAN OBSERVED ARC | | ° | ′ | ″ | | ″ |
| | | | W | 077 | 07 | 15.18 | ± | 0.06 |
| REMARKS | | ECCENTRIC REDUCTION | E | | | 02.54 | | |
| Reduced to $UT_o$ | | ASTRONOMIC LONGITUDE | W | 077 | 07 | 12.64 | ± | 0.06 |

| COMPUTER | DATE | CHECKER | DATE |
|---|---|---|---|
| LES (AMS) | 22 July 1963 | ORN (AMS) | 9 Dec 1963 |

### AZIMUTH

| OBSERVER AND ORGANIZATION | INSTRUMENT | CHRONOMETER | DATE OBSERVED |
|---|---|---|---|
| F.E.W. - AMS | Wild T-3 53010 | 12460 | 10 Apr, 23 May 63 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| MARK | | MEAN OBSERVED AZIMUTH | | ° | ′ | ″ | | ″ |
| MAP (AMS, 1958) | | | | 008 | 25 | 06.43 | ± | 0.19 |
| ELEVATION OF MARK | 348.78 Ft | DIURNAL ABERRATION | + | | | 00.32 | | |
| OBSERVATIONS ACCEPTED | 32 pos | Eccentric Reduction<br>ELEVATION CORRECTION | + | | 03 | 02.93<br>00.00 | | |
| OBSERVATIONS REJECTED | 0 | ASTRONOMIC AZIMUTH (MEASURED FROM South ) | | 008 | 28 | 09.68 | ± | 0.19 |

REMARKS

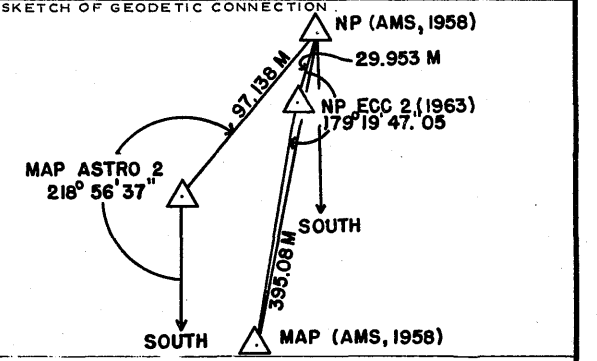
| COMPUTER | DATE | CHECKER | DATE |
|---|---|---|---|
| LES (AMS) | 2 Oct 1963 | ORN (AMS) | 11 Dec 1963 |

| GEODETIC LATITUDE | GEODETIC LONGITUDE |
|---|---|
| N 38° 57′ 05.″159 | W 007° 07′ 16.″394 |
| DATUM: 1927 NAD | NOTE: NORTH LATITUDES AND EAST LONGITUDES POSITIVE |

| | | ″ |
|---|---|---|
| DEFLECTION IN MERIDIAN | $(\phi_A - \phi_G)$ | -01.519 |
| DIFFERENCE IN LONGITUDE | $(\lambda_A - \lambda_G)$ | +03.754 |
| PRIME VERTICAL DEFLECTION | $(\lambda_A - \lambda_G) \cos \phi$ | +02.919 |
| LAPLACE CORRECTION | $(\lambda_A - \lambda_G) \sin \phi_G$ | +02.360 |

| LAPLACE AZIMUTH | ° | ′ | ″ |
|---|---|---|---|
| $(\alpha_G)$ | 008 | 28 | 07.32 |

SKETCH OF GEODETIC CONNECTION




NOTE: $\alpha_G = \alpha_A$ - LAPLACE CORR (SUBTRACT ALGEBRAICALLY)

DA FORM 2850, 1 OCT 64

*Figure 53. Astronomic results.*

# CHAPTER 4

# DISTANCE MEASUREMENTS

## Section I. TAPE MEASUREMENTS

### 38. Data Required

*a.* In the establishment and extension of horizontal control, the distance between control points is a primary requirement. In triangulation, selected sides designated as baselines are obtained by precise measurement, and these are used with observed angles of connecting polygons to obtain all other lengths of a triangulation net. In trilateration, the sides are measured by electronic methods, and taped distances are seldom involved. In traverse, the distances between stations are generally measured directly. One method of determining these distances is by measurement with standardized metallic tapes, using special tapes, with a small coefficient of thermal expansion, for baselines, and steel tapes for other measurements.

*b.* Base line computations should be made on DA Form 1914, Computation of Base Line (fig. 54). The first five columns and the temperature column are filled in from the data in the field records. These six columns are used to record the section measured, the date of measurement, the direction of measurement (forward or backward), the number of the tape used for each measurement in the section, the number of supports used for the tape for each measurement, and the average temperature of the forward and rear thermometers (a mean of the temperature can be used for all the full-tape lengths). The column "Uncorrected Length" is used to record the total number of tape lengths and the corresponding total length of each section. Odd tape lengths, such as half- or quarter-tapes, or measurements with a steel tape are also recorded in this column. Set-up or set-back measurements of less than a meter are entered as corrections in the column provided.

*c.* Computation of other tape measurements should be made on DA Form 1939 (Reduction of Taped Distances) (fig. 55). All field data is entered on the form in the space provided.

### 39. Corrections to Tape Measurements

*a.* The field measurement of any line must be corrected for temperature, tape and catenary, inclination, and elevation above or below sea-level.

*b.* The information for the temperature correction and the tape and catenary corrections is found on either a tape standardization certificate furnished by the National Bureau of Standards (fig. 56) or from results of field comparison of tapes with previously standardized tapes (para. 41). The NBS certificate contains the coefficient of thermal expansion of the tape and the standardized length of the tape under several support conditions. Every tape used in a base measurement should be standardized by the National Bureau of Standards, Washington, D.C., both before and after the base is measured. The average lengths of the two standardizations are used for the computation of the tape and catenary correction.

(1) The temperature correction is found by multiplying the coefficient of thermal expansion (shown on tape certificate) by the number of degrees difference between the temperature at the time of measurement and the temperature of standardization, usually 25° C. for invar tapes and 20° C. (68° F.) for steel tapes, times the measured length. The thermal expansion as given on the tape certificate usually states a certain expansion per tape length per degree Celsius (centigrade). For example, a tape may have a thermal expansion of +0.020 millimeters per 50 meters per degree Celsius.

(2) As an illustration of a temperature correction computation, Tape No. 5123

PROJECT I-442 | NAME OF BASE Pisula | COMPUTATION OF BASE LINE (TM 5-237)

LOCATION USA | ORGANIZATION A&S | ORDER OF BASE Third | FIELD BOOK NR 2 | PAGE NR 33 | PAGE NR 1 | NR OF PAGES 1

| SECTION | DATE | DIR. OF MEAS. | TAPE NR | TAPE SUPPORT | UNCORRECTED LENGTH: TAPE LENGTHS | UNCORRECTED LENGTH: METERS | TEMP. °C | CORRECTIONS: TEMP. (Meters) | CORRECTIONS: TAPE AND CATENARY (Meters) | CORRECTIONS: SET-UP SET-BACK (Meters) | INCLINATION (Meters) | SEA LEVEL (Meters) | REDUCED LENGTH (Meters) | ADOPTED LENGTH (Meters) | (V) (mm.) | P. E. (mm.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A to B | 17 May 54 | F | 5123 | 3 | 6 | 300.000 | 30° | +0.0012 | +0.0040 | — | -0.2396 | -0.1024 | 624.6648 | | 6.8 | |
| | | | | 4 | 6 | 300.000 | | | +0.0013 | — | | | | | | |
| | | | | 2 | | 25.000 | 29° | +0.00004 | +0.0003 | — | | | | 624.6716 | | 4.59 |
| | | B | 5124 | 3 | 12 | 600.000 | 28° | +0.0010 | +0.0091 | — | -0.2297 | -0.1024 | 624.6784 | | 6.8 | |
| | | | | 2 | | 25.000 | 28° | — | +0.0004 | — | | | | | | |

COMPUTED BY Rdhmitel | DATE 25 Jan 56 | CHECKED BY W. C. Ames | DATE 25 Jan 56

DA FORM 1914

*Figure 54. Computation of a Base Line (DA Form 1914).*

(fig. 56) has a thermal expansion of 0.020 millimeters per 50 meters per degree Celsius standardized at 25° C. Section A of a base line was measured in the forward direction with this tape. There were 12 full tape lengths and 1 half length. The average temperature of the front and rear thermometers for the 12 full tape lengths was 30° C., and the temperature for the half tape length was 29° C. The correction for the 12 full tape lengths is:

$12 \times 0.020 \times (30-25) = +1.2$ millimeters

This correction is plus because the measurement temperature was higher than the standardization temperature; therefore, the tape was actually longer than recorded. When the measurement temperature is lower than the standardization temperature, the correction is minus. The correction for the half-tape length is—

$\frac{1}{2} \times 0.020 \times (29-25) = +0.04$ millimeter

Corrections are ordinarily entered only to $\frac{1}{10}$ millimeter on the base line computation form.

*c.* When the tape is supported in the same manner as when standardized, the tape and catenary corrections are combined. If the tape is supported at different points than it was at standardization, it is necessary to compute the tape and catenary corrections separately.

(1) Using Tape No. 5123 as an example and the same section of the base which has 12 full tape lengths and 1 half tape length, the tape and catenary corrections are computed using the standardization data. For the 6 tape lengths with 3 supports, the correction is:

(6) $(+0.00067) = +0.0040$ meter

For the 6 tape lengths with 4 supports, the correction is:

(6) $(+0.00021) = +0.0013$ meter

The half tape length is supported at 2 points, and the correction is:

(1) $(+0.00028) = +0.0003$ meter

The example illustrates a very simple case where standardization was available for all the tape lengths used. A more

| PROJECT | BOOK NR | REDUCTION OF TAPED DISTANCES (TM 5-237) | | | |
|---|---|---|---|---|---|
| Test Traverse | Vol. 30 | | | | |
| LOCATION | TRAVERSE NO. | FROM STA. | | TO STA. | |
| Missouri | 3TT-1 | LENOX 1947 | | ANUTT 1947 | |
| ORGANIZATION | TAPE | CAL. LENGTH | TEMP. | TENSION | SUPPORT T |
| A&S | 6602 | | | | |
| DATE | | CAL. LENGTH | TEMP. | TENSION | SUPPORT 2 |
| 13 June 54 | | 199.956 | 68° | 20# | |

| SECTION | SUPPORT | MEASURED DISTANCE | TEMP. | TEMP. CORRECTION | CALIBRATION CORRECTION | DIFFERENCE OF ELEVATION % | SLOPE CORRECTION | CORRECTED FIELD DISTANCE |
|---|---|---|---|---|---|---|---|---|
| LENOX | 2 | 200.000 | 57° | | | 4.0 | -0.160 | LENOX |
| to | | 200.000 | | | | 4.0 | -0.160 | to #1 |
| #1 | | 200.000 | | | | — | | |
| | | 200.000 | | total | total | — | total | |
| | | 78.000 | | -0.062 | -0.176 | — | -0.320 | 877.442 |
| | | | | | | | | |
| #1 | 2 | 200.000 | 60.5 | | | — | | #1 |
| to | | 200.000 | | | | 2.0 | -0.040 | to |
| #2 | | 200.000 | | total | total | — | total | #2 |
| | | 150.865 | | -0.036 | -0.148 | — | -0.040 | 750.641 |
| | | | | | | | | |
| #2 | 2 | 200.000 | 60.5 | | | 2.0 | -0.040 | #2 |
| to | | 200.000 | | | | 2.4 | -0.058 | to |
| #3 | | 200.000 | | total | total | 2.2 | -0.048 | #3 |
| | | 89.100 | | -0.033 | -0.132 | | -0.146 | 688.789 |
| | | | | | | | | |
| #3 | 2 | 200.000 | 59.5 | | | — | | |
| to | | 200.000 | | | | — | | #3 |
| #4 | | 200.000 | | | | — | | to |
| | | 200.000 | | | | — | | #4 |
| | | 200.000 | | total | total | — | total | |
| | | 190.000 | | -0.065 | -0.257 | — | — | 1189.678 |
| | | | | | | | | |
| #4 | 2 | 200.000 | 57.0 | | | — | | #4 |
| to | | 200.000 | | | | 2.2 | -0.048 | to |
| #5 | | 200.000 | | total | total | — | total | #5 |
| | | 42.810 | | -0.046 | -0.130 | — | -0.048 | 642.586 |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| W. C. A. | 17 June 54 | RAS | 17 June 54 |

DA FORM 1 FEB 57 1939

*Figure 55. Computation of Taped Distances (DA Form 1939).*

757-381 O - 65 - 7

Form 1
11-1784

UNITED STATES DEPARTMENT OF COMMERCE
WASHINGTON

# National Bureau of Standards

## Certificate

FOR

50-METER IRON-NICKEL ALLOY TAPE
(Low Expansion Coefficient)

Maker: Keuffel & Esser Co. NBS No. 7871
No. 5123

Submitted by

Inter-American Geodetic Survey Liaison Office
c/o Army Map Service
6500 Brooks Lane
Washington 25, D. C.

This tape has been compared with the standards of the United States under a horizontal tension of 15 kilograms. The intervals indicated have the following lengths at 25° centigrade under the conditions given below:

| Interval (meters) | Points of Support (meters) | Length (meters) |
|---|---|---|
| 0 to 50 | 0, 25, and 50 | 50.00067 |
| 0 to 50 | 0, 12 1/2, 37 1/2, and 50 with the 12 1/2- and 37 1/2-meter points 6 inches above the plane of the 0 and 50-meter points. | 50.00021 |
| 0 to 12 1/2 | 0 and 12 1/2 | 12.50088 |
| 0 to 25 | 0 and 25 | 25.00028 |
| 0 to 37 1/2 | 0 and 37 1/2 | 37.49701 |
| 0 to 37 1/2 | 0, 25, and 37 1/2 | 37.50134 |

For the interval 0 to 50 meters, thermometers weighing 45 grams each were attached at the 1-meter and 49-meter points. One thermometer weighing 45 grams was attached at the 1-meter point for all other intervals.

Test No. 2.4/G-15354

*Figure 56. Tape standardization certificate.*

NBS Certificate continued - 2 - NBS No. 7871

These comparisons were made on the section of the lines near the end on the edge of the tape marked with small dots near the graduation.

The weight per meter of this tape, previously determined, is 25.8 grams.

The values for the lengths of the intervals 0 to 25 meters and 0 to 50 meters are not in error by more than 1 part in 500,000; the probable error does not exceed 1 part in 1,500,000. The values for the lengths of the intervals 0 to 12 1/2 meters and 0 to 37 1/2 meters are not in error by more than 1 part in 250,000; the probable error does not exceed 1 part in 750,000.

The values for the lengths were obtained from measurements made at 20.2° centigrade, and in reducing to 25° centigrade, the thermal expansion of +0.020 millimeter per 50 meters per degree centigrade was used.

For the Director
National Bureau of Standards

Lewis V. Judson
Chief, Length Section
Optics and Metrology Division

Test No. 2.4/G-15354
Date: September 10, 1954

*Figure 56*—Continued.

difficult computation is necessary if standardization is not available for broken tape lengths. Under these circumstances it is necessary to make use of the formula for correction due to sag (catenary correction) which is:

$$C_s = -\frac{n}{24}\left(\frac{w}{t}\right)^2 l^3$$

in which:

$n$=number of sections into which the tape is divided by equidistant supports
$l$=the length of a section in meters
$w$=the weight of the tape in grams per meter
$t$=tension in grams

The minus sign in the formula presupposes that the correction is to be applied to a tape standardized under conditions of full support throughout. In other words, the length measured by a fully supported tape is shortened by the effect of sag.

(2) Since the Bureau of Standards does not ordinarily standardize tapes fully supported, it is often necessary to reduce a tape standardized with 3 or 4 supports to the value with full support. This reduction is made by applying the correction for sag to one of the standardized lengths. It is generally easiest to use the standardization with three supports, at 0, 25, and 50 meters. The computation of the sag correction is greatly simplified by using tables found in USC&GS Sp. Pub. 247 and TM 5-236. Both tables required that $t$= 15,000 grams. TM 5-236 gives the catenary corrections for various lengths

and weights of tape, while SP 247 gives the value of the quantity $\frac{1}{24}\left(\frac{w}{t}\right)^2 \times 10^{10}$ for each $\frac{1}{10}$ gram from 20 grams to 30 grams.

*d.* To illustrate the application of the catenary correction, the standardized tape, for which the certificate is shown in figure 56, will be reduced to full support and then various odd lengths and supports computed.

(1) From the certificate, the length of the tape interval, from 0 to 50 meters when supported at 3 points, 0, 25, and 50, is 50.00067. The weight of the tape is 25.8 grams per meter. From TM 5–236, the catenary correction for a 25-meter interval for that weight tape is 1.93 millimeters. For two 25-meter intervals, the correction is $2 \times 1.93 = 3.86$ millimeters. The length of the tape when supported throughout is then—

$$50.00067 + 0.00386 = 50.00453 \text{ meters}$$

Starting with this standard length as supported throughout, the correction for an odd distance can be found. Assuming a length was measured as 37½ meters with 3 supports at 0, 25, and 37½, the correct measured length is:

¾ times 50.00453 = 37.50340 minus the catenary corrections for 25 meters (1.93 mm) and 12½ meters (0.24 mm) which gives a corrected measured length of 37.50123 meters.

The catenary correction for 12½ meters could be computed from the formula or taken as ⅛ the correction for 25 meters. The fraction ⅛ comes from the fact that the correction varies as the cube of the length.

(2) Comparing the computed value for the 37½ meter length over 3 supports with the value on the certificate, a discrepancy of 0.11 mm is found. This discrepancy is negligible and occurs because of the markings or irregular stretching of the tape. By using this method, the correct standard length can be obtained for any odd measurement or support arrangement.

*e.* The inclination (or slope) correction reduces all measurements to a horizontal plane. In order to make this reduction to horizontal, the difference of elevation of the ends of the tape must be known. Also very important are the elevations of any intermediate tape supports if they differ greatly from the grade of the end supports. The required differences of elevation are usually determined by spirit leveling. This leveling will be discussed later in the text.

(1) The inclination correction can be computed from the formula:

$$C_G = -(l - \sqrt{l^2 - h^2})$$

or

$$C_G = -\frac{h^2}{2l} - \frac{h^4}{8l^3} - \frac{h^6}{16l^5} \cdots$$

in which $C_G$ = inclination correction (grade correction), $l$ = inclined length, and $h$ = difference in elevation of the ends of the inclined length.

(2) For short lengths or steep grades, use the formula:

$$C_G = -(l - \sqrt{l^2 - h^2}).$$

For 50-meter lengths and differences of elevation of less than 7.5 meters, TM 5–236 can be used, or the series formula:

$$C_G = -\frac{h^2}{2l} - \frac{h^4}{8l^3} \cdots$$

No more than two terms are necessary.

(3) Differences in elevations as abstracted from the level books and inclination corrections as computed or as taken from tables are recorded on DA Form 1915 (Abstract of Levels and Computation of Inclination Corrections) (fig. 57). In the second column of the form, the distances between the points in the first column are entered. The mean differences of elevation between the points are written in the third column. Be sure to indicate the units of measure for the differences of elevation by crossing out either meters or feet at the top of the column. The inclination correction is entered in millimeters or 0.01 foot for each length. The sum of the correction is obtained for the section and recorded at the end of the column. Elevations are computed for sufficient supports so that a mean of the

## ABSTRACT OF LEVELS AND COMPUTATION OF INCLINATION CORRECTIONS

(TM 5-237)

PROJECT: I-442 | DATE: 23 Jan 56

LOCATION: U. S. A. | ORGANIZATION: A&S, Inc.

| POINT | DISTANCE (Meters) | MEAN DIFFERENCE OF ELEVATION (Meters or ~~feet~~) | INCLINATION CORRECTION (mm or 0.01 ft) | ELEVATION (Meters or ~~feet~~) | MEAN ELEVATION (Meters or ~~feet~~) | REMARKS |
|---|---|---|---|---|---|---|
| A | | | | 1042.41 | | |
| 1 | 50 | +0.55 | 3.0 | | | |
| 2 | 50 | +1.14 | 13.0 | 1044.10 | | |
| 3 | 50 | -0.23 | 0.5 | | | |
| 4 | 50 | -0.83 | 6.9 | 1043.04 | | |
| 5 | 50 | +1.01 | 10.2 | | | |
| 6 | 50 | +1.09 | 11.9 | | | |
| 7 | 50 | +1.16 | 13.5 | 1046.30 | | |
| 8 | 50 | -1.07 | 11.4 | | | |
| 9 | 50 | -1.89 | 35.7 | 1043.34 | | |
| 10 | 50 | +1.68 | 28.2 | | | |
| 11 | 50 | +3.14 | 98.7 | 1048.16 | | |
| 12 | 50 | -0.74 | 5.5 | | | |
| B | 25 | -0.23 | 1.1 | 1047.19 | 1044.93 | |
| | | | 239.6 | | | |

COMPUTED BY: W. C. A. | DATE: 7 Feb. 56 | CHECKED BY: RAB | DATE: 10 Feb. 56

DA FORM 1 FEB 57 1915

*Figure 57. Computation of inclination corrections (DA Form 1915).*

elevations will provide a mean elevation for the section which is accurate to within 2 meters. Use of this mean elevation in computing the reduction to sea level of a section is explained in the following paragraph. The sum of the inclination corrections for the section is entered on DA Form 1914. Any odd tape lengths, such as 12½ or 37½ meters, or setups and setbacks, should show their individual inclination corrections merely for the sake of convenience in checking.

*f.* Each section of a base line must also be reduced from the measured length to the length at sea level. It is for this purpose that the mean elevation of the section is computed on the Abstract of Levels form. The formula for reducing the base to sea level is:

$$C = -S\frac{h}{r} + S\frac{h^2}{r^2} - S\frac{h^3}{r^3} + \ldots$$

in which C = correction to reduce a length, S, to sea level; $h$ = mean elevation of the section; and $r$ = radius of curvature of the earth's surface for that section. Only the first term of the formula is needed, except for first-order base lines at high altitudes. The value of log $r$ can be found in table IV, appendix III, using the mean latitude of the ends of the base and the azimuth of the base as the arguments.

(1) For this example, the mean latitude is 25° N and the azimuth of the line is 67°. The value of log $r$ from the table is 6.80459. The mean elevation of the section, as found on the Abstract of Levels form, is 1044.9 meters. The approximate length of the section is 625 meters. The reduction to sea level is computed as follows:

| | | |
|---|---|---|
| log 625 | = | 2.79588 |
| log 1044.9 | = | 3.01907 |
| colog $r$ | = | 3.19541–10 |
| log C | = | 9.01036–10 |
| C | = | −0.1024 meter |

(2) Inclination corrections always shorten the measured length. Sea-level corrections shorten the measured length if the base is above mean sea level, and lengthen the measured length if the base is below mean sea level.

*g.* As the subject of setups and setbacks is sometimes confusing, additional discussion is needed. Setups and setbacks can take either of two forms. They can be partial tape lengths, such as 20 meters, 15 meters, and so on, or they can be short measurements of the order of 5 or 10 millimeters measured with a scale made to bring the tape end onto the marking strip. Special care must be taken in the computation of temperature, tape, and inclination corrections for setups and setbacks.

## 40. Final Length and Probable Error

*a.* The reduced length of the section is now obtained by applying the corrections to the recorded length.

*b.* The adopted length is the mean of the reduced length of the forward and backward measurements of the section, provided the discrepancy between the measurements is less than 25 mm $\sqrt{K}$ (K is the length of the section in kilometers). The limit of 25 mm $\sqrt{K}$ applies to third-order base measurements. Closer limits are placed on first and second-order bases.

*c.* In the sample base computation, the reduced lengths of the forward and backward measurements are 624.6648 meters and 624.6784 meters, respectively. The allowable difference in measurements is 25 mm $\sqrt{0.625} = 19.76$ mm. The actual difference is 13.6 mm, which is within the allowable limits, and the 2 reduced lengths are meaned to obtain the adopted length of 624.6716 meters for the section A to B.

*d.* The value in the $v$ column is for use in finding the probable error of the section and the base. The residual $v$ is the difference between the reduced length and the adopted length.

*e.* The last column on the form is headed P.E. and may be used for either $v^2$ (residual squared) or for the probable error of the section. The probable error is computed, using the following formula:

$$P.E. = 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}$$

in which $v$ is the residual, and $n$ is the number of acceptable measurements made of the section. Where a section is measured only twice, the probable error is merely 0.6745 times ½ the difference between the two measurements.

*f.* The probable error of the entire base is the square root of the sum of the squares of the prob-

able errors of the individual sections. The probable error is also expressed as a fraction with a numerator of 1, such as 1/1,000,000, meaning an error of 1 part in 1 million. For third-order base lines, the computed probable error should not exceed 1 part in 250,000. For first- and second-order bases, the computed probable errors should not exceed 1 part in 1,000,000 and 1 part in 500,000 respectively.

*g.* All results are entered on DA Form 2851 (Baseline—Abstract of Results) (fig. 58).

## 41. Standardization of Field Tapes

When the tape used in the field has been compared to a standardized tape, a length correction table is computed for use in the reduction of lengths measured with the field tape.

*a.* In order to have a true comparison of the field and standardized tapes, certain reductions are made to the comparison measurements of the two tapes.

(1) The standardized tape is reduced to its true length for the conditions of measurement.

(2) Both tapes are corrected for the difference in temperature of observation and temperature of standardization, unless both tapes have the same coefficient of expansion.

(3) The tension must be the same for both tapes.

(4) The true lengths of the field tape are then tabulated (fig. 59).

*b.* The length correction table for the field tape (fig. 60) is computed and tabulated as follows:

(1) The correction is computed for tape segments which are multiples of 10 feet, or 5 meters depending on the units of the tape.

(2) The catenary factor, $L_3$, is computed. This factor is the cube of the segment length divided by the total length of the tape cubed.

$$L_3=\left(\frac{\text{segment length}}{\text{total length}}\right)^3$$

(3) The tape correction, $C_t$, is considered directly proportional to the length of the tape. In figure 59, the correction for 100 feet is +0.002, therefore the correction for 10 feet is +0.0002. Between

| | | | | | |
|---|---|---|---|---|---|
| BASELINE: PISULA | | REDUCED LENGTH | 624.6716 METERS | | |
| LOCATION: USA | DATE: 17 May 1954 | PROBABLE ERROR | ±4.59 mm | RATIO: 1/136 094 | |
| OBSERVER AND ORGANIZATION: C. PISULA - AMS | | LOGARITHM OF LENGTH (Meters) | 2.79565176 | | |
| FROM STATION (Master): A | ELEVATION: M | LOG ARC-SIN CORR | 0 | | |
| TO STATION (Remote): B | ELEVATION: M | NO. OF OBSERVATIONS: 2 | | REJECTIONS: 0 | |

**TAPE STANDARDIZATION**

| TYPE | MANUFACTURER | MFG'R. NO. | N. B. S. NO. | DATE |
|---|---|---|---|---|
| INVAR | K & E | 5123 | 7871 | 10 Sept 54 |
| INVAR | K & E | 5124 | | 16 May 54 |
| | | | | |

**ELECTRONIC INSTRUMENT**

| MASTER (Type and No.) | DATE CALIBRATED | REMOTE (Type and No.) | DATE CALIBRATED |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

REMARKS: This baseline does not meet third-order specifications, and should be used as a check base only. Tape K & E 5123 was standardized after use in the field.

MICRO FILM NO.:

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| AMS - R.A. Smith | 25 Jan 56 | AMS - W.C. Aumen | 25 Jan 56 |

DA FORM 2851, 1 OCT 64 — BASE-LINE ABSTRACT OF RESULTS (TM 5-237)

*Figure 58. Baseline—Abstract of Results (DA Form 2851).*

100 and 200 feet the correction is $+0.0050-0.0020=+0.0030$ and the correction for 110 feet is $+0.0023$.

(4) The catenary correction, $C_s$, is determined by multiplying the catenary factor ($L_3$) by the total catenary correction for the tape. The total catenary correction is found by subtracting the suspended length from the fully supported length of the tape. In figure 59 this correction is $200.005-199.929=0.076$ for the 200 foot length. The check at the 100 foot mark shows the computed catenary correction of $-0.0095$ is very close to the measured correction of $-0.009$. This check should always be made.

(5) The tape and catenary corrections are combined to form the $C_t+C_s$ corrections.

(6) These corrections are applied to each segment to give a table of corrections for use with any lengths measured with the field tape.

## 42. Broken Base

*a. Description.* A broken base is a base consisting of more than one horizontal tangent. No portion of the base with considerable length should be inclined at an angle of more than 20° to the final projected length of the base and the maximum should be kept down to 12° if possible.

*b. Computation.* To reduce the broken base to a single horizontal tangent, the law of cosines is used.

$$a^2=b^2+c^2-2bc\cos A$$

where $a$ is the single horizontal tangent to be determined, $b$ and $c$ are the two measured segments of the base, and A is the angle at the intersection of the two measured segments.

RESULTS OF TAPE COMPARISON

LOCATION: Ellensburg, Washington — DATE: 17 October 1957

TIME: 12:30 p.m. — TENSION: 20 lbs.

STANDARD TAPE NO: K&E 8159 — FIELD TAPE NO: 8161

OBSERVER: R. C. Campbell — CHIEF OF PARTY: J. E. Norton

SUPPORTED ON A HORIZONTAL FLAT SURFACE

| INTERVAL | LENGTH |
|---|---|
| 0 to 100 feet | 100.002 |
| 100 to 200 feet | 100.001 |
| 0 to 200 feet | 200.005 |

SUPPORTED AT THE ENDS OF THE INTERVALS INDICATED BELOW

| INTERVALS | LENGTH |
|---|---|
| 0 to 100 feet | 99.993 |
| 100 to 200 feet | 99.992 |
| 0 to 200 feet | 199.929 |

*Figure 59. Results of field tape comparison.*

LENGTH CORRECTION TABLE

FIELD TAPE NO: K&E 8161 DATE COMPARED: 17 Oct 1957

| LENGTH (feet) | CATENARY FACTOR $L_3$ | COMPUTATIONS $C_t$ | $C_s$ | $C_t+C_s$ | TOTAL CORRECTION W/TWO (2) SUPPORTS |
|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) |
| 10 | .000125 | +.0002 | -.0000 | +.0002 | .000 |
| 20 | .001000 | +.0004 | -.0001 | +.0003 | .000 |
| 30 | .003375 | +.0006 | -.0003 | +.0003 | .000 |
| 40 | .008000 | +.0008 | -.0006 | +.0002 | .000 |
| 50 | .015625 | +.0010 | -.0012 | -.0002 | .000 |
| 60 | .027000 | +.0012 | -.0021 | -.0009 | -.001 |
| 70 | .042875 | +.0014 | -.0033 | -.0019 | -.002 |
| 80 | .064000 | +.0016 | -.0049 | -.0033 | -.003 |
| 90 | .091125 | +.0018 | -.0069 | -.0051 | -.005 |
| 100 | .125000 | | -.(0095) | -.(0075) | |
| | | +.0020 | -.009 | -.007 | -.007 |
| 110 | .166375 | +.0023 | -.0126 | -.0103 | -.010 |
| 120 | .216000 | +.0026 | -.0164 | -.0138 | -.014 |
| 130 | .274625 | +.0029 | -.0209 | -.0180 | -.018 |
| 140 | .343000 | +.0032 | -.0261 | -.0229 | -.023 |
| 150 | .421875 | +.0035 | -.0321 | -.0286 | -.029 |
| 160 | .512000 | +.0038 | -.0389 | -.0351 | -.035 |
| 170 | .614125 | +.0041 | -.0467 | -.0426 | -.043 |
| 180 | .729000 | +.0044 | -.0554 | -.0510 | -.051 |
| 190 | .857375 | +.0047 | -.0652 | -.0605 | -.060 |
| 200 | 1.000000 | +.0050 | -.0760 | -.0710 | -.071 |

*Figure 60. Length correction table (field tape).*

## Section II. TACHYMETRY MEASUREMENTS

### 43. Methods Used

Tachymetry is generally construed as the measurement of distance by optical means. Included within this category are the stadia methods and the subtense methods. Each method makes use of a base of some type. The stadia methods utilize a vertical base which depends on the length of rod subtended between crosshairs within the instrument. The subtense methods utilize an outside base of fixed length, across which horizontal angles are measured.

### 44. Stadia Methods

The stadia methods determine distance by the length of a vertical rod intercepted between two fixed crosshairs within the instrument. The method may be used for lower order mapping where the corrections applied are very rough.

It is used as the means of determining distances for leveling for which the stadia constant is determined regularly and with considerable care. The self-reducing tacheometer uses an intercept on the vertical staff which is always 1/100th of the distance.

## 45. Use of the Self Reducing Tacheometer

*a.* The RDS tacheometer equipment consists of a tacheometer and a vertical staff. The tacheometer is constructed in such a way that the stadia crosswires always intercept that portion of the staff that is 1/100th of the distance, regardless of the inclination of the telescope. The telescope also displays the appropriate curve (of four) and the difference of elevation factor that is required to determine the difference of elevation at hand. By use of this method, it is possible to determine the difference in elevation, direction, and distance from the same observations.

*b.* The distance is determined by reading the rod intercept between the lower and upper stadia crosshairs, considering the millimeters of the intercept to be whole numbers and multiplying by 0.1. The difference in elevation is determined by multiplying the rod intercept, between the lower stadia crosshair and the cutting point of elevation curve across rod, by the elevation factor displayed beside the elevation curve. In some cases, two elevation curves and two elevation factors appear in the telescope at one time. In such cases each rod intercept and its corresponding elevation factor will be handled as described, but the differences in elevation for both must be the same. The signs of the elevation factors must be altered to conform to the elevations in the direction of progress.

*c.* The tacheometer should not be used in cases where the required tolerance in elevation is less than 2 meters.

## 46. Subtense Method

*a.* Subtense distances, as the name implies, are measured by the angle subtended by a known length. The length of a subtense bar is usually 2 meters. The directions for the use of a subtense bar state that the midpoint of the bar must be centered over the station, the bar must be level, and the bar must be perpendicular to the line of sight of the instrument. Figure 61 illustrates the conditions for a subtense measurement.

*b.* A and B, figure 61, are stations, $d$ is the subtense bar of known length, $a$ is the subtended angle which is measured by the transit or theodolite, and D is the required length from A to B. Since the figure is an isosceles triangle, the perpendicular bisector (D) of the base ($d$) also bisects the vertex angle ($a$) and forms two right triangles in which an acute angle ($\frac{1}{2}a$) and a side ($\frac{1}{2}d$) are known. By plane trigonometry, $\cot \frac{1}{2}a = \frac{D}{\frac{1}{2}d}$ or $D = (\frac{1}{2}d)(\cot \frac{1}{2}a)$, but the subtense bar is 2 meters long which makes $\frac{1}{2}d = 1$ meter, and the formula becomes $D = \cot \frac{1}{2}a$ in meters. This formula is useful in emergencies, but very accurate tables are required for the cotangent functions.

*c.* The best method for solving subtense distances is by using tables XLII and XLIII in TM 5–236, which have the subtended angle ($a$) as the argument and give the distance immediately in meters (table XLII) or feet (table XLIII). The tables are valid only with a 2-meter subtense bar. The whole subtended angle is used as the argument in the tables.

*d.* It is possible to obtain third-order accuracy by the subtense method by using the proper equipment and careful observation.

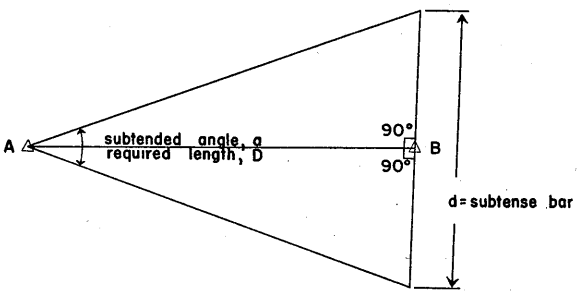



*Figure 61. Sketch, subtense measurement.*

# Section III. MEASUREMENTS USING LIGHT WAVES

## 47. Geodimeter-Model 2

*a.* The Model 2 series Geodimeter measures distances indirectly by measuring the time required for a light beam to pass from the Geodimeter to a reflector and back to the Geodimeter. The Model 2 Geodimeter is primarily a first order baseline instrument. Its size and weight limits its use to drive stations and makes its use on lower order surveys impractical.

*b.* According to the manufacturer, the Model 2A Geodimeter will determine distances up to 30 miles with an instrumental error of 1 cm ± 1 ppm (part per million). The approximate distance to be measured must be known within 1,000 meters.

*c.* As with all of the electronic distance measuring equipment, the atmospheric unknowns will probably introduce more error into the lines measured than the inherent accuracy of the instrument. To reduce these unknown conditions to a minimum, only calibrated thermometers, psychrometers, and altimeters should be used.

## 48. Measurement Reduction

For the purpose of explaining the computation of the length of a line, it is assumed a Model 2–A Geodimeter was used. A calibration was previously performed (fig. 62). The field observa-

CALIBRATION CONSTANTS
GEODIMETER NO.141
Calibrated by Army Map Service, 9 August 1961

| | |
|---|---|
| Zero Correction (Z) | 1.1526 M |
| Light Conductor Length (L) | .7960 M |
| Transmitter Mirror Lens Focus | 7.0 mm |
| Receiver Mirror Lens Focus | 7.8 mm |
| Photocell Wave Length | 5550 A° |
| Refractive Index (0°C & 760 mm Hg) | 1.0003042 |
| Calibration Temperature | 27.0°C |
| Unit Length, $F_1$ @ 0°C & 760 mm Hg | 7.4925333 M |
| Unit Length, $F_2$ @ 0°C & 760 mm Hg | 7.4553801 M |
| Unit Length, $F_3$ @ 0°C & 760 mm Hg | 7.2746155 M |
| Frequency, $F_1$ | 10,000,000 CPS |
| Frequency, $F_2$ | 10,049,834 CPS |
| Frequency, $F_3$ | 10,299,559 CPS |
| Prism Eccentricity Correction (19 prism bank) | -.0480 M |
| Prism Eccentricity Correction (54 prism bank) | -.0450 M |
| Velocity of Light in Vacuum | 299,792.5 KMS |
| Coefficient of Expansion of Aluminum | 0.000022 M/°C |

*Figure 62. Calibration constants, geodimeter model 2.*

tions have been recorded on DA Form 2852 (Geodimeter (Model 2) Observations) (fig. 63), including the Mirror Constant, Approximate Length of Line, Transmitter and Receiver Focus, and Eccentricity of the Mirror and the Geodimeter.

*a. Determination of the Fine Delay (LC).* For each frequency, the electrical length is reduced to a physical length. A space near the bottom of the observation sheet has been provided for this computation. The formula may be written:

$$LC = LC_1 - \left(\frac{LC_{1m} - M_m}{LC_{1m} - LC_{2m}}\right)(LC_1 - LC_2)$$

Where: $LC$ = Fine Delay in Meters
$LC_1$ = First Light Conductor Setting
$LC_2$ = Second Light Conductor Setting
$LC_{1m}$ = Mean of First Light Conductor Readings
$LC_{2m}$ = Mean of Second Light Conductor Readings
$M_m$ = Mean of the two Mean Mirror Readings

Watch the signs carefully. Obviously if the $LC_2$ setting is larger than the $LC_1$ setting, the resulting LC will be larger than $LC_1$.

*b. Light Conductor, Coarse (nL).* On the observation form, under the column headed LC, are numbers 7–68. These numbers indicate that seven light conductors were used or more accurately six light conductors and a portion of another. Since the exact length of the light conductors is shown on the calibration sheet, multiply the number of whole light conductors by this length ($6 \times 0.7960 = 4.7760$) to obtain the number of meters. This value is entered on DA Form 2853 (Geodimeter (Model 2) Computations) (fig. 64).

**GEODIMETER (Model 2) OBSERVATIONS** (TM 5-237)

| | | | |
|---|---|---|---|
| PROJECT | GEODIMETER NO. 141 | OBSERVATION NO. 1 | |
| LOCATION CALIFORNIA | MIRROR CONSTANT −0.0480 | BASELINE AMBATO | APPROXIMATE LENGTH 10 Km. |
| ORGANIZATION USAMS | DATE 21 SEPT. 1961 | TRANSMITTER FOCUS 0.9 mm | RECEIVER FOCUS 1.2 mm / RECORDER HOFFDAHL |
| GEODIMETER STATION CANAL, AMS 1961 | GEODIMETER HEIGHT 0.705 m | ELEVATION 56.030 m | GEODIMETER ECC 0.010 m (INSIDE) / OBSERVER WITTER |
| MIRROR STATION JUNCTION, USC & GS 1934 | MIRROR HEIGHT 5.608 m | ELEVATION 42.572 m | MIRROR ECC − / MIRROR TENDER MURPHY |

| FREQ | DELAY | MIRROR | LC | MIRROR | LC |
|---|---|---|---|---|---|
| 1 | 4 | | 7-68 | | 7-64 |
| SIGN | | + | − | + | − |
| 1 | | 26 5 | 26 0 | 25 0 | 24 2 |
| 2 | | 29 7 | 29 1 | 29 0 | 27 4 |
| 3 | | 29 7 | 32 0 | 29 0 | 29 7 |
| 4 | | 23 2 | 21 0 | 22 7 | 19 0 |
| SUM | | 109 1 | 108 1 | 105 7 | 100 3 |
| MEAN | | 27 28 | 27 02 | 26 42 | 25 08 |
| | | GEOD | MIRROR | GEOD | MIRROR |
| TIME | | 2037 | 2035 | 2046 | 2045 |
| TEMP | | 22.4 °C | 22.2 °C | 22.2 °C | 22.0 °C |
| PRESS | | 100.0 m | 90.0 m | 99.0 m | 89.8 m |
| HUM W | | 58.5 °F | 57.5 °F | 58.0 °F | 57.5 °F |
| HUM D | | 73.0 °F | 72.0 °F | 72.5 °F | 72.0 °F |

LC = .6800 − (27.02 − 26.85) / (27.02 − 25.08) × .04
= .6765

| | TEMP | PRESS | W | D |
|---|---|---|---|---|
| MEAN | 22.20 °C | 751.51 * | 57.88 °F | 72.38 °F |

| FREQ | DELAY | MIRROR | LC | MIRROR | LC |
|---|---|---|---|---|---|
| 2 | 5 | | 7-72 | | 7-68 |
| SIGN | | − | − | − | − |
| 1 | | 28 0 | 22 1 | 28 0 | 21 0 |
| 2 | | 20 2 | 26 5 | 20 2 | 24 9 |
| 3 | | 22 2 | 28 4 | 22 3 | 27 0 |
| 4 | | 22 7 | 17 6 | 23 0 | 15 5 |
| SUM | | 93 1 | 94 6 | 93 5 | 88 4 |
| MEAN | | 23 28 | 23 65 | 23 38 | 22 10 |
| | | GEOD | MIRROR | GEOD | MIRROR |
| TIME | | 2057 | 2055 | 2104 | 2103 |
| TEMP | | 22.0 °C | 21.8 °C | 21.9 °C | 21.8 °C |
| PRESS | | 98.0 m | 88.0 m | 98.0 m | 87.0 m |
| HUM W | | 58.0 °F | 57.5 °F | 57.5 °F | 57.0 °F |
| HUM D | | 72.0 °F | 71.5 °F | 71.5 °F | 71.5 °F |

LC = .7200 − (23.65 − 23.33) / (23.65 − 22.10) × .04
= .7117

| | TEMP | PRESS | W | D |
|---|---|---|---|---|
| MEAN | 21.88 °C | 751.68 * | 57.50 °F | 71.62 °F |

| FREQ | DELAY | MIRROR | LC | MIRROR | LC |
|---|---|---|---|---|---|
| 3 | 4 | | 8-54 | | 8-50 |
| SIGN | | + | − | + | − |
| 1 | | 78 1 | 78 8 | 78 2 | 75 9 |
| 2 | | 86 8 | 86 9 | 85 8 | 84 1 |
| 3 | | 85 8 | 89 8 | 86 0 | 87 8 |
| 4 | | 73 7 | 69 5 | 73 9 | 67 4 |
| SUM | | 324 4 | 325 0 | 323 9 | 315 2 |
| MEAN | | 81 10 | 81 25 | 80 98 | 78 80 |
| | | GEOD | MIRROR | GEOD | MIRROR |
| TIME | | 2112 | 2111 | 2120 | 2120 |
| TEMP | | 21.8 °C | 21.6 °C | 21.7 °C | 21.4 °C |
| PRESS | | 97.0 m | 88.0 m | 96.0 m | 87.0 m |
| HUM W | | 57.5 °F | 57.0 °F | 57.5 °F | 57.0 °F |
| HUM D | | 71.5 °F | 71.0 °F | 71.0 °F | 71.0 °F |

LC = .5400 − (81.25 − 81.04) / (81.25 − 78.80) × .04
= .5366

| | TEMP | PRESS | W | D |
|---|---|---|---|---|
| MEAN | 21.62 °C | 751.75 * | 57.25 °F | 71.12 °F |

NOTE: Give units for temperature and pressure; give pressure reduced for inst constant (300m, 1000 ft)

REMARKS 19 PRISM BANK USED

* PRESSURE UNITS ARE MM Hg

DA FORM 2852, 1 OCT 64

*Figure 63. Geodimeter (model 2) observations (DA Form 2852).*

| PROJECT | OBSERVATION NO. | GEODIMETER (MODEL 2) COMPUTATIONS (TM 5-237) | |
|---|---|---|---|
| | 1 | | |
| LOCATION | GEODIMETER NO. | DATE | BASELINE |
| CALIFORNIA | 141 | 21 Sept. '61 | AMBATO |
| ORGANIZATION | OBSERVER | GEODIMETER STATION | MIRROR STATION |
| USAMS | WITTER | CANAL AMS 1961 | JUNCTION USC&GS 1934 |

| DETERMINATION OF PARTIAL UNIT LENGTH | | | |
|---|---|---|---|
| LIGHT CONDUCTOR, COARSE (nL) | 4.7760 | 4.7760 | 5.5720 |
| LIGHT CONDUCTOR, FINE (Lc) | .6765 | .7117 | .5366 |
| ZERO CORRECTION (Z) | 1.1526 | 1.1526 | 1.1526 |
| ECCENTRICITY, GEODIMETER | + .0100 | + .0100 | + .0100 |
| ECCENTRICITY, MIRROR | — | — | — |
| CONSTANT, MIRROR | - .0480 | - .0480 | - .0480 |
| FOCUS CORRECTIONS | + .0127 | + .0127 | + .0127 |
| TEMP CORR., LIGHT CONDUCTOR | - .0005 | - .0005 | - .0007 |
| SUM OF ABOVE (K) | $K_1$ 6.5793 | $K_2$ 6.6145 | $K_3$ 7.2352 |
| **DETERMINATION OF QUARTER WAVE LENGTH** | | | |
| TEMPERATURE (t) (°C) | 22.20 | 21.88 | 21.62 |
| PRESSURE (P) (mm Hg.) | 751.51 | 751.68 | 751.75 |
| HUMIDITY (%) | 40 | 42 | 42 |
| $N'_a$ | 1.00027821 | 1.00027858 | 1.00027885 |
| HUMIDITY CORRECTION | - .00000041 | - .00000042 | - .00000041 |
| REFRACTIVE INDEX ($N_a$) | 1.00027780 | 1.00027816 | 1.00027844 |
| FREQUENCY (f) cps | $f_1$ 10 000 000 | $f_2$ 10 049 834 | $f_3$ 10 299 559 |
| QUARTER WAVE LENGTH (U) | $U_1$ 7.492731019 | $U_2$ 7.455574213 | $U_3$ 7.274802954 |

| DETERMINATION OF NUMBER OF UNITS | | | | | |
|---|---|---|---|---|---|
| PHASE SIGN, $f_1$ | — | PHASE SIGN, $f_2$ | + | PHASE SIGN, $f_3$ | — |
| APPROX LENGTH ($L_A$) | 10 000 m. | $L_A - K_2$ | | 9993.3855 | m. |
| $L_A - K_1$ | 9993.4207 m. | $N'_2 = L_A - K_2 / U_2$ | | 1340 4/ | |
| $N'_1 = L_A - K_1 / U_1$ | 1333 7/ | $L'_2 = (N'_2 \times U_2) + K_2$ | | 9997.0847 | m. |
| $L'_1 = (N'_1 \times U_1) + K_1$ | 9994.3898 m. | $U_1 - U_2$ | | 0.037156283 | m. |
| $L'_2 - L'_1$ | 2.6949 | $\Delta N = L'_2 - L'_1 / U_1 - U_2$ | | 72 5/ | |
| $N_1 = N'_1 + \Delta N$ | 1405 | $N_2 = N'_2 + \Delta N$ | 1412 | $N_3 = N_1(1.03)$ | 1447 |

| DETERMINATION OF SLOPE LENGTH | | | |
|---|---|---|---|
| UNIT LENGTH (U×N) | 10527.2871 | 10527.2708 | 10526.6399 |
| SLOPE LENGTH (U×N)+K | 10533.8664 | 10533.8853 | 10533.8751 |

$$N_g = 1 + \left( 2876.4 + 3\left(\frac{16.288}{\lambda^2}\right) + 5\left(\frac{0.136}{\lambda^4}\right) \right) 10^{-7} \qquad N'_a = 1 + \left(\frac{N_g - 1}{1 + t/273}\right) \frac{P}{760}$$

$\lambda$ = Wave length of light in microns. Hum. Corr. $= \frac{5.5 \times 10^{-8} e}{1 + t/273}$ $\qquad U = \frac{299792500}{4\,(f)\,(N_a)}$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
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DA FORM 2853, 1 OCT 64

*Figure 64. Geodimeter (model 2) computations (DA Form 2853).*

*c. Light Conductor, Fine (LC).* See *a* above, for determination of this value.

*d. Zero Correction.* This value is sometimes called the calibration constant and is determined during calibration. The value is the distance from the back centering point of the Geodimeter to the electrical center of the Geodimeter and can only be determined by calibration procedure. This value will change for each instrument and will change if any major components are replaced or repaired.

*e. Mirror Constant.* The reflex used with the Geodimeter must have its constant computed. This value is furnished on the sheet of Calibration constants or may be computed. Since the refractive index of glass is about 1.57 as compared to air, the thickness of the prisms from the front to the apex at the back must be multiplied by 0.57 to get the Mirror constant. The correction is negative as the light travels farther when going through the glass due to the index of refraction being 1.57 for glass as compared with 1.00 for air.

*f. Focus Corrections.* The focus correction is the algebraic sum of the Transmitter and Receiver Focus as calibrated, minus the sum of the Transmitter and Receiver Focus as determined during observations. Watch the sign. The formula may be written:

$$FC=(Tf_c+Rf_c)-(Tf_o+Rf_o)$$

Where: FC=Focus Correction
$Tf_c$=Transmitter Focus Calibrated Value
$Rf_c$=Receiver Focus Calibrated Value
$Tf_o$=Transmitter Focus Observed Value
$Rf_o$=Receiver Focus Observed Value

*g. Temperature Correction of Light Conductor* The difference in observation temperature and the calibration temperature will cause an expansion or contraction of the light conductor tubes. This correction can be determined by the formula:

$$nL(0.000022)(t_o-t_c)$$

Where:

$nL$=Light Conductor, Course (*b* above)
0.000022=Coefficient of Expansion of Aluminum (meters per degree Celsius)
$t_o$=Observation Temperature (Celsius)
$t_c$=Calibration Temperature (Celsius)
$(t_o-t_c)$ determines the sign of the correction.

*h. Temperature (t) (°C.).* The mean temperature (°C.) of the air for each frequency is brought forward from the observation sheet. If temperature is in °F., convert to °C. using table XV, appendix III.

*i. Pressure (P) (mm Hg).* The mean pressure is brought forward from the observation sheet and if necessary converted from altimeter readings to millimeters of mercury. Conversion tables are furnished in table XVI, appendix III, or computed from the following formula:

$$\frac{P_z}{P_o}=\left(\frac{288-0.0065h}{288}\right)^{5.256}$$

Where: $P_z$=Barometric Pressure
$P_o$=Barometric Pressure at sea level (760 mm Hg)
$h$=Altitude (Meters)

*j. Relative Humidity.* The relative humidity may be determined from tables supplied with the psychrometer or altimeter or by a Humidity Slide Rule (Short and Mason) using the wet and dry bulb temperatures as arguments.

*k. Refractive index ($N_a$).* Before proceeding further it is necessary to compute the refractive index so that the length of a quarter wavelength at each frequency can be determined. The formulas are:

$$N_a=1+\left(\frac{N_g-1}{1+\frac{t}{273.2}}\right)\left(\frac{P}{760}\right)-\frac{5.5\times10^{-8}e}{1+\frac{t}{273.2}}$$

and:

$$N_g=1+\left[2876.4+3\left(\frac{16.288}{\lambda^2}\right)+5\left(\frac{0.136}{\lambda^4}\right)\right]10^{-7}$$

Where: $N_a$=Refractive Index
$N_g$=Refractive Index for group velocity
$t$=Temperature in degrees Celsius
P=Pressure in mm Hg
$e$=Humidity in mm Hg
$\lambda$=Wavelength of light in microns ($10^{-6}$m)

(1) $N_g$ values are furnished in table XVII, appendix III, for a series of values of $\lambda$. The photocell wavelength ($\lambda$) must be furnished on the calibration data and will be different for each instrument and/or each photocell.

(2) $N'_a$ is determined from the first portion of the $N_a$ formula

$$N'_a = 1 + \left(\frac{N_g - 1}{1 + \frac{t}{273.2}}\right)\left(\frac{P}{760}\right)$$

$N'_a$ is the refractive index without the humidity correction.

(3) Humidity Correction—The humidity correction is taken from the correction for humidity graph (chart 4, app. II) using Temperature Celsius and Relative Humidity as the arguments. This correction is subtracted from $N'_a$ to produce the refractive index ($N_a$).

(4) Frequency ($f$)—The frequencies for $f_1$, $f_2$, and $f_3$ are determined during calibration and are given in cycles per second. These values are assumed to remain constant until a new calibration is performed.

(5) Quarter Wavelength (U)—The quarter wavelength is determined by the following formula:

$$U = \frac{299{,}792{,}500}{4(f)(N_a)}$$

Where: 299,792,500 ± 400 meters per second is the velocity of light in a vacuum as adopted by the International Union of Geodesy and Geophysics and the International Scientific Radio Union at their international meeting at Toronto, Canada in 1957.

*l. Determination of Number of Units.* With the value for a quarter wavelength at each frequency of modulation corrected for the temperature, pressure, and humidity at the time of observation, the number of whole quarter-wavelengths at each frequency in the line being measured can be determined.

(1) From the observation sheet determine the phase signs from the four signs of each frequency at the top of the page. If the four signs for the frequency are alike, the phase sign is positive; if they are unlike, the sign is negative.

(2) From the observation sheet, enter the approximate distance on the computation form. This length must be within 1,000 meters of the correct length or an incorrect result will be obtained. If the approximate distance cannot be determined by any other method, a short base with all angles turned should be observed by the field party to provide data for an approximate length.

(3) The sum of the corrections for frequency 1 and frequency 2 ($K_1$ and $K_2$) are subtracted from the approximate length.

$$L_A - K_1 \text{ and } L_A - K_2$$

(4) $N'_1$ and $N'_2$ are determined from the formula:

$$N'_1 = \frac{L_A - K_1}{U_1} \text{ and } N'_2 = \frac{L_A - K_2}{U_2}$$

$N'_1$ and $N'_2$ are rounded to agree with the phase sign of each frequency. For example, the phase sign of $f_1$ is negative and $N'_1$ computes out as 1333.7. According to the phase sign, $N'_1$ must be an odd number, therefore 1333 is entered on the form.

(5) $L'_1$ and $L'_2$ are determined from the formulas:

$$L'_1 = (N'_1 \times U_1) + K_1 \text{ and } L'_2 = (N'_2 \times U_2) + K_2$$

and then $L'_1$ is subtracted from $L'_2$. If $L'_1$ is larger than $L'_2$ the resulting value will be negative.

(6) $U_2$ is subtracted from $U_1$ and divided into $L'_2 - L'_1$ to produce $\Delta N$ which is the value to apply to $N'_1$ to obtain the total number of quarter-wavelengths.

(7) The resulting $N_1$ and $N_2$ should agree with the phase sign for each frequency:

Phase sign −, N is an odd number
Phase sign +, N is an even number

A relationship exists between the N values and is 100, 100.5 and 103. $N_3$ is determined by multiplying $N_1$ by 1.03.

*m. Determination of Slope Length.* The unit length is determined by multiplying the corrected quarter wavelength (U) for each frequency by the N value for that frequency. The quarter wavelength has been corrected for temperature, pressure, and humidity and will change for each frequency. The slope length is determined by adding the internal corrections (K) for each frequency to the unit length (U×N) for each frequency.

*n. Electronic Distance Measurement Summary.*

(1) The slope lengths for a series of measurements over a line are entered on DA Form 2854 (Electronic Distance Measurement Summary) (fig. 65). This form is provided for listing of the measurements, determination of statistics, and reduction to a geodetic distance on the reference spheroid. After listing the measurements, rejection of doubtful observations should be made using rejection limits determined by Chauvenet's Formula (table XI). The arithmetic mean distance is determined and residuals ($v$'s) computed by subtracting the observed distance from the mean distance. The formulas for computing the probable errors are given on the form. The ratio is the mean distance divided by the probable error.

(2) The observed slope distance must be reduced to a geodetic distance on the reference spheroid. Care should be used when abstracting the elevations of the instruments as observations are made over several days and the height of the instrument will vary.

*o. Horizontal Distance.* The horizontal distance (H) is computed using the Pythagorean theorem:

$$H=\sqrt{(\text{SLOPE DISTANCE})^2-(\text{DIFFERENCE IN ELEVATION})^2}$$

(1) The difference in elevation ($d$) is obtained from the following formula:

$$d=(h_a+HI_a)-(h_b+HI_b)$$

Where: $h_a$=Elevation of occupied station
$h_b$=Elevation of distant station
$HI_a$=Height of instrument at occupied station
$HI_b$=Height of instrument at distant station

(2) The elevations of the stations are normally determined by either differential leveling or trigonometric leveling. When computing the elevations by trigonometric leveling, the reduction to line formula is:

$$\text{Red.}=\frac{(t-o)(206265)\tan\zeta}{H}$$

when H is a horizontal distance and:

$$\text{Red.}=\frac{(t-o)(206265)\sin\zeta}{T}$$

when T is a slope distance and $\zeta$ is the observed zenith distance. When using slope lengths, the formulas for $h_2-h_1$ are as follows:

$h_2-h_1=T\sin\frac{1}{2}(\zeta_2-\zeta_1)$,
for reciprocal observations

$h_2-h_1=T\sin(90°-\zeta_1+k)$,
for nonreciprocal observations

(3) If nonreciprocal observations are used, a value for (0.5−M) or 0.429 should be used in the computations.

(4) The use of altimeter elevations to determine differences of elevation is permissible if more accurate methods of determination are not required. The altimeters must be carefully calibrated both relatively and absolutely, and fairly stable air conditions must exist between the stations.

*p. Chord-Arc Correction.* The Chord-Arc Correction (K) is applied to the horizontal distance to change it from a chord distance to an arc distance on the surface of the spheroid. This correction is computed using the formula:

$$K=\frac{H^3}{24\rho^2}$$

or by the approximate formula:

$$K=1.027H^3\times10^{-15}$$

Where: H=Horizontal distance
$\rho$=Mean radius of curvature from table XX (app. III), using azimuth of the line and mean latitude as the arguments.

## 49. Geodimeter Model 4

*a. Method of Use.* The Model 4 Geodimeter measures distances indirectly by measuring the time required for a light beam to pass from the Geodimeter to a reflector and back to the Geodimeter. The approximate distance must be known to within 1,000 meters. According to the

| PROJECT | ELECTRONIC DISTANCE MEASUREMENT SUMMARY (TM 5-237) | |
|---|---|---|
| YUMA - YUCCA TRAVERSE | | |
| LOCATION: ARIZONA | ORGANIZATION: USAMS | DATE: FEB. 1964 |
| TYPE OF LINE MEASURED: ☐ BASE ☐ TRAVERSE ☐ TRILATERATION ☐ TRIANGULATION | | BASELINE |

| MASTER STATION | INST. (Type and No.) | H. I. | STATION ELEV. | INST. ELEV. | OBSERVER |
|---|---|---|---|---|---|
| CANAL AMS 1961 | 141 | 0.705 M. | 56.030 | 56.735 | WITTER |
| **REMOTE STATION** | **INST. (Type and No.)** | **H. I.** | **STATION ELEV.** | **INST. ELEV.** | **OBSERVER** |
| JUNCTION USC&GS 1934 | 19 BANK PRISM | 5.608 M. | 42.572 | 48.180 | MURPHY |

| OBS. NO. | (Slope) ~~(Geodetic)~~ DISTANCE METERS | RESIDUAL (V) |
|---|---|---|
| 1 | 10533.8664 | + .0111 |
| 2 | .8853 | - .0078 |
| 3 | .8751 | + .0024 |
| 4 | .8832 | - .0057 |
| 5 | .8786 | - .0011 |
| 6 | .8681 | + .0094 |
| 7 | .8751 | + .0024 |
| 8 | .8786 | - .0011 |
| 9 | .8772 | + .0003 |
| 10 | .8780 | - .0005 |
| 11 | .8703 | + .0072 |
| 12 | .8695 | + .0080 |
| 13 | .8825 | - .0050 |
| 14 | .8823 | - .0048 |
| 15 | .8782 | - .0007 |
| 16 | .8822 | - .0047 |
| 17 | .8817 | - .0042 |
| 18 | .8827 | - .0052 |
| 19 | | |
| 20 | | |
| 21 | | |
| 22 | | |
| 23 | | |
| 24 | | |
| MEAN DISTANCE | 10533.8775 | $\Sigma V^2$ .00055032 |

| PE(OBS) | PE (Mean) | RATIO |
|---|---|---|
| ± 3.84 | ± 0.91 | 1: 11,600,000 |

$PE(OBS) = \pm 0.6745\sqrt{\Sigma V^2/(n-1)}$ n = NO. OF OBSERVATIONS

$PE(Mean) = \pm 0.6745\sqrt{\Sigma V^2/n(n-1)}$ $\Sigma V^2$ = SUM OF RESIDUALS SQUARED

| DISTANCE REDUCTION | |
|---|---|
| DIFF. OF ELEVATION (d) | 8.555 ~~FEET~~ METERS |
| MEAN ELEVATION (h) | 52.458 ~~FEET~~ METERS |
| AZ. OF LINE (α) | 90° 00' 00" |
| MEAN LATITUDE ($\Phi_m$) | 33° 00' 00" |
| MEAN RADIUS OF CURVATURE (ρ) | 6384619 METERS |
| SLOPE DISTANCE (T) | 10533.8775 METERS |
| HORIZONTAL DISTANCE (H) | - 0.0035 |
| ECCENTRICITY CORRECTION* | — |
| CHORD-ARC CORRECTION (K) | + 0.0012 |
| SEA LEVEL REDUCTION (C) | - 0.0865 |
| GEODETIC DISTANCE (S) | 10533.7887 METERS |
| REFERENCE SPHEROID | CLARKE 1866 |

$K = \frac{H^3}{24\rho^2} \cdots$ or $K = 1.027H^3 \times 10^{-15}$

$H = \sqrt{T^2 - d^2}$ or $H = T - \frac{d^2}{2T} - \frac{d^4}{8T^3} \cdots$

$C = -H\frac{h}{\rho} + H\frac{h^2}{\rho^2}$ $S = H + K + C$

*Apply Eccentricity to H before computing C

REMARKS

Observations 1 thru 9 taken on 21 Sept. 1961.

Observations 10 thru 18 taken on 22 Sept. 1961.

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|---|---|---|---|
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DA FORM 2854, 1 OCT 64

*Figure 65. Electronic Distance Measurement Summary (DA Form 2854).*

manufacturer, the Model 4 Geodimeter will determine distances within 1 cm. ±5 ppm (parts per million). The Model 4B is listed as having a night range up to 3,000 meters, but it has been used successfully on 8,000 meter lengths under ideal conditions. The Model 4D employs a mercury vapor lamp and the manufacturer claims a night range of 25 miles and a daylight range of 3 miles.

(1) A quarter wavelength for any Geodimeter may be determined from the following formula:

$$\frac{\lambda}{4}=\frac{C}{4(F)(N_a)}$$

Where: $\frac{\lambda}{4}$=Quarter wavelength

C=Velocity of light
F=Modulating frequency
$N_a$=Index of refraction

(2) The Model 4 Geodimeter has been designed to have a quarter wavelength of 2.5 meters for Frequency 1, at a temperature of −6 degrees Celsius and a pressure of 760 mm of mercury. If a color sensitivity of 5,650 angstroms is adopted for the photomultiplier tube, the refractive index for light waves at this temperature and pressure is 1.0003104. These values were used by the manufacturer to produce the tables furnished with the instrument, but the manufacturer used a value of 299,792,900 meters per second for the speed of light. The velocity of 299,792,500 meters per second has been adopted internationally.

(3) In order to use the velocity of light of 299,792,500 meters per second with the tables furnished with the Model 4 Geodimeter, it is necessary to change the modulating frequencies from F 1=29,970,000 cycles per second; F 2=30,044,920 c/sec, F 3=30,468,500 c/sec as furnished by the manufacturer to the following: F 1=29,969,947 c/sec, F 2=30,044,872 c/sec, F 3=30,468,445 c/sec. If the frequencies are not changed from the manufacturer's values, then the distance measured can be multiplied by 0.9999986657 to correct the computed distance for the velocity of light difference.

(4) When the color sensitivity of the photomultiplier tube is known, a new refractive index should be computed and applied as a refractive index deviation. Computation of the refractive index and the deviation is taken up later in this section.

*b. Geodimeter Reductions.* The Model 4 Geodimeter measurements are recorded on DA Form 2855 (Geodimeter (Model 4) Observations and Computations) (fig. 66) which also serves as a computation form for field and office use. For purposes of explaining the computation procedure it is assumed that the headings have been completely filled out including elevations, the approximate distance, and the calibration date. A sheet containing the most recent calibration (delay line) data must be furnished the computer. A sample sheet containing the calibration data is shown in figure 67.

*c. Determination of Meters From Delay Line Data.* Phase 1, 2, 3, and 4 of Frequencies 1, 2 and 3 with the sign of each are recorded in the field. The signs for $F_1$, $F_2$, and $F_3$ for the Reflex and Geodimeter are determined by the sign of the initial setting for Phase 1 of $F_1$, $F_2$, and $F_3$. These signs are determined by the direction the null indicator moves in relation to the movement of the delay line control. The four phase readings are meaned in each of the six columns. Using these mean values as the argument, the meters are interpolated from the calibration sheet and are entered on the form to the nearest millimeter. Subtract the meters in the Geodimeter column from the meters in the Reflex column for each frequency. The resulting value must be positive in each case. If a subtraction is impossible, add $U_1$, $U_2$, or $U_3$ to the reflex meters under $F_1$, $F_2$, and $F_3$ as needed. The signs to be entered in the Reflex-Geodimeter column are determined from the signs at the top of the six columns of readings and are paired for each frequency. For example, if the signs for Frequency 1 are both positive or both negative then the Reflex-Geodimeter sign is positive (even); if the signs are unlike, the Reflex-Geodimeter sign will be negative (odd). However, if $U_1$, $U_2$, or $U_3$ is added to the meters so a subtraction can be performed, then the sign of the reflex minus Geodimeter will change. The $L_1$, $L_2$, and $L_3$ are the Reflex-Geodimeter values. However, $L_2$ and $L_3$ must be larger than $L_1$, and therefore $U_2$ and/or $U_3$ are added if necessary and the signs changed again if they are used.

| PROJECT | GEODIMETER (Model 4) OBSERVATIONS AND COMPUTATIONS (TM 5-237) | | | |
|---|---|---|---|---|
| GIMRADA | | | | |
| DATE | APPROX. DISTANCE | METHOD OF DETERMINATION | OBSERVER | |
| 28 FEB. 1962 | 3.6 KM | SCALED | H. G. KING | |
| INSTRUMENT NO. | REFLECTOR TYPE | CALIBRATION DATE | λ (Angstroms) | RECORDER |
| 47 | 7 PRISM | 23 July 1961 | 5500 | J. M. WHITE |

| GEODIMETER STATION | H.I. | ELEVATION | INST. ELEV. | AZIMUTH OF LINE |
|---|---|---|---|---|
| HORSE, AMS 1962 | 1.38 M | 379.78 M | 381.16 M | 320° |
| **REFLECTOR STATION** | **H.I.** | **ELEVATION** | **INST. ELEV.** | **MEAN LATITUDE** |
| BASE "A", AMS 1962 | 1.56 M | 439.10 M | 440.66 M | 33° 00′ |

| $F_1$ | REFLEX | | GEODIMETER | | $F_2$ | REFLEX | | GEODIMETER | | $F_3$ | REFLEX | | GEODIMETER | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| PHASE | SIGN + | | SIGN + | | PHASE | SIGN + | | SIGN + | | PHASE | SIGN − | | SIGN + | |
| 1 | 197 | + | 53 | + | 1 | 92 | + | 54 | + | 1 | 235 | − | 55 | + |
| 2 | 195 | − | 52 | − | 2 | 88 | − | 53 | − | 2 | 237 | + | 53 | − |
| 3 | 192 | + | 51 | + | 3 | 84 | + | 52 | + | 3 | 237 | − | 53 | + |
| 4 | 191 | − | 49 | − | 4 | 81 | − | 52 | − | 4 | 239 | + | 50 | − |
| MEAN | 193.75 | | 51.25 | | MEAN | 86.25 | | 52.75 | | MEAN | 237.00 | | 52.75 | |
| METERS | 2.137 | | 0.629 | | METERS | 1.004 | | 0.647 | | METERS | 2.623 | | 0.439 | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| REFLEX - GEOD. (+ $U_1$ if required) | 1.508 M | E | REFLEX - GEOD. (+ $U_2$ if required) | 0.357 M | E | REFLEX - GEOD. (+ $U_3$ if required) | 2.184 M | O |
| $L_1$ | 1.508 M | E | $L_2$ (+ $U_2$ if req.) | 2.851 M | O | $L_3$ (+ $U_3$ if req.) | 2.184 M | O |
| $(L_2 - L_1) \times (4)$ | 5.372 | HUNDREDS OF METERS | $\frac{(L_3 - L_1)(21)}{100}$ = | .642 | | $N_1 = L_A \div U_1$ = | 1426 | |
| $(L_3 - L_1) \times (21)$ | 64.196 | APPROXIMATE TENS AND UNITS OF METERS | | | | $N_2 = L_A \div U_2$ = | 1429 | |
| ADD 50 METERS IF SIGNS OF $L_3$ AND $L_1$ ARE NOT THE SAME | | | | | | $N_3 = L_A \div U_3$ = | 1497 | |

| | | | | | |
|---|---|---|---|---|---|
| $L_A$ | 3565.000 M | $U_2 \times N_2$ | 3563.592 M | $U_3 \times N_3$ | 3564.285 M |
| $L_1$ | 1.508 M | $L_2$ | 2.851 M | $L_3$ | 2.184 M |
| $D_1$ | 3566.508 M | $D_2$ | 3566.443 M | $D_3$ | 3566.469 M |

| | | | METEOROLOGICAL READINGS | | | |
|---|---|---|---|---|---|---|
| | | | PRESSURE OR ALTIMETER | TEMPERATURE DRY | TEMPERATURE WET | HUMIDITY |
| $.99999866\left(\frac{D_1 + D_2 + D_3}{3}\right)$ = | 3566.469 | METERS | | | | |
| GEODIMETER CONSTANT | + 0.235 | METERS | | | | |
| REFLECTOR CONSTANT | − 0.030 | METERS | | | | |
| TEMP.-PRESSURE CORRECTION | | METERS | 28.78 | 05.0 | 02.2 | 60 % |
| HUMIDITY CORRECTION | + 0.001 | METERS | 28.78 | 04.4 | 02.2 | 68 % |
| REFRACTIVE INDEX CORR. (RC) | + 0.087 | METERS | 28.68 | 07.6 | 03.2 | 45 % |
| SLOPE DISTANCE | 3566.762 | METERS | 28.68 | 07.6 | 03.2 | 45 % |
| HORIZONTAL DIST. OR CORR. | −0.493 | METERS | MEAN 28.73 | MEAN 06.2 | MEAN 02.7 | MEAN 54 % |
| ECCENTRICITY CORRECTION | | METERS | FACTOR (From Nomogram) | 25.2 | × $10^{-6}$ | |
| CHORD-ARC CORRECTION | | METERS | ASSUMED REFRACTIVE INDEX | | 1.0003104 | |
| SEA-LEVEL REDUCTION | −0.229 | METERS | COMPUTED REFRACTIVEINDEX ($N_g'$) | | 1.0002859 | |
| GEODETIC DISTANCE | 3566.039 | METERS | REFRACTIVE INDEX DEVIATION (RD) | | .0000245 | |

$$N_g = 1 + \left[2876.4 + 3\left(\frac{16.288}{\lambda^2}\right) + 5\left(\frac{0.136}{\lambda^4}\right)\right] 10^{-7}$$

$$N_g' = 1 + \frac{0.359474\,(N_g - 1)\,P}{273.2 + t}$$

RC = D × RD

USE °C FOR TEMPERATURE AND MM/$H_g$ FOR PRESSURE.

*NOTE:*

*If the refractive index correction is used omit the temp-pressure correction.*

| COMPUTED BY | DATE | CHECKED BY | DATE | PAGE |
|---|---|---|---|---|
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DA FORM 2855, 1 OCT 64

*Figure 66. Geodimeter (Model 4) Observations and Computations (DA Form 2855).*

CALIBRATION TABLES FOR NASM-4-47 JULY 1961

| Nominal frequencies | | Unit lengths at -6 C/760 mm Hg | |
|---|---|---|---|
| F-1 | 29,970.000 kc/s | U-1 | 2.500.000 meters |
| F-2 | 30,044.920 kc/s | U-2 | 2.493.766 meters |
| F-3 | 31,468.500 kc/s | U-3 | 2.380 952 meters |
| Inst. Constant | = | 0.235 m | |
| Mirror Constant | | 3 & 7 Prism = -0.030 m. | |
| | | Plastic Refl. = +0.004 m. | |

DELAY LINE DATA

| F-1 | | | F-2 | | | F-3 | | |
|---|---|---|---|---|---|---|---|---|
| Div. | | diff. | Div. | | diff. | Div. | | diff. |
| 0 | - | | 0 | - | | 0 | - | |
| 10 | 0.102 | 102 | 10 | 0.103 | 103 | 10 | 0.082 | 082 |
| 20 | 0.206 | 104 | 20 | 0.208 | 105 | 20 | 0.164 | 082 |
| 30 | 0.348 | 142 | 30 | 0.350 | 142 | 30 | 0.250 | 086 |
| 40 | 0.490 | 142 | 40 | 0.488 | 138 | 40 | 0.340 | 090 |
| 50 | 0.615 | 125 | 50 | 0.616 | 128 | 50 | 0.418 | 078 |
| 60 | 0.730 | 115 | 60 | 0.730 | 114 | 60 | 0.494 | 076 |
| 70 | 0.843 | 113 | 70 | 0.841 | 111 | 70 | 0.570 | 076 |
| 80 | 0.951 | 108 | 80 | 0.943 | 102 | 80 | 0.654 | 084 |
| 90 | 1.048 | 097 | 90 | 1.041 | 098 | 90 | 0.742 | 088 |
| 100 | 1.142 | 094 | 100 | 1.134 | 093 | 100 | 0.840 | 098 |
| 110 | 1.236 | 094 | 110 | 1.230 | 096 | 110 | 0.958 | 118 |
| 120 | 1.332 | 096 | 120 | 1.332 | 102 | 120 | 1.094 | 136 |
| 130 | 1.430 | 098 | 130 | 1.432 | 100 | 130 | 1.248 | 154 |
| 140 | 1.530 | 100 | 140 | 1.532 | 100 | 140 | 1.408 | 160 |
| 150 | 1.632 | 102 | 150 | 1.635 | 103 | 150 | 1.570 | 162 |
| 160 | 1.737 | 105 | 160 | 1.745 | 110 | 160 | 1.730 | 160 |
| 170 | 1.848 | 111 | 170 | 1.860 | 115 | 170 | 1.888 | 158 |
| 180 | 1.964 | 116 | 180 | 1.977 | 117 | 180 | 2.035 | 147 |
| 190 | 2.088 | 124 | 190 | 2.108 | 131 | 190 | 2.163 | 128 |
| 200 | 2.218 | 130 | 200 | 2.234 | 126 | 200 | 2.279 | 116 |
| 210 | 2.346 | 128 | 210 | 2.360 | 126 | 210 | 2.388 | 109 |
| 220 | 2.474 | 128 | 220 | 2.486 | 126 | 220 | 2.480 | 092 |
| 230 | 2.602 | 128 | 230 | 2.611 | 125 | 230 | 2.568 | 088 |
| 240 | 2.734 | 132 | 240 | 2.734 | 123 | 240 | 2.647 | 079 |
| 250 | 2.862 | 128 | 250 | 2.854 | 120 | 250 | 2.717 | 070 |
| 260 | 2.985 | 123 | 260 | 2.976 | 122 | 260 | 2.788 | 071 |
| 270 | 3.108 | 123 | 270 | 3.102 | 126 | 270 | 2.864 | 076 |
| 280 | 3.228 | 120 | 270 | 3.218 | 116 | 280 | 2.942 | 078 |
| 290 | 3.334 | 106 | 290 | 3.318 | 100 | 290 | 3.030 | 088 |
| 300 | 3.440 | 106 | 300 | 3.418 | 100 | 300 | 3.134 | 104 |

*Figure 67. Calibration data, Model 4 Geodimeter.*

*d. Determination of a Length for Frequency 1.* Thousands of meters are determined from the approximate distance. Multiply 4 times $(L_2-L_1)$ to determine the hundreds of meters. $\frac{(L_3-L_1)(21)}{100}$ should approximately equal the decimal part of $(L_2-L_1)\times 4$ and is used as a check only. In the example (fig. 66), we have 3,500 meters at this point (approx. distance 3.6 km and $L_2-L_1$ times $4=5.372$). To determine the approximate tens and units of meters:

$$(L_3-L_1)(21)$$

If the *final* signs of $F_1$ and $F_3$ are unlike, add 50 meters to $(L_3-L_1)(21)$. If the final signs of $F_1$ and $F_3$ are alike, the value of $(L_3-L_1)(21)$ is entered in the column. To determine the correct tens and units of meters, the *final* sign of $L_1$ is used. If the *final* sign of $L_1$ is even, the tens and units of meters are rounded to the nearest multiple of 2.5 meters ending in 0 or 5. If the *final* sign of $L_1$ is odd, the tens and units of meters are rounded to the nearest multiple of 2.5 meters ending in 2.5 or 7.5 meters. In the example, $(L_3-L_1)(21)=14.196$; however, $L_3$ is odd and $L_1$ is even so 50 meters must be added making the result 64.196 meters. The sign of $L_1$ is even and 64.196 is rounded to 65 meters. The resulting length ($L_A$) in the example is 3565.000 meters for Frequency 1.

*e. Determination of D.*

(1) The number of unit lengths (N) for each frequency may be determined by the following formula:

$$N_1=L_A\div U_1$$

$$N_2=L_A\div U_2$$

$$N_3=L_A\div U_3$$

or

$$N_1=L_A(0.4)$$

$N_2=N_1+1$ for each 1,000 meters

$$N_3=N_1+\frac{N_1(0.1)}{2}$$

$N_1$, $N_2$, and $N_3$ must be rounded to agree with the final sign condition of $L_1$, $L_2$, and $L_3$.

(2) $N_2$ and $N_3$ are multiplied by $U_2$ and $U_3$ respectively and $L_1$, $L_2$, and $L_3$, which were determined previously, are re-entered on the form and added to produce three uncorrected distances. These three lengths should agree very closely with each other if all frequencies were operating correctly and there is not an error in the computations. For ordinary use when only one or two sets of measurements are taken (one set is one length for each frequency), the three lengths can be meaned. However, if a number of sets of measurements are made, it may be desirable to consider each length measurement separately.

(3) At this point in the computations, the mean of the uncorrected distances is multiplied by 0.9999986657 to correct for the velocity of light. This was discussed in *a* above.

*f. Geodimeter Constant.* The Geodimeter constant is furnished by the manufacturer and will be different for each instrument. This constant is the distance between the centering point and the electrical center of the Geodimeter. This value would normally be checked or redetermined during calibration of the instrument and should be furnished with the calibration data.

*g. Reflector Constant.* The reflector constant varies with the type of reflector used. The constant for each reflector is stamped on the reflector housing. The three reflectors most commonly used are as follows:

3 Prism
7 Prism
Plastic Reflector

*h. Temperature-Pressure Correction.* The nomogram for the Temperature-Pressure Correction (chart 5, app. II) may be used for determining this correction. The value obtained from the nomogram is in units of the sixth decimal and must be multiplied by the distance to obtain the correction. The formula for temperature correction is—

$$C_t=(N_g-1)\left(1-\frac{273.2}{273.2+t}\right)$$

The formula for pressure correction is—

$$C_p=\frac{1}{273.2+t}\times(760-P)\times\frac{(Ng-1)\,(273.2)}{760}$$

Where: $Ng$ = Index of refraction (group velocity)
$t$ = temperature (Celsius)
P = pressure (mm of mercury)

*Note.* If the angstrom units of the photomultiplier are known and the refractive index is computed, omit the temperature-pressure correction.

*i. Humidity Correction.* The humidity correction taken from the correction for humidity graph (appendix II) is negative when applied to the refractive index. However, in the Model 4 Geodimeter computation, the value obtained from the nomogram is multiplied by the distance to obtain the correction and is always positive when applied in this manner. The nomogram tables are derived from the following formula:

$$\text{Hum. Corr.} = -\frac{5.5\times10^{-8}e}{1+\frac{t}{273.2}}$$

Where: $e$ = humidity in mm of mercury
$t$ = temperature (Celsius)

*j. Refractive Index Correction (RC).* The assumed refractive index is based on a temperature of −6° Celsius and a pressure of 760 mm of mercury. The manufacturer accepted 5,650 angstroms as a mean value for the photomultiplier tube. Then, from the formula:

$$N'_a = 1+\left(\frac{Ng-1}{1+\frac{t}{273.2}}\right)\left(\frac{P}{760}\right)$$

Simplifying:

$$N'_a = 1+\frac{0.359474(Ng-1)P}{273.2+t}$$

therefore:

$$N'_a = 1+\frac{0.359474(0.00030361)760}{273.2+(-6°)}$$

$$N'_a = 1.0003104$$

When the angstrom value for the photomultiplier tube is known, a new refractive index must be computed using the mean temperature and pressure at the time of the observation. $Ng$ values are tabulated for a series of angstrom units ($\lambda$) in table XVII (app. III). $N'_a$ is the refractive index formula without the portion concerning the humidity. It is more convenient to compute $N'_a$ and apply the humidity correction separately as described in *i* above. In the example, the angstrom units are 5,500, mean temperature 6.2° Celsius, and the pressure 28.73 inches of mercury or 729.74 mm of mercury. Therefore:

$$N'_a = 1+\frac{0.359474(Ng-1)P}{273.2+t}$$

$$N'_a = 1+\frac{0.359474(0.00030453)729.74}{273.2+6.2}$$

$$N'_a = 1.0002859$$

The difference between the computed refractive index and the assumed refractive index is 0.0000245 which is called the refractive index deviation. Refractive Index Correction (RC) equals the refractive index deviation times the length of the line being measured.

RD = Assumed Refractive Index − $Na'$
RC = RD × Length
RC = 0.0000245(3566.473)
RC = +0.087 m.

*k. Slope Distance.* The slope distance is the sum of Mean Geodimeter Distance, Geodimeter Constant, Reflector Constant, Temperature Pressure Correction, or Refractive Index Correction, and the Humidity Correction.

*l. Geodetic Distance.* The corrections applied to the slope distance to obtain geodetic distance have already been discussed under Model 2A Geodimeter (par. 48).

## Section IV. MEASUREMENTS USING ELECTROMAGNETIC WAVES

### 50. Tellurometer—MRA1—MRA2

*a. Readings.* Tellurometer measurements are indicated in units of time called millimicroseconds (10,740 mμs, approximately, are required for radio waves to travel 1 mile, out and back). Slope distances up to 9⅓ miles are read directly from the instrument by use of four major pattern frequencies (A, B, C, D). For distances greater than 9⅓ miles, the measuring cycle is repeated and the number of full 9⅓-mile intervals must be determined and added to the instrument measurement before reducing it to a linear value. A negative "A" reading is provided to elimate zero errors; readings are made with the "A" frequency in a reverse direction to eliminate centering errors. These readings, A+, A−, A+R, A−R, B, C, and D are used to obtain the distance in time units.

*b. Interpretation of Coarse Readings.*

| Estimated Mileage | A+ 13 B 14 | A+ 13 C 14 | A+ 13 D 40 | A+ 13 A− 86 |
|---|---|---|---|---|
| 28 | 99 | 99 | 73 | 27 |

(1) Subtract the B, C, D, and A− readings from the A+ reading to yield unadjusted differences, adding 100 to the A+ reading wherever it is smaller than the subtrahend. The coarse reading is composed of the adjusted tens digit of estimated mileage, the B, C, and D differences, and all the digits of one-half the "A" difference. This coarse reading interpretation should completed in the field by the recorder before the frequency (cavity tune) setting is changed. All coarse readings for a measurement should agree with each other within a few millimicroseconds (units of the A scale). The procedure for adjusting the difference is described in (2) through (6) below; (7) below, discusses the adjustments for two lines to clarify the procedures.

(2) Divide the A+A− difference by 2, adding 100 to this difference if necessary to keep the quotient of the same order of magnitude as the original A+ reading. The quotient gives the tens and units (and perhaps decimal) digits of the coarse reading; these should then be recorded.

(3) Adjust the A+D difference slightly if, necessary (adding or subtracting not more than 3 units) to make its units digit identical with the tens digit of the A figures already set down. The tens digit of the adjusted D difference is the hundreds digit of the coarse reading. This should then be recorded.

(4) Similarly adjust the A+C difference, if necessary, to make its units digit identical with the tens digit of the adjusted D difference. The tens digit of the adjusted C difference is the thousands digit of the coarse reading and should be so recorded.

(5) Likewise adjust the A+B difference, if necessary, so that its units digit corresponds to the tens digit of the adjusted C difference. Use the tens digit of the adjusted B difference for the tens-of-thousands digit of the coarse reading.

(6) Scale from a map, or estimate the total distance in miles. Adjust this estimate if necessary, by the method described for the different modulation pattern differences, so that the units digit of the mileage corresponds to the tens digit of the adjusted B difference. The tens digit of the adjusted mileage becomes the hundreds-of-thousands digit, which must be supplied to complete the coarse reading.

(7) Lines 1 and 2 below illustrate the method of interpretation. The distance scaled from the map is 28 miles.

(*a*) Referring to line one, the figures 13.5 or ½ the A difference, are set down as the final three digits of the interpreted reading. The D difference is adjusted to 71 and digit 7 is set down. The C difference is adjusted to 97 and digit 9 is set down. As the B difference does not require adjustment, the 9 digit is recorded. The mileage (28) must be adjusted to 29 and the digit supplied is 2. Note that for distances less than 10 miles, there will be no tens digit in the mileage figure and the digit supplied to the interpreted reading will be zero.

(*b*) Referring to line two, one-half the A difference, or 13.5, is set down. The D difference is adjusted to 71 and the 7 set down. The C difference is adjusted to 07 and the 0 digit is recorded. The B difference is adjusted to 100 (=00) and the 0 digit is recorded. The mileage must be adjusted to 30 and the three digit supplied.

*Line 1:*

| | Mileage | B | C | D | A | |
|---|---|---|---|---|---|---|
| Difference | 28 | 99 | 99 | 73 | 27 | |
| Adjusted | 29 | 99 | 97 | 71 | 13. 5 | (not recorded) |
| Interpreted Reading | | 299 | | 713. 5 | | |

*Line 2:*

| | Mileage | B | C | D | A | |
|---|---|---|---|---|---|---|
| Difference | 28 | 99 | 06 | 73 | 27 | |
| Adjusted | 30 | 00 | 07 | 71 | 13. 5 | (not recorded) |
| Interpreted Reading | | 300 | | 713. 5 | | |

*c. Determination of Fine Readings.*

(1) Subtract A− from A+, first adding 100 to A+ if it is smaller than A−. Similarly, subtract A−R from A+R. These two differences should be approximately equal and roughly twice the original A+ reading. Add 100 if necessary; add differences and divide by 4. The quotient must be of the same order of magnitude as the A+ reading.

(2) Do the same for each fine reading at the different cavity tune settings, comparing corresponding readings differences, and means for all readings. Any unusual deviation from the trend should be examined for rejection.

(3) Average all the accepted mean differences and substitute the averaged mean difference of the fine readings for the halved A difference of the coarse readings to obtain the uncorrected transit time (UTT). This value is in terms of millimicroseconds (m$\mu$s).

*d. Determination of Auxiliary Readings.*

(1) Compute the mean of the crystal temperature readings (MRA–1) at the master station. The units of measurement are normally microamperes.

(2) Compute the mean of the altimeter or barometer readings at both stations. Most altimeters are set to read 1,000 feet at sea level. The field man normally makes this correction before recording the readings.

(3) Compute the means of the dry-bulb and wet-bulb temperature readings at both stations.

*e. Field Sheet Entries.* The field readings on DA Form 5–137 are shown in figure 68. The line is approximately 34 miles in length. As shown on the form, the computations are completed through the uncorrected transit time (UTT) by the field party. This is done using the methods outlined in the preceding paragraphs. These computations should be checked in the office before proceeding with the Tellurometer reduction.

*f. Reduction.* DA Form 5–138 (Tellurometer Reduction) (fig. 69) is used to reduce Tellurometer lengths. The following field data is entered from the field sheet:

(1) Project, date of observation, location, and organization.

(2) Names of stations, instrument numbers, heights of instruments (HI), eccentricity of instruments, and elevations of stations.

(3) Mean values for altimeter or barometer reading, dry-bulb and wet-bulb temperatures, and crystal temperature.

(4) Uncorrected transit time.

*g. Corrected Transit Time.*

(1) To determine the corrected transit time (CTT), scale the frequency deviation (FD) in parts per million cycles per second from the frequency-temperature curve graph (fig. 70). (Each master instrument has its own graph.)

(2) Compute the crystal correction (CC) from the following formula:

$$CC=UTT \times -FD \times 10^{-6}$$

(3) Apply the correction algebraically to the UTT to obtain the CTT.

*h. Index of Refraction.*

(1) The following explanations and formulas are based on the use of barometric pressures measured in inches of mercury and temperatures measured in degrees Fahrenheit. If other units are used in the observations, the conversion to inches of mercury and degrees Fahrenheit may be made as follows:

(*a*) To convert degrees Celsius to degrees Fahrenheit, use table XV (app. III).

(*b*) To convert pressure in millibars to inches of mercury, multiply the observed value by 0.02953.

(*c*) To convert pressure in millimeters to inches of mercury, multiply the observed value by 0.03937.

(2) The formula for the index of refraction, $\eta$, is:

$$\eta=1+10^{-6}N$$

$$N=\left(\frac{4730}{459.688+t}\right)\left(P+\frac{8540e}{459.688+t}\right)$$

Where:

$\eta$ = Index of refraction

P = Mean barometric pressure, in in. Hg.

$e$ = Mean vapor pressure, in in. Hg.

$t$ = Dry-bulb temperature, in ° F.

## FIELD SHEET, TELLUROMETER DATA ENTRIES

(TM 5-237)

PAGE NUMBER | NUMBER OF PAGES

| OPERATION | APPROXIMATE DISTANCE | DATE | METHOD OF DETERMINATION |
|---|---|---|---|
| TEST | 34 MILES | 20 SEPT. 1957 | TRIANGULATION |

| MASTER STATION | INST. NO. | H.I. | ECCENTRICITY | ELEVATION |
|---|---|---|---|---|
| THOMPSON, 1846 | MA-25 | 1.524 M | 0 | 83 M |

| REMOTE STATION | INST. NO. | H.I. | ECCENTRICITY | ELEVATION |
|---|---|---|---|---|
| BLUEHILL, 1845 | RA-26 | 1.524 M | +1.900 M | 194 M |

| MASTER OPERATOR | REMOTE OPERATOR | RECORDER | COMPUTER |
|---|---|---|---|
| Mc CALL | WILSON | WILLIAMS | |

### COARSE READINGS

| | | | | |
|---|---|---|---|---|
| INITIAL | A+ 70.5 | A+ 70.5 | A+ 70.5 | A+ 70.5 |
| | B 05.5 | C 33.0 | D 02.0 | A− 28.0 |
| | 65.0 | 37.5 | 68.5 | 2 \| 142.5 / 71.25 |
| FINAL | A+ 71.0 | A+ 71.0 | A+ 71.0 | A+ 71.0 |
| | B 06.5 | C 34.0 | D 04.0 | A− 28.0 |
| | 64.5 | 37.0 | 67.0 | 2 \| 143.0 / 71.5 |

INITIAL COARSE FIGURE 363671.25 MUS

FINAL COARSE FIGURE 363671.5 MUS

### METEOROLOGICAL READINGS

| | XTAL. TEMP. | PRESSURE ALT. | TEMPERATURE DRY | TEMPERATURE WET |
|---|---|---|---|---|
| MASTER INITIAL | 65 | 110 FT. | 79.5 °F | 67.5 °F |
| REMOTE INITIAL | | 455 | 85.5 | 69.0 |
| MASTER FINAL | 78 | 111 | 79.5 | 67.5 |
| REMOTE FINAL | | 455 | 84.3 | 68.5 |
| SUM | 143 | 1131 | 328.8 | 272.5 |
| MEAN | 71.5 MA | 282.8 FT | 82.20 | 68.12 |

### COMPUTATION

| | |
|---|---|
| UNCORRECTED TRANSIT TIME | MUS |
| XTAL. FREQ. CORR. _____ PPM | |
| CORRECTED TRANSIT TIME | MUS |
| DISTANCE IN FEET/METERS | |
| VAPOR PRESSURE IN. HG. (P) | |
| EQUIVALENT PRESS. IN. HG. (E) | |
| TOTAL EFFECTIVE PRESS (P+E) | |
| REFRACTIVE INDEX (N) | |
| REFRACTIVE INDEX CORR. | |
| ZERO CORRECTION | |
| SLOPE DISTANCE | |
| COS. VERTICAL ANGLE | |
| HORIZONTAL DISTANCE | |
| ECCENTRICITY | |
| CORR. HORIZ. DIST. | |

### FINE READINGS

| NO. | FREQ. DIAL | A+ / A− DIFF. | A+R / A−R DIFF. | MEAN DIFF. |
|---|---|---|---|---|
| 1 | 7 | 71.0 / 27.0 / 144.0 | 23.5 / 82.5 / 141.0 | 71.250 |
| 2 | 8 | 71.0 / 27.0 / 144.0 | 22.5 / 82.0 / 140.5 | 71.125 |
| 3 | 9 | 71.0 / 27.0 / 144.0 | 23.0 / 81.0 / 142.0 | 71.500 |
| 4 | 10 | 70.5 / 28.0 / 142.5 | 22.5 / 81.0 / 141.5 | 71.000 |
| 5 | 11 | 71.0 / 28.0 / 143.0 | 23.5 / 81.0 / 142.3 | 71.375 |
| 6 | 12 | 71.0 / 27.5 / 143.5 | 23.0 / 81.0 / 142.0 | 71.375 |
| 7 | 13 | 70.5 / 27.5 / 143.0 | 23.0 / 81.0 / 142.0 | 71.250 |
| 8 | 14 | 70.5 / 28.0 / 142.5 | 23.5 / 81.0 / 142.5 | 71.250 |
| 9 | 15 | 70.5 / 27.5 / 143.0 | 23.0 / 81.5 / 141.5 | 71.125 |
| 10 | 16 | 70.5 / 27.0 / 143.5 | 23.0 / 81.0 / 142.0 | 71.375 |
| 11 | 17 | 71.0 / 28.0 / 143.0 | 24.0 / 81.0 / 143.0 | 71.500 |
| 12 | 19 | 71.0 / 27.5 / 143.5 | 24.0 / 81.0 / 143.0 | 71.625 |
| 13 | 21 | 71.5 / 27.5 / 144.0 | 24.0 / 81.0 / 143.0 | 71.750 |
| 14 | | | | |
| | | | SUM | 927.500 |
| | | | MEAN | 71.35 |

DA FORM 1 JUN 60 5-137

*Figure 68. Field sheet, Tellurometer MRA–1 or MRA–2 readings (DA Form 5–137).*

| PROJECT | DATE | TELLUROMETER REDUCTION (TM 5-232) |
|---|---|---|
| TEST | 20 SEPT. '57 | |
| LOCATION: MASS. | ORGANIZATION: USAMS | |

| MASTER STATION | INST. NO. | HI | ECCENTRICITY | ELEVATION |
|---|---|---|---|---|
| THOMPSON 1846 | MA-25 | 1.524 M. | — METERS | 83 M. |
| REMOTE STATION | INST. NO. | HI | ECCENTRICITY | ELEVATION |
| BLUE HILL 1845 | RA-26 | 1.524 M. | +1.900 METERS | 194 M. |

| MEAN ALTIMETER READING | DRY BULB TEMP. (t) | WET BULB TEMP. (t') | DEPRESSION (t−t') | CRYSTAL TEMP. |
|---|---|---|---|---|
| 282.8 FEET | 82.20 °F | 68.12 °F | 14.08 °F | 71.5 MA |

| | | | | | |
|---|---|---|---|---|---|
| UNCORRECTED TRANSIT TIME (UTT) | 363671.35 MμS | | FREQUENCY DEVIATION (FD) | +3.4 | PPM |
| CRYSTAL CORRECTION (CC) | − 1.24 MμS | | e' | 0.6931 | "HG |
| CORRECTED TRANSIT TIME (CTT) | 363670.11 MμS | | Δe | − 0.1564 | "HG |
| BAROMETRIC PRESSURE (P) | 29.62 "HG | | e | 0.5367 | "HG |
| B: 8.729 | A: 137.6 | | N | 332.4 | |
| TELLUROMETER DISTANCE (T) | 54494.672 METERS | | η | 1.0003324 | |
| HORIZONTAL DISTANCE (H) | 54494.559 METERS | | DIFF. OF ELEV (d) | 111 ~~FEET~~ METERS | |
| ECCENTRICITY CORRECTION | − 1.900 METERS | | MEAN ELEVATION (h) | 140 ~~FEET~~ METERS | |
| CHORD-ARC CORRECTION (K) | + 0.166 METERS | | α | MEAN φ: 36° 42° | |
| SEA LEVEL REDUCTION (C) | − 1.197 METERS | | MEAN RADIUS OF CURVATURE (ρ) | 6372205 METERS | |
| GEODETIC DISTANCE (S) | 54491.628 METERS | | | | |

| MASTER STATION | INST. NO. | HI | ECCENTRICITY | ELEVATION |
|---|---|---|---|---|
| | | | METERS | |
| REMOTE STATION | INST. NO. | HI | ECCENTRICITY | ELEVATION |
| | | | METERS | |

| MEAN ALTIMETER READING | DRY BULB TEMP. (t) | WET BULB TEMP. (t') | DEPRESSION (t−t') | CRYSTAL TEMP. |
|---|---|---|---|---|
| FEET | °F | °F | °F | MA |

| | | | | | |
|---|---|---|---|---|---|
| UNCORRECTED TRANSIT TIME (UTT) | MμS | | FREQUENCY DEVIATION (FD) | | PPM |
| CRYSTAL CORRECTION (CC) | MμS | | e' | | "HG |
| CORRECTED TRANSIT TIME (CTT) | MμS | | Δe | | "HG |
| BAROMETRIC PRESSURE (P) | "HG | | e | | "HG |
| B | A | | N | | |
| TELLUROMETER DISTANCE (T) | METERS | | η | | |
| HORIZONTAL DISTANCE (H) | METERS | | DIFF. OF ELEV (d) | FEET METERS | |
| ECCENTRICITY CORRECTION | METERS | | MEAN ELEVATION (h) | FEET METERS | |
| CHORD-ARC CORRECTION (K) | METERS | | α | MEAN φ: ° ° | |
| SEA LEVEL REDUCTION (C) | METERS | | MEAN RADIUS OF CURVATURE (ρ) | METERS | |
| GEODETIC DISTANCE (S) | METERS | | | | |

$CC = UTT \times -FD \times 10^{-6}$ $\quad e = e' - \Delta e$ $\quad \eta = 1 + 10^{-6} N$ $\quad T = 0.14989625 \times \frac{CTT}{\eta}$ $\quad C = -H\frac{h}{\rho} + H\frac{h^2}{\rho^2}$ $\quad K = \frac{H^3}{24\rho^2} \cdots$

$CTT = UTT + CC$ $\quad N = BP + Ae$ $\quad H = \sqrt{T^2 - d^2}$ or $H = T - \frac{d^2}{2T} - \frac{d^4}{8T^3} \cdots$ $\quad S = H + K + C$ $\quad$ or $K = 1.027H^3 \times 10^{-15}$

$B = \frac{4730}{459.688 + t}$ $\quad A = \frac{8540}{459.688 + t} \times \frac{4730}{459.688 + t}$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.R. Smith - AMS | DEC. '63 | J.A. Andrews - AMS | JAN. '64 |

DA FORM 1 JUN 60 5-138

*Figure 69. Tellurometer Reduction (DA Form 5-138).*

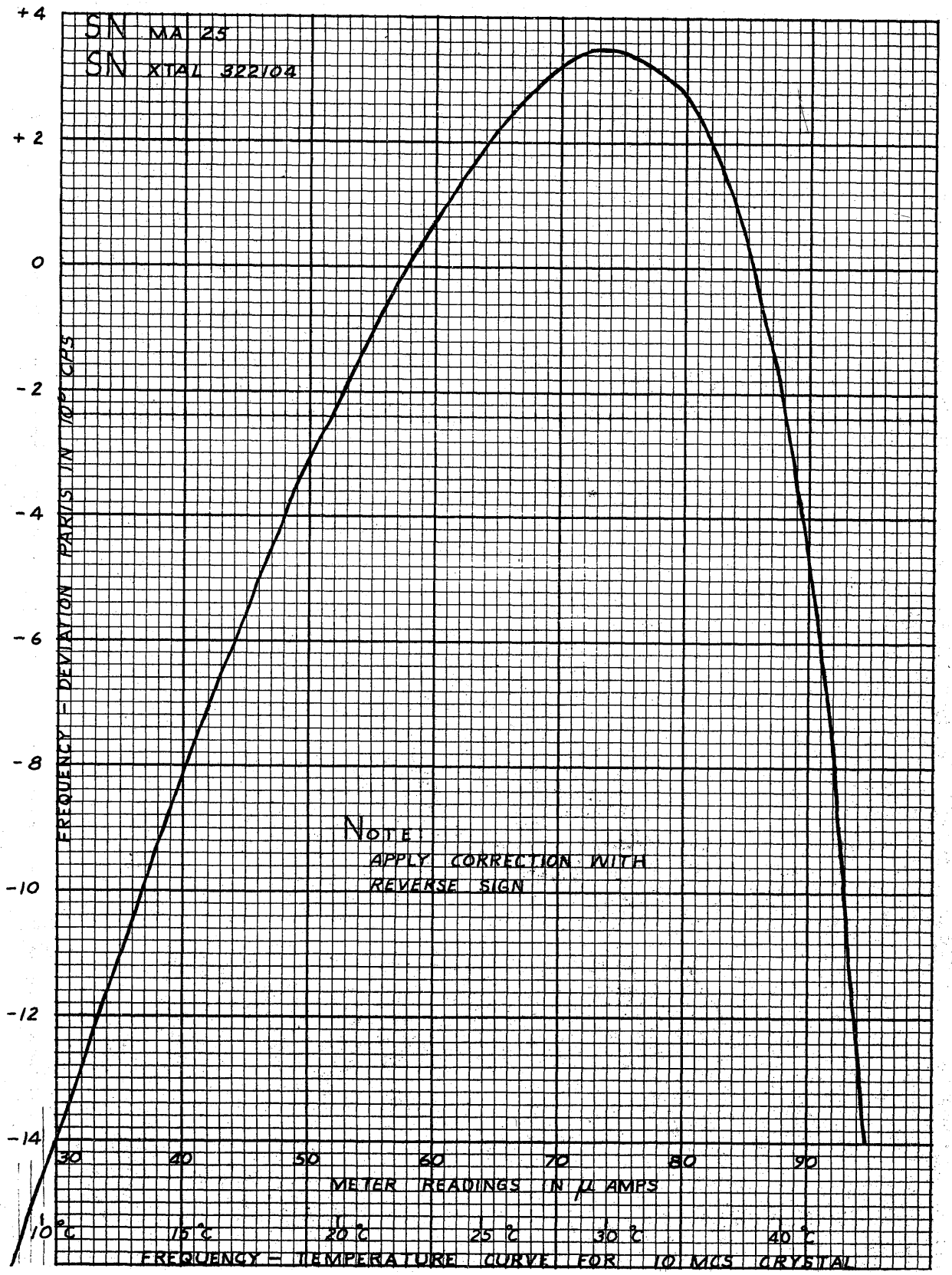



*Figure 70. Frequency-temperature correction curve.*

Substituting in the formula:

$$A=\frac{8540}{459.688+t}\times\frac{4730}{459.688+t}$$

$$B=\frac{4730}{459.688+t}$$

the formula becomes:

$$N=BP+Ae$$

(3) The mean barometric pressure (P) in inches of mercury is either observed directly or computed from formulas or tables.

(*a*) P is found by using table XVI, appendix III, with the mean reduced altimetric elevation (ft) of the line as the argument.

(*b*) To determine P from formulas use:

$$P=P_o\left(\frac{288-0.00198h}{288}\right)^{5.256}$$

Where:

$h$=Mean reduced altimetric elevation (ft) of line

$P_o$=Atmospheric pressure at mean sea level=29.9212 in. Hg.

(4) A and B are taken from table XVIII, appendix III, using the dry-bulb temperature $t$ (°F.) as the argument, or computed from formulas.

(5) The mean vapor pressure ($e$) is determined from tables or computed from formulas:

(*a*) To determine $e$ from tables, abstract $e'$ in inches of mercury from table XIX, appendix III, using the wet-bulb temperature ($t'$) (°F.) as the argument. At the same time, abstract $\Delta e$ for 1 inch of mercury and 1 °F. from table XIX using $t'$ as the argument. Multiply $\Delta e$ by $P(t-t')$ to obtain the total $\Delta e$ and subtract the total $\Delta e$ from $e'$ to obtain $e$.

(*b*) To determine $e$ from formulas, use:

$$e=e'-0.000367P(t-t')\left(1+\frac{t'-32}{1571}\right)$$

Where:

$t$=Dry-bulb temperature, °F.

$t'$=Wet-bulb temperature, °F.

P=Mean barometric pressure, in in. Hg.

$e'$=Saturation vapor pressure of water in in. Hg. at the temperature $t'$.

(6) The index of refraction ($\eta$) is now computed using the formulas.

*i. Tellurometer Distance.*

(1) The Tellurometer slope distance (T) in meters is determined by dividing the corrected transit time by the index of refraction and multiplying the quotient by the constant 0.14989625.

$$T\text{ (meters)}=0.14989625\times\frac{CTT}{\eta}$$

(2) The constant 0.14989625 was determined using 299,792.5 kilometers per second as the velocity of an electromagnetic wave in a vacuum.

*j. Horizontal Distance.*

(1) The horizontal distance (H) is computed using the Pythagorean theorem:

$$H=\sqrt{T^2-d^2}$$

or by the formula:

$$H=T-\frac{d^2}{2T}-\frac{d^4}{8T^3}-\frac{d^6}{16T^5}\cdots\cdot$$

where $d$ is the difference in elevation between the master and remote stations.

(*a*) The difference in elevation ($d$) is obtained from the formula:

$$d=(h_m+HI_m)-(h_r+HI_r)$$

Where: $h_m$=Elevation of master station

$h_r$=Elevation of remote station

$HI_m$=Height of Tellurometer at master station

$HI_r$=Height of Tellurometer at remote station

(*b*) The elevations of the stations are normally determined by either differential or trigonometric leveling. The trigonometric leveling method is explained in chapter 11. Since only the Tellurometer slope distance is normally available, certain changes must be made in the computation. The formula for the correction to the zenith

distances for the reduction to line joining stations becomes:

$$\text{Reduction (sec)} = \frac{-(t-o)\sin\zeta}{\text{T}\sin 1''} = \frac{-206265(t-o)\sin\zeta}{\text{T}}$$

Where $\zeta$ is the observed zenith distance. The formula for the difference in elevation becomes:

$$h_2 - h_1 = \text{T}\sin\tfrac{1}{2}(\zeta_2 - \zeta_1).$$

If nonreciprocal observations are made, a value for $(0.5-m)$ of 0.429 should be used in the computation and

$$h_2 - h_1 = \text{T}\sin(90° - \zeta + k)$$

(c) The use of altimeter elevations to determine differences of elevation is permissible if more accurate methods of determination are not required. For accurate measurements, the altimeters must be carefully calibrated both relatively and absolutely; and fairly stable air conditions must exist between the stations.

(2) The accuracy required in the difference of elevation depends upon the *horizontal* distance, accuracy required in the horizontal distance, and difference of elevation itself. The allowable error may be computed from the following formula:

$$\Delta_d = \frac{\text{H}(\Delta\text{H})}{d}$$

Where: $\Delta d$ = Error allowed in difference of elevation
H = Horizontal distance
$\Delta$H = Error allowed in horizontal distance
$d$ = Difference of elevation

*Example:* H = 5000 meters
$\Delta$H = 1/50,000 = 5000/50,000 = 0.10 meters
$d$ = 100 meters

$$\Delta d = \frac{\text{H}(\Delta\text{H})}{d} = \frac{5000(0.10)}{100} = \pm 5.0 \text{ meters}$$

(3) If zenith distances are measured to obtain the difference of elevation, the error allowed in the reduced angle can be determined from the formula:

$$\Delta\zeta \text{ (in sec.)} \frac{(\Delta\text{T})\tan\frac{1}{2}(\zeta_2-\zeta_1)}{\text{T}\sin 1''} = \frac{206265(\Delta\text{T})\tan\frac{1}{2}(\zeta_2-\zeta_1)}{\text{T}}$$

Where: $\frac{1}{2}(\zeta_2-\zeta_1)$ = Reduced zenith distance from reciprocal observations
$\Delta\zeta$ = Allowable error in zenith distance
T = Measured slope distance
$\Delta$T = Allowable error in slope distance

*Example:* T = 5000 meters
$\Delta$T = 1/50,000 − 5000/50,000 = 0.10 meters
$\frac{1}{2}(\zeta_2-\zeta_1) = 88°51'00''$
$\tan\frac{1}{2}(\zeta_2-\zeta_1) = 49.81573$

$$\Delta\zeta = \frac{206265(\Delta\text{T})\tan\frac{1}{2}(\zeta_2-\zeta_1)}{\text{T}} = \frac{206265(0.10)49.81573}{5000}$$

$$\Delta\zeta = \pm 206'' = \pm 00°03'26''$$

*k. Mean Elevation of Line.*

(1) In determination of the mean elevation ($h$) of the line, add the antenna heights (HI) to the elevations, if the elevations of the stations are known, and then compute the mean of the values.

(2) If the elevations are not known, scale the elevations from a map, if available, then compute the mean.

(3) When the elevations and map are unavailable, use the mean corrected altimeter readings.

*l. Geodetic Distance.*

(1) The horizontal distance is reduced to the geodetic distance (S) by applying the sea level (C) and the chord-arc (K) corrections. If either instrument was eccentric during the measurement, an additional correction for eccentricity must be applied.

(2) The C correction reduces the horizontal distance at the mean elevation of the stations to the horizontal distance at mean sea level. This correction is nega-

tive if the stations are above sea level. The formula for C is:

$$C=-H\frac{h}{\rho}+H\frac{h^2}{\rho^2}\ldots.$$

Where: H = Horizontal distance
$h$ = Mean elevation of the stations
$\rho$ = Mean radius of curvature of the spheroid, and is taken from table XX, using the mean latitude ($\varphi$) of the stations and the azimuth ($\alpha$) between the stations as the arguments, or is computed from the formula:

$$\rho=\frac{RN}{R\sin^2\alpha+N\cos^2\alpha}$$

Where: R = Radius of curvature of the spheroid in the meridian.
N = Radius of curvature of the spheroid in the prime vertical.
$\alpha$ = Azimuth of line

(3) The K correction is applied to the horizontal distance to change it from a chord distance to an arc distance on the surface of the spheroid. This is computed using the formula:

$$K=\frac{H^3}{23\rho^2}$$

or by the approximate formula:

$$K=1.027H^3\times10^{-15}$$

For distances less than 5 miles, K is negligible.

(4) The application of these corrections—eccentricity, sea level (normally negative), and chord-arc (always positive)—results in a geodetic distance at sea level on the spheroid of reference.

*m. Zero Correction.* The zero correction is determined during calibration of the instrument by making repeated measurements over a known distance using paired instruments. This correction may be as large as several centimeters. If the form has no provision for a zero correction, it may be combined with the eccentricity and entered on the form under eccentricity correction.

## 51. Tellurometer MRA–3

*a. Capabilities.* The Tellurometer MRA–3 consists of two identical units which measure the phase delay of a radio wave transmitted between the two units. According to the manufacturer, the Tellurometer MRA–3 will determine distances ranging from 100 meters to over 60 kilometers with an overall measuring error of 2 centimeters $\pm3$ ppm. As with all electronic distance measuring equipment, the atmospheric unknowns introduce more error into the measured lines than the inherent accuracy of the instrument. To reduce these unknown conditions to a minimum, only calibrated thermometers, psychrometers and altimeters should be used.

*b. Measurement Reduction.*

(1) *Determination of uncorrected distance.*

(*a*) The field observations, consisting of meteorological readings, initial and final coarse readings, and a series of fine readings are entered on DA Form 2856 (Field Sheet, Tellurometer Data Entries (MRA–3)) (fig. 71). The headings should be completely filled out including an approximate distance.

(*b*) The initial and final coarse readings are resolved from the readings A, E, D, C, and B in that order. The second digit of each value should agree within three units with the third digit of the succeeding value (para. 50b). Should these differ by more than three, the coarse readings should be repeated. The resolved values for the initial coarse and final coarse readings should agree within approximately one-tenth of a meter under normal conditions.

(*c*) A sufficient number of fine readings should be taken over the entire range of carrier frequencies to provide a smooth sine curve when graphically representing the readings against the frequencies. The "A Forward" and "A Reverse" readings are meaned and replace the units, decimeters, centimeters and millimeters in the resolved coarse reading to obtain the uncorrected distance. Comparison of the final mean of the fine readings with the individual fine readings will yield an indication of the relative accuracy of the observations. An excessive deviation of any one of these readings

**FIELD SHEET, TELLUROMETER DATA ENTRIES** *(MRA3 MKII)*
*(TM 5-237)*

PAGE NO. | NO. OF PAGES

| OPERATION | APPROXIMATE DISTANCE | DATE | METHOD OF DETERMINATION |
|---|---|---|---|
| | 3 mi. | 27 Feb. 1964 | MAP SCALE |

| | MASTER STATION | INST NO. | H. I. | ECCENTRICITY | ELEVATION |
|---|---|---|---|---|---|
| | ROOF | MRA. 188 | 1.54 M. | — | 99.3 M. |
| | **REMOTE STATION** | **INST NO.** | **H. I.** | **ECCENTRICITY** | **ELEVATION** |
| | G.W. PARKWAY | MRA. 180 | 1.46 M. | — | 33.5 M. |

| MASTER OPERATOR | REMOTE OPERATOR | RECORDER | ORGANIZATION |
|---|---|---|---|
| R.E. RUSSMAN | W.A. HOFFDAHL | W. D. NUTT | USAMS |

**COARSE READINGS**

| TIME | CAVITY | TIME | CAVITY |
|---|---|---|---|
| 14:15 | 20 | 15:00 | 200 |

| INITIAL | | FINAL | |
|---|---|---|---|
| A | 829 | A | 820 |
| E | 480 | E | 462 |
| D | 069 | D | 063 |
| C | 516 | C | 488 |
| B | 049 | B | 041 |
| | 05048 29 | | 05048 20 |

**FINE READINGS**

| RDG. NO. | CAVITY | A FORWARD | A REVERSE | RDG. NO. | CAVITY | A FORWARD | A REVERSE |
|---|---|---|---|---|---|---|---|
| 1 | 20 | 829 | 835 | 13 | 140 | 820 | 826 |
| 2 | 30 | 824 | 832 | 14 | 150 | 836 | 832 |
| 3 | 40 | 819 | 827 | 15 | 160 | 810 | 817 |
| 4 | 50 | 827 | 824 | 16 | 170 | 817 | 824 |
| 5 | 60 | 816 | 820 | 17 | 180 | 820 | 818 |
| 6 | 70 | 829 | 832 | 18 | 190 | 816 | 817 |
| 7 | 80 | 820 | 826 | 19 | 200 | 820 | 823 |
| 8 | 90 | 818 | 830 | 20 | | | |
| 9 | 100 | 821 | 820 | 21 | | | |
| 10 | 110 | 815 | 815 | 22 | | | |
| 11 | 120 | 825 | 828 | 23 | | | |
| 12 | 130 | 815 | 822 | 24 | | | |
| SUM | | | | SUM | | 155.97 | 156.68 |
| MEAN | | | | MEAN | | 8.209 | 8.246 |
| FINAL MEAN | | | | | | 8.228 | |

**METEOROLOGICAL READINGS**

| | PRESSURE ALT | TEMPERATURE DRY (t) | TEMPERATURE WET (t') |
|---|---|---|---|
| MASTER INITIAL | 29.80" | 39.9 °F | 33.2 °F |
| REMOTE INITIAL | 29.91 | 38.2 | 29.2 |
| MASTER FINAL | 29.80 | 39.9 | 33.2 |
| REMOTE FINAL | 29.91 | 35.7 | 32.0 |
| SUM | 119.42 | 153.7 | 127.6 |
| MEAN | 29.86 | 38.42 | 31.90 |

| | | | | | |
|---|---|---|---|---|---|
| BAROMETRIC PRESSURE (P) | 29.86 "HG | | DEPRESSION (t − t') | 6.52 °F | |
| B | A | 9.496 | 162.807 | e' | 0.1797 "HG |
| UNCORRECTED DISTANCE (UD) | 5048.228 METERS | | Δe | 0.0715 "HG | |
| ZERO CORRECTION (Z) | — METERS | | e | 0.1082 "HG | |
| REFRACTIVE INDEX CORR. (RC) | + 0.120 METERS | | N | 301.2 | |
| SLOPE DISTANCE (T) | 5048.348 METERS | | REFRACTIVE INDEX DEVIATION (RD) | + 23.8 PPM | |
| HORIZONTAL DISTANCE (H) | 5047.919 METERS | | DIFF OF ELEV (d) | 65.8 ~~FEET~~ METERS | |
| ECCENTRICITY CORRECTION | — METERS | | MEAN ELEVATION (h) | 66.4 ~~FEET~~ METERS | |
| CHORD ARC CORRECTION (K) | — METERS | | a | MEAN φ | 335° 39° |
| SEA LEVEL REDUCTION (C) | − 0.053 METERS | | MEAN RADIUS OF CURVATURE (ρ) | 6365253 METERS | |
| GEODETIC DISTANCE (S) | 5047.866 METERS | | REMARKS (Use reverse) | | |

$B = \frac{4730}{459.688 + t}$ $A = \frac{8540}{459.688 + t} \times \frac{4730}{459.688 + t}$ $H = \sqrt{T^2 - d^2}$ or $H = T - \frac{d^2}{2T} - \frac{d^4}{8T^3} \cdots$ $K = \frac{H^3}{24\rho^2} \cdots$ or $K = 1.027H^3 \times 10^{-15}$ $C = -H\frac{h}{\rho} + H\frac{h^2}{\rho^2} \cdots$

$e = e' - \Delta e$ $N = BP + Ae$ $RD = 325 - N$ $RC = UD \times RD \times 10^{-6}$ $CT = UD + Z + RC$ $S = H + K + C$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.E. Russman - AMS | MAR. '64 | J.A. Andrews - AMS | MAR. '64 |

DA FORM 2856, 1 OCT 64

*Figure 71. Field Sheet, Tellurometer Data Entries (MRA–3) (DA Form 2826).*

from the final mean indicates that possibly one or more readings should be rejected.

(2) *Determination of refractive index of radio waves.* Refer to paragraph 50*i*.

(3) *Zero correction.* Refer to paragraph 50*m*.

(4) *Refractive index correction.*

(*a*) The preset refractive index for the Tellurometer MRA–3 is 1.000325.

(*b*) The refractive index correction is determined by subtracting the computed refractive index from the preset refractive and multiplying this deviation by the uncorrected distance. The formula is as follows:

RC=(Preset Refractive Index$-\eta$) x UD

(5) *Horizontal and geodetic distance.* Refer to paragraph 50*j*, *k*, *l*, and *m*.

## 52. Micro-Chain (MC–8)

*a. Capabilities.* The Micro-Chain MC–8 distance measuring equipment consists of two identical units which measure the phase delay of a radio wave transmitted between any two units. According to the manufacturer, the Micro-Chain will determine distances ranging from 200 meters to 50,000 meters at an accuracy better than 1.5 centimeters ± 4 ppm. As with all of the electronic distance measuring equipment, the atmospheric unknowns will introduce more error into the lines measured than the inherent accuracy of the instrument. To reduce the unknown conditions to a minimum, only calibrated thermometers, psychrometers, and altimeters should be used.

*b. Determination of Uncorrected Distance.*

(1) For the purpose of explaining the computations necessary to determine the length of the line measured, it is assumed that the headings on DA Form 2857 (Field Sheet, Micro-Chain Data Entries) (fig. 72) have been completed, including the date of calibration (zero correction) and the approximate length of the line. Also on the field sheet are the field observations consisting of the coarse channel readings for frequencies 1 and 9 (channels 3 through 6), the fine channel readings for frequencies 1 through 9 (channels 1 and 2), and the meterological data.

(2) The difference between each channel reading and the channel 1 reading provides the distance data. Subtract channel 1 data from each of the other channels. In each case, use the channel 1 data that was observed at each particular frequency setting.

(3) Total the 2 minus 1 column and divide by 10 to obtain the mean M2. The two sets of coarse readings are meaned and the results are transferred to the offset blocks M6, M5, M4, M3, and M2.

(4) If the channel 1 reading is greater than any one or all of the other channel readings so that a direct subtraction is impossible, add 1000 to the smaller channel readings so that a subtraction can be performed.

(5) The mean M2 value is considered correct and is used to resolve the M3 data which is correct only to within ±50 parts. In the sample computation (fig. 72), the M3 data is 368±50. Therefore, the true number is somewhere between 318 and 418. Since the first figures of the "correct" M2 value are 43, the correct number for M3 must be 343. The remainder of the distance is resolved as above and produces an uncorrected distance in millimeters.

(6) Comparison of the mean 2–1 value with individual 2–1 readings will yield an indication of the relative accuracy of the observations. The digits in the 2–1 channel are meters, decimeters, centimeters, and millimeters. An excessive deviation of any one of these readings from the mean indicates that possibly one or more readings should be rejected.

*c. Determination of Refractive Index of Radio Waves.* Refer to paragraph 50*i*.

*d. Zero Correction.* Refer to paragraph 50*m*.

*e. Refractive Index Correction.* Refer to paragraph 51*b*. The preset refractive index is the same as the MRA–3.

*f. Horizontal and Geodetic Distance.* Refer to paragraph 50 *j*, *k*, *l*, and *m*.

## 53. Electro-Tape DM–20

*a.* The Electro-Tape DM–20 equipment consists of two identical units which measure the phase delay of a radio wave transmitted between

| PROJECT | FIELD SHEET, MICRO-CHAIN DATA ENTRIES (TM 5-237) | | |
|---|---|---|---|
| TEST | | | |
| ORGANIZATION: C. of E. | DATE: 10 JUNE '63 | APPROX. DISTANCE: 40 KM. | |
| AZ. OF LINE: 179° | MEAN LATITUDE: N 38°30' | CALIBRATION DATE: JUNE '63 | OBSERVER: J. MARKOVICH | RECORDER: J. BUCZAK |
| MASTER STATION: TAPP ECC. | H. I.: 1.54 M. | ELEV.: 197.97 M. | ELEV. INST.: 199.51 | ECCENTRICITY: — | INST. NO.: 1 |
| REMOTE STATION: CLARKE | H. I.: 1.54 M. | ELEV.: 324.19 M. | ELEV. INST.: 325.73 | ECCENTRICITY: — | INST. NO.: 2 |

**METEOROLOGICAL READINGS**

| | TIME | PRESS. ALT. MM | TEMPERATURE DRY (t) | TEMPERATURE WET (t') |
|---|---|---|---|---|
| MASTER INITIAL | 0925 | 736 | 88 °F | 74 °F |
| REMOTE INITIAL | 0935 | 726 | 84 | 74.5 |
| MASTER FINAL | 0945 | 736 | 89 | 75 |
| REMOTE FINAL | 0945 | 727 | 85 | 75 |
| SUM | | 2925 | 346 | 298.5 |
| MEAN | | 731.2 | 86.5 | 74.6 |

| | | | |
|---|---|---|---|
| BAROMETRIC PRESSURE (P) | | 28.79 | "HG |
| B | A | 8.660 | -135.4 |
| e' | | 0.8636 | "HG |
| Δe | | 0.1292 | "HG |
| e | | 0.7344 | "HG |
| N | 348.8 | η | 1.000 3488 |
| PRESET REFRACTIVE INDEX | | 1.000325 | |
| REFRACTIVE INDEX DEVIATION (RD) | | -0.0000238 | |
| DIFF. OF ELEV. (d) | | 126.22 | ~~FEET~~ METERS |
| MEAN ELEV. (h) | | 262.22 | FEET METERS |
| MEAN RADIUS OF CURVATURE (ρ) | | 6360083 | METERS |

**CHANNEL**

| | 6 | 5 | 4 | 3 | 2 | 1 | 2 - 1 | FREQ |
|---|---|---|---|---|---|---|---|---|
| MASTER INITIAL | 508 | 246 | 536 | 460 | 522 | 096 | 426 | 1 HI |
| REMOTE INITIAL | 096 | 096 | 096 | 096 | 527 | 097 | 430 | 2 HI |
| MASTER FINAL | 412 | 150 | 440 | 364 | 528 | 097 | 431 | 3 HI |
| REMOTE FINAL | | | | | 530 | 097 | 433 | 4 HI |
| | | | | | 535 | 098 | 437 | 5 HI |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | 528 | 088 | 440 | 5 LO |
| | | | | | 529 | 089 | 440 | 6 LO |
| | | | | | 528 | 095 | 433 | 7 LO |
| | 6 | 5 | 4 | 3 | 526 | 091 | 435 | 8 LO |
| | 510 | 245 | 529 | 464 | 531 | 093 | 438 | 9 LO |
| | 093 | 093 | 093 | 093 | | SUM | 4343 | |
| | 417 | 152 | 436 | 371 | | MEAN | M2 434.3 | |
| | 829 | 302 | 876 | 735 | SUM | | | |
| | M6 414 | M5 151 | M4 438 | M3 368 | MEAN | | | |

1. INDICATE °C OR °F TEMPERATURE.
2. RECORD CORRECTED ALTIMETER READINGS IF USED.
3. ALWAYS ZERO THE *NULL INDICATOR* WITH A CLOCKWISE ROTATION OF THE DIAL.

| | | | |
|---|---|---|---|
| M6 | 4 | 1 | 4 |
| M5 | 1 | 5 | 1 |
| M4 | 4 | 3 | 8 |
| M3 | 3 | 6 | 8 |
| M2 | 4 | 3 | 4 | 3 |

| | | | |
|---|---|---|---|
| UNCORRECTED DISTANCE | (UD) | 41434.3434 | METERS |
| ZERO CORRECTION* | (Z) | - 0.165 | METERS |
| REFRACTIVE INDEX CORR. | (RC) | - 0.986 | METERS |
| SLOPE DISTANCE | (T) | 41433.192 | METERS |
| HORIZONTAL DISTANCE | (H) | 41433.000 | METERS |
| ECCENTRIC CORRECTION | | — | METERS |
| CHORD-ARC CORRECTION | (K) | + 0.073 | METERS |
| SEA-LEVEL REDUCTION | (C) | - 1.711 | METERS |
| GEODETIC DISTANCE | (S) | 41431.362 | METERS |

$B = \frac{4730}{459.688+t}$ $A = \frac{8540}{459.688+t} \times \frac{4730}{459.688+t}$

$e = e' - \Delta e$ $N = BP + Ae$ $\eta = 1 + 10^{-6} N$

$RD = 1.000325 - \eta$ $RC = UD \times RD$

$T = UD \pm Z \pm RC$ $H = \sqrt{(T^2) - (d^2)}$

$K = \frac{H^3}{24\rho^2} \cdots$ $C = -H\frac{h}{\rho} + H\frac{h^2}{\rho^2}$

or

$K = 1.027H^3 \times 10^{-15}$ $S = H + K + C$

* *Obtained from Instrument Calibration.*

*NOTE: Apply eccentric correction to H before computing K and C.*

REMARKS

| COMPUTED BY | DATE | CHECKED BY | DATE | PAGE |
|---|---|---|---|---|
| J.A. Andrews - AMS | FEB. '64 | G. Tennis - AMS | FEB. '64 | OF |

DA FORM 2857, 1 OCT 64

*Figure 72. Field Sheet, Micro-Chain Data Entries (DA Form 2857).*

any two units. According to the manufacturer, the DM–20 will determine distances ranging from 50 meters to 50,000 meters with an accuracy of 1 centimeter ±3 ppm.

*b*. The computations for the DM–20 are identical to those for the Micro-Chain MC–8 with the one exception that the preset refractive index for the DM–20 is 1.000320, while for the Micro-Chain is 1.000325.

*c*. Field observations and office computations may be completed on the same form as the Micro-Chain.

# CHAPTER 5

# TRIANGULATION

## Section I. PREPARATION OF DATA FOR ADJUSTMENT

### 54. Introduction

*a.* Adjusted survey data is the result of a continuous process of planning, selection, and adjustment. The field survey parties, since they are most familiar with the field methods and equipment used, should be the first to examine the field data. The final selection and adjustment of the survey data is performed by experienced computers.

*b.* Definite rules in regard to acceptance or rejection of field data cannot always be stated. However, a great many years of experience and testing have produced methods and rules that are almost as reliable as basic specifications. These methods and rules may not be the complete answer in all cases but the exceptions are rare. Many of these methods and rules have been included in this chapter.

### 55. Abstract of Horizontal Directions

*a.* An Abstract of Directions is prepared for every station at which horizontal angles or directions have been observed. DA Form 1916, Abstract of Horizontal Directions (fig. 73), is used for abstracting directions from field record books. This form provides spaces for recording readings of 16 separate positions of the instrument circle. After the field books are checked, the readings should be entered opposite the proper circle position as noted in the field book. For example, 8 readings may be taken at positions 1, 3, 5, 7, 9, 11, 13, and 15 of the instrument circle. The degrees and minutes for each direction are entered only once, at the top of each column, and only the seconds are entered for each circle position. All readings are recorded on DA Form 1916 opposite the appropriate position number with the exception of those specifically rejected in the field book.

*Note.* All field rejected values must be accompanied by a "statement of reason" in the field record book, i.e., bumped tripod, wrong light.

*b.* After all observations not rejected in the field book have been abstracted, any observation differing from the others so greatly that it is an obvious blunder in observing or recording should be rejected immediately. The remaining observations are now meaned. Any readings varying from this mean by more than $\pm 4''$ for first-order, $\pm 5''$ for second-order, or $\pm 5''$ for third-order are rejected. (These rejection limits apply basically to observations made with direction theodolites on which the micrometers can be read to $\pm 1''$.) The rejection limit for third-order triangulation can be extended to $\pm 15''$ when a transit type instrument is used for the observations. For any position still having more than one reading, a mean value is determined for that position. At this point, there should be only one value for each position, and no values for those positions whose readings have been rejected. A new mean value (of the remaining readings) is computed and the rejection limit is applied again. This procedure is repeated until all the values that remain are within the rejection limit. Once a value has been rejected, it cannot be used again, even though it may fall within the rejection limit of a subsequently determined mean. The mean value of the remaining position readings is the final value of the observed direction.

*c.* In the example shown (fig. 73), station Burdell is the occupied station. Directions are turned to stations Red and Hicks with an initial pointing on station Lincoln. The first eight settings of the instrument circle were used. The first observation of Red for position 2 was $06''.7$ which the observer considered too high, and the position was reobserved after completion

## ABSTRACT OF HORIZONTAL DIRECTIONS

(TM 5-237)

| LOCATION | ORGANIZATION | STATION |
|---|---|---|
| California | 29th Engr. | Burdell (29th Engr) |
| **OBSERVER** | **DATE** | **INST. (TYPE) (NO.)** |
| Cpl. P.E. Smith | 22 Aug. 1938 | Wild T-2 #146875 |

| POSITION NO. | STATIONS OBSERVED | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | LINCOLN (USC+GS) | Red (29th Engr) | HICKS (USC+GS) | | | | | |
| | (INITIAL) 0° 00′ | ° ′ 294 48 | ° ′ 337 10 | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 0. 00 | 01.6 | 34.0 | | | | | |
| 2 | 0. 00 | (06.7) R 00.4 {03.6 | 29.4 | | | | | |
| 3 | 0. 00 | 57.8 | 31.8 | | | | | |
| 4 | 0. 00 | 00.2 | 34.9 | | | | | |
| 5 | 0. 00 | (07.6) R 04.8 | 28.2 / 32.2 {30.2 | | | | | |
| 6 | 0. 00 | 59.1 | 36.1 | | | | | |
| 7 | 0. 00 | 03.4 | 33.0 | | | | | |
| 8 | 0. 00 | 02.3 | 33.8 | | | | | |
| 9 | 0. 00 | | | | | | | |
| 10 | 0. 00 | | | | | | | |
| 11 | 0. 00 | | | | | | | |
| 12 | 0. 00 | | | | | | | |
| 13 | 0. 00 | | | | | | | |
| 14 | 0. 00 | | | | | | | |
| 15 | 0. 00 | | | | | | | |
| 16 | 0. 00 | | | | | | | |
| Sum, | | 09.6 +2.8 23.4 | 263.2 | | | | | |
| Mean, | | 1.2 1.6 2.9 | 32.9 | | | | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| W. C. A. | 25 Aug 38 | W. L. N. | 27 Aug 38 |

DA, FORM 1 FEB 57 1916

*Figure 73. Abstract of horizontal directions (DA Form 1916).*

of the set and a second value of 00″.4 was read. The first reading for position 5 was 07″.6 which the field man also considered too high, and the position was reobserved at the same time as position 2. This time the reading was 04″.8. The two values for positions 2 and 5 are included on the abstract, and a mean of all ten values is found to be 02″.4. The 07″.6 reading for position 5 falls outside the 5″ limit and is rejected. Now the values for position 2 are meaned. A new mean of the remaining values (1″.6) is computed. The 06″.7 reading for position 2 is rejected at this time. The mean of the eight values remaining (1 for each position) is found to be 01″.2, and all readings are within the limits. The final value of the observed direction to Red is 294°48′01″.2. Notice the values for positions 3 and 6 which are recorded as $\overline{57.8}$ and $\overline{59.1}$ respectively. The bar over the numbers is used to indicate that this value of seconds goes with an angle 1 minute less than that recorded at the top of the column. The angle observed for position 3 was actually 294°47′57″.8 and position 6 was 294°47′59″.1. When summing up the column, these values should be treated as minus 02″.2 and minus 00″.9 respectively. The direction to Hicks shows a set of nine observations (for the eight positions), all of which are within 5″ of the mean, including the mean at position 5. None are rejected. The final value of the direction is 337°10′32″.9.

## 56. List of Directions

*a.* A list of directions must be prepared for each occupied station. The directions to *all* objects observed appear on the list (DA Form 1917, List of Directions). This list is used to obtain the angles for all subsequent computations and adjustments. It must be complete and accurate.

*b.* In some cases, more than one initial direction is used for the observations at a station. When this occurs, a preliminary list of directions (fig. 74) is prepared and refers to the respective initial directions. From this list, the various directions are consolidated into a final List of Directions (fig. 75). All of the directions are referred to a single initial, and one direction is shown to each observed object. Normally, these directions are listed with the initial direction being the one most counterclockwise in the triangulation scheme or network, with the others following in order of their increasing directions. This procedure allows the internal angles of the scheme to be readily obtained as the differences between desired directions.

*c.* Whenever observations of directions have been made using different initials, or where the values vary too greatly to be meaned by normal procedures, it may be necessary to compute a station adjustment before consolidating the directions. Generally this is done only for first- and second-order observations. A station adjustment is a least squares solution of all of the directions observed from a station, with proper weights applied to each. Statistically, it provides the most probable direction to each observed object. A sample station adjustment is presented in paragraph 61.

*d.* Any eccentricity of instrument or object should be clearly explained by sketch and descriptive information on the list of directions. The computation of the eccentric reduction provides the correction which, when properly applied to the observed direction, reduces it to the value it would have been if the instrument or target or both had not been eccentric. A sample computation of an eccentric reduction is presented in paragraph 58.

*e.* The sea-level correction or reduction computation is determined by the formula in paragraph 14*b*(12), but the numerical value of the correction may be taken from the "Sea Level Reduction Chart" (chart 3, app. II). The arguments used in the chart are: latitude of the occupied station, elevation of the observed station, and the azimuth of the observed line.

*f.* The eccentric reduction and the sea level reduction are listed in their appropriate spaces on DA Form 1917, and the application of these two corrections provides the "Corrected Direction with Zero Initial". The values in this column are used in all subsequent computations and adjustments.

*g.* The final column on DA Form 1917, "Adjusted Direction", is completed only after a final adjustment of the network has been accomplished.

## 57. Triangle Computation

*a. Basic Factors.*

(1) The basic computation in triangulation is the solution of a triangle in which at least two angles and one side are known. (Ordinarily all three angles in the triangle are observed). The solution of this problem is made by the law of sines for a plane triangle. Since the observed

| PROJECT | ORGANIZATION | LIST OF DIRECTIONS |
|---|---|---|
| I-243 | 29th Engrs. | Preliminary (TM 5-237) |

| LOCATION | STATION |
|---|---|
| California | Burdell (29th Engrs) |

| OBSERVER | INST. (TYPE) (NO.) | DATE |
|---|---|---|
| Cpl. P.E. Smith | Wild T-2 # 146875 | 22 Aug 55 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| LINCOLN (USC+GS) | 0 00 00.00 | | | 0 00 00.00 | |
| Red (29th Engrs) | 294 48 01.2 | | | | |
| HICKS (USC+GS) | 337 10 32.9 | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| Red | 0 00 00.0 | | | | |
| Ref. No.1, d = 7.342m | 6 27 37 | | | | |
| Hilman Water Tank | 21 14 36.3 | | | | |
| Ref. No.2, d = 6.761m | 84 13 21 | | | | |
| Ecc. Light to HICKS, d = 0.03m | 96 42 | | | | |

* These columns are for office use and should be left blank in the field.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 25 Aug 55 | W.C.A. | 27 Aug 55 |

DA FORM 1 FEB 57 1917

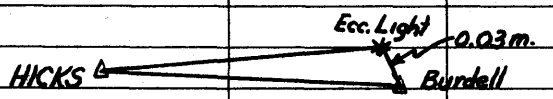
*Figure 74. List of directions (preliminary) (DA Form 1917).*

| PROJECT | ORGANIZATION | LIST OF DIRECTIONS (TM 5-237) |
|---|---|---|
| I-243 | 29th Engrs | |

| LOCATION | STATION |
|---|---|
| California | Burdell |

| OBSERVER | INST. (TYPE) (NO.) | DATE |
|---|---|---|
| Cpl. P.E. Smith | Wild T-2 #146875 | 22 Aug 55 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| Red | 0 00 00.00 | | | 0 00 00.00 | |
| Ref. No.1, d=7,342m | 6 27 37 | | | | |
| Hilman Water Tank | 21 14 36.3 | | | | |
| HICKS | 42 22 31.7 | | | | |
| LINCOLN | 65 11 58.8 | | | | |
| Ref. No.2, d=6.751m | 84 13 21 | | | | |
| Ecc. Light to HICKS, d=0.03 | 96 42 | | | | |




* These columns are for office use and should be left blank in the field.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 28 Aug 55 | W. C. A. | 28 Aug 55 |

DA FORM 1 FEB 57 1917

*Figure 75. List of Directions (Final) (DA Form 1917).*

angles are not plane angles, but spherical angles, the sum of the three angles of these triangles is slightly larger than the 180° associated with plane triangles. The amount that this sum is larger than the plane triangle sum is known as the spherical excess ($\epsilon$), the computation of which is explained in *f* below. It has been proven that the spherical triangle can be solved by using plane angle formulas if one-third of the spherical excess is subtracted from each of the spherical angles. Therefore, one use of this method of triangulation computation is as a preliminary step to finding the spherical excess in the triangle.

(2) Since the true total of the angles in a triangle is known to be $180°+\epsilon$, it follows that the observed angles must add up to this total. Seldom, if ever, does the occasion arise when the observed angles do total exactly $180°+\epsilon$ in any triangle. Therefore, some correction must be made to each observed angle to perfect the total. Various methods are used to find the values of the corrections, such as a most probable value by least-squares, or approximate corrections as three equal values. Whatever method is used, the algebraic sum of the corrections must equal numerically, but with opposite algebraic sign, the sum of the observed angles minus $(180°+\epsilon)$. This difference between the sum of the observed angles and $180°+\epsilon$ is known as the triangle closure. In the approximate adjustment of the triangle, one-third of the triangle closure is applied, with opposite sign, to each of the observed angles. If the triangle closure is not exactly divisible by 3, the odd value correction is applied to the angle nearest 90°. Applying corrections to the observed angles produces the spherical angles which total exactly $180°+\epsilon$. One-third of the total spherical excess of the triangle is subtracted from each of the spherical angles to obtain the plane angles. If the spherical excess is not exactly divisible by 3, the odd value correction is applied to the angle nearest 90°.

(3) Figure 76 illustrates the triangle numbering system. For computation, the vertices of the triangle are numbered in clockwise order, starting with number 1 at the vertex to which the lengths are to be computed. This means the ends of the known side are numbered 2 and 3. The station names and the angles observed at the station are entered beside the appropriate number. By applying the corrections and spherical excess to the observed angles, the plane angles are obtained.

*b. Computation By Logarithms.* For computation by logarithms, DA Form 1918 (Computation of Triangles) is used. Enter the log of the known distance on the line 2–3 and the log sines of the plane angles in the column headed "Logarithms". The computation process for the unknown sides is then:

(1) Log (2–3) minus log sin (1) plus log sin (2) equals log (1–3).

(2) Log (2–3) minus log sin (1) plus log sin (3) equals log (1–2).

(3) A numerical example is shown in figure 77. In the example, Burdell is station 1, Hicks station 2, and Lincoln station 3. The observed angles are 22°49′27″.1 at Burdell, 49°00′55″.2 at Hicks and 108°09′31″.4 at Lincoln. The distance Hicks-Lincoln is 4,870.241 meters (log dist. 3.687 55045) and the spherical excess is 0″.1 for the triangle. The names of the stations are listed in the station column on the form—(1) Burdell, (2) Hicks, and (3) Lincoln. Then the observed angles are listed in the observed angle colum beside the station where they were observed. Total the angles (179°59′53″.7)

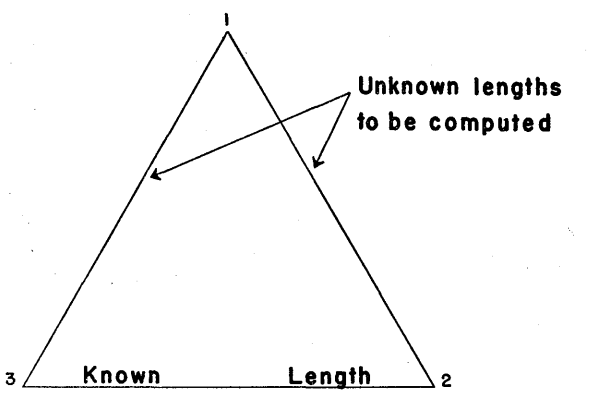



*Figure 76. Triangle diagram.*

| PROJECT | DATE | COMPUTATION OF TRIANGLES (TM 5-237) |
|---|---|---|
| 1-243 | 12 Aug 55 | |

| LOCATION | ORGANIZATION |
|---|---|
| California | 29th Engr. |

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM OR NATURAL |
|---|---|---|---|---|---|---|---|
| | | | | | | | LOGARITHM |
| | 2-3 HICKS - LINCOLN | | | | | | 3.68755045 |
| | 1 Burdell | 22 49 27.1 | +2.1 | 29.2 | 0.0 | 29.2 | 9.58873569 |
| | 2 HICKS | 49 00 55.2 | +2.1 | 57.3 | 0.0 | 57.3 | 9.87788471 |
| | 3 LINCOLN | 108 09 31.4 | +2.2 | 33.6 | -0.1 | 33.5 | 9.97781212 |
| | 1-3 Burdell-LINCOLN | | | | | | 3.97669947 |
| | 1-2 Burdell - HICKS | | | | | | 4.07662688 |
| | | 179 59 53.7 | +6.4 | 00.1 | -0.1 | 0.00 | |
| | 2-3 | | | | | | |
| | 1 | | | | | | |
| | 2 | | | | | | |
| | 3 | | | | | | |
| | 1-3 | | | | | | |
| | 1-2 | | | | | | |
| | | | | | | | NATURAL FUNCTION |
| | 2-3 HICKS - LINCOLN | | | | | | 4870.241 M |
| | 1 Burdell | 22 49 27.1 | +2.1 | 29.2 | 0.0 | 29.2 | 0.38791421 |
| | 2 HICKS | 49 00 55.2 | +2.1 | 57.3 | 0.0 | 57.3 | 0.75489180 |
| | 3 LINCOLN | 108 09 31.4 | +2.2 | 33.6 | -0.1 | 33.5 | 0.95019364 |
| | 1-3 Burdell-LINCOLN | | | | | | 9477.624 |
| | 1-2 Burdell - HICKS | | | | | | 11929.627 |
| | | 179 59 53.7 | +6.4 | 00.1 | -0.1 | 00.0 | |
| | 2-3 | | | | | | |
| | 1 | | | | | | |
| | 2 | | | | | | |
| | 3 | | | | | | |
| | 1-3 | | | | | | |
| | 1-2 | | | | | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 21 Aug 55 | W.C.A. | 25 Aug 55 |

DA FORM 1 FEB 57 1918

*Figure 77. Computation of a triangle using three angles and a known side (DA Form 1918).*

and subtract $(180°+\epsilon)$ from the sum to find the triangle closure.

$(179°59'53''.7) - (180°+0''.1) = -6''.4$

The correction (+6.4) is divided into thirds (+2.1, +2.1, and +2.2) and applied to the angles; the odd part to the angle nearest 90° (large angle), in the correction column, to give the spherical angles in the spherical angle column. Then the spherical excess is divided into thirds (0''.0, 0''.0 and 0''.1) and placed in the spherical excess column. These values are subtracted from the spherical angles to give the plane angles in the plane angle column. Finally, the logarithms of the plane angle and the logarithm of the distance (2–3) are listed in the logarithms columns, and the distances (1–2) and (1–3) are found using the law of sines. The logarithm of (1–2) is 4.07662688, and the logarithm of (1–3) is 3.97669947.

*c. Computation By Natural Functions.* For computation by natural functions, the same DA Form 1918 is used. Enter the length of the known distance (2–3) and the *natural* sines of the plane angles in the column labeled Logarithms (strike out Logarithms when using natural functions). The computation process is then—

(1) Length (2–3) divided by sin (1), then multiplied by sin (2) equals length (1–3).

(2) Length (2–3) divided by sin (1), then multiplied by sin (3) equals length (1–2).

*d. Computation Using Two Sides and Included Angle.*

(1) On DA Form 1919, Triangle Computation Using Two Sides and Included Angle (fig. 78), sides $a$ and $b$ and angle $C$ are known. The problem is to find side $c$, and angles $A$ and $B$.

(2) Call the longer known side $a$ and enter log $a$, log $b$, and the measured angle $C_s$ on the form. If the triangle is large and a precise solution is required, the spherical excess in the triangle should be computed by adding log $m$ (from table of log $m$ in TM 5–236), log sin $C_s$, log $a$, and log $b$. This sum will be log spherical excess. This computation is illustrated in the upper right-hand corner of the form. Subtract ⅓ of the spherical excess from $C_s$ to obtain $C_\rho$ the plane angle. Subtract $\frac{1}{2}C_\rho$ from 90° to find $\frac{1}{2}(A_\rho+B_\rho)$. Subtract log $b$ from log $a$, the result being log tan $(45°+\phi)$. From a table of logarithms of trigonometric functions, extract the angle $(45°+\phi)$. From this angle, subtract 45° to obtain angle $\phi$. From the table, find log tan $\phi$ and log tan $\frac{1}{2}(A_\rho+B_\rho)$. Add these logs to find log tan $\frac{1}{2}(A_\rho-B_\rho)$. From the table, find angle $\frac{1}{2}(A_\rho-B_\rho)$. Add angle $\frac{1}{2}(A_\rho+B_\rho)$ and angle $\frac{1}{2}(A_\rho-B_\rho)$ to obtain angle $A_\rho$. Subtract angle $\frac{1}{2}(A_\rho-B_\rho)$ from angle $\frac{1}{2}(A_\rho+B_\rho)$ to obtain angle $B_\rho$. The sum of angles $A_\rho$, $B_\rho$, and $C_\rho$ should be 180°. Side $c$ is now computed by the sine law. Log $a$ plus log sin $C\rho$, minus log sin $A\rho$, equals log $c$. Since the solution for angles $A_\rho$ and $B_\rho$ can be in error and the sum of the angles $A_\rho$, $B_\rho$, $C_\rho$ still equal 180°, a further check must be made on the solution. This check can be made by solving the triangle twice by the sine law, using first the fixed length $a$ and then the fixed length $b$ as the starting line. Agreement of the computed lengths in these two computations will prove the value of $A_\rho$ and $B_\rho$.

*e. Machine Computation of Triangles.*

(1) DA Form 1920, Triangle Computation (For Calculating Machine) was designed especially for use with the calculating machine (fig. 79). All four common cases for triangle solution can be solved; and a solution of the three-point problem can be made on this form.

(2) The four cases to be solved:

(*a*) Given one side and either two or three angles.

(*b*) Given two sides and an angle opposite one of them.

(*c*) Given two sides and the included angle.

(*d*) Given three sides.

(3) Since there is space for four triangle computations on the form, one case will be computed in each space. The three-point problem will be solved separately. All of the problems are taken from the triangulation scheme shown herein.

(4) A general explanation of the form is given to explain a few of the headings that might not be too clear. The Symbol column is for use when the triangulation is to be adjusted and the angles are designated by some kind of symbol.

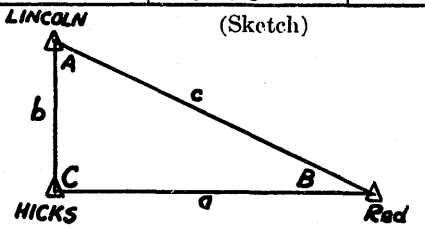
| PROJECT | TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE (TM 5-237) | |
|---|---|---|
| 1-243 | | |
| LOCATION: California | ORGANIZATION: 29th Engr. | DATE: 22 Aug 38 |

$\left[\frac{a}{b}=\tan(45°+\phi)\right]$ (Call longer side $a$). $\tan\frac{1}{2}(A_p-B_p)=\tan\phi\tan\frac{1}{2}(A_p+B_p)$: $\left[c=\frac{a\sin C_p}{\sin A_p}\right]^*$

| | | | | | |
|---|---|---|---|---|---|
| $C_s$ | 91° 29′ 08.6″ | Log $a$ | 3.90704529 | Log $m$ | $\bar{1}$.40471 |
| Sph. excess / 3 | 0.1 | Log $b$ | 3.68755045 | Log sin $C_s$ | 9.99985 |
| $C_p$ | 91 29 08.5 | Log tan $(45°+\phi)$ | 0.21949484 | Log $a$ | 3.90705 |
| $\frac{1}{2}C_p$ | 45 44 34.25 | $(45°+\phi)$ | 58° 53′ 56.626″ | Log $b$ | 3.68755 |
| $90°-\frac{1}{2}C_p=\frac{1}{2}(A_p+B_p)$ | 44 15 25.75 | $\phi$ | 13 53 56.626 | Log sph. ex. | 8.99916 |
| $\frac{1}{2}(A_p-B_p)$ | 13 33 23.56 | Log tan $\phi$ | 9.39350083 | Sph. excess | 0.09987 |
| Sum $=A_p$ | 57 48 49.31 | Log tan $\frac{1}{2}(A_p+B_p)$ | 9.98873737 | | |
| Diff $=B_p$ | 30 42 02.19 | Log tan $\frac{1}{2}(A_p-B_p)$ | 9.38223820 | | |
| $C_p$ | 91 29 08.5 | | | | |

| | |
|---|---|
| Log $a$ | 3.90704529 |
| Log sin $C_p$ | 9.99985398 |
| Colog sin $A_p$ | 0.07246512 |
| Log $c$ | 3.97936439 |

(Sketch)




CHECK COMPUTATION

| No. | STATION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | LOGARITHM |
|---|---|---|---|---|---|
| | 2-3 Red - HICKS | | | | 3.90704529 |
| | 1 LINCOLN | 57 48 49.31 | — | 49.31 | 9.92753488 |
| | 2 Red | 30 42 02.19 | — | 02.19 | 9.70804002 |
| | 3 HICKS | 91 29 08.6 | 0.1 | 08.5 | 9.99985398 |
| | 1-3 LINCOLN - HICKS | | | | 3.68755043 |
| | 1-2 LINCOLN - Red | | | | 3.97936439 |
| | | | 0.1 | | |
| | 2-3 HICKS - LINCOLN | | | | 3.68755045 |
| | 1 Red | 30 42 02.19 | | 02.19 | 9.70804002 |
| | 2 HICKS | 91 29 08.6 | 0.1 | 08.5 | 9.99985398 |
| | 3 LINCOLN | 57 48 49.31 | | 49.31 | 9.92753488 |
| | 1-3 Red - LINCOLN | | | | 3.97936441 |
| | 1-2 Red - HICKS | | | | 3.90704531 |

*The subscripts $s$ and $p$ on this form refer to spherical and plane angles respectively.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 25 Aug 38 | W. C. A. | 28 Aug 38 |

DA FORM 1 FEB 57 1919

*Figure 78. Computation of a triangle using two sides and the included angle (DA Form 1919).*

PROJECT 1-243

**TRIANGLE COMPUTATION** (FOR CALCULATING MACHINE) (TM 5-237)

LOCATION California | ORGANIZATION 29th Engrs | DATE 19 Aug 55

| SYMBOL | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE | SINE | DISTANCE | | SIDE |
|---|---|---|---|---|---|---|---|---|---|---|
| Case I | 1 Burdell | 22° 49′ 27.″1 | +2.7 | 29.8 | 0.0 | 29.8 | 0.38791690 | 4870.241 | 2–3 | HICKS-LINCOLN |
| | 2 HICKS | 49 00 55.2 | +0.8 | 56.0 | 0.0 | 56.0 | 0.75488767 | 9477.506 | 1–3 | Burdell-LINCOLN |
| | 3 LINCOLN | 108 09 31.4 | +2.9 | 34.3 | 0.1 | 34.2 | 0.95019259 | 11929.532 | 1–2 | Burdell-HICKS |
| | | | | | | | | D=Ratio, side/sine | | 12554.8565 |
| Case II | | | | | | | | | | |
| | 1 Red | 30 41 59.7 | — | 59.7 | 0.0 | 59.7 | 0.51054167 | 4870.241 | 2–3 | HICKS-LINCOLN |
| | 2 HICKS | (91 29 22.8) | — | 22.8 | 0.1 | 22.7 | 0.99966204 | 9536.136 | 1–3 | Red-LINCOLN |
| | 3 LINCOLN | 57 48 37.6 | — | 37.6 | 0.0 | 37.6 | 0.84629018 | 8073.067 | 1–2 | Red-HICKS |
| | | | | | | | | D=Ratio, side/sine | | 9539.3604 |
| Case III | | | | | | | | | | |
| | 1 Red | 38 50 24.6 | — | 24.6 | 0.0 | 24.6 | 0.62715001 | 6428.344 | 2–3 | BLACK-HICKS |
| | 2 BLACK | (51 57 49.6) | — | 49.6 | 0.0 | 49.6 | 0.78762137 | 8073.192 | 1–3 | Red-HICKS |
| | 3 HICKS | 89 11 45.9 | — | 45.9 | 0.1 | 45.8 | 0.99990156 | 10249.081 | 1–2 | Red-BLACK |
| | tan A = 0.80517532 | | | | | cos B = +0.01403102 | | D=Ratio, side/sine | | 10250.0899 |
| Case IV | | | | | | denominator = 7982.9958 | | | | |
| | 1 Burdell | 22 49 29.8 | — | 29.8 | 0.0 | 29.8 | 0.38791690 | 4870.241 | 2–3 | HICKS-LINCOLN |
| | 2 HICKS | 49 00 56.0 | — | 56.0 | 0.0 | 56.0 | 0.75488764 | 9477.506 | 1–3 | Burdell-LINCOLN |
| | 3 LINCOLN | 108 09 34.3 | — | 34.3 | 0.1 | 34.2 | 0.95019262 | 11929.532 | 1–2 | Burdell-HICKS |
| | 1.92169437 | | | | | 2s = 2.6277279 | | D=Ratio, side/sine | | 12554.8565 |
| | Cos A = 0.92169437 | | | | | bc = 1.1306221 | | (s-a) = 0.82683985 | | |

Case I a/sin A=b/sin B=c/sin C Given: 3 angles, 1 side

Case II sin B=b sin A/a Given: 2 sides and an angle opposite

Case III tan A=a sin B/c−a cos B Given: 2 sides and included angle

Case IV cos A=[2s(s−a)/bc]−1 s=1/2(a+b+c) Given: 3 sides

COMPUTED BY RAS DATE 23 Aug 55

CHECKED BY W.C.A DATE 25 Aug 55

DA FORM 1 FEB 57 1920

*Figure 79. Computation of triangles (DA Form 1920).*

The Corr'n column contains the correction to be applied to each angle as determined by an adjustment. The Side column provides space to label the lengths found by computation by name as well as by number. The ratio, D, of side to sine is the length of a side divided by the sine of the angle opposite that side. The ratios of the sides to sines should be constant in a triangle. For instance, in case (1) side 2–3 (4,870.241 meters) divided by the sine of angle 1 (sine 22° 49′ 29″.8 = 0.38791690) is a ratio, D, of 12,554.8565. Now side 1–3 divided by the sine of angle 2 should be almost exactly the same as the ratio 2–3/sin 1, and it is 12,554.8560. The same hold true for the ratio of 1–2/sin 3 = 12,554.8569. These ratios can be used as a check on the computations.

(5) For case (1), one side and either two or three angles in the triangle are given. If only two angles are given the third is concluded, and if three are given they must be corrected to close the triangle. For the illustration of case (1), the triangle Burdell-Hicks-Lincoln was used with the three angles and side Hicks-Lincoln (4,870.241 meters) given. The corrections to the given angles are from an adjustment; the spherical excess is computed as explained in paragraph 57*a* and is applied to the spherical angles to obtain the plane angles. The sines of the plane angles are entered in the column so headed. The given distance (2–3) of 4,870.241 meters is entered in the distance column. The triangles are written as previously explained for the triangulation net. The given distance

is divided by the sine of angle 1 (the angle opposite), 4,870.241 ÷ 0.38791690 to find the ratio, D. The ratio is multiplied by sin (2) to obtain the length 1–3, and again by sin (3) to obtain the length 1–2. The computation of the triangle for case (1) is now complete.

(6) In case (2), two sides and an angle opposite one of those sides are given. For this illustration, the triangle Red-Hicks-Lincoln will be solved with the known components of the triangle being the angle at Red (30°41′59″.7) and the sides Hicks-Lincoln (4,870.241) and Red-Hicks (8,073.067). The general formula for solving this case is—

$$\sin B = \frac{b \sin A}{a},$$

in which angle A and sides $a$ and $b$ are known. Angle B is found from its sine and the third angle in the triangle is concluded. In this example the ratio, D, is found by dividing side 2–3 by sin (1) to get 9,539.3604. The sine of angle (3) is now obtained by dividing side 1–2 by D. Angle (3) is found from its sine. Angle (2) is concluded, its sine obtained, and side 1–3 found by multiplying D times sine (2). The triangle is now completely solved. Notice the first two cases use only three lines of the five provided for each triangle. The extra two lines are needed only for cases (3) and (4).

(7) Case (3) is a triangle is which two sides and the included angle are given. The general formula for this solution is:

$$\tan A = \frac{a \sin B}{c - a \cos B};$$

$a$ and $c$ being the given sides and angle B the angle between them. The triangle chosen to illustrate the case is Red-Black-Hicks, and the known values are those obtained from the angle method of adjusting the triangulation. This same triangle was solved on DA Form 1919 as part of the angle method of adjustment. The known data are sides Black-Hicks, Red-Hicks, and the angle at Hicks. Numerically, these values are 6,428.344 meters, 8,073.192 meters, and 89°11′45″.9, respectively. The first step in the solution is to obtain the sine and cosine of the known angle from tables. The correct algebraic sign of the cosine is essential to the correct solution. The cosine for angles between 90° and 180° is negative. The cosine is entered on the form on the fourth line of the sine column. To avoid confusion, the word "cos" may be written beside the value if desired. In this case, cos (3) = 0.01403102. For this solution, it is usually better to call the shorter known side $a$. In this example, then, $a$ = (2–3) = 6,428.344, $c$ = 8,073.192, B = 89°11′45″.9. The denominator of the fraction in the formula, $c - a \cos B$, is solved and written below cosine B and can be labeled "Denom." The numerator of the fraction, $a \sin b$, is multiplied on the machine and divided by the denominator previously written down to obtain tan A, which is the tangent of the angle opposite the side $a$. Notice that the product, $a \sin b$, need not be written in the Observed Angle column on line 4 (0.80517532). The angle A [in this example, angle (1) at Red] is taken out for tan A and entered in the "Plane Angle" column. Remember that all the angles used in these triangle computations are plane angles. Only the seconds of the plane angle are written in that column; the degrees and minutes are written in the Observed Angle column. Angle (2), at Black, is now concluded. The sines of angles (1) and (2) are found and the ratio, D, obtained, and the missing length found by multiplying D times the sine of (3). The ratio of the side to sines should now be used to check the computed and concluded angles. The solution of the triangle is now complete. A comparison of this solution with the solution made on DA Form 1919 in the angle method of adjustment of the same triangle, shows that exactly the same results are obtained.

(8) Case (4) is the solution of a triangle when three sides are given. For this example, the triangle used for case (1), Burdell-Hicks-Lincoln, was used with the lengths

found by solving case (1) becoming the given data for case (4). The general formula for solving case (4) is—

$$\cos A = \frac{2s(s-a)}{bc} - 1$$

in which $s = \frac{1}{2}(a+b+c)$; $a$, $b$, and $c$ being the given lengths. The angle A is opposite side $a$. In this example, side $a$ is Hicks-Lincoln (4,870.241), $b$ is Burdell-Lincoln (9,477.506) and $c$ is Burdell-Hicks (11,929.532). Side $a$ should not be a very short line as compared to the other two sides. For the sake of convenience throughout the computation, the decimal point in the given lengths may be moved the same number of places to the left in each to keep the numbers near unity. A four-place shift was sufficient in this example. No change in the shape of the triangle or size of the angles will be incurred by this shift of the decimals. The lengths are now used as 0.4870241, 0.9477506, and 1.1929532, although they are not written down as such. The solution of the formula is now begun by finding $2s$, $bc$, and $(s-a)$. $2s$ is found simply by adding $a+b+c$. This result is 2.6277279 and is written on line 4 of the form. It can be labeled "$2s$" if desired. One-half of $2s$ can be found mentally and from this quantity subtract $a$. The result is $(s-a)$ and is labeled and entered on the form in any convenient space on lines 4 or 5. The product $(bc)$ is found and entered on the form. Numerical values for these three items in this example are: $2s = 2.6277279$, $(s-a) = 0.82683985$, $bc = 1.13062211$. Cos A is now obtained by dividing the product of $2s \times (s-a)$ by $bc$ and then subtracting 1 from the result. Angle A is now found using cos A as the argument. If cos A is positive, angle A is less than 90°; if cos A is negative, angle A is between 90° and 180°. In this example, cos A = +0.92169437 (positive, so less than 90°) and angle A = 22°49′29″.8. Next find sin A, and from this the ratio D. Using D and the given sides, the sines of the other two angles in the triangle are found by the relation, $\text{sine} = \frac{\text{side}}{D}$ from which $\sin B = \frac{b}{D}$ and $\sin C = \frac{c}{D}$. From the sines the angles may be found. If angle A is less than 90°, it is possible that either angle B or C may be more than 90°. The angle over 90° can never be opposite the short side. In the illustration, D is found to be 12554.8565; sin B (sin 2) equals 0.75488764, sin C (sin 3) equals 0.95019262. From sin B, angle B could be 49° or 130°. Using the statement concerning the short side, the correct angle for B in this triangle is 49°00′56″.0, as side $b$ is shorter than side $c$ (9,477 to 11,929). Now from sin C, angle C could be 71° or 108°. Since the three angles in the triangle must total 180°, it is easy to see that the correct value for angle C in this triangle is 108°09′34″.2. The solution is now complete.

(9) Comparing the angles computed for the triangle by solving case (4) with the angles used in case (1), it is evident that the case (4) solution is correct, since the computed angles are exactly the same as the original angles of case (1).

*f. Spherical Excess Computation.*

(1) It is known from spherical trigonometry that the three angles of a spherical triangle add up to slightly more than 180°. The difference between 180° and the total of the angles in the triangle is called the spherical excess ($\epsilon$). As this spherical excess amounts to approximately 1 second for every 75 square miles of area, it is evident that for lower-order triangulation or for triangles covering a small area the amount of spherical excess is negligible and need not be computed.

(2) The formula for computing spherical excess is:

$$\epsilon = \frac{a_1 b_1 \sin C_1 (1 - e^2 \sin^2 \phi)^2}{2a^2(1-e^2) \sin 1''}$$

In the formulas for the spherical excess, $a_1$, $b_1$, and $C_1$ are two sides and the included angle of the triangle; $e^2$ is the eccentricity squared; $a$ the semimajor axis of the spheroid of reference; and $\phi$ the mean latitude of the vertices of the triangle. Since $e^2$ and $a$ are constant

values for each spheroid it has been possible to tabulate a factor $m$ for $\frac{(1-e^2 \sin^2 \phi)^2}{2a^2(1-e^2) \sin 1''}$ using the mean latitude as an argument. Thus the formula for spherical excess becomes $\epsilon = a_1 b_1 \sin C_1 m$. A table for log $m$ for the Clarke 1866 spheroid expressed in meters is given in TM 5-236 and USC&GS Sp. Pub. 138 and 247. Sp. Pub. 247 and TM 5-236 have tables of the natural value of $m$ for the Clarke 1866 spheroid. USC&GS Sp. Pub. 200 has a table of log $m$ for the International Ellipsoid. For an equilateral triangle with 200-kilometer sides, the above formula will give a value of the spherical excess too small by 1/100 of a second. Therefore, this formula is entirely adequate for all normal triangulation.

(3) Since spherical excess is a function of the area on the sphere, the total spherical excess in a triangulation figure is constant. For example, the sum of the spherical excess in each pair of triangles making up a quadrilateral should be equal.

(4) Figure 80 illustrates the computations and application of spherical excess. Given the quadrilateral Lincoln-Burdell-Red-Hicks:

| | Log |
|---|---|
| $a_1$=Lincoln-Burdell | =3.97670 |
| $b_1$=Lincoln-Hicks | =3.68755 |
| $C_1$=Hicks-Lincoln-Burdell | =108° 09′ 31″ |
| log sin $C_1$ | =9.97781 |

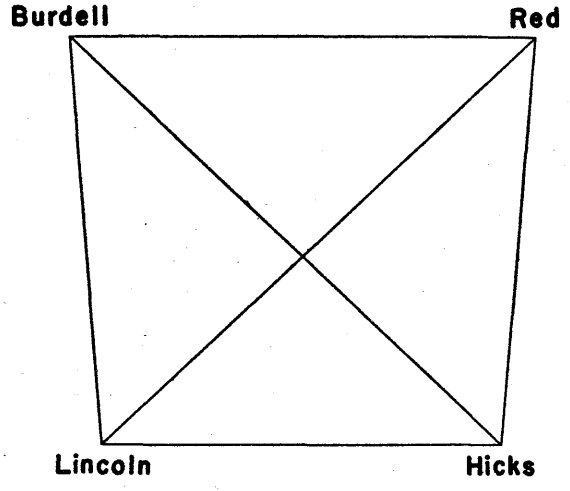



*Figure 80. Quadrilateral diagram for spherical excess.*

Mean latitude of Lincoln, Hicks, and Burdell is 38°08′

Log $m$ (Clarke 1866) =1.40471−10

The spherical excess of only one triangle (Burdell-Hicks-Lincoln) is illustrated, since the computation for the other three triangles in the quadrilateral is merely a repetition of the process. Using the formulas: $\epsilon = a_1 b_1 \sin C_1 m$, and arranging the given data in columnar form:

| | |
|---|---|
| log $a_1$ | =3.97670 |
| log $b_1$ | =3.68755 |
| log sin $C_1$ | =9.97781 |
| log $m$ | =1.40471 |
| log $\epsilon$ | =9.04677 |
| $\epsilon$ | =0.111 sec. |

This computation can be made on a regular triangle computation sheet as a preliminary step. Logarithms need be carried only to five places. Splitting the quadrilateral into two pairs of triangles to facilitate recording the spherical excess for each triangle, shows the sums for each pair of triangles to be equal (fig. 81).

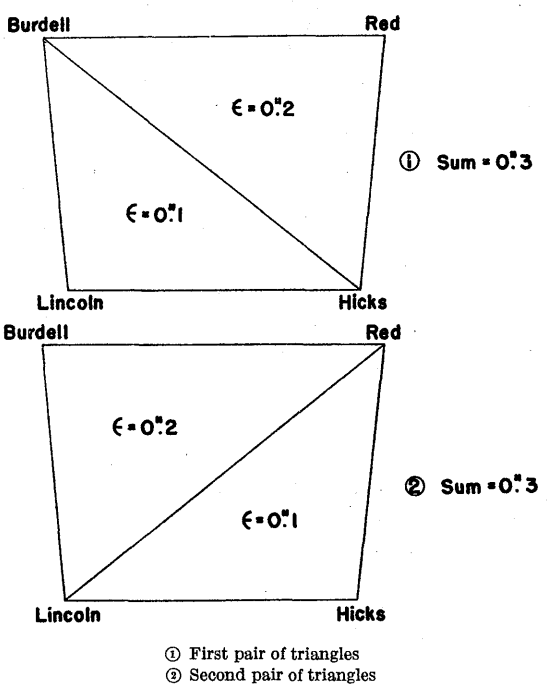



*Figure 81. Equality of spherical excess.*

## 58. Reduction to Center

*a. Need for Reduction.* If, when observing horizontal directions, the instrument or the signal is not set exactly over the triangulation station, the instrument or the signal is said to be eccentric to the true station. Directions measured from an eccentric instrument, or to an eccentric signal, must be corrected to the directions that would have been observed if the instrument or object had been centered over the station. The computation necessary to correct these directions is called *reduction to center* or *eccentric reduction.*

*b. Computation.*

(1) The formula for computing the reduction to center (symbolized $c$) is as follows:

$$c=\frac{d \sin \alpha}{s \sin 1''}$$

and is derived from the law of sines. Figure 82 illustrates the general form of the problem in which—

$d$=measured distance from eccentric to true station.

$\alpha$=clockwise angle from the true station to the distant station as measured at the eccentric station.

$s$=distance from the true station to the distant station from preliminary triangle computation. At times, it may be possible to obtain only an approximation of $s$ which means that successive computations must be made until no change occurs in the computed value of $c$.

From figure 82 and the law of sines, the relation can be written as follows:

$$\frac{d}{\sin c}=\frac{s}{\sin \alpha}$$

from which:

$$\sin c=\frac{d \sin \alpha}{s}$$

Since the basic nature of an eccentric is such that the eccentric distance, $d$, is always very small as compared to the distance, $s$, it is evident that angle $c$, the eccentric reduction, will always be very small. Under these conditions, it is possible to use the approximation that the sine of a small angle is very nearly directly proportional to the angle. Therefore, the value of a small angle in seconds can be found by dividing the sine of the angle by the sine of 1 second, thus:

$$c=\frac{\sin c}{\sin 1''}$$

Using this approximation, the equation,

$$\sin c=\frac{d \sin \alpha}{s}$$

is converted to the formula for eccentric reduction by dividing both sides by $\sin 1''$,

$$\frac{\sin c}{\sin 1''}=\frac{d \sin \alpha}{s \sin 1''}$$

and then substituting $c$ for $\frac{\sin c}{\sin 1''}$ which gives:

$$c=\frac{d \sin \alpha}{s \sin 1''}$$

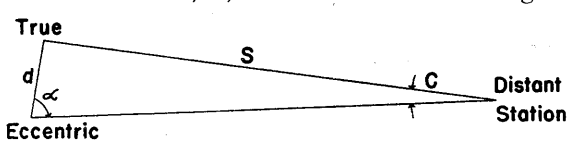



*Figure 82. Reduction to center diagram.*

(2) In the development of the formula, $\alpha$ was called a clockwise angle from the true station to the distant station as measured at the eccentric station, but it would be just as correct to call a direction from the eccentric station to the distant station by observing the rule that the direction from the eccentric to the true station is always zero. Following this rule, it can be stated that for all directions ($\alpha$) less than 180°, $c$ is positive; and for directions more than 180°, $c$ is negative.

(3) When an eccentric instrument setup is used to measure horizontal directions, the initial direction of the list of directions receives an eccentric correction as well as do the rest of the directions on the list. The correction to the initial direction naturally affects all the directions referred to the initial direction. In order to retain the initial direction as zero, the correction to the initial direction is applied, with opposite sign, to all the other directions on the list.

(4) To illustrate the reduction for an eccentric instrument, the sketches in figure 83

were drawn showing the two cases of the direction ($\alpha$) greater and less than 180°. In each case, a dashed line has been drawn through the eccentric, parallel to the line from the true station to the distant station. It is the direction of this dashed line that is computed, as its direction is the same as that from the true station to the distant station. In figure 83 ① in which $\alpha$ is more than 180°, it can be seen that direction $\alpha$ is larger than it should be by the angle $c'$ (which equals $c$). Therefore, the reduction, $c'$, is minus when applied to the observed direction. In figure 83 ②, $\alpha$ is less than 180°, and is smaller than it should be by the angle $c'$ (equals $c$); therefore, the reduction, $c'$, is plus when applied to the observed direction.

(5) Figure 83 also illustrates the condition when an eccentric signal is observed from the distant station. The only differences to be pointed out are that the eccentric reduction in this case is angle $c$ and the reduction is applied to the direction observed at the distant station even though it is computed at the eccentric.

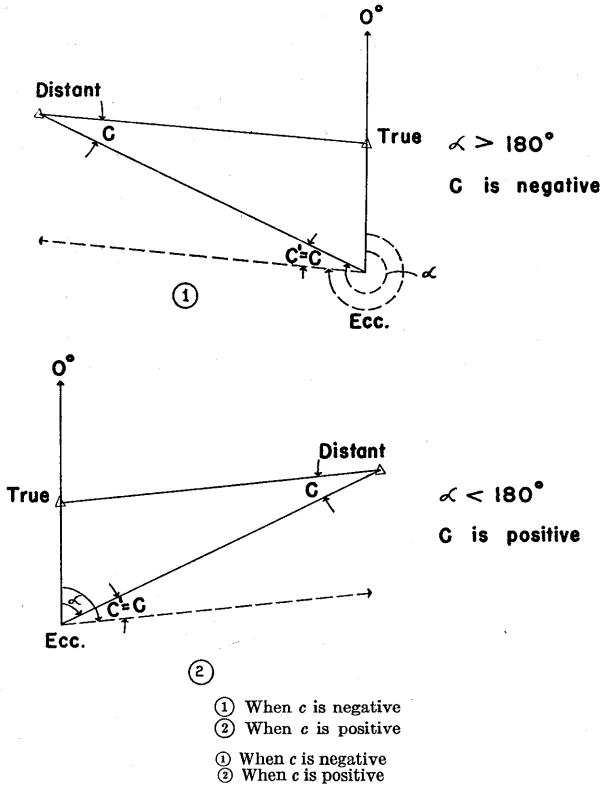



*Figure 83. Eccentric reduction, sign of* c.

The directions used in this computation are the one observed at the true station with the initial direction changed to the eccentric. Following the rule stated previously, the direction from the true to the eccentric must be 180°. Therefore, to prepare all directions on the list of directions for use in the computation of the reduction, add 180° to the direction to the eccentric as given on the list, and then subtract this sum from each of the other directions on the list, for lines on which an eccentric reduction is required.

(6) The situation sometimes arises where an eccentric instrument observes an eccentric object. The solution of this problem is merely a combination of the two previously discussed reductions. A reduction is computed for each eccentric separately, and then the two reductions are combined algebraically on the list of directions.

(7) The eccentric reduction may be computed on DA Form 1921. (Reduction to Center) (fig. 84). Notice that $d$ and $s$ are labeled "meters" on the form. It is not absolutely necessary that $d$ and $s$ be in meters, but they must both be in the same unit of measure. The form is set up for use with logarithms, and the colog of sin 1″ is printed on the form for convenience. Use only 5 decimals in the logarithms in the computations, and the directions ($\alpha$) to the nearest minute.

(*a*) The procedure in the use of the form is first to enter all the given data: distance $d$, station names, direction to stations ($\alpha$), and log $s$ from the preliminary triangle computations. From logarithmic tables, obtain log $d$ and the log sines of the directions, and enter in the appropriate spaces. Add log $d$ and colog sin 1″, and record the sum. Subtract log $s$ from log sin $\alpha$, and enter the results in the column headed $\log\left(\frac{\sin\alpha}{s}\right)$. Add the sum of log $d$ plus colog sin 1″ to the values in the $\log\left(\frac{\sin\alpha}{s}\right)$ column. The results are the logarithms of the reductions (in seconds) for each direction. Antilog

| PROJECT | REDUCTION TO CENTER (TM 5-237) |
|---|---|
| 37-553 | |

LOCATION: District of Columbia

TYPE OF STATION: ☒ ECCENTRIC STATION Ken
☐ ECCENTRIC OBJECT AT STATION

ORGANIZATION: A & S, Inc

DATE: 21 Jan. 56

Log $d$ = 0.54020
Colog sin 1″ = 5.31443
Sum = 5.85463

Distance ($d$) (*meters*): 3.469

| STATION | $a$ | Log sin $a$ | Log $s$ ($s$ in meters) | Log $\left(\frac{\sin a}{s}\right)$ | Logarithm of Reduction in Seconds | Reduction $= c$ |
|---|---|---|---|---|---|---|
| Center | ° 0 00 / ~~180 00~~ | | | | | ″ |
| Chevy | 183 18 | 8.76015 | 3.75097 (100) | 5.00918 (5) | 0.86381 (78) | −7.31 |
| Tank west of Δ Dulce | 212 22 | 9.72863 | 3.73904 (12) | 5.98959 (1) | 1.84422 (14) | −69.86 (5) |
| Forest Glen Standpipe | 136 43 | 9.83608 | 3.43240 (65) | 6.40368 (43) | 2.25831 (06) | +181.26 (16) |
| Home | 149 50 | 9.70115 | 4.05161 (3) | 5.64954 (2) | 1.50417 (5) | + 31.93 |
| Bu. of Stand. wireless pole | 175 35 | 8.88654 | 3.98487 | 4.90167 | 0.75630 | + 5.71 |
| Reno | 180 47 | 8.13581 | 3.92632 (5) | 4.20949 (6) | 0.06412 (09) | − 1.16 |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 23 June 56 | W. C. A. | 24 June 56 |

DA FORM 1 FEB 57 1921

*Figure 84. Computations of a Reduction to Center (Eccentric Station) (DA Form 1921).*

these results to obtain the reduction in seconds. Attach the algebraic sign to the reductions according to the rules previously stated.

(*b*) Apply the reductions to the list of directions, and recompute the preliminary triangles using the reduced directions to obtain the angles. From this set of preliminary triangles, take the values of log *s* required on the reduction to center. Using these new values of log *s*, recompute the eccentric reductions and repeat this process until no change occurs in the reductions. If the recomputations of the preliminary triangles produce no change or only an insignificant change in log *s*, it is not necessary to recompute the eccentric reductions.

(8) An illustration of the reduction for an eccentric instrument is described below. The example shows the original list of directions observed at Ken Ecc. (fig. 85), the preliminary triangle computations (fig. 86), and the eccentric reduction computation (fig. 84). The figures for the second computation of the preliminary triangles and the second computation of the eccentric reduction are written above the original figures.

(*a*) For the first computation of the preliminary triangles, Ken-Home-Reno and Ken-Home-Chevy, the angles at Ken are concluded. This will give a better approximation of the distance *s* than would be obtained by using the observed angle at Ken Ecc. For the other triangles, the directions at Ken

| PROJECT 37-553 | ORGANIZATION A&S. Inc. | | | LIST OF DIRECTIONS (TM 5-237) | |
|---|---|---|---|---|---|
| LOCATION District of Columbia | | | STATION Ken Ecc. | | |
| OBSERVER G.L. Berkin | INST. (TYPE) (NO.) Wild T-2 # 145974 | | | | DATE 1 Dec. 55 |
| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| Chevy | 0 00 00.00 | − 7.31 | | 0 00 00.00 | |
| Tank west of △ Dulce | 29 03 37.0 | −1 09.8 | | 29 02 34.4 5 | |
| Ken (center) 3.469 m. | 176 42 | | | | |
| Forest Glen Standpipe | 313 24 53.0 | +3 01.3 2 | | 313 28 01.6 5 | |
| Home | 326 31 30.21 | + 31.93 | | 326 32 09.45 | |
| Bur. of Stand. wireless pole | 352 17 20.8 | + 5.7 | | 352 17 33.8 | |
| Reno | 357 28 48.63 | − 1.16 | | 357 28 54.78 | |

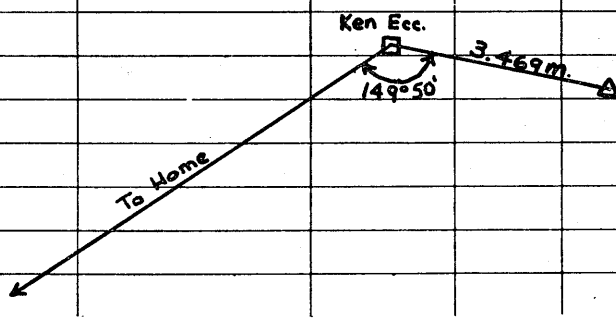



*Figure 85. List or directions at eccentric station (DA Form 1917).*

PROJECT 37-553 | DATE 18 Jan 56 | COMPUTATION OF TRIANGLES (TM 5-237) Preliminary

LOCATION Maryland | ORGANIZATION A & S, Inc.

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM |
|---|---|---|---|---|---|---|---|
| | 2-3 Home - Reno | | | | | | 3.772214 |
| | 1 Ken | (30 56 ~~55.6~~ 45.3) | +3.5 | 48.8 | | | 9.711192 (69) |
| | 2 Home | 47 09 57.7 | +3.4 | 61.1 | | | 9.865298 (304) |
| | 3 Reno | 101 53 06.7 | +3.4 | 10.1 | | | 9.990588 (7) |
| | 1-3 Ken - Reno | | | | | | 3.926320 (49) |
| | 1-2 Ken - Home | | | | | | 4.051610 (32) |
| | | 49.7 | | | | | |
| | 2-3 Home - Chevy | | | | | | 3.860903 |
| | 1 Ken | (33 27 ~~57.3~~ 50.6) | +2.3 | 52.9 | | | 9.741499 (85) |
| | 2 Home | 25 20 54.6 | +2.2 | 56.8 | | | 9.631569 (78) |
| | 3 Chevy | 121 11 08.1 | +2.2 | 10.3 | | | 9.932217 (4) |
| | 1-3 Ken - Chevy | | | | | | 3.750973 (96) |
| | 1-2 Ken - Home | | | | | | 4.051621 (32) |
| | | 53.3 | | | | | |
| | 2-3 Ken - Reno | | | | | | 3.926320 (49) |
| | 1 Tank near Dulce | (111 07 ~~06~~ 53.8 46.2) | | | | | 9.969821 (768) |
| | 2 Ken | 31 34 ~~33~~ 39.6 47.2 | | | | | 9.719070 (8839) |
| | 3 Reno | 37 18 26.6 | | | | | 9.782538 |
| | 1-3 Tank - Reno | | | | | | 3.675568 (420) |
| | 1-2 Tank - Ken | | | | | | 3.739036 (119) |
| | 2-3 Reno - Ken | | | | | | 3.926320 (49) |
| | 1 Forest Glen Standpipe | (119 ~~49~~ 46 22.6 19.0) | | | | | 9.938524 (303) |
| | 2 Reno | 16 09 44.2 | | | | | 9.444605 |
| | 3 Ken | 44 03 ~~00~~ 53.2 56.8 | | | | | 9.842287 (1887) |
| | 1-3 For. Glen. - Ken | | | | | | 3.432401 (651) |
| | 1-2 For. Glen - Reno | | | | | | 3.830083 (29933) |
| | 2-3 Reno - Ken | | | | | | 3.926320 (49) |
| | 1 Bur. of Standards Wireless pole | (31 22 14.8 06.8) | | | | | 9.716455 (83) |
| | 2 Reno | 143 26 24.2 | | | | | 9.775001 |
| | 3 Ken | 5 11 21.0 29.0 | | | | | 8.956566 (381) |
| | 1-3 Wireless pole - Ken | | | | | | 3.984866 (7) |
| | 1-2 Wireless pole - Reno | | | | | | 3.166431 (247) |

*Figure 86. Preliminary computations of triangles for reduction to center (DA Form 1918).*

Ecc. must be used because in each case, one of the stations was not occupied. For the second computation of the preliminary triangle, the reduced directions at Ken should be used to obtain all angles at Ken, and the triangles should be corrected where necessary to close to 180°.

(*b*) Attention is now directed to the change in the list of directions. Although the list was originally labeled Ken Ecc., the application of the eccentric reductions has changed the list so that the directions in the Corrected Direction with Zero Initial column are at station Ken.

(9) A second illustration shows the reduction to center for an eccentric object observed. In this illustration, the lists of directions for all the stations involved and the reduction to center computation are shown in figures 87 and 88, but the preliminary triangles are not shown as no new principle is involved in computing the triangles. The two important points to be brought out in this illustration are the methods of obtaining the directions for the eccentric reduction computation and the application of the reductions to the appropriate lists of directions.

(10) To obtain the directions ($\alpha$'s) for the reduction to center form, subtract 180° from the direction from Home to Home Ecc. as referred to Park on the list of directions for Home (222°55′20″−180° =42°55′20″). Since this direction (Home Ecc. to Home) must be 0° according to the rules stated previously, the procedure is to subtract the direction Home Ecc. to Home (42°55′20″) from each direction on the list of directions for Home. The results of this operation are the directions from Home Ecc. to each station on the list and referred to Home as 0°. These are the directions entered on the reduction to center form. For example, on the list of directions for Home, the direction to Park is 0°00′00.″00 as it is the initial; subtracting 42°55′20″ from it gives the direction 317°04′40″, from Home Ecc. to Park as referred to Home. This value is rounded off to 317°05′ for use on the reduction to center form. The direction from Home Ecc. to Cedar is 47°07′49.″6−42°55′20″ =4°12′. The rest of the directions are found in the same manner.

(11) The eccentric reductions are now computed as explained previously, and the algebraic signs attached (fig. 88). The reductions are applied to the individual list of directions. The reduction for Cedar of +3.″18 is applied to the direction to Home Ecc. on the list of directions for Cedar; the reduction for Gerst of +44.″84 is applied to the direction to Home Ecc. on the list for Gerst, and so on. The application of the eccentric reductions automatically changes the observed station from Home Ecc. to Home on the various lists.

(12) In all cases, a sketch of the eccentric conditions should be shown on the list of directions and on the reduction to center computation form.

## 59. Strength of Figure

*a. Introduction.*

(1) For computation purposes, it is often necessary to know the relative strength of the figures involved in the triangulation net. The strength of a figure depends on the size of the length angles in the triangles through which the length is computed, on the number of directions observed in the figure, and on the number of conditions to be satisfied in the figure.

(2) The strength of a figure is designated by the letter R and is computed from the equation:

$$R=\left(\frac{D-C}{D}\right)\sum[\delta_A^2+\delta_A\delta_B+\delta_B^2]$$

in which D=the number of directions observed in the figure; C=the number of conditions to be satisfied in the figure; $\delta_A$ and $\delta_B$ are the logarithmic differences in the sines for 1″ change in the distance angles A and B of a triangle.

(3) In table XLI, TM 5-236, the values tabulated are $[\delta_A^2+\delta_A\delta_B+\delta_B^2]$ in units of the sixth place of logarithms. The two arguments of the table are the distance angles in degrees with the smaller distance angle being given at the top of the table.

| PROJECT 37-553 | ORGANIZATION A & S Inc. | LIST OF DIRECTIONS (TM 5-237) |
|---|---|---|
| LOCATION Maryland | | STATION Cedar |
| OBSERVER F. T. A. | INST. (TYPE) (NO.) Wild T-2 No. 137946 | DATE 16 Jan 56 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| Wallace | 0 00 00.00 | — | | 0 00 00.00 | |
| Home Ecc. | 43 02 07.4 | + 3.2 | | 43 02 10.6 | |
| Takoma | 58 59 17.6 | — | | 58 59 17.6 | |
| Park | 59 29 28.5 | — | | 59 29 28.5 | |

| PROJECT 37-553 | ORGANIZATION A&S, Inc. | LIST OF DIRECTIONS (TM 5-237) |
|---|---|---|
| LOCATION Maryland | | STATION Garfield |
| OBSERVER T. K. E. | INST. (TYPE) (NO.) Wild T-2 # 137946 | DATE 17 Jan 56 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| Insane | 0 00 00.00 | — | | 0 00 00.00 | |
| Home Ecc. | 62 57 25.9 | + 51.3 | | 62 58 17.2 | |
| Stanton | 88 40 09.0 | — | | 88 40 09.0 | |
| Potomac | 93 11 08.1 | — | | 93 11 08.1 | |

| PROJECT 37-553 | ORGANIZATION A&S, Inc. | LIST OF DIRECTIONS (TM 5-237) |
|---|---|---|
| LOCATION Maryland | | STATION Gerst |
| OBSERVER F. T. A. | INST. (TYPE) (NO.) Wild T-2 No. 137946 | DATE 17 Jan 56 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| Garfield | 0 00 00.00 | — | | 0 00 00.00 | |
| Stanton | 12 13 22.0 | — | | 12 13 22.0 | |
| Home Ecc. | 46 20 53.2 | + 44.8 | | 46 21 38.0 | |

| PROJECT 37-553 | ORGANIZATION A & S, Inc. | LIST OF DIRECTIONS (TM 5-237) |
|---|---|---|
| LOCATION Maryland | | STATION Park |
| OBSERVER TKE | INST. (TYPE) (NO.) Wild T-2 No. 137946 | DATE 18 Jan 56 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| Cedar | 0 00 00.00 | — | | 0 00 00.00 | |
| Wallace | 44 45 54.8 | — | | 44 45 54.8 | |
| Home Ecc. | 116 26 12.1 | -1 33.6 | | 116 24 38.5 | |
| Reno | 173 33 28.9 | — | | 173 33 28.9 | |

*Figure 87. Corrected lists of directions (DA Form 1917).*

| PROJECT 37-553 | ORGANIZATION A & S, Inc. | LIST OF DIRECTIONS (TM 5-237) |
|---|---|---|
| LOCATION Maryland | | STATION Home |
| OBSERVER TKE | INST. (TYPE) (NO.) Wild T-2 No. 137946 | DATE 18 Jan 56 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| Park | 0 00 00.00 | | | 0 00 00.00 | |
| Cedar | 47 07 49.6 | | | | |
| Gerst | 137 42 15.9 | | | | |
| Garfield | 152 25 42.7 | | | | |
| Home Ecc. 2.79 m. | 222 55 20 | | | | |

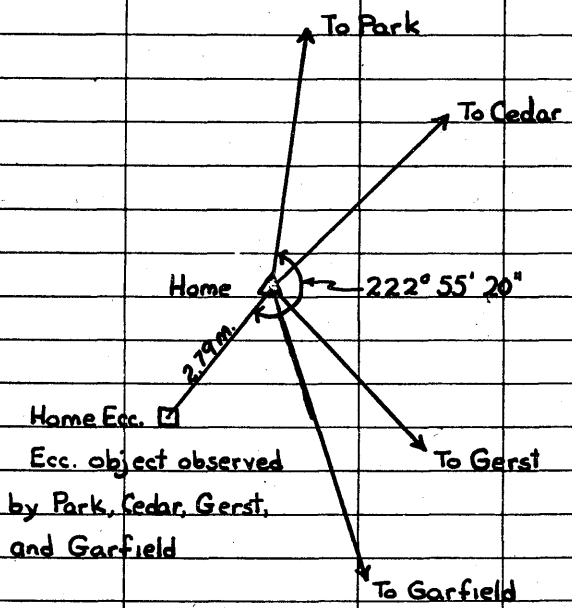



*Figure 87*—Continued.

*b. Use of Table XLI in TM 5–236.* To compare two alternative figures, either quadrilaterals or central-point figures, so far as the strength with which the length is carried is concerned, proceed as follows:

(1) For each figure, take out the distance angles (to the nearest degree if possible) for the best and second-best chains of triangles through the figure. These chains are to be selected at first by estimation, and the estimate is to be checked later by the results of comparison.

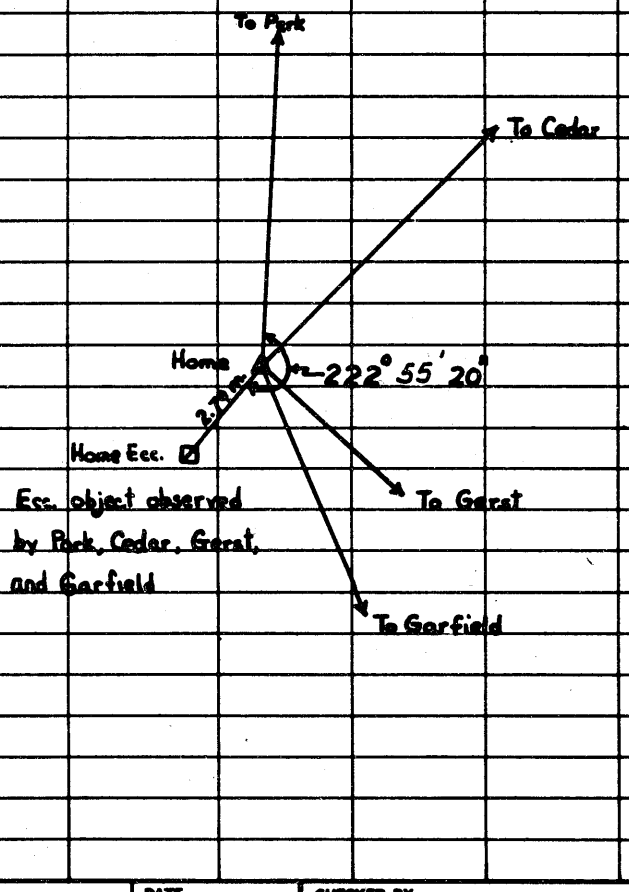
(2) For each triangle in each chain, enter the table with the distance angles as the two arguments and take out the tabular value.

(3) For each chain, the best and second-best, through each figure, take the sum of the tabular values.

(4) Multiply each sum by the factor $\frac{D-C}{D}$ for that figure, where D is the number of directions observed and C is the number of conditions to be satisfied in the figure.

| PROJECT | 37-553 | REDUCTION TO CENTER (TM 5-237) |
|---|---|---|
| LOCATION | District of Columbia | TYPE OF STATION: ☐ ECCENTRIC STATION ☒ ECCENTRIC OBJECT AT STATION Home |
| ORGANIZATION | A&S, Inc. | Log $d$ = 0.44560<br>Colog sin 1" = 5.31443<br>Sum = 5.76003 |
| DATE | 21 Jan. 56 | Distance ($d$) (*meters*) 2.79 |

| STATION | $\theta$ | Log sin $\theta$ | Log $s$ ($s$ in meters) | Log $\left(\frac{\sin\theta}{s}\right)$ | Logarithm of Reduction in Seconds | Reduction $=c$ |
|---|---|---|---|---|---|---|
| Center | ° ′ ~~0 00~~ {180 00} | | | | | " |
| Cedar | 4 12 | 8.86474 | 4.12189 | 4.74285 | 0.50288 | + 3.18 |
| Gerst | 94 47 | 9.99848 | 4.10683 | 5.89165 | 1.65168 | +44.84 |
| Garfield | 109 30 | 9.97435 | 4.02427 | 5.95008 | 1.71011 | +51.30 |
| Park | 317 05 | 9.83310 | 3.62200 | 6.21110 | 1.97113 | -93.57 |




| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RHS | 28 June 56 | W. CA. | 30 Jun 56 |

DA FORM 1 FEB 57 1921

*Figure 88. Computation of a reduction to center (eccentric object) (DA Form 1921).*

The quantities so obtained, namely:

$$\left(\frac{D-C}{D}\right)\sum[\delta_A^2+\delta_A\delta_B+\delta_B^2]$$

will for convenience be called $R_1$ and $R_2$ for the best and second-best chains, respectively.

(5) The strength of the figure is dependent mainly upon the strength of the best chain through it, hence the smaller the $R_1$ the greater the strength of the figure. The second-best chain contributes somewhat to the total strength, and the other weaker and progressively less independent chains contribute still smaller amounts. In deciding between figures, they should be classed according to their best chains, unless the best chains are very nearly of equal strength and their second best chains differ greatly.

*c. Some Values of the Quantity* $\frac{D-C}{D}$.

(1) The starting line is supposed to be completely fixed.

(*a*) For a single triangle, $\frac{4-1}{4}=0.75$.

(*b*) For a completed quadrilateral, $\frac{10-4}{10}=0.60$.

(*c*) For a quadrilateral with one station on the fixed line unoccupied, $\frac{8-2}{8}=0.75$.

(*d*) For a quadrilateral with one station not on the fixed line unoccupied, $\frac{7-2}{7}=0.71$.

(*e*) For a three-sided, central-point figure, $\frac{10-4}{10}=0.60$.

(*f*) For a three-sided, central-point figure with one station on the fixed line unoccupied, $\frac{8-2}{8}=0.75$.

(*g*) For a three-sided, central-point figure with one station not on the fixed line unoccupied, $\frac{7-2}{7}=0.71$.

(*h*) For a four-sided, central point figure, $\frac{14-5}{14}=0.64$.

(*i*) For a four-sided, central-point figure with one corner station on the fixed line unoccupied, $\frac{12-3}{12}=0.75$.

(*j*) For a four-sided, central-point figure with one corner station not on the fixed line unoccupied, $\frac{11-3}{11}=0.73$.

(*k*) For a four-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{10-2}{10}=0.80$.

(*l*) For a four-sided, central-point figure with one diagonal also observed, $\frac{16-7}{16}=0.56$.

(*m*) For a four-sided, central-point figure with the central station not on the fixed line unoccupied and one diagonal observed, $\frac{12-4}{12}=0.67$.

(*n*) For a five-sided, central-point figure, $\frac{18-6}{18}=0.67$.

(*o*) For a five-sided, central-point figure with a station on a fixed outside line unoccupied, $\frac{16-4}{16}=0.75$.

(*p*) For a five-sided, central-point figure with an outside station not on the fixed line unoccupied, $\frac{15-4}{15}=0.73$.

(*q*) For a five-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{13-2}{13}=0.85$.

(*r*) For a six-sided, central-point figure, $\frac{22-7}{22}=0.68$.

(*s*) For a six-sided, central-point figure with one outside station on the fixed line unoccupied, $\frac{20-5}{20}=0.75$.

(*t*) For a six-sided, central-point figure with one outside station not on the fixed line unoccupied, $\frac{19-5}{19}=0.74$.

(*u*) For a six-sided, central-point figure with the central station not on the fixed line unoccupied, $\frac{16-2}{16}=0.88$.

(2) To illustrate the application of the strength table, the $R_1$ and $R_2$ for figure 102 will be considered. Let it be assumed that the direction of progress is

from the bottom line toward the top line. It will be found that the smallest R, called $R_1$ for this figure, will be obtained by computing through the three best-shaped triangles around the central point. The next best R, called $R_2$, will be obtained by computing through the two triangles formed by the diagonal. The $R_2$ is easily computed as follows: From the known side to the diagonal, the distance angles are 89° and 27°. Using these angles as arguments in the strength table, the factor 17.5 is obtained. Similarly, from the diagonal to the top line, the distance angles are 91° and 26°, and the corresponding factor is 18.8. The sum of the two factors is 36.3. If the central point of the figure is an occupied station, $\frac{D-C}{D}=0.56$ (see above), and $R_2=36.3\times0.56=20$. If the control point is unoccupied, as shown in figure 102, $\frac{D-C}{D}=0.67$, and $R_2=36.3\times0.67=24$, as given opposite the figure.

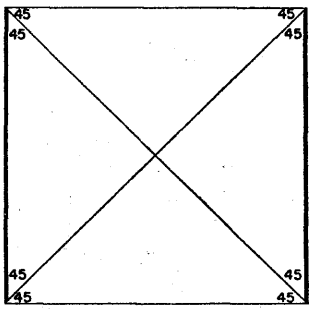
(3) The $R_1$ may be computed in a similar manner by using the distance angles in the three best-shaped triangles around the central point.

*d. Examples of Various Triangulation Figures.*

(1) Figures 89 through 102 show some of the principles involved in the selection of strong figures and illustrate the use of the strength table.

(2) In every figure the line which is supposed to be fixed in length and the line of which the length is required are represented by heavy lines. Either of these two heavy lines may be considered to be the fixed line and the other the required line. Opposite each figure, $R_1$ and $R_2$, as given by the table, are shown. The smaller the value of $R_1$, the greater the strength of the figure. $R_2$ need not be considered in comparing two figures unless the two values of $R_1$ are equal, or nearly so.

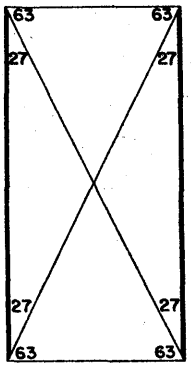
(3) Compare figures 89, 90, and 91. Figure 89 is a square quadrilateral, figure 90 is a rectangular quadilateral which is ½ as long in the direction of progress as it is wide, and figure 91 is a rectangular quadrilateral twice as long in the di-




① All stations occupied. $R_1=5$ $R_2=5$

② Any one station not occupied. $R_1=6$ $R_2=6$

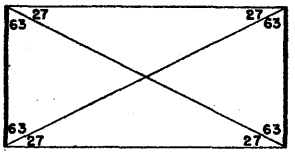
*Figure 89. Strength of figure diagram.*




① All stations occupied. $R_1=1$ $R_2=1$

② Any one station not occupied. $R_1=2$ $R_2=2$

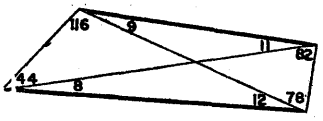
*Figure 90. Strength of figure diagram.*




① All stations occupied. $R_1=22$ $R_2=22$

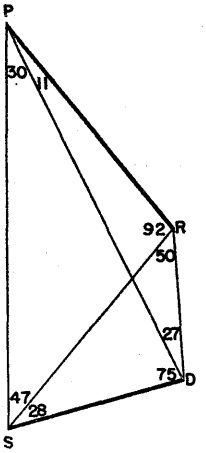
② Any one station on fixed line not occupied. $R_1=27$ $R_2=27$

*Figure 91. Strength of figure diagram.*




All stations occupied. $R_1=1$ $R_2=2$

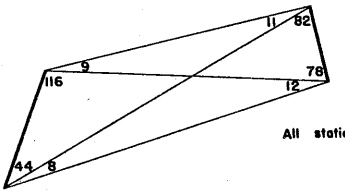
*Figure 92. Strength of figure diagram.*




All stations occupied. $R_1=10$ $R_2=12$

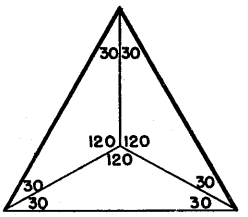
*Figure 93. Strength of figure diagram.*

rection of progress as it is wide. The comparison of the values of $R_1$ in figures 89 and 90 shows that shortening a rectangular quadrilateral in the direction of progress increases its strength. A comparison of figures 89 and 91 shows that extending a rectangular quadrilateral in the direction of progress weakens it. Figure 92, like figure 90, is short in the direction of progress.




All stations occupied. $R_1 = 164$ (approx.) $R_2 = 176$ (approx.)

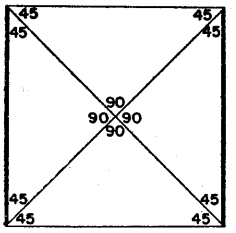
*Figure 94. Strength of figure diagram.*




① All stations occupied. $R_1 = 2$ $R_2 = 12$

② One outside station on fixed line not occupied. $R_1 = 3$ $R_2 = 15$

*Figure 95. Strength of figure diagram.*

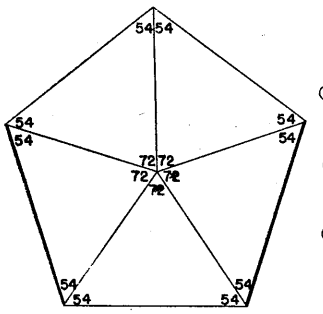



① All stations occupied. $R_1 = 13$ $R_2 = 15$

② One corner station not occupied. $R_1 = 16$ $R_2 = 16$

③ Central station not occupied. $R_1 = 17$ $R_2 = 17$

*Figure 96. Strength of figure diagram.*

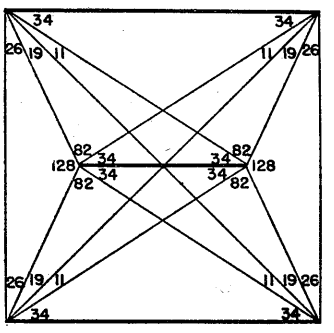



① All stations occupied. $R_1 = 10$ $R_2 = 15$

② Any one outside station not occupied. $R_1 = 11$ $R_2 = 16$

③ Central station not occupied. $R_1 = 13$ $R_2 = 19$

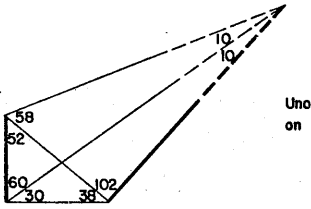
*Figure 97. Strength of figure diagram.*




All stations occupied. $R_1 = 5$ $R_2 = 5$

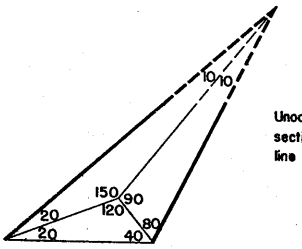
$$\frac{D-C}{D} = \frac{28-16}{28} = 0.43$$

*Figure 98. Strength of figure diagram.*



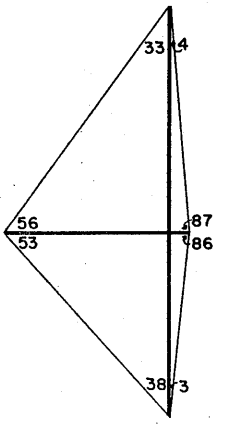

Unoccupied station not on fixed line. $R_1 = 36$ $R_2 = 102$

*Figure 99. Strength of figure diagram.*




Unoccupied station at intersection of fixed line and line to be determined. $R_1 = 4$ $R_2 = 20$

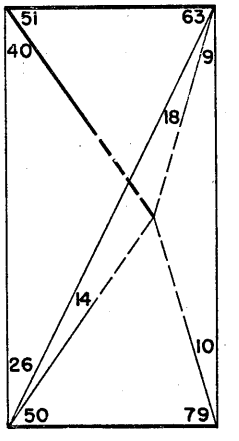
*Figure 100. Strength of figure diagram.*




All stations occupied. $R_1 = 9$ $R_2 = 9$

(A strong and quick expansion figure.)

*Figure 101. Strength of figure diagram.*




Central station not occupied. $R_1 = 18$ $R_2 = 24$

*Figure 102. Strength of figure diagram.*

Such short quadrilaterals are in general very strong, even though badly distorted from the rectangular shape, but they are not economical as progress with them is slow. Figure 93 is badly distorted from a rectangular shape, but is still a moderately strong figure. The best pair of triangles for carrying the length through this figure are DSR and RSP. As a rule, one diagonal of the quadrilateral is common to the two triangles forming the best pair, and the other diagonal is common to the second-best pair. In the unusual case illustrated in figure 93, a side line of the quadrilateral is common to the second-best pair of triangles. Figure 94 is an example of a quadrilateral so much elongated, and therefore so weak, that it is not allowable in any class of triangulation. Figure 95 is the regular three-sided, central-point figure. It is extremely strong. Figure 96 is the regular four-sided, central-point figure. It is much weaker than figure 89, the corresponding quadrilateral. Figure 97 is the regular five-sided, central-point figure. Note that it is much weaker than any of the quadrilaterals shown in figures 89, 90, or 91. Figure 98 is a good example of a strong, quick expansion from a base. The expansion is in the ratio of 1 to 2. Figures 99 and 100 are given as a suggestion of the manner in which, in second- and third-order triangulation, a point A, difficult or impossible to occupy, may be used as a concluded point common to several figures.

(4) Many of the figures given are too weak to be used on first-order triangulation, but for convenience or reference and to illustrate the principles involved, they are included with the figures which can be used.

## 60. Side Equation Test

*a.* Experience has shown that the requirement for triangle closures is not always sufficient, and the agreement in length of the various lines in the figure as computed through the two best chains of triangles must be checked before leaving the station. This is done by a logarithmic computation of the triangle sides, and a comparison of the log-lengths of those for which there is a double determination. In a quadrilateral, these will be the three exterior sides other than the known side. For first-order, the log-lengths should agree within about one and one-half to two times the logarithmic difference for one second in the sine of the smallest angle involved in the computation of the length; for second order, 2 to 4 times the difference; and 10 to 12 times the difference for third order.

*b.* When a quadrilateral (fig. 103) has unsatisfactory triangle closures, it will be necessary to make an inspection of the closures to determine what stations should be reobserved.

(1) Usually, it will be found that the triangle closures (fig. 104) will show two triangles with large triangle closures which have a common line. This indicates that poor

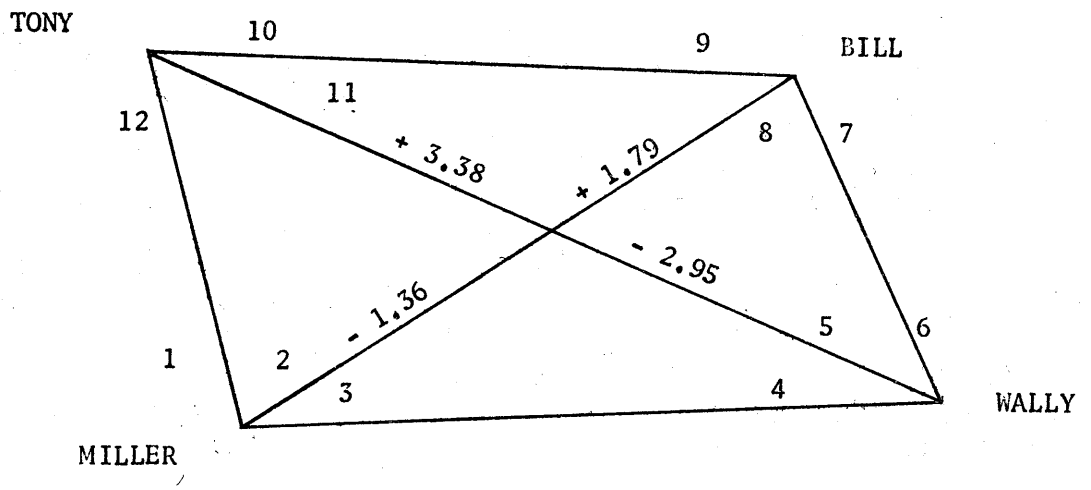



*Figure 103. Quadrilateral for side-equation test.*

| PROJECT | | DATE | COMPUTATION OF TRIANGLES (TM 5-237) |
|---|---|---|---|
| PETCHY | | JAN. '64 | |
| LOCATION | DERBY | ORGANIZATION | USAMS |

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM OR NATURAL |
|---|---|---|---|---|---|---|---|
| | | | | | *COLOG | | |
| | 2–3 | | | | | | 4.3066864 |
| -1+2 | 1 MILLER | 56 14 39.34 | | | | | 0.0801827 * |
| -10+12 | 2 TONY | 76 13 21.69 | | | | | 9.9873215 |
| -8+9 | 3 BILL | 47 32 02.17 | | | | | 9.8678664 |
| | 1–3 | | | | | | 4.3741906 |
| | 1–2 | | | | | | 4.2547355 |
| | | 03.20 | | | 0.90 | | |
| | 2–3 | | | | | | 4.3741906 |
| -4+6 | 1 WALLY | 85 46 27.75 | | | | | 0.0011822 * |
| -2+3 | 2 MILLER | 38 22 35.98 | | | | | 9.7929717 |
| -7+8 | 3 BILL | 55 50 53.32 | | | | | 9.9177956 |
| | 1–3 | | | | | | 4.1683445 |
| | 1–2 | | | | | | 4.2931684 |
| | | 57.05 | | | 0.73 | | |
| | 2–3 | | | | | | 4.3066864 |
| -5+6 | 1 WALLY | 45 25 13.12 | | | | | 0.1473524 * |
| -10+11 | 2 TONY | 31 11 52.28 | | | | | 9.7143256 |
| -7+9 | 3 BILL | 103 22 55.49 | | | | | 9.9880452 |
| | 1–3 | | | | | | 4.1683644 |
| | 1–2 | | | | | | 4.4420840 |
| | | 00.89 | | | 0.74 | | |
| | 2–3 | | | | | | 4.4420840 |
| -1+3 | 1 MILLER | 94 37 15.32 | | | | | 0.0014140 * |
| -11+12 | 2 TONY | 45 01 29.41 | | | | | 9.8496732 |
| -4+5 | 3 WALLY | 40 21 14.63 | | | | | 9.8112459 |
| | 1–3 | | | | | | 4.2931712 |
| | 1–2 | | | | | | 4.2547439 |
| | | 59.36 | | | 0.89 | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
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*Figure 104. Triangles for test quadrilateral.*

observations were made at one or both of the stations at the ends of the line.

(2) Applying a trial correction to the observations at one of the stations to improve the closure and by recomputing, the triangle may give better side checks, which definitely establishes the fact that this station must be reoccupied. If the side check is not improved but made worse, then the same procedure should be followed at the other station. In the majority of cases, the above procedure will show which station should be occupied.

(3) Now and then the closures will be so distributed with poor side checks that the above procedure will not be conclusive. In this case, it will be necessary to make a side equation test to determine what station or stations must be reobserved. This example of making a side equation test is for first-order triangulation.

(4) The numbers on the sketch are measured directions. For example, 1 at MILLER means the direction from MILLER to TONY; 2 is the direction from MILLER to BILL; 3 is the direction from MILLER to WALLY, etc. Referring to the angle at MILLER from TONY to BILL, we use the expression $-1+2$, the left hand direction is always given a negative sign. At WALLY, the angle from TONY to BILL will be expressed as $-5+6$. If we consider the triangles in which directions were measured at station MILLER, then the following relationship can be expressed:

$$\frac{\text{MILLER-TONY}}{\text{MILLER-BILL}} \times \frac{\text{MILLER-BILL}}{\text{MILLER-WALLY}} \times \frac{\text{MILLER-WALLY}}{\text{MILLER-TONY}} = 1$$

(5) Since the equation was written around MILLER, it is therefore called the *pole*. Equations similar to the above can be written for the other stations when they are used as *poles*. The sides in any triangle are proportional to the sines of the opposite angles and, therefore, we may write the following equation for the above:

$$\frac{\text{Sine } (-8+9)}{\text{Sine } (-10+12)} \times \frac{\text{Sine } (-4+6)}{\text{Sine } (-7+8)} \times \frac{\text{Sine } (-11+12)}{\text{Sine } (-4+5)} = 1$$

or, using logarithms, it will become—

$$\text{Log Sine } (-8+9) + \text{Log Sine } (-4+6) + \text{Log Sine } (-11+12) - \text{Log Sine } (-10+12) - \text{Log Sine } (-7+8) - \text{Log Sine } (-4+5) = 0$$

(6) The above relationship would be exact if the measurements were perfect, but since there is always a small error in the direction measurements, there will be a residual. This residual is the indication of the errors so far as the equation is concerned, and is referred to as the constant term of the equation.

(7) This constant term, divided by the sum of the tabular log difference for one second of the sines of the angles involved, will give a quotient which is the average correction to be applied to the angles in the equation so as to eliminate the residual. By experience of many years, it has been found that for triangulation, this quotient must be less than 0.7″ for first-order, and 2″ to 4″ for second-order.

(8) If there are large closures and it appears that the error is at one end or both ends of a common line, the test should be applied by using both stations as poles, and if the quotient is greater than 0.7″ for one of the equations, the error in angular measurements will be found at the opposite end of the line from the selected pole. This is true because the angles at the station selected for the pole do not enter into the equation, and the source of trouble must be at the station at the opposite end of the line from the pole.

(9) A side equation test is given for the quadrilateral MILLER-TONY-BILL-WALLY. A study of the triangle closures shows that errors are present at two or more stations. Therefore, equations are solved with each station as a pole, below.

*Select Pole at MILLER*

| *Angle* | *Value* | *Log Sine* | *Log Diff. 1″* |
|---|---|---|---|
| −8 +9 | 47–32–02.17 | 9.867 8664 | +19.3 |
| −4 +6 | 85–46–27.75 | 9.998 8178 | + 1.5 |
| −11+12 | 45–01–29.41 | 9.849 6732 | +21.1 |
| | | +9.716 3574 | |
| −10+12 | 76–13–21.69 | 9.987 3215 | + 5.2 |
| −7 +8 | 55–50–53.32 | 9.917 7956 | +14.3 |
| −4 +5 | 40–21–14.63 | 9.811 2459 | +24.8 |
| | | −9.716 3630 | 86.2 |
| | | +9.716 3574 | |
| | | −56 | |

Average correction $\frac{-56}{86.2} = -0.65''$

*Select Pole at WALLY*

| *Angle* | *Value* | *Log Sine* | *Log Diff. 1″* |
|---|---|---|---|
| −11+12 | 45–01–29.41 | 9.849 6732 | +21.1 |
| −7 +9 | 103–22–55.49 | 9.988 0452 | − 5.0 |
| −2 +3 | 38–22–35.98 | 9.792 9717 | +26.6 |
| | | +9.630 6901 | |
| −1 +3 | 94–37–15.32 | 9.998 5860 | − 1.7 |
| −10+11 | 31–11–52.28 | 9.714 3256 | +34.8 |
| −7 +8 | 55–50–53.32 | 9.917 7956 | +14.3 |
| | | −9.630 7072 | 103.5 |
| | | +9.630 6901 | |
| | | −171 | |

Average correction $\frac{-171}{103.5} = -1.65''$

*Select Pole at TONY*

| *Angle* | *Value* | *Log Sine* | *Log Diff. 1″* |
|---|---|---|---|
| −5+6 | 45–25–13.12 | 9.852 6476 | +20.7 |
| −1+3 | 94–37–15.32 | 9.998 5860 | − 1.7 |
| −8+9 | 47–32–02.17 | 9.867 8664 | +19.3 |
| | | +9.719 1000 | |
| −7+9 | 103–22–55.49 | 9.988 0452 | − 5.0 |
| −4+5 | 40–21–14.63 | 9.811 2459 | +24.8 |
| −1+2 | 56–14–39.34 | 9.919 8173 | +14.1 |
| | | −9.719 1084 | 85.6 |
| | | +9.719 1000 | |
| | | −84 | |

Average correction $\frac{-84}{85.6} = -0.98''$

*Select Pole at BILL*

| *Angle* | *Value* | *Log Sine* | *Log Diff. 1″* |
|---|---|---|---|
| −2 +3 | 38–22–35.98 | 9.792 9717 | +26.6 |
| −10+12 | 76–13–21.69 | 9.987 3215 | + 5.2 |
| −5 +6 | 45–25–13.12 | 9.852 6476 | +20.7 |
| | | +9.632 9408 | |
| −4 +6 | 85–46–27.75 | 9.998 8178 | + 1.5 |
| −1 +2 | 56–14–39.34 | 9.919 8173 | +14.1 |
| −10+11 | 31–11–52.28 | 9.714 3256 | +34.8 |
| | | −9.632 9607 | 102.9 |
| | | +9.632 9408 | |
| | | −199 | |

Average correction $\frac{-199}{102.9} = -1.93''$

A summary, therefore, shows the following results:

Pole at MILLER—average correction is −0.65″
Pole at TONY —average correction is −0.98″
Pole at BILL —average correction is −1.93″
Pole at WALLY —average correction is −1.65″

(10) The equations show that the angles at MILLER enter into the three which give an average correction greater than the specified value of 0.70″ while the angles at MILLER do not enter into the test where the average correction is less than that amount. It is definite that the directions measured at MILLER are probably in error. A test should be made before actually reoccupying the station by assuming a change in angles. This can be done very easily by multiplying the tabular log difference for 1 second by the number of seconds the angle is changed and adding this value to the sine or subtracting according to sign. Suppose the direction at MILLER to BILL to be in error. The angle at MILLER from TONY to BILL may be decreased with a corresponding increase in the angle BILL to WALLY. If we assume a decrease of 3″ from TONY to BILL and the corresponding increase from BILL to WALLY, then the side equation test will result as follows:

Pole at TONY—average correction—

$$\frac{-42}{85.6} = -0.49''$$

Pole at BILL—average correction—

$$\frac{-78}{102.9} = -0.76''$$

Pole at WALLY—average correction—

$$\frac{-92}{103.5} = -0.89''$$

(11) The correction for pole at MILLER will remain −0.65 since the angles at MILLER do not enter the test. An examination shows that the change at MILLER has improved the corrections and station MILLER should be RE-OBSERVED.

(12) The average corrections with poles at BILL and WALLY are still too large, while with the pole at TONY, a satisfactory value was obtained. The angles measured at TONY do not enter into the equation with the pole at TONY, but do enter into the tests for the poles at BILL and WALLY. It is probable that the angles at TONY are in error. If we assume a change in direction to WALLY from TONY of 2″ in order to balance the closures of either side of the diagonal, i.e., decrease the angle from BILL to WALLY at TONY by 2″ and increase the angle from WALLY to MILLER by the same amount, the test will give the following results:

Pole at BILL—correction will be—

$$\frac{-08}{102.9} = -0.08''$$

Pole at WALLY—correction will be—

$$\frac{+10}{103.5} = +0.10''$$

Pole at MILLER—correction will be—

$$\frac{-14}{86.2} = -0.16''$$

(13) The correction at TONY will remain −0.49″ since the angles do not enter into the test when it is selected as the pole. Tests show that changes at TONY will result in improved corrections and TONY should be reobserved. In this example, both MILLER and TONY must be reobserved to obtain satisfactory results.

(14) The side equation test will be effective provided that either the angles are approximately equal or that the error involves fairly small angles. If the bad angles are close to 90°, the test will not be conclusive because the tabular log difference for 1 second of the sine is very small, and an error of several seconds in such an angle might still give a quotient for the equation which will be less than 0 70″.

## 61. Station Adjustment

*a.* Many times, during high-order triangulation observations, duplicate directions are observed to a station from two or more initials. These duplicated directions cause condition equations which can be properly satisfied only by a least square solution in which each observed direction is weighted according to the number of sets involved in its determination. This least squares solution is referred to as a station adjustment. In the case of a station adjustment containing only one condition, the solution is referred to as a weighted mean. The station adjustment provides statistically the most probable value for the direction to each observed object.

*b.* An example of a station adjustment is presented with the intention of clarifying the computation, and to emphasize the value of its application. The mathematics involved in the computation are presented in USC&GS Special Publication 138 (pages 8–16) and are not repeated here. The station chosen for this example is within a complex first-order triangulation network and has a total of 27 directions referred to five different initials (fig. 105). These directions were each determined by meaning a set of not less than 12 circle position readings observed using a direction theodolite on which the horizontal circle micrometer may be read to within ±0.2″. The readings were abstracted from the field books according to the methods outlined in paragraph 55.

*c.* Following is the procedure for preparing the condition equations:

(1) A preliminary list of directions (fig. 106) is made from the abstracts of directions. All directions are evaluated and those

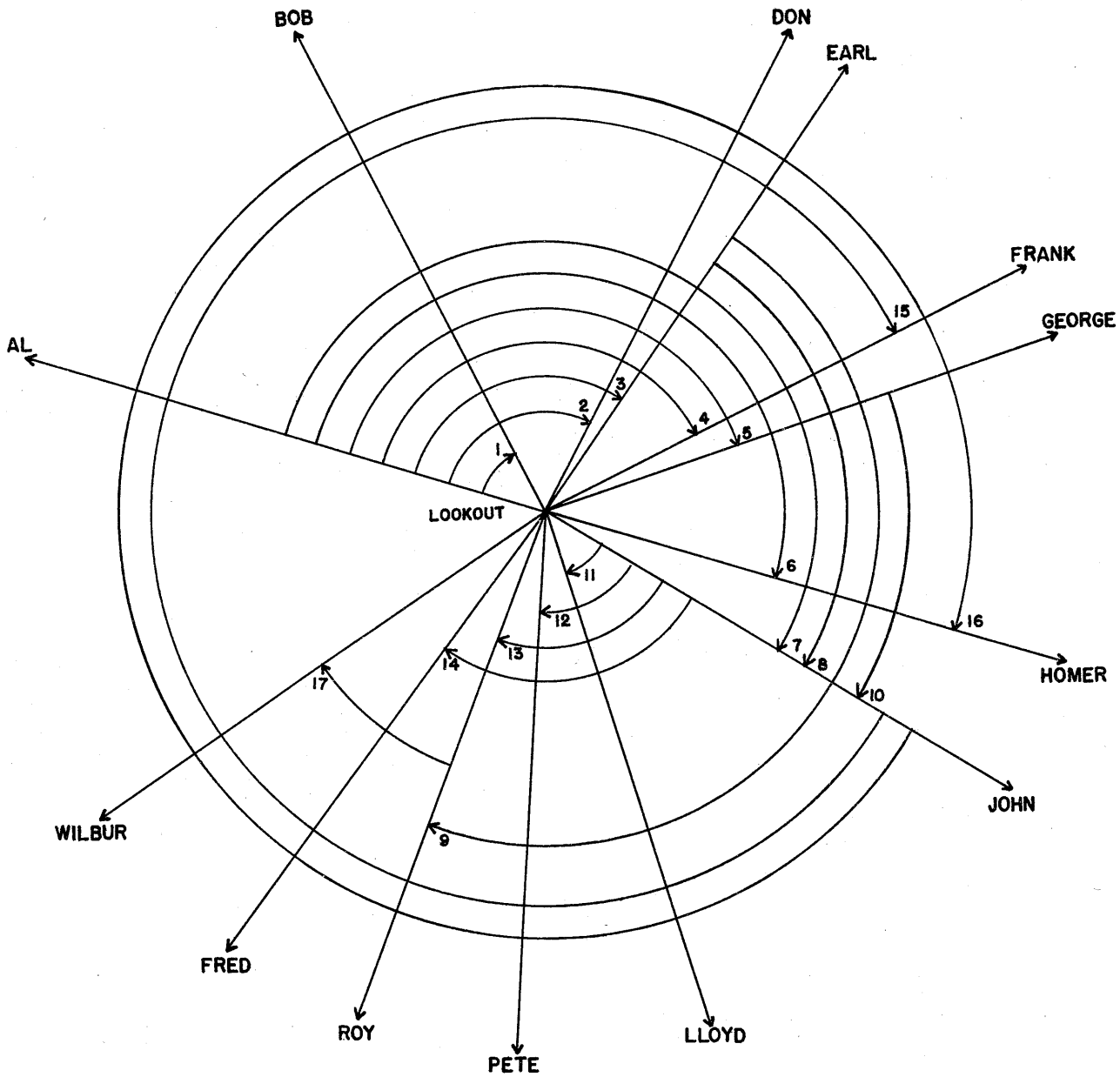



*Figure 105. Diagram of observed directions.*

PRELIMINARY

| PROJECT TM | ORGANIZATION USAMS | LIST OF DIRECTIONS (TM 5-237) | |
|---|---|---|---|
| LOCATION KENTUCKY | | STATION LOOKOUT | |
| OBSERVER F. WILSON | INST. (TYPE) (NO.) WILD T-3 NO. 12345 | | DATE MAR. '60 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| AL | 0 00 00.00 | | | 0 00 00.00 | |
| BOB | 46 54 45.51 | | | | |
| | 45.55 | | | | |
| DON | 101 14 32.49 | | | | |
| | 31.09 | | | | |
| EARL | 108 40 58.86 | | | | |
| | 58.71 | | | | |
| FRANK | 137 42 27.56 | | | | |
| GEORGE | 145 53 40.88 | | | | |
| | 40.98 | | | | |
| HOMER | 180 02 56.63 | | | | |
| | 57.49 | | | | |
| JOHN | 195 26 49.88 | | | | |
| | 50.60 | | | | |
| | 49.85 | | | | |
| | | | | | |
| | | | | | |
| EARL | 00° 00′ 00.″00 | NEW | INITIAL | | |
| JOHN | 86 45 50.88 | | | | |
| | 51.78 | | | | |
| ROY | 166 21 29.39 | | | | |
| | | | | | |
| | | | | | |
| GEORGE | 00° 00′ 00.″00 | NEW | INITIAL | | |
| JOHN | 49 33 08.57 | | | | |
| | | | | | |
| | | | | | |
| ROY | 00° 00′ 00.″00 | NEW | INITIAL | | |
| WILBUR | 35 56 24.20 | | | | |
| | | | | | |
| JOHN | 00° 00′ 00.″00 | NEW | INITIAL | | |
| LLOYD | 41 01 45.21 | | | | |
| | 45.18 | | | | |
| PETE | 62 19 51.91 | | | | |
| ROY | 79 35 36.69 | | | | |
| FRED | 95 56 55.17 | | | | |
| | 54.87 | | | | |
| FRANK | 302 15 37.54 | | | | |
| HOMER | 344 36 06.92 | | | | |

* These columns are for office use and should be left blank in the field.

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|---|---|---|---|
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*Figure 106. List of directions (preliminary) for station adjustment.*

directions which cause excessively large condition equations are rejected and not shown on this list.

(2) An adjustment list of observed angles (fig. 107) is made repeating all the angles from the preliminary list. Means are determined for all of the angles having more than one value and a column headed WT. (weight) lists the number of values used in determining each mean angle. Another column headed $v$ shows the identifying number at each correction ($v$) which will be applied to the observed angles upon completion at the station adjustment. The final column headed adjusted final seconds is left blank at this stage and will be completed when the computed corrections are applied.

(3) The list of directions for adjustment (fig. 108) shows a direction to every observed station, all referenced to a single initial with the $v$'s associated with the angles used to determine those directions. This list is completed utilizing the minimum number of $v$'s required to refer all directions to the single initial. Again, the final seconds column (adjusted directions) is left blank and will be completed when the computed $v$'s are applied.

(4) Every $v$ which was not used in the "list of directions for adjustment" creates a condition. Each unused observed angle and its associated $v$ is listed in turn and compared to the corresponding angle from the "list of directions for adjustment." In this example, the list angles have been subtracted in each case from the observed angles in order that the algebraic sign in the condition equations will be correct. This algebraic difference is set equal to zero in forming the condition equation. Since there were five angles not used in the sample adjustment list, there are five condition equations (fig. 109).

*d.* The correlate equations are prepared from the condition equations. In figure 110, column 2, headed $a/p$, $a$ is a selected constant, and $p$ is the weight of the particular $v$. Normally, $a$ is chosen as the least common multiple of all the weights, $p$, in order that the values of $a/p$ be integers. In this example, $a$ equals 6.

*e.* The normal equations (fig. 111) are obtained by taking the algebraic sums of $a/p$ times the products of the various columns in the correlate equations.

*f.* The Doolittle method is used in the solution of the normal equations (fig. 112). This method of solving normal equations is covered in paragraph 65.

*g.* After the C's are determined, the $v$'s are computed by substituting the values of the C's into the correlate equations taking into account the weights in the $a/p$ column.

PROJECT TM

TABULATION OF GEODETIC DATA (TM 5-237)

LOCATION KENTUCKY

ORGANIZATION USAMS

STATION LOOKOUT

| OBSERVED STATIONS FROM - TO | OBSERVED ANGLES | WT. (P) | V | ADJUSTED FINAL SECONDS |
|---|---|---|---|---|
| AL - BOB | 46° 54' 45.51" | | | |
| | 45.55 | | | |
| | MN. 45.53 | 2 | 1 | 45.53 |
| AL - DON | 101 14 32.49 | | | |
| | 31.09 | | | |
| | MN. 31.79 | 2 | 2 | 31.79 |
| AL - EARL | 108 40 58.86 | | | |
| | 58.71 | | | |
| | MN. 58.78 | 2 | 3 | 58.62 |
| AL - FRANK | 137 42 27.56 | 1 | 4 | 27.60 |
| AL - GEORGE | 145 53 40.88 | | | |
| | 40.98 | | | |
| | MN. 40.93 | 2 | 5 | 41.13 |
| AL - HOMER | 180 02 56.63 | | | |
| | 57.49 | | | |
| | MN. 57.06 | 2 | 6 | 57.04 |
| AL - JOHN | 195 26 49.88 | | | |
| | 50.60 | | | |
| | 49.85 | | | |
| | MN. 50.11 | 3 | 7 | 50.09 |
| EARL - JOHN | 86 45 50.88 | | | |
| | 51.78 | | | |
| | MN. 51.33 | 2 | 8 | 51.47 |
| EARL - ROY | 166 21 29.39 | 1 | 9 | 28.78 |
| GEORGE - JOHN | 49 33 08.57 | 1 | 10 | 08.96 |
| JOHN - LLOYD | 41 01 45.21 | | | |
| | 45.18 | | | |
| | MN. 45.20 | 2 | 11 | 45.20 |
| JOHN - PETE | 62 19 51.91 | 1 | 12 | 51.91 |
| JOHN - ROY | 79 35 36.69 | 1 | 13 | 37.31 |
| JOHN - FRED | 95 56 55.17 | | | |
| | 54.87 | | | |
| | MN. 55.02 | 2 | 14 | 55.02 |
| JOHN - FRANK | 302 15 37.54 | 1 | 15 | 37.51 |
| JOHN - HOMER | 344 36 06.92 | 1 | 16 | 06.95 |
| ROY - WILBUR | 35 56 24.20 | 1 | 17 | 24.20 |

TABULATED BY G. Tennis - AMS DATE APR. '60 CHECKED BY J.A. Andrews - AMS DATE APR. '60

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*Figure 107. List of angles for adjustment.*

| PROJECT | ORGANIZATION | LIST OF DIRECTIONS (TM 5-237) |
|---|---|---|
| TM | USAMS | |

| LOCATION | STATION |
|---|---|
| KENTUCKY | LOOKOUT |

| OBSERVER | INST. (TYPE) (NO.) | DATE |
|---|---|---|
| F. WILSON | WILD T-3 NO. 12345 | MAR. '60 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| AL | 0 00 00.00 | | | 0 00 00.00 | 00 00.00 |
| BOB | 46 54 45.53 | $+ V_1$ | | | 54 45.53 |
| DON | 101 14 31.79 | $+ V_2$ | | | 14 31.79 |
| EARL | 108 40 58.78 | $+ V_3$ | | | 40 58.62 |
| FRANK | 137 42 27.56 | $+ V_4$ | | | 42 27.60 |
| GEORGE | 145 53 40.93 | $+ V_5$ | | | 53 41.13 |
| HOMER | 180 02 57.06 | $+ V_6$ | | | 02 57.04 |
| JOHN | 195 26 50.11 | $+ V_7$ | | | 26 50.09 |
| LLOYD | 236 28 35.31 | $+ V_7 + V_{11}$ | | | 28 35.29 |
| PETE | 257 46 42.02 | $+ V_7 + V_{12}$ | | | 46 42.00 |
| ROY | 275 02 26.80 | $+ V_7 + V_{13}$ | | | 02 27.40 |
| FRED | 291 23 45.13 | $+ V_7 + V_{14}$ | | | 23 45.11 |
| WILBUR | 310 58 51.00 | $+ V_7 + V_{13} + V_{17}$ | | | 58 51.60 |

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|---|---|---|---|
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*Figure 108. List of directions for adjustment.*

PROJECT: TM

**TABULATION OF GEODETIC DATA** (TM 5-237)

LOCATION: KENTUCKY

ORGANIZATION: USAMS

| STATION | | | | |
|---|---|---|---|---|
| LOOKOUT | | | | |
| EARL - JOHN | OBS. ANGLE | 86° 45' 51.33" | $+V_8$ | |
| | LIST. ANGLE | 86 45 51.33 | $-V_3+V_7$ | |
| | CONDITION 1 | 0 = 0.00 | $+V_3-V_7+V_8$ | |
| EARL - ROY | OBS. ANGLE | 166 21 29.39 | $+V_9$ | |
| | LIST. ANGLE | 166 21 28.02 | $-V_3+V_7+V_{13}$ | |
| | CONDITION 2 | 0 = +1.37 | $+V_3-V_7+V_9-V_{13}$ | |
| GEORGE - JOHN | OBS. ANGLE | 49 33 08.57 | $+V_{10}$ | |
| | LIST. ANGLE | 49 33 09.18 | $-V_5+V_7$ | |
| | CONDITION 3 | 0 = -0.61 | $+V_5-V_7+V_{10}$ | |
| JOHN - FRANK | OBS. ANGLE | 302 15 37.54 | $+V_{15}$ | |
| | LIST. ANGLE | 302 15 37.45 | $+V_4-V_7$ | |
| | CONDITION 4 | 0 = +0.09 | $-V_4+V_7+V_{15}$ | |
| JOHN - HOMER | OBS. ANGLE | 344 36 06.92 | $+V_{16}$ | |
| | LIST. ANGLE | 344 36 06.95 | $+V_6-V_7$ | |
| | CONDITION 5 | 0 = -0.03 | $-V_6+V_7+V_{16}$ | |

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CHECKED BY: J. A. Andrews - AMS DATE: APR. '60

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*Figure 109. Condition equations—station adjustment.*

Sta. LOOKOUT

Correlate Equations

| V's | a/p | 1 | 2 | 3 | 4 | 5 | Σc | V | ADOPTED V |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | | | | | | | 0.000 | 0.00 |
| 2 | 3 | | | | | | | 0.000 | 0.00 |
| 3 | 3 | +1 | +1 | | | | +2 | -0.164 | -0.16 |
| 4 | 6 | | | | -1 | | -1 | +0.034 | +0.04 |
| 5 | 3 | | | +1 | | | +1 | +0.196 | +0.20 |
| 6 | 3 | | | | | -1 | -1 | -0.017 | -0.02 |
| 7 | 2 | -1 | -1 | -1 | +1 | +1 | -1 | -0.021 | -0.02 |
| 8 | 3 | +1 | | | | | +1 | +0.143 | +0.14 |
| 9 | 6 | | +1 | | | | +1 | -0.614 | -0.61 |
| 10 | 6 | | | +1 | | | +1 | +0.392 | +0.39 |
| 11 | 3 | | | | | | | 0.000 | 0.00 |
| 12 | 6 | | | | | | | 0.000 | 0.00 |
| 13 | 6 | | -1 | | | | -1 | +0.614 | +0.62 |
| 14 | 3 | | | | | | | 0.000 | 0.00 |
| 15 | 6 | | | | +1 | | +1 | -0.034 | -0.03 |
| 16 | 6 | | | | | +1 | +1 | +0.034 | +0.03 |
| 17 | 6 | | | | | | | 0.000 | 0.00 |

*Figure 110. Correlate equations—station adjustment.*

Sta. LOOKOUT

Normal Equations

| | 1 | 2 | 3 | 4 | 5 | n | Σn | c |
|---|---|---|---|---|---|---|---|---|
| 1 | +8 | +5 | +2 | -2 | -2 | 0.00 | +11.00 | +0.04757 |
| 2 | | +17 | +2 | -2 | -2 | +1.37 | +21.37 | -0.10227 |
| 3 | | | +11 | -2 | -2 | -0.61 | +10.39 | +0.06540 |
| 4 | | | | +14 | +2 | +0.09 | +10.09 | -0.00572 |
| 5 | | | | | +11 | -0.03 | +6.97 | +0.00571 |

*Figure 111. Normal equations—station adjustment.*

Sta. LOOKOUT

Solution of Normals

| 1 | 2 | 3 | 4 | 5 | n | Σn | c's |
|---|---|---|---|---|---|---|---|
| +8. | +5. | +2. | -2. | -2. | 0.00 | +11.00 | |
| $C_1$ = | -0.62500 | -0.25000 | +0.25000 | +0.25000 | 0.00000 | -1.37500 | +0.04757 |
| | +17. | +2. | -2. | -2. | +1.37 | +21.37 | |
| | +13.87500 | +0.75000 | -0.75000 | -0.75000 | +1.37000 | +14.49500 | |
| | $C_2$ = | -0.05405 | +0.05405 | +0.05405 | -0.09874 | -1.04468 | -0.10227 |
| | | +11. | -2. | -2. | -0.61 | +10.39 | |
| | | +10.45946 | -1.45946 | -1.45946 | -0.68406 | +6.85649 | |
| | | $C_3$ = | +0.13953 | +0.13953 | +0.06540 | -0.65553 | +0.06540 |
| | | | +14. | +2. | +0.09 | +10.09 | |
| | | | +13.25582 | +1.25582 | +0.06861 | +14.58023 | |
| | | | $C_4$ = | -0.09474 | -0.00518 | -1.09991 | -0.00572 |
| | | | | +11. | -0.03 | +6.97 | |
| | | | | +10.13684 | -0.05790 | +10.07894 | |
| | | | | $C_5$ = | +0.00571 | -0.99429 | +0.00571 |

*Figure 112. Solution of normal equations—station adjustment.*

## Section II. QUADRILATERAL ADJUSTMENT (LEAST-SQUARES METHOD)

### 62. Introduction

*a.* The most common figure occurring in triangulation is the quadrilateral with both diagonals observed, because it embodies both strength and simplicity. A thorough understanding of the computation and adjustment of such a quadrilateral is basic to the understanding of a net adjustment. For this reason, a quadrilateral

adjustment will be explained and an example computation shown. Because this is to be background material, the quadrilateral in the example will be adjusted by the least squares method. DA Form 1925, Quadrilateral Adjustment (Least Squares Method), has been set up for this solution.

*b.* Field observations always contain small errors which cannot be distributed in any way except by the laws of probability. Because of these errors, certain geometric properties of a quadrilateral are not satisfied by the field observations. It is the purpose of the least-squares adjustment to find the most probable values of the field observations that will satisfy the geometric conditions. If the following conditions are satisfied, all the geometric properties of the quadrilateral will also be satisfied. The conditions are as follows:

(1) In the four triangles in the quadrilateral, the sum of the three angles in each triangle must total 180° plus the spherical excess.

(2) The length of the common sides of adjacent triangles must be the same no matter which of the two adjacent triangles is used to compute the length.

*c.* By solving certain equations known as conditions equations, small corrections are determined which are applied to the observed angles. These corrected angles will then satisfy the above conditions.

*d.* The two types of equations occurring in a quadrilateral are angle equations and side equations. In a completed quadrilateral (fig. 113), three angle equations and one side equation are needed. The number of angle and side equations in any figure can be determined by the formula in the section on triangulation adjustment.

## 63. Direction Method

*a.* This illustration will be made using the direction method in which an angle is considered to be made up of two directions. The corrections, therefore, will be found for each of the two directions making up the angle.

*b.* The adjustment is begun by numbering the observed directions on the sketch (fig. 113), starting at the left end of the fixed line looking into the quadrilateral. Any directions not observed are not numbered, and such directions are shown by a dashed line on the sketch. The numbers on the lines are used as the subscripts

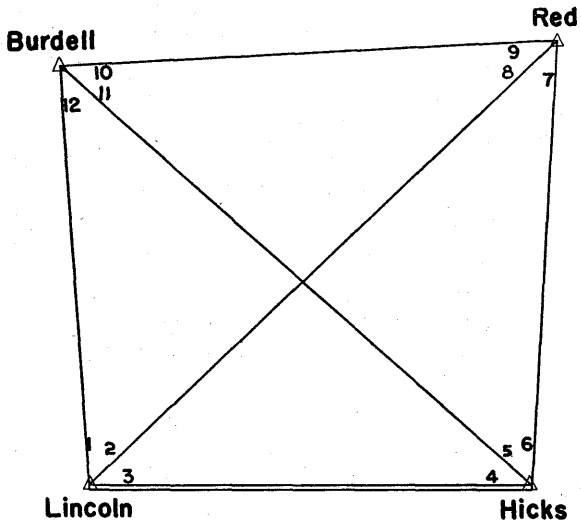



*Figure 113. Quadrilateral sketch.*

on the symbol $v$ used to designate the corrections to the observed directions. The designation of the corrections is then $v_1$, $v_2$, $v_3$, and so on. For convenience, the symbol $v$ is usually not written, thus the subscript is written as (1) and the symbol is understood.

*c.* Each angle is designated by the two directions forming the angle. Always considering the angle as being measured clockwise, the first direction is negative and the second direction is positive. For example, the angle at Lincoln from Burdell to Red is designated $-1+2$, while the angle at Hicks from Lincoln to Burdell is $-4+5$. In this way, not only the angle, but also the correction to the angle is designated. For instance, the correction to the angle in the first example above is $-v_1+v_2$ and when numerical values are found for $v_1$ and $v_2$, they are algebraically added as $(-v_1+v_2)$.

*d.* DA Form 1918 (fig. 114) is used for the computation of the triangles in this example.

*e.* In order to be certain that all the triangles in a net are written on the triangle computation sheets, some sort of order should be established for writing the triangles. The system outlined here will establish a pattern which will continue through all subsequent computations. In the quadrilateral Lincoln-Burdell-Red-Hicks, start with the fixed line Hicks-Lincoln and proceed clockwise around the figure. The first station encountered is Burdell which will be number 1 in the first section of the triangle computation sheet. At Burdell, find the first clockwise line to a fixed station which will be the line to Hicks. Station Hicks then is number 2 in the first triangle. Still

| PROJECT | | DATE | | COMPUTATION OF TRIANGLES (TM 5-237) | | | | |
|---|---|---|---|---|---|---|---|---|
| 1-243 | | 12 Aug 55 | | | | | | |
| LOCATION | California | ORGANIZATION | 29th Engr. | | | | | |
| | STATION | OBSERVED ANGLE | | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM ~~OR NATURAL~~ |
| | 2-3 HICKS - LINCOLN | | | | | | | 3.68755045 |
| -11+12 | 1 Burdell | 22 49 | 27.1 | +1.3 | 28.4 | 0.0 | 28.4 | 9.58873169 |
| -4+5 | 2 HICKS | 49 00 | 55.2 | +1.9 | 57.1 | 0.0 | 57.1 | 9.87788434 |
| -1+3 | 3 LINCOLN | 108 09 | 31.4 | +3.2 | 34.6 | 0.1 | 34.5 | 9.97781144 |
| | 1-3 Burdell - LINCOLN | | | | | | | 3.97670310 |
| | 1-2 Burdell - HICKS | | | | | | | 4.07663020 |
| | | | 53.7 | +6.4 | 00.1 | 01 | 00.0 | |
| | 2-3 HICKS - LINCOLN | | | | | | | 3.68755045 |
| -7+8 | 1 Red | 30 41 | 59.7 | +2.5 | 02.2 | 0.0 | 42' 02"2 | 9.70804006 |
| -4+6 | 2 HICKS | 91 29 | 06.2 | +3.9 | 10.1 | 0.1 | 10.0 | 9.99985390 |
| -2+3 | 3 LINCOLN | 57 48 | 45.3 | +2.5 | 47.8 | 0.0 | 47.8 | 9.92753288 |
| | 1-3 Red - LINCOLN | | | | | | | 3.97936429 |
| | 1-2 Red - HICKS | | | | | | | 3.90704327 |
| | | | 51.2 | +8.9 | 00.1 | 0.1 | 00.0 | |
| | 2-3 HICKS - Burdell | | | | | | | 4.07663020 |
| -7+9 | 1 Red | 95 09 | 11.0 | +3.2 | 14.2 | 0.0 | 14.2 | 9.99824056 |
| -5+6 | 2 HICKS | 42 28 | 11.0 | +2.0 | 13.0 | 0.1 | 12.9 | 9.82943713 |
| -10+11 | 3 Burdell | 42 22 | 31.7 | +1.3 | 33.0 | 0.1 | 32.9 | 9.82865377 |
| | 1-3 Red - Burdell | | | | | | | 3.90782677 |
| | 1-2 Red - HICKS | | | | | | | 3.90704341 |
| | | | 53.7 | +6.5 | 00.2 | 0.2 | 00.0 | |
| | 2-3 LINCOLN - Burdell | | | | | | | 3.97670310 |
| -8+9 | 1 Red | 64 27 | 11.3 | +0.7 | 12.0 | 0.1 | 11.9 | 9.95531925 |
| -1+2 | 2 LINCOLN | 50 20 | 46.1 | +0.7 | 46.8 | 0.1 | 46.7 | 9.88644309 |
| -10+12 | 3 Burdell | 65 11 | 58.8 | +2.6 | 01.4 | 0.0 | 12' 01"4 | 9.95798077 |
| | 1-3 Red - Burdell | | | | | | | 3.90782694 |
| | 1-2 Red - LINCOLN | | | | | | | 3.97936462 |
| | | | 56.2 | +4.0 | 00.2 | 0.2 | 00.0 | |
| COMPUTED BY | RAS | DATE | 18 Aug 55 | CHECKED BY | W.C.A. | DATE | 20 Aug 55 | |

DA FORM FEB 57 1918

*Figure 114. Computation of triangles for quadrilateral.*

at Burdell, find the next clockwise line which will be the line to Lincoln. Lincoln is number 3. The triangle thus defined is Burdell-Hicks-Lincoln. Now move to the next clockwise station which is Red. This is number 1 in the second triangle. The first clockwise line is to Hicks. Hicks is number 2. The next clockwise line is to Lincoln which is again number 3. The triangle Red-Hicks-Lincoln is now defined. Still at Red, the first clockwise line is to Hicks; the next clockwise line not previously used is to Burdell. The triangle now completed is Red-Hicks-Burdell. All the triangles with Red-Hicks as the first clockwise line are now written, so shift to the next clockwise line which is Red-Lincoln. Lincoln is now number 2 in the fourth triangle. The next clockwise line is Red-Burdell. The fourth triangle is Red-Lincoln-Burdell. Notice that station Burdell is number 1 in the first triangle, and station Red is number 1 in the second, third, and fourth triangles.

*f.* In the left-hand margin of the form, the designations of the angles are shown. The observed angles are from the list of directions. (In this example, the angles are from the list shown in the section on triangulation adjustment.) The sum of the observed angles is recorded for each triangle. Only the seconds of the sum need be written on the form. A bar over the seconds indicates that the sum is less than 180°. For example, in the first triangle in figure 114 the three angles total 179°59′53″7 and the seconds are recorded as $\overline{53.7}$. The three angles in a triangle should total 180° plus the spherical excess ($\epsilon$). (Spherical excess computation is discussed in paragraph 57.) $180°+\epsilon$ minus the sum of the three angles is the error of closure of the triangle. The algebraic sum of the corrections to the three angles (each angle correction made up of directions corrections) must equal the error of closure of the triangle. This statement leads to the condition equation known as the angle equation, which can be stated as: The algebraic sum of the $v$'s in a triangle must equal the triangle closure. As an example, the angle equation for the triangle Burdell-Hicks-Lincoln is—

$$-(1)+(3)-(4)+(5)-(11)+(12)=+6.4$$

remembering that (1), (3), (4), and so on are the subscripts of $v$'s.

*g.* An angle equation can be written for each of the four triangles in a quadrilateral, but values for the $v$'s which will satisfy three of the equations will also satisfy the fourth, because the fourth equation is a combination of the other three. When choosing the three-angle equations to be solved, triangles with small angles should be avoided if possible. In the example, therefore, the angle equations for the second, third, and fourth triangles are selected. The three angle equations are as follows:

$$0=-8.9-(2)+(3)-(4)+(6)-(7)+(8)$$
$$0=-6.5-(5)+(6)-(7)+(9)-(10)+(11)$$
$$0=-4.0-(1)+(2)-(8)+(9)-(10)+(12)$$

The equations are numbered in ascending order of $v$'s for convenience in the solution. The equations are shown in correct order on the example form.

*h.* The condition of side agreement is satisfied by including a side equation in the solution. One way to set up this equation is to select a station as a pole and write the product of the ratios of the lines running to that pole as equal to 1. By the laws of sines, the sines of the angles opposite the sides can replace the sides. By replacing natural sines by logarithms, the expression can be reduced to one which can be solved by addition and subtraction.

*i.* In this example, the side equation is written using the pole at Lincoln in order to include the small angles at Burdell and Red. The ratio of the lines intersecting at Lincoln is—

$$\frac{\text{Lincoln-Burdell}}{\text{Lincoln-Red}}\times\frac{\text{Lincoln-Red}}{\text{Lincoln-Hick}}\times\frac{\text{Lincoln-Hicks}}{\text{Lincoln-Burdell}}=1$$

Substituting the sines of the angles opposite these sides and using the symbol designation of the angles, this ratio becomes—

$$\frac{\sin(-8+9)}{\sin(-10+12)}\times\frac{\sin(-4+6)}{\sin(-7+8)}\times\frac{\sin(-11+12)}{\sin(-4+5)}=1$$

Replacing natural sines by log sines:

$$\log\sin(-8+9)+\log\sin(-4+6)$$
$$+\log\sin(-11+12)-\log\sin(-10+12)$$
$$-\log\sin(-7+8)-\log\sin(-4+5)=0$$

When written on a side equation form, the log sines of the angles in the numerators are entered in the left-hand column and the log sines of the angles in the denominators are entered in the right-hand column.

*j.* For each angle, the tabular difference is entered in the column headed "Tab. Diff." (fig. 115) beside the angle. The tabular difference is the

PROJECT: I-243

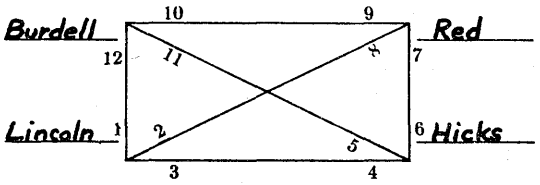
**QUADRILATERAL ADJUSTMENT (Least Squares Method)** (TM 5-237)

LOCATION: California

ORGANIZATION: 29th Engr.

DATE: 15 Aug 55

SKETCH




ANGLE EQUATIONS

(1.) $0 = -4.0 - (1) + (2) - (8) + (9) - (10) + (12)$

(2.) $0 = -8.9 - (2) + (3) - (4) + (6) - (7) + (8)$

(3.) $0 = -6.5 - (5) + (6) - (7) + (9) - (10) + (11)$

SIDE EQUATION

| Symbol | Angle | | | Log. Sine | Tab. Diff | Symbol | Angle | | | Log. Sine | Tab. Diff |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -8+9 | 64 | 27 | 11.3 | 9.95531865 | +1.00 | -10+12 | 65 | 11 | 58.8 | 9.95797824 | +0.97 |
| -4+6 | 91 | 29 | 06.2 | 9.99985411 | -0.06 | -7+8 | 30 | 41 | 59.7 | 9.70803120 | +3.55 |
| -11+12 | 22 | 49 | 27.1 | 9.58872519 | +5.00 | -4+5 | 49 | 00 | 55.2 | 9.87788087 | +1.83 |
| (4.) | | | | 9.54389795 | | | | | | 9.54389031 | |

$0 = +7.64 + 1.89(4) - 1.83(5) - 0.06(6) + 3.55(7) - 4.55(8) + 1.00(9) + 0.97(10) - 5.00(11) + 4.03(12)$

CORRELATE EQUATIONS

| | 1 | 2 | 3 | 4 | $\Sigma_c$ | v | Adopt. v | $v^2$ | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | | | | -1.0 | -1.2843 | -1.3 | 1.69 | 1 |
| 2 | +1 | -1 | | | 0.0 | -0.6335 | -0.6 | 0.36 | 2 |
| 3 | | +1 | | | +1.0 | +1.9178 | +1.9 | 3.61 | 3 |
| 4 | | -1 | | +1.89 | +0.89 | -1.9008 | -1.9 | 3.61 | 4 |
| 5 | | | -1 | -1.83 | -2.83 | -0.0426 | -0.0 | — | 5 |
| 6 | | +1 | +1 | -0.06 | +1.94 | +1.9434 | +2.0 | 4.00 | 6 |
| 7 | | -1 | -1 | +3.55 | +1.55 | -1.9120 | -1.9 | 3.61 | 7 |
| 8 | -1 | +1 | | -4.55 | -4.55 | +0.5926 | +0.6 | 0.36 | 8 |
| 9 | +1 | | +1 | +1.00 | +3.00 | +1.3194 | +1.3 | 1.69 | 9 |
| 10 | -1 | | -1 | +0.97 | -1.03 | -1.3017 | -1.3 | 1.69 | 10 |
| 11 | | | +1 | -5.00 | -4.00 | -0.0189 | -0.0 | — | 11 |
| 12 | +1 | | | +4.03 | +5.03 | +1.3206 | +1.3 | 1.69 | 12 |

NORMAL EQUATIONS

| | 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_\eta$ | |
|---|---|---|---|---|---|---|---|
| 1 | +6 | -2 | +2 | +8.61 | -4.0 | +10.61 | 1 |
| 2 | | +6 | +2 | -10.05 | -8.9 | -12.95 | 2 |
| 3 | | | +6 | -6.75 | -6.5 | -3.25 | 3 |
| 4 | | | | +83.4114 | +7.64 | +82.8614 | 4 |

SOLUTION OF NORMALS

| 1 | 2 | 3 | 4 | $\eta$ | $\Sigma_\eta$ | C | |
|---|---|---|---|---|---|---|---|
| +6.0 | -2.0 | +2.0 | +8.61 | -4.0 | +10.61 | | |
| $C_1$ | +0.3333 | -0.3333 | -1.4350 | +0.6667 | -1.7683 | +1.2843 | $C_1$ |
| | +6.0 | +2.0 | -10.05 | -8.9 | -12.95 | | |
| | +5.3333 | +2.6667 | -7.1800 | -10.2333 | -9.4133 | | |
| | $C_2$ | -0.5000 | +1.3463 | +1.9187 | +1.7650 | +1.9178 | $C_2$ |
| | | +6.0 | -6.75 | -6.5 | -3.25 | | |
| | | +4.0000 | -6.0298 | -0.0500 | -2.0798 | | |
| | | $C_3$ | +1.5075 | +0.0125 | +0.5200 | +0.0261 | $C_3$ |
| | | | +83.4114 | +7.64 | +82.8614 | | |
| | | | +52.2997 | -0.4714 | +51.8283 | | |
| | | | $C_4$ | +0.0090 | -0.9910 | +0.0090 | $C_4$ |

PROBABLE ERROR OF OBSERVED DIRECTION

$d = 0.6745\sqrt{\frac{\Sigma v^2}{c}}$

$d = \pm 1.''6$

c = 4 = NUMBER OF CONDITIONS

COMPUTED BY: P.A. Smith — DATE: 21 Aug 55

CHECKED BY: W. C. Aumen — DATE: 22 Aug 55

DA FORM 1 FEB 57 1925

*Figure 115. Computation of Quadrilateral adjustment.*

change in the logarithm for 1 second change in the angle. Express the tabular difference in units of the sixth decimal place of the logarithm. Notice that the tabular difference for an angle over 90° is minus. When this tabular arrangement of the side equation is used, the constant term of the equation is found by subtracting the sum of the log sines in the right-hand column from the sum of the log sines in the left-hand column and pointing the difference off in units of the sixth decimal. If the right-hand sum is larger than the left-hand sum, the constant term has a minus sign. The constant term is written with unchanged algebraic sign on the same side of the equation as the $v$ terms. The tabular differences are used as the coefficient of the $v$'s as shown in the Symbol column. If a $v$ appears on both sides of the form, the coefficients for that $v$ must be combined in the equation. The tabular differences at the right-hand side reverse the algebraic sign for this operation. The side equation in the example, before coefficients of $v$ are combined, is as follows:

$$\begin{aligned}0=&+7.64+(-0.06)\ (-4)+(-1.83)\ (-4)\\&+(-1.83)\ (5)+(-0.06)\ (6)\\&+(-3.55)\ (-7)+(1.00)\ (-8)\\&+(-3.55)\ (8)+(1.00)\ (9)\\&+(-0.97)\ (-10)+(5.00)\ (-11)\\&+(5.00)\ (12)+(-0.97)\ (12)\end{aligned}$$

When the coefficients of the $v$'s are combined, the equation is as follows:

$$\begin{aligned}0=&+7.64+1.89\ (4)-1.83\ (5)-0.06\ (6)\\&+3.55\ (7)\\&-4.55\ (8)+1.00\ (9)+0.97\ (10)-5.00\ (11)\\&+4.03\ (12)\end{aligned}$$

*k.* At this point, a check should be made to insure that the sum of the coefficients of corrections to directions ($v$'s) radiating from any one station equals zero.

*l.* After the condition equations are formed, they are tabulated in correlates. For each equation, there is a numbered column in the arrangement of the correlates. On the horizontal lines at the sides of the correlates are the numbers of the $v$'s. The coefficients of the $v$'s in each equation are written in the correct column on the appropriate numbered line. (Coefficients in equation 1 are written in column 1, and so on.) The column headed "$\Sigma_c$" contains the quantities obtained by adding algebraically the coefficients on the same horizontal line in the four columns. The columns headed "$v$", "Adopt $v$", and "$v^2$", are filled in after the solution of the normal equations. The correlates are used to form the normal equations.

*m.* The normal equations are arranged using a column for each condition equation (corresponding to the numbered columns for the correlates) plus a column headed "$\eta$" and a column headed "$\Sigma\eta$". The $\eta$ column contains the constant terms of the condition equations, while the values in the $\Sigma_\eta$ column are used as a check corresponding to the values in the $\Sigma_c$ column of the correlates. The normal equations themselves are numbered on the horizontal lines.

(1) The normal equations are formed by finding the algebraic sums of the products of the values in one column of the correlates multiplied by the values in the various other columns in the correlates. The products are found only for values on the same horizontal line. In other words, a value on line 1 is multiplied only by other values on line 1 and not by values on any other line.

(2) The first normal is found as the summation of the products of the values in column 1 of the correlates, multiplied by—

- (*a*) The values in column 1 of the correlates.
- (*b*) The values in column 2 of the correlates.
- (*c*) The values in column 3 of the correlates.
- (*d*) The values in column 4 of the correlates.
- (*e*) Plus $\eta_1$, the constant term of the first condition equation.
- (*f*) The values in column $\Sigma_c$ of the correlates (after the summation of this multiplication, add $\eta_1$).

(3) The values obtained in (2) above, are tabulated on the form for the normals as follows: (*a*) in column 1, (*b*) in column 2, (*c*) in column 3, (*d*) in column 4, (*e*) in column $\eta$, and (*f*) in column $\Sigma_\eta$; all on the first line. The value in the $\Sigma$ column is [$\eta_1$+column 2 times $\Sigma_c$]. As a check, the sum of the values in columns 1, 2, 3, 4, and $\eta$ should equal the value in the $\Sigma_\eta$ column.

(4) The second normal is found as the summation of the products of the values in column 2 of the correlates multiplied by:

- (*a*) The values in column 1 of the correlates.

(*b*) The values in column 2 of the correlates.
(*c*) The values in column 3 of the correlates.
(*d*) The values in column 4 of the correlates.
(*e*) Plus $\eta_2$, the constant term of the second condition equation.
(*f*) The values in column $\Sigma_c$ of the correlates (after the summation of this multiplication, add $\eta_2$).

(5) The values obtained are tabulated in the same order as explained for normal equation 1.

(6) Notice that item (*a*) in (4) above, for normal 2 is exactly the same as item (*b*) for normal 1. If both these items were written in the normal equation, there would be a repetition of the value in line 1 column 2, and line 2 column 1. A repetition of this type occurs in each normal for all values to the left of the diagonal term. The diagonal term is the value obtained when a column in the correlates is multiplied by itself in the formation of the normals. Each normal, therefore, can be formed by finding the diagonal term and the terms to the right. The terms to the left of the diagonal term appear in the column above the diagonal term, in order, reading down from the top. The complete normal reads down the column to the diagonal and then across the line to the right. When checking the equation as explained for normal 1, *remember to add down and across*. Each term that falls in a numbered column in the tabulation of the normal equation is the coefficient of a constant (or C) corresponding to the column. The C's also correspond to the same numbered column in the correlates. (That is, $C_1$ is the constant for column 1 of the correlates and the normals.)

*n*. The normal equations in the example could be written as follows:

$$\textcircled{1}\quad +6C_1-2C_2+2C_3+8.61C_4-4.0=0$$
$$\textcircled{2}\quad -2C_1+6C_2+2C_3-10.05C_4-8.9=0$$
$$\textcircled{3}\quad +2C_1+2C_2+6C_3-6.75C_4-6.5=0$$
$$\textcircled{4}\quad +8.61C_1-10.05C_2-6.75C_3+83.4114C_4+7.64=0$$

These are 4 simultaneous equations. (Notice that the terms to the left of the diagonal term are included.)

## 64. Solution of Normal Equations by Successive Substitution

*a*. The solution of the example normal equations (par. 63) by successive substitution is as follows: Equation ① is solved for $C_1$ in terms of $C_2$, $C_3$, $C_4$, and a constant.

(1) $C_1=+0.3333C_2-0.3333C_3-1.4350C_4+0.6667$

This value of $C_1$ is substituted in equation ②:

$$-2(0.3333C_2-0.3333C_3-1.4350C_4+0.6667)+6C_2+2C_3-10.05C_4-8.9=0$$

which is:

$$-0.6667C_2+0.6667C_3+2.8700C_4-1.3334+6C_2+2C_3-10.05C_4-8.9=0$$

Collecting terms, equation ② becomes:

$$+5.3333C_2+2.6667C_3-7.1800C_4-10.2333=0$$

Solve this equation for $C_2$ in terms of $C_3$, $C_4$, and a constant:

$$C_2=\frac{-2.6667C_3+7.1800C_4+10.2333}{5.3333}$$

(2) $C_2=-0.5000C_3+1.3463C_4+1.9187$

Substituting the value of $C_1$ from ① and the value of $C_2$ from ② in equation ③; reduced equation ③ is found to be:

$$4.0000C_3-6.0298C_4-0.0500=0$$

Solving this equation for $C_3$ in terms of $C_4$ and a constant:

(3) $C_3=+1.5075C_4+0.0125$

Substituting the reduced values for $C_1$, $C_2$, and $C_3$, from above formulas in normal equation ④ and solving the resulting equation for $C_4$ as a constant, the forward solution of the normals is complete. The reduced equation for $C_4$ is:

$$+52.2997C_4-0.4714=0$$

Solving for $C_4$:

(4) $C_4=\frac{+0.4714}{+52.2997}=+0.0090$

The numerical value of $C_3$ is obtained by substituting the value of $C_4$ into equation (3). The numerical value of $C_2$ is obtained by substituting $C_3$ and $C_4$ into equation (2), and $C_1$ is found from equation (1). This process is known as the back solution.

*b.* The solution of the normals can be checked by substituting the numerical values of the C's into the original normal equations. The $v$'s are now found by substituting the C's into the correlate equations. For example, in this problem,

$$v_1=(-1)\times C_1$$

or

$$v_1=(-1)(+1.2843)=-1.2843.$$

The correlate equation for $v_2$ is—

$$v_2=(+1)(C_1)+(-1)(C_2),$$

or

$$v_2=(+1)(1.2843)+(-1)(+1.9178)$$
$$=+1.2843-1.9178=-0.6335.$$

The value for each $v$ is found in the same manner. Carry these values of the $v$'s to at least 1 more decimal than is required in the final result. Substitute the values of the $v$'s into the original condition equation as a check on the whole solution. After this check has been made, the $v$'s can be rounded off to the desired number of decimals. The process of rounding off may disrupt the closure of the angle equations by 1 or 2 in the last decimal place. If this occurs, 1 or 2 of the $v$'s should be arbitrarily raised or lowered 1 in the last decimal to insure the closure of the angle equations.

## 65. Solution of Normal Equations by the Doolittle Method

*a.* Although the discussion in paragraph 64 covers what actually is taking place in the solution of normal equations, the solution should be made by the Doolittle method. The Doolittle method, a form of successive substitution, is preferred as being the easiest for longer solutions due to the deletion of the diagonal terms in the equations. The form which the solution takes is shown at the bottom of the example form in figure 115.

*b.* For this example, where there are four normals to be solved, the form has four horizontal spaces, columns corresponding to the normal equation arrangement, plus an additional column in which the numerical value of the C's can be written. The top horizontal space contains two lines, and the other three spaces have three lines each. On the top line of each space the normal equations are entered directly as formed from the correlates. On the bottom line of each space are the divided equations corresponding to equations ①, ②, ③, and ④ as illustrated in paragraph 64*a*. The middle lines in the second, third, and fourth spaces are for the reduced equations found by the successive substitution of the C's.

*c.* The Doolittle method simplifies the reduction of the normal equations greatly, as successive substitution rapidly becomes too laborious to be practical. In the Doolittle method, use is made of the coefficients from the reduced equations as well as from the divided equations.

*d.* In the following explanation of the Doolittle method, the solution of the normals as shown in the example (fig. 115) will be used as reference, and equations ①, ②, ③, and ④ are used here with the terms to the left of the diagonal terms removed. The first normal is written on the form on the first line. This equation is then divided by minus the diagonal term which is $-(+)6$. The result is a divided equation giving $C_1$ in terms of $C_2$, $C_3$, $C_4$, and a constant. Normal equation ② is written on the first line of the second space. This equation is reduced by the product of the coefficient of $C_2$ in equation ① (which is $-2$), times each of the divided coefficients of equation ①, algebraically added to each of the coefficients of equation ②. Products are added only to coefficients in the same column as the divided coefficient making the product. For example, the diagonal term in equation ② $(+6)$ is reduced by the product of $(-2)$ $(+0.3333)$ which gives a reduced diagonal of $+5.3333$. The second term of normal 2 $(+2)$ is reduced by the product $(-2)$ $(-0.3333)$ which gives a reduced value of $+2.6667$. The third term $(-10.05)$ is reduced by $(-2)$ $(-1.4350)$ to give $-7.1800$, and so on for $\eta$ and $\Sigma_\eta$. As a check on the solution, the algebraic sum of the coefficients and $\eta$ in the reduced equation should equal the value in the $\Sigma_\eta$ column with the possible exception of 1 or 2 units in the last decimal. Always change the value in the $\Sigma_\eta$ column to agree with the addition of the coefficients if the difference is in only 1 or 2 units. To check the division, the algebraic sum of the coefficients and $\eta$ in the divided equations should equal the value in the $\Sigma_\eta$ column. Remember to include a $-1$ as the coefficient of the C found by dividing the equation. The reduced normal 2 is now divided by minus the reduced diagonal, $[-(+5.3333)]$, which gives $C_2$ in terms of $C_3$, $C_4$, and a constant. Normal equation ③ is written on the first line of the third space. This equation is reduced by the product of the coefficient of $C_3$ in normal 1 (which is $+2$), times each of the divided coefficients of normal 1, plus the product of the coefficient of $C_3$ in reduced normal 2 (which is $+2.6667$), times each of the divided coefficients of normal 2.

Both products are added algebraically to the coefficients of equation ③ in the same column as the divided coefficients making the products. Thus the diagonal term of normal 3 (+6) is reduced by the product of (+2) (−0.3333) plus the product of (+2.6667) (−0.5000) which gives a reduced diagonal of +4.0000. The second term in normal 3 (−6.75) is reduced by (+2) (−1.4350) plus (+2.6667) (+1.3463) which gives a reduced term of −6.0298. The third term (−6.5) is reduced by (+2) (+0.6667) plus (+2.6667) (+1.9187) which gives −0.0500 for the reduced third term. The same procedure is used to obtain the reduced $\Sigma_\eta$ term. The reduced equation is checked by adding across and then dividing by minus the reduced diagonal term which is [−(+4.0000)]. Repeating the process for normal 4 produces a numerical value for $C_4$ of +0.0090. The solution of the C's and $v$'s is made as previously explained.

*e.* After the $v$'s have been computed and checked through the equations, the adopted $v$'s are applied to the angles on the computation of triangles sheet in the column headed "Corr'n". Applying the correction to the observed angle produces the spherical angle from which the spherical excess is subtracted to obtain the plane angle. The triangles are then computed as explained in paragraphs 57 through 61, and the geographic positions computed as explained in paragraphs 67 or 68.

*f.* As part of the adjustment of a quadrilateral, the probable error of an observed direction is computed as shown on the form in figure 115.

## Section III. GEOGRAPHIC POSITION

### 66. Introduction

*a.* When the geographic position of a station is unknown, but the azimuth and distance to the station from a station of known position are available, the unknown position can be determined. There are many acceptable formulas, varying in accuracy, for this computation. In this manual, the USC&GS formulas were selected for all computations. These formulas are sufficient for all triangulation lengths in normal latitudes.

*b.* Normally, two known stations are used to give a check on the position of the unknown station. The procedure generally followed is to solve a triangle, with the unknown station as 1 and the known stations 2 and 3, after correcting the observed spherical angles by some type of adjustment. Then using the azimuths between the known stations, and the spherical angles of the triangle, determine the azimuths to the unknown station from the known stations. Finally, the formulas are solved for the position of the unknown station and azimuths from the unknown station to the known stations. Due to the convergence of the meridians on the earth's surface, the back (or reverse) azimuth of a geodetic line is not exactly 180° different from the forward azimuth. The difference between these azimuths is known as convergence and is dependent on the difference in longitude of the ends of the line.

*c.* It sometimes becomes necessary to compute the geographic azimuths and length of a line joining two stations which are fixed in position, but have not been directly connected by the observations. In order to compute this line, an inverse or back computation must be made. The mathematical basis of the inverse position computation is exactly the same as that of the position computation. The computation is based upon the solution of the right spherical traingle formed by the line connecting the two known stations, and the line representing $\Delta\phi$ and $\Delta\lambda$.

### 67. Direct Position Computation, Logarithmic Solution

*a.* The formulas used in the solution of this problem are as follows:

$$-\Delta\phi = s\cos\alpha\cdot B + s^2\sin^2\alpha\cdot C + (\delta\phi)^2 D - hs^2\sin^2\alpha\cdot E - \tfrac{1}{2}s^2 k E + (3/2)s^2\cos^2\alpha\cdot kE + \tfrac{1}{2}s^2\cos^2\alpha\sec^2\phi A'^2 k\sin^2 1'';$$

$$h = s\cos\alpha\cdot B;$$

$$-\delta\phi = s\cos\alpha\cdot B + s^2\sin^2\alpha\cdot C - hs^2\sin^2\alpha\cdot E;$$

$$k = s^2\sin^2\alpha\cdot C;$$

$$\sin\Delta\lambda = \sin\frac{s}{N'}\sec\phi'\sin\alpha;$$

$$\log\Delta\lambda = \log s + C_{\log\Delta\lambda} - C_{\log s} + \log\sin\alpha + \log A' + \log\sec\phi';$$

$$-\tan\tfrac{1}{2}(\Delta\alpha) = \tan\tfrac{1}{2}(\Delta\lambda)\frac{\sin\frac{1}{2}(\phi'+\phi)}{\cos\frac{1}{2}(\phi'-\phi)};$$

$$-\Delta\alpha = \Delta\lambda\sin\tfrac{1}{2}(\phi'+\phi)\sec\tfrac{1}{2}(\Delta\phi) + (\Delta\lambda)^3 F.$$

Where: C (log $\Delta\lambda$) and C (log $s$) are the arc-sine corrections, the arguments log $\Delta\lambda$ and log $s$ being indicated in parentheses.

A logarithmic solution of a first-order position computation is shown in figure 116. The dis-

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| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 2 MT. NEBO | to 3 TUSHAR | 20 | 05 | 36.08 | $\alpha$ | 3 TUSHAR | to 2 MT. NEBO | 199 | 41 | 08.58 |
| 2d ∠ | & | | +48 | 04 | 05.50 | 3d ∠ | & | | -88 | 16 | 30.92 |
| $\alpha$ | 2 MT. NEBO | to 1 WHEELER PEAK | 68 | 09 | 41.58 | $\alpha$ | 3 TUSHAR | to 1 WHEELER PEAK | 111 | 24 | 37.66 |
| $\Delta\alpha$ | | | - 1 | 37 | 01.48 | $\Delta\alpha$ | | | - 1 | 11 | 20.22 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| $\alpha'$ | 1 WHEELER PEAK | to 2 MT. NEBO | 246 | 32 | 40.10 | $\alpha'$ | 1 WHEELER PEAK | to 3 TUSHAR | 290 | 13 | 17.44 |
| | First Angle of Triangle | | 43 | 40 | 37.34 | | | | | | |

| | ° | ′ | ″ | | | ° | ′ | ″ | | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | 39 | 48 | 38.316 | 2 MT. NEBO | $\lambda$ | 111 | 45 | 56.235 | $\phi$ | 38 | 25 | 09.551 | 3 TUSHAR | $\lambda$ | 112 | 24 | 42.168 |
| $\Delta\phi$ | - | 49 | 29.300 | | $\Delta\lambda$ | + 2 | 32 | 50.783 | $\Delta\phi$ | + | 33 | 59.467 | | $\Delta\lambda$ | + 1 | 54 | 04.851 |
| $\phi'$ | 38 | 59 | 09.016 | 1 WHEELER PEAK | $\lambda'$ | 114 | 18 | 47.018 | $\phi'$ | 38 | 59 | 09.018 | 1 WHEELER PEAK | $\lambda'$ | 114 | 18 | 47.019 |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $s$ | Logarithms 5.3761505 | (1) | ″ +2867.8000 | $\frac{1}{2}$ | Logs 9.699 | $\frac{1}{2}(\phi+\phi')$ | ° ′ ″ 39 23 53.67 | $s$ | Logarithms 5.2478407 | (1) | ″ -2094.8860 | $\frac{1}{2}$ | Logs 9.699 | $\frac{1}{2}(\phi+\phi')$ | ° ′ ″ 38 42 09.28 |
| cos $\alpha$ | 9.5705323 | (2) | + 103.0505 | $s^2$ | 10.752 | Logarithms | | cos $\alpha$ | 9.5623485 | (2) | + 54.6500 | $s^2$ | 10.496 | Logarithms | |
| B | 8.5108661 | (4) | - 1.7599 | K | 2.013 | $s$ | 5.3761505 | B | 8.5109712 | (4) | + 0.6708 | K | 1.738 | $s$ | 5.2478407 |
| (1)=h | 3.4575489 | $\delta\phi$ | +2969.0906 | E | 6.100 | sin $\alpha$ | 9.9676586 | (1)=h | 3.3211604 | $\delta\phi$ | -2039.5652 | E | 6.072 | sin $\alpha$ | 9.9689446 |
| $s^2$ | 10.75230 | (3) | + 0.2140 | (5) | 8.564 | A′ | 8.5091440 | $s^2$ | 10.49568 | (3) | + 0.1000 | (5) | 8.005 | A′ | 8.5091440 |
| sin² $\alpha$ | 9.93532 | (5) | - 0.0366 | 3 | 0.477 | sec $\phi'$ | 0.1094105 | sin² $\alpha$ | 9.93789 | (5) | - 0.0102 | 3 | 0.477 | sec $\phi'$ | 0.1094105 |
| C | 1.32543 | (6) | + 0.0152 | cos² $\alpha$ | 9.141 | Sum | 3.9623636 | C | 1.30402 | (6) | + 0.0040 | cos² $\alpha$ | 9.125 | Sum | 3.8353398 |
| (2)=K | 2.01305 | (7) | + 0.0167 | (6) | 8.182 | Arc−sin corr. | + 428 | (2)=K | 1.73759 | (7) | + 0.0046 | (6) | 7.607 | Arc−sin corr. | + 242 |
| $(\delta\phi)^2$ | 6.9452 | $-\Delta\phi$ | 2969.2999 | (colog) E | 13.900 | $\frac{\Delta\lambda}{2}$ + | 3.9624064 | $(\delta\phi)^2$ | 6.6191 | $-\Delta\phi$ | -2039.4668 | (colog) E | 13.928 | $\Delta\lambda$ | 3.8353640 |
| D | 2.3852 | $\frac{\Delta\phi}{2}$ | 1484.65 | $\frac{A^2 arc^2 1''}{3}$ | 5.912 | $\sin\frac{1}{2}(\phi+\phi')$ | 9.8025732 | D | 2.3807 | $\frac{\Delta\phi}{2}$ | 1019.73 | $\frac{A^2 arc^2 1''}{3}$ | 5.912 | $\sin\frac{1}{2}(\phi+\phi')$ | 9.7960730 |
| (3) | 9.3304 | | | sec²$\phi$ | 0.229 | sec $\frac{\Delta\phi}{2}$ | 113 | (3) | 8.9998 | | | sec²$\phi$ | 0.212 | sec $\frac{\Delta\phi}{2}$ | 53 |
| −h | 3.4575 | | | (7) | 8.223 | (approx.) $-\Delta\alpha$ | 3.7649909 | −h | 3.3212 | | | (7) | 7.659 | (approx.) $-\Delta\alpha$ | 3.6314423 |
| $s^2\sin^2\alpha$ | 10.6876 | Arc−sin corr. | | | | do + | 5820.″910 | $s^2\sin^2\alpha$ | 10.4336 | Arc−sin corr. | | | | do | 4279.″985 |
| E | 6.1004 | for $s$ | - 1003 | $(\Delta\lambda)^3$ | 11.887 | (8) | + 0.″571 | E | 6.0718 | for $s$ | - 555 | $(\Delta\lambda)^3$ | 11.506 | (8) | + 0.″240 |
| (4) | 0.2455 | for $\Delta\lambda$ | + 1431 | F | 7.870 | $-\Delta\alpha$ + | 5821.″481 | (4) | 9.8266 | for $\Delta\lambda$ | + 797 | F | 7.874 | $-\Delta\alpha$ | 4280.″225 |
| | | Total | + 428 | (8) | 9.757 | $\Delta\lambda$ + | 9170.″7828 | | | Total | + 242 | (8) | 9.380 | $\Delta\lambda$ | 6844.″8511 |

| COMPUTED BY | R.A. Smith - AMS | DATE | MAY '56 | CHECKED BY | W.C. Aumen - AMS | DATE | MAY '56 | NOTE: For log $s$ to 3.9, omit terms below heavy black line NOT in heavy bold type or underlined. |
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**POSITION COMPUTATION, FIRST ORDER TRIANGULATION (Logarithmic)**
(TM 5-237)

*Figure 116. Position computation, first-order triangulation (logarithmic) ( DAForm 1922).*

tances used are the sides of a triangle, and the azimuths are the angles the vertical section makes with the meridian, measured clockwise from the south. The solution is made using the Clarke 1866 Spheroid, and all factors are taken from USC&GS SP 8. For the International Spheroid, SP 200 would be used to obtain the necessary factors. Wheeler Peak, the unknown station, is number 1. Mt. Nebo and Tushar, the known stations, are numbered 2 and 3, respectively. The azimuths ($\alpha$) from 2 to 3 and from 3 to 2 are entered on the form, and the second and third angles from the triangle computation are applied to these azimuths, giving the azimuths from the known stations to the unknown station. The log distances ($s$) are taken from the triangle computation, log sin $\alpha$ 2–1 (3–1), and log cos $\alpha$ 2–1 (3–1) are taken from the tables to seven decimal places, and the factors B, C, D, E, and F are taken from USC&GS Sp. Pub. 8 using the latitude ($\phi$) of the known station as the argument. The sum of log $s$, log cos $\alpha$, and B is the log of (1), the sign being determined by the sign of cos $\alpha$. In the Northern Hemisphere, the cosine is plus in the first (0°—90°) and fourth (270°—360°) quadrants, and minus in the second (90°—180°) and third (180°—270°) quadrants; in the Southern Hemisphere the reverse is true. The sum of log $s^2$, log $\sin^2 \alpha$, and C is the log of (2), and the sum of log $(-h)$, log $s^2 \sin^2 \alpha$, and E is the log of (4). Find the antilogs of (1) (2) and (4) and add algebraically to get ($\delta\phi$). The sum of log $(\delta\phi)^2$ and D is the log of (3). Log (5) is the sum of log ½, log $s^2$, log K, and log E. Log (6) is the sum of log (5), log 3, and log $\cos^2 \alpha$. Log (7) is the sum of log (6), colog E, log $\frac{A^2 \text{ arc}^2 1''}{3}$, and log $\sec^2 \phi$.

*Note.* If the distance ($s$) is small, (4), (5), (6), and (7) will be small and the characteristics must be carefully watched to avoid error.

Find the antilogs of (3), (5), (6), and (7) and add these algebraically to the sum of (1), (2), and (4) to get $-\Delta\phi$ in seconds. Apply $-\Delta\phi$ to the latitude ($\phi$) of the known station to obtain the latitude ($\phi'$) of the unknown station. $\phi'$ is used to interpolate A′, from Sp. Pub. 8, and log sec $\phi'$ from the tables. Add log $s$, log sin $\alpha$, A′, and log sec $\phi'$. This sum is used as an approximate $\Delta\lambda$ to find the arc-sin correction, and takes its sign from sin $\alpha$. In the Western Hemisphere, the sine of $\alpha$ is plus in the first (0°–90°) and second (90°–180°) quadrants, and minus in the third (180°–270°) and fourth (270°–360°) quadrants; in the Eastern Hemisphere, the reverse is true. The arc-sin correction is equal to the correction for $\Delta\lambda$ minus the correction for $s$. Both of these corrections can be found in Sp. Pub. 8, page 17, or computed from the formulas in Sp. Pub. 8, page 18. After the arc-sin correction is computed, it is added numerically to the previous sum to obtain the log of $\Delta\lambda$. Applying $\Delta\lambda$ algebraically to the longitude ($\lambda$) of the known station gives the longitude ($\lambda'$) of the unknown station. The sum of log $(\Delta\lambda)^3$ and F is the log of (8), the sign being the same as the sign of $\Delta\lambda$. The sum of log $\Delta\lambda$, log sin $\frac{1}{2}(\phi+\phi')$, and log sec $\frac{\Delta\phi}{2}$ is the log of $-\Delta\alpha$ (approx.), the sign being the same as the sign of $\Delta\lambda$ in the Northern Hemisphere, west of Greenwich, and in the Southern Hemisphere, east of Greenwich, and opposite the sign of $\Delta\lambda$ in the Northern Hemisphere, east of Greenwich, and the Southern Hemisphere, west of Greenwich. Add algebraically the antilogs of $-\Delta\alpha$ (approx) and (8) for $-\Delta\alpha$ in seconds. Apply $-\Delta\alpha$ to the azimuth ($\alpha$) from 2 to 1 (3 to 1) to get the azimuth ($\alpha'$) from 1 to 2 (1 to 3). These steps are used on both sides of the form. The positions computed for station 1 should check within 1- or 2-thousandths of a second for both latitude and longitude, and the azimuth should check within 1-hundredth when the first angle of the triangle is applied to either side of the computation.

*b.* Figure 117 is a logarithmic solution of a third-order position computation. The unknown station is Parson, numbered 1, and the known stations are Outer and Hard, numbered 2 and 3, respectively. The known azimuths are entered on the form, and the second and third angles of the triangle are applied to give the azimuths from the known stations to the unknown station. The distances ($s$) are taken from the triangle computation; log sin $\alpha$, and log cos $\alpha$ to six decimal places, are taken from the tables; and factors B, C, and D are taken from USC&GS Sp. Pub. 8, using the latitude ($\phi$) as the argument, and all are entered on the form. Log $s$, log cos $\alpha$, and B are added to find $h$, which is the first term in $-\Delta\phi$. The sign of $h$ is determined by the sign of cos $\alpha$. The sum of log $s^2$, log $\sin^2 \alpha$, and C is the second term, and the sum of log $h^2$ (enter in $(\delta\phi)^2$ space) and D is the log of the third term of $-\Delta\phi$. Add the three terms algebraically to find $-\Delta\phi$. Apply $-\Delta\phi$ to the latitude ($\phi$) of the known station to

get the latitude ($\phi'$) of the unknown station. Use $\phi'$ as the argument for A′ in Sp. Pub. 8, and find the log sec $\phi'$ from the tables to six decimal places. Adding log $s$, log sin $\alpha$, A′, and log sec $\phi'$ gives the log of $\Delta\lambda$. The sign of $\Delta\lambda$ is determined by the sign of sin $\alpha$. Apply $\Delta\lambda$ to the longitude ($\lambda$) of the known station to get the longitude ($\lambda'$) of the unknown station. Add log $\Delta\lambda$, and log sin $\frac{1}{2}(\phi+\phi')$ for the log of $-\Delta\alpha$. Apply $-\Delta\alpha$ to the azimuth ($\alpha$) from the known station to the unknown station to get the azimuth ($\alpha'$) from the unknown station to the known station. The positions computed for the unknown station should not differ by more than 1-thousandth of a second

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 2 OUTER | to 3 HARD | 144 | 43 | 42.5 | $\alpha$ | 3 HARD | to 2 OUTER | 324 | 42 | 28.9 |
| 2d ∠ | | & | +23 | 51 | 16.1 | 3d ∠ | | & | −52 | 56 | 25.1 |
| $\alpha$ | 2 OUTER | to 1 PARSON | 168 | 34 | 58.6 | $\alpha$ | 3 HARD | to 1 PARSON | 271 | 46 | 03.8 |
| $\Delta\alpha$ | | | − | | 20.7 | $\Delta\alpha$ | | | + | | 52.9 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| $\alpha'$ | 1 PARSON | to 2 OUTER | 348 | 34 | 37.9 | $\alpha'$ | 1 PARSON | to 3 HARD | 91 | 46 | 56.7 |
| | First Angle of Triangle | | 103 | 12 | 18.8 | | | | | | |

| | ° | ′ | ″ | | | ° | ′ | ″ | | ° | ′ | ″ | | | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | 40 | 35 | 18.742 | 2 OUTER | $\lambda$ | 73 | 36 | 33.964 | $\phi$ | 40 | 37 | 20.514 | 3 HARD | $\lambda$ | 73 | 38 | 27.008 |
| $\Delta\phi$ | + | 1 | 59.853 | | $\Delta\lambda$ | + | | 31.763 | $\Delta\phi$ | − | | 01.919 | | $\Delta\lambda$ | − | 1 | 21.281 |
| $\phi'$ | 40 | 37 | 18.595 | 1 PARSON | $\lambda'$ | 73 | 37 | 05.727 | $\phi'$ | 40 | 37 | 18.595 | 1 PARSON | $\lambda'$ | 73 | 37 | 05.727 |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $s$ | Logarithms 3.576 526 | (1) | − 119.8541 | $\frac{1}{2}$ | Logs 9.699 | $\frac{1}{2}(\phi+\phi')$ | 40 36 18.7 | $s$ | Logarithms 3.281 346 | (1) | + 1.9115 | $\frac{1}{2}$ | Logs 9.699 | $\frac{1}{2}(\phi+\phi')$ | 40 37 19.6 |
| cos $\alpha$ | 9.991 320 | (2) | + 0.0012 | $s^2$ | | Logarithms | | cos $\alpha$ | 8.489 222 | (2) | + 0.0079 | $s^2$ | | Logarithms | |
| B | 8.510 807 | (4) | | K | | $s$ | 3.576 526 | B | 8.510 804 | (4) | | K | | $s$ | 3.281 346 |
| (1)=h | 2.078 653 | $\frac{\delta\phi}{}$ | | E | | sin $\alpha$ | 9.296 554 | (1)=h | 0.281 372 | $\frac{\delta\phi}{}$ | | E | | sin $\alpha$ | 9.999 793 |
| $s^2$ | 7.153 05 | (3) | + 0.0004 | (5) | | A′ | 8.509 103 | $s^2$ | 6.562 69 | (3) | + 0 | (5) | | A′ | 8.509 103 |
| sin² $\alpha$ | 8.593 11 | (5) | − | 3 | 0.477 | sec $\phi'$ | 0.119 745 | sin² $\alpha$ | 9.999 59 | (5) | − | 3 | 0.477 | sec $\phi'$ | 0.119 745 |
| C | 1.337 31 | (6) | + | cos² $\alpha$ | | Sum | | C | 1.337 83 | (6) | + | cos² $\alpha$ | | Sum | |
| (2)=K | 7.083 47 | (7) | + | (6) | | Arc−sin corr. | | (2)=K | 7.900 11 | (7) | + | (6) | | Arc−sin corr. | |
| $(\frac{\delta\phi}{})^2$ | 4.1573 | $-\frac{\Delta\phi}{}$ | − 119.8525 | (colog) E | | $\frac{\Delta\lambda}{}$ | 1.501 928 | $(\frac{\delta\phi}{})^2$ | 0.5627 | $-\frac{\Delta\phi}{}$ | + 1.9194 | (colog) E | | $\frac{\Delta\lambda}{}$ | 1.909 987 |
| D | 2.3872 | $\frac{\Delta\phi}{2}$ | | $A^2\text{arc}^2 1'' \div 3$ | 5.912 | $\sin\frac{1}{2}(\phi+\phi')$ | 9.813 476 | D | 2.3873 | $\frac{\Delta\phi}{2}$ | | $A^2\text{arc}^2 1'' \div 3$ | 5.912 | $\sin\frac{1}{2}(\phi+\phi')$ | 9.813 626 |
| (3) | 6.5445 | | | sec²$\phi$ | | $\sec\frac{\Delta\phi}{2}$ | | (3) | 2.9500 | | | sec²$\phi$ | | $\sec\frac{\Delta\phi}{2}$ | |
| −h | | | | (7) | | (approx.) $-\Delta\alpha$ | 1.315 404 | −h | | | | (7) | | (approx.) $-\Delta\alpha$ | 1.723 613 |
| $s^2\sin^2\alpha$ | | Arc−sin corr. | | | | do | " | $s^2\sin^2\alpha$ | | Arc−sin corr. | | | | do | " |
| E | | for $s$ | − | $(\Delta\lambda)^3$ | | (8) | " | E | | for $s$ | − | $(\Delta\lambda)^3$ | | (8) | " |
| (4) | | for $\Delta\lambda$ | + | F | | $-\Delta\alpha$ | 20.67 | (4) | | for $\Delta\lambda$ | + | F | | $-\Delta\alpha$ | 52.92 |
| | | Total | | (8) | | $\Delta\lambda$ | 31.7635 | | | Total | | (8) | | $\Delta\lambda$ | 81.2806 |

COMPUTED BY R.A.S. - AMS | DATE MAY '56 | CHECKED BY W.C.A. - AMS | DATE MAY '56 | NOTE: For log $s$ to 3.9, omit terms below heavy black line NOT in heavy bold type or underlined.

DA FORM 1 OCT 64 1922 REPLACES DA FORM 1922, 1 FEB 57, WHICH IS OBSOLETE. **POSITION COMPUTATION, Third (TM 5-237) ORDER TRIANGULATION (Logarithmic)**

*Figure 117. Position computation, third-order triangulation (logarithmic) (DA Form 1922).*

in either latitude or longitude. This method meets third-order requirements when the triangle sides do not exceed 25,000 meters (approx).

*c.* Examples shown are for the Northern Hemisphere, west of Greenwich, and the following variations should be noted:

(1) For computations in the Southern Hemisphere, the sign of the cosine is reversed.

(2) For computations in the Eastern Hemisphere, the sign of the sine is reversed.

(3) $\Delta\alpha$ is applied according to the following rules:

(*a*) In the Northern Hemisphere, if $\alpha$ is less than 180°, $\Delta\alpha$ is minus; if $\alpha$ is greater than 180°, $\Delta\alpha$ is plus.

(*b*) In the Southern Hemisphere, if $\alpha$ is less than 180°, $\Delta\alpha$ is plus; if $\alpha$ is greater than 180°, $\Delta\alpha$ is minus.

## 68. Direct Position Computation, Natural Function Solution

*a.* The formulas used in the solution of the problem are as follows:

$$y_2 = y_0 + [y + (yfa \cdot 10^{-7})] - Va + K\left(\frac{Va}{1{,}000}\right)^2$$

$$\Delta\lambda'' = \left[x - \left(\frac{xfb}{2} \cdot 10^{-7}\right)\right] H + \left[x'\left(\frac{V(Va)}{15}\right) \cdot 10^{-7}\right]$$

$$\Delta\alpha'' = \Delta\lambda'' \frac{\sin\phi + \sin\phi'}{1 + \cos\Delta\phi} + F(\Delta\lambda'')^3$$

A solution, using natural functions, of a first-order position computation is shown in figure 118. The solution is made using the Clarke 1866 Spheroid, therefore all factors are taken from USC&GS Sp. Pub. 241. For the International Spheroid, USC&GS G–58 would be used to obtain the necessary factors. The station of unknown position is Hayford, numbered 1, and the known stations are Pioche and Burger, numbered 2 and 3, respectively. The azimuths between the known stations are entered on the form, and the second and third angles in the triangle computation are applied to these azimuths to get the azimuths from the known stations to the unknown station. The distances (*s*) are taken from the triangle computation (if the triangle computation was logarithmic, enter both the log distance and the

PROJECT 8-6886 — POSITION COMPUTATION, First ORDER TRIANGULATION (For calculating machine computation) (TM 5-237)

LOCATION Nevada — ORGANIZATION A & S, Inc. — DATE 20 May 56

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 PIOCHE | To 3 BURGER | | 326 | 17 | 48.84 | α | 3 BURGER | To 2 PIOCHE | | 146 | 37 | 59.97 |
| 2nd ∠ | & | | | +68 | 41 | 14.78 | 3rd ∠ | & | | | −82 | 12 | 53.48 |
| α | 2 PIOCHE | To 1 HAYFORD | | 34 | 59 | 03.62 | α | 3 BURGER | To 1 HAYFORD | | 64 | 25 | 06.49 |
| Δα | | | | − | 41 | 47.81 | Δα | | | | − | 01 | 21.66 |
| | | | | 180 | 00 | 00.00 | | | | | 180 | 00 | 00.00 |
| α′ | 1 HAYFORD | To 2 PIOCHE | | 214 | 17 | 15.81 | α′ | 1 HAYFORD | To 3 BURGER | | 243 | 23 | 44.83 |
| | | First Angle of Triangle | | 29 | 06 | 29.03 | | | | | | | |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 37 | 59 | 10.102 | 2 PIOCHE | λ | 114 | 03 | 03.921 | φ | 37 | 19 | 31.852 | 3 BURGER | λ | 113 | 30 | 01.480 |
| | | | | s = 179,171.942m | Δλ | + 1 | 08 | 55.942 | | | | | s = 168,468.884 | Δλ | + 1 | 41 | 58.383 |
| φ′ | 36 | 39 | 27.946 | 1 HAYFORD | λ′ | 115 | 11 | 59.863 | φ′ | 36 | 39 | 27.946 | 1 HAYFORD | λ′ | 115 | 11 | 59.863 |

| | | | |
|---|---|---|---|
| Δφ | 1° 19′ 42.″2 | (log s = 5.2532700 b = (y/10,000)² 215.494) | |
| sin α | + 0.57335251 | x cor. = −½fb | 884.3 |
| cos α | + 0.81930880 | x′ | + 102,719.599 |
| x = s sin α | + 102,728.683m | H | 0.040263424 |
| y = −s cos α | − 146,797.149 | Hx′ = (approx. Δλ″) | + 4135.8428 |
| a = (x′/10,000)² | 105.5132 | Arc−sin cor = + V(Va)/15 | 238.9 |
| y cor. = +fa | 866.0 | Δλ″ | + 4135.9416 |
| $y_0$ | 4,205,753.823 | sin φ | 0.61547083 |
| y′ | − 146,809.862 | sin φ′ | 0.59703393 |
| $y_1$ | 4,058,943.961 | 1 + cos Δφ | 1.99973125 |
| Va | − 614.963 | (sin φ + sin φ′)/(1 + cos Δφ) | 0.60633386 |
| $y_2$ | 4,058,329.033 | −Δα″ (approx.) | + 2507.761 |
| V | 5.82830 | + F (Δλ″)³ | 0.053 |
| K (Va/1,000)² + | 0.035 | −Δα″ | + 2507.814 |

| | | | |
|---|---|---|---|
| Δφ | 40′ 03.″9 | (log s = 5.2263197 b = (y/10,000)² 52.917) | |
| sin α | + 0.90197176 | x cor. = −½fb | 217.2 |
| cos α | + 0.43179502 | x′ | + 151,950.876 |
| x = s sin α | + 151,954.176 | H | 0.040263424 |
| y = −s cos α | − 72,744.025 | Hx′ = (approx. Δλ″) | + 6118.0625 |
| a = (x′/10,000)² | 230.8908 | Arc−sin cor = + V(Va)/15 | 523.1 |
| y cor. = +fa | 1895.3 | Δλ″ | + 6118.3825 |
| $y_0$ | 4,132,432.698 | sin φ | 0.60634257 |
| y′ | − 72,757.812 | sin φ′ | 0.59703393 |
| $y_1$ | 4,059,674.886 | 1 + cos Δφ | 1.99993209 |
| Va | − − 1,346.024 | (sin φ + sin φ′)/(1 + cos Δφ) | 0.60170868 |
| $y_2$ | 4,058,329.030 | −Δα″ (approx.) | + 3681.484 |
| V | 5.82970 | + F (Δλ″)³ | 0.172 |
| K (Va/1,000)² + | 0.168 | −Δα″ | + 3681.656 |

COMPUTED BY RAS — DATE 27 May 56 — CHECKED BY W.C.A. — DATE 29 May 56

NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1 FEB 57 1923

*Figure 118. Position computation, first-order, natural functions (DA Form 1923).*

distance on the form). Extract sin $\alpha$ and cos $\alpha$ from the tables (eight-place natural functions must be used to meet the accuracy requirements). Next, multiply the length by the proper functions to determine $x$ and $y$. The $x$ value takes the algebraic sign of sine $\alpha$, while the $y$ value takes the algebraic sign opposite that of cosine $\alpha$. The algebraic signs of sine and cosine are explained in the log computation. Next, compute the $b$ value by moving the decimal point in $y$ four places to the left and squaring the result. Multiply $b$ by $\frac{1}{2}f$ to find the $x$ correction factor. $f$ is found at the bottom of the page in USC&GS Sp. Pub. 241, using the latitude of the known station as the argument. The minus sign before the correction factor means $x$ is reduced numerically. The correction factor is in units of the seventh place of decimals; therefore, in the example on the left-hand side, $x$ would be multiplied by 1.0—0.00008843 or 0.99991157 to get $x'$. (For other methods of finding $x'$, see page 81 of USC&GS Sp. Pub. 241.) Now use the value of $x'$ to compute $a$ in the same manner as the computation for $b$, taking $a$ to 4 decimal places. Multiply $f$ by $a$ for the $y$ correction factor. The plus sign indicates $y$ must be increased numerically. The correction factor is in units of the seventh place of decimals, so in the example, $y$ is multiplied by 1.0+0.00008660 or 1.00008660 to get $y'$. $y_0$ is taken from Sp. Pub. 241 under meridional arcs (meters) using the latitude of the known station as the argument. $y'$ is applied to $y_0$ to get the $y_1$ value. Now using $y_1$ as the argument, the value for V and K are found. Note the correction for the second differences of V. The correction is in units of the fifth decimal place. (No space is provided on the form for K.) Multiply V by $a$ and K by $(Va/1{,}000)^2$ for the two corrections to be applied to $y_1$ to get $y_2$. The value of $y_2$ should check on both sides of the form. The $Va$ correction is always minus and the $K(Va/1{,}000)^2$ correction is always plus. Use $y_2$ to interpolate for the latitude ($\phi'$) of the unknown station. After $\phi'$ is determined, $\Delta\phi$ is taken out to tenths of seconds and entered on the form. Using $\phi'$ as the argument, the value of H is interpolated in the table. (Note the correction for second difference in H.) Multiply H by $x'$ for an approximate $\Delta\lambda''$. Next, compute the arc-sine correction factor which is $\frac{1}{15}$ of the product of V and $Va$. The plus sign indicates the approximate $\Delta\lambda''$ is increased numerically. Like the other corrective factor, the arc-sine factor is in units of the seventh place of decimals. Apply the arc-sine correction to get $\Delta\lambda''$, which is applied to the longitude ($\lambda$) of the known station to get the longitude ($\lambda'$) of the unknown station. The longitude should check on both sides of the form. From a table of natural functions, interpolate the cosine of $\Delta\phi$. The sines of $\phi$ and $\phi'$ can be obtained from Sp. Pub. 241. Add 1 to the cosine of $\Delta\phi$ before entering on the form. Sin $\phi$ and sin $\phi'$ are added and the sum divided by the $1+\cos\Delta\phi$ value. Multiply this value by $\Delta\lambda''$ to get the approximate value of $-\Delta\alpha''$. The small correction $F(\Delta\lambda'')^3$ must be computed and added numerically to $-\Delta\alpha''$. The argument for F is $\phi_m$. The F factor as tabulated contains the factor $10^{-12}$. To effect this factor, move the decimal point in $\Delta\lambda''$ four places to the left and then cube the result. It is sufficient to take $\Delta\lambda''$ to the nearest second for the computation of the correction. Notice that this correction is not the factor type such as the $x$ and $y$ corrections. In this case, the approximate $-\Delta\alpha''$ is numerically increased by the quantity $F(\Delta\lambda'')^3$. Apply the final $-\Delta\alpha''$ to the appropriate azimuth ($\alpha$) to obtain the back azimuth ($\alpha'$). On the computation form, the sum of the azimuth from 1 to 2 plus the first angle in the triangle should equal the azimuth from 1 to 3. In this computation, emphasis is put on the fact that the plus or minus before a corrective term indicates, respectively, a *numerical* increase or decrease of the term to be corrected, irrespective of the algebraic sign of the term.

*b.* Figure 119 is a solution of a third-order position computation. For lines under 8,000 meters in length, none of the terms involving the use of $f$ or $b$ has a significant effect. As both lines here are well under this 8,000-meter limit, no corrective terms are needed. Seven decimal places are sufficient for sin $\alpha$ and cos $\alpha$. Both $x'$ and $y'$ may be taken the same as $x$ and $y$, respectively. V need be taken out only to the same number of significant figures as $a$ or perhaps one more. In the example shown, V may be interpolated by inspection to the number of figures needed. The arc-sine correction is not needed, and $\Delta\lambda''$ is the product of $x'$ and H. Five-place sines are sufficient for the computation of $-\Delta\alpha''$.

*c.* Examples shown are for the Northern Hemisphere, west of Greenwich, and the following variations should be noted:

(1) For computations in the Southern Hemisphere, the sign of the cosine is reversed.
(2) For computations in the Eastern Hemisphere, the sign of the sine is reversed.
(3) $\Delta\alpha$ is applied according to the following rules:
  (*a*) In the Northern Hemisphere, if $\alpha$ is less than 180°, $\Delta\alpha$ is minus; if $\alpha$ is greater than 180°, $\Delta\alpha$ is plus.
  (*b*) In the Southern Hemisphere, if $\alpha$ is less than 180°, $\Delta\alpha$ is plus; if $\alpha$ is greater than 180°, $\Delta\alpha$ is minus.

## 69. Direct Position Computation, Natural Function Solution (Any Spheroid)

The formulas used in the solution of the problem are those listed in paragraph 67*a*. Tables for the A, B, C, D, E, and F factors for all major spheroids were prepared by the Army Map Service and are published under the TM 5-241-series (i.e., TM 5-241-18, Latitude functions: Clarke 1866 spheroid).

## 70. Inverse Position Computation, Logarithmic

*a*. Figure 120 is a logarithmic solution of the inverse position computation on DA Form 1924 (Inverse Position Computation). The positions of the stations, arbitrarily numbered 1 and 2, are entered at the top of the form. Next, $\Delta\phi$, $\frac{\Delta\phi}{2}$, $\phi_m$, $\Delta\phi$ (seconds), $\Delta\lambda$, $\frac{\Delta\lambda}{2}$, and $\Delta\lambda$ (seconds) are computed, completing the top block of the form. Log $\Delta\phi$ and log $\Delta\lambda$ are extracted from the tables, to seven decimal places. The arc-sin corrections are interpolated from table XXVI in TM 5–236, or from USC&GS Sp. Pub. 8, page 15, and subtracted from log $\Delta\phi$ and log $\Delta\lambda$ giving log $\Delta\phi_1$. Log cos $\frac{\Delta\lambda}{2}$ and log cos $\phi_m$ are computed. $\phi_m$ is used as the argument to find log A′ and log B in USC&GS Sp. Pub. 8. The cologs are computed and entered as colog $B_m$ and colog $A_m$. Log $\Delta\phi_1$, log cos $\frac{\Delta\lambda}{2}$, and colog $B_m$ are added to

PROJECT 5-6886 — POSITION COMPUTATION, Third ORDER TRIANGULATION (For calculating machine computation) (TM 5-237)

LOCATION Georgia — ORGANIZATION A&S, Inc. — DATE 19 May 56

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 2 ATWOOD | To 3 MARY | | | 298 | 30 | 51.2 | $\alpha$ | 3 MARY | To 2 ATWOOD | | | 118 | 31 | 52.3 |
| 2d ∠ | & | | | | +21 | 13 | 56.4 | 3d ∠ | & | | | | −112 | 20 | 18.5 |
| $\alpha$ | 2 ATWOOD | To 1 LITTLE SAPELO | | | 319 | 44 | 47.6 | $\alpha$ | 3 MARY | To 1 LITTLE SAPELO | | | 6 | 11 | 33.8 |
| $\Delta\alpha$ | | | | | | + | 57.3 | $\Delta\alpha$ | | | | | | − | 3.8 |
| | | | | | 180 | 00 | 00.00 | | | | | | 180 | 00 | 00.00 |
| $\alpha'$ | 1 LITTLE SAPELO | To 2 ATWOOD | | | 139 | 45 | 44.9 | $\alpha'$ | 1 LITTLE SAPELO | To 3 MARY | | | 186 | 11 | 30.0 |
| | First Angle of Triangle | | | | 46 | 25 | 45.1 | | | | | | | | |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | 31 | 27 | 35.461 | 2 ATWOOD | $\lambda$ | 81 | 20 | 09.455 | $\phi$ | 31 | 26 | 40.900 | 3 MARY | $\lambda$ | 81 | 18 | 12.351 |
| | | | | s=4492.584 | $\Delta\lambda$ | − | 1 | 49.919 | | | | | s=1758.997 | $\Delta\lambda$ | | + | 7.185 |
| $\phi'$ | 31 | 25 | 44.120 | 1 LITTLE SAPELO | $\lambda'$ | 81 | 18 | 19.536 | $\phi'$ | 31 | 25 | 44.120 | 1 LITTLE SAPELO | $\lambda'$ | 81 | 18 | 19.536 |

| | | | |
|---|---|---|---|
| $\Delta\phi$ | | (log $\frac{s}{b=(y/10{,}000)^2}$ 3.6524962) | $\Delta\phi$ | | (log $\frac{s}{b=(y/10{,}000)^2}$ 3.2452651) |
| sin $\alpha$ | −0.6461699 | x cor. $=-\frac{1}{2}fb$ | sin $\alpha$ | +0.1078731 | x cor. $=-\frac{1}{2}fb$ |
| cos $\alpha$ | +0.7631936 | x′ (same as x) | cos $\alpha$ | +0.9941647 | x′ (same as x) |
| x=s sin $\alpha$ | −2902.973 | H 0.037864399 | x=s sin $\alpha$ | +189.748 | H 0.037864399 |
| y=−s cos $\alpha$ | −3428.711 | Hx′=(approx. $\Delta\lambda''$) | y=−s cos $\alpha$ | −1748.733 | Hx′=(approx. $\Delta\lambda''$) |
| a=$(x'/10{,}000)^2$ | 0.0843 | Arc−sin cor $=+\frac{V\,(Va)}{15}$ | a=$(x'/10{,}000)^2$ | 0.0004 | Arc−sin cor $=+\frac{V\,(Va)}{15}$ |
| y cor. $=+fa$ | | $\Delta\lambda''$ −109.9193 | y cor. $=+fa$ | | $\Delta\lambda''$ +7.1847 |
| $y_o$ | 3,481,773.777 | sin $\phi$ | $y_o$ | 3,480,093.396 | sin $\phi$ |
| y′ | −3,428.711 | sin $\phi'$ | y′ | −1,748.733 | sin $\phi'$ |
| $y_1$ | 3,478,345.066 | 1 + cos $\Delta\phi$ | $y_1$ | 3,478,344.663 | 1 + cos $\Delta\phi$ |
| Va − | 0.403 | ~~sin $\phi$ + sin $\phi'$ / 1 + cos $\Delta\phi$~~ or sin $\phi_m$ 0.52167 | Va − | 0.002 | ~~sin $\phi$ + sin $\phi'$ / 1 + cos $\Delta\phi$~~ or sin $\phi_m$ 0.52156 |
| $y_2$ | 3,478,344.663 | $-\Delta\alpha''$ (approx.) | $y_2$ | 3,478,344.661 | $-\Delta\alpha''$ (approx.) |
| V | 4.786 | + F $(\Delta\lambda'')^3$ | V | 4.800 | + F $(\Delta\lambda'')^3$ |
| K $(Va/1{,}000)^2$+ | | $-\Delta\alpha''$ −57.34 | K $(Va/1{,}000)^2$+ | | $-\Delta\alpha''$ +3.75 |

COMPUTED BY RAS — DATE 28 May 56 — CHECKED BY W.C.A. — DATE 31 May 56

NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1 FEB 57 1923

*Figure 119. Position computation, third-order, natural functions (DA Form 1923).*

PROJECT 23-105

**INVERSE POSITION COMPUTATION** (TM 5-237)

LOCATION Oregon

ORGANIZATION A&S Inc.

$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$ | $s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$ | $-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^3$

in which log $\Delta\lambda_1$=log $(\lambda'-\lambda)$—correction for arc to sin; log $\Delta\phi_1$=log $(\phi'-\phi)$—correction for arc to sin; and log s=log $s_1$+correction for arc to sin.

| | | NAME OF STATIONS | | |
|---|---|---|---|---|
| 1. $\phi$ | 43° 59′ 00.715 | SPENCER | $\lambda$ | 123° 05′ 41.248 |
| 2. $\phi'$ | 44 30 38.293 | PETERSON | $\lambda'$ | 122 58 05.537 |
| $\Delta\phi$ $(=\phi'-\phi)$ | + 31 37.578 | | $\Delta\lambda$ $(=\lambda'-\lambda)$ | - 7 35.711 |
| $\frac{\Delta\phi}{2}$ | + 15 48.789 | | $\frac{\Delta\lambda}{2}$ | - 3 47.856 |
| $\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$ | 44 14 49.504 | | | |
| $\Delta\phi$ (secs.) | + 1897.″578 | | $\Delta\lambda$ (secs.) | - 455.″711 |
| log $\Delta\phi$ | 3.278 1996 | | log $\Delta\lambda$ | 2.658 6895 n |
| cor. arc—sin | — 15 | | cor. arc—sin | — 1 |
| log $\Delta\phi_1$ | 3.278 1981 | | log $\Delta\lambda_1$ | 2.658 6894 n |
| log $\cos\frac{\Delta\lambda}{2}$ | 9.999 9997 | | log cos $\phi_m$ | 9.855 1177 |
| colog $B_m$ | 1.489 4742 | | colog $A_m$ | 1.490 9902 |
| log $\{s_1 \cos(\alpha+\frac{\Delta\alpha}{2})\}$ | 4.767 6720 (opposite in sign to $\Delta\phi$) | | log $\{s_1 \sin(\alpha+\frac{\Delta\alpha}{2})\}$ | 4.004 7973 n |
| | | | log $\{s_1 \cos(\alpha+\frac{\Delta\alpha}{2})\}$ | 4.767 6720 n |
| log $\Delta\lambda$ | 2.658 6895 | 3 log $\Delta\lambda$ 7.976 | log tan $(\alpha+\frac{\Delta\alpha}{2})$ | 9.237 1253 |
| log sin $\phi_m$ | 9.843 7023 | log F 7.848 | $\alpha+\frac{\Delta\alpha}{2}$ | 189° 47′ 40.″69 |
| log sec $\frac{\Delta\phi}{2}$ | 0.000 0046 | log b 5.824 | log sin $(\alpha+\frac{\Delta\alpha}{2})$ | 9.230 7483 |
| log a | 2.502 3964n | | log cos $(\alpha+\frac{\Delta\alpha}{2})$ | 9.993 6230 |
| a | -317.98 | | log $s_1$ | 4.774 0490 |
| b | 0.00 | | cor. arc—sin | + 16 |
| $-\Delta\alpha$ (secs.) | - 317.″98 | | log s | 4.774 0506 |
| $-\frac{\Delta\alpha}{2}$ | - 158.″99 | | s | 59436.15 m. |
| | ° - 2 38.″99 | | | |
| $\alpha+\frac{\Delta\alpha}{2}$ | 189 47 40.″69 | | | |
| $\alpha$ (1 to 2) | 189 45 01.″70 | | | |
| $\Delta\alpha$ | + 5 17.″98 | | | |
| | 180 | | | |
| $\alpha'$ (2 to 1) | 9 50 19.″68 | | | |

NOTE.—For log s up to 4.0 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 3′, omit all terms below the heavy line except those printed (in whole or in part) in heavy type or those underscored, if using logarithms to 7 decimal places.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 12 Nov 54 | W. C. A. | 17 Nov 54 |

DA FORM 1 FEB 57 1924

*Figure 120. Inverse position computation, logarithmic (DA Form 1924).*

obtain the log $\left[s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]$ which is opposite in sign to $\Delta\phi$. Log $\Delta\lambda_1$, log cos $\phi_m$, and colog $A_m$ are added to obtain log $\left[s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]$ which takes the sign of $\Delta\lambda$. Subtract log $\left[s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]$ from log $\left[s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]$ to obtain log tan $\left(\alpha+\frac{\Delta\alpha}{2}\right)$. $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ is extracted from the tables using the log tan $\left(\alpha+\frac{\Delta\alpha}{2}\right)$, and this value is used to interpolate for log sin $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ and log cos $\left(\alpha+\frac{\Delta\alpha}{2}\right)$. Log $s_1$ is obtained by subtracting log sin $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ from $\left[s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]$, and checked by subtracting log cos $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ from log $\left[s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right]$. The arc-sin correction is added to log $s_1$ to get log $s$, and $s$ is extracted from the tables. Log $\Delta\lambda$, log sin $\phi_m$, and log sec $\frac{\Delta\phi}{2}$ are added, the sum being log $a$, which has the same sign as $\Delta\lambda$. 3 log $\Delta\lambda$ and log F are added, the sum being log $b$, which also has the same sign as $\Delta\lambda$. The argument for F is $\phi$. The values for $a$ and $b$ are extracted from the tables and added to obtain $-\Delta\alpha''$ (seconds). Then $\frac{-\Delta\alpha}{2}$ is found and applied to $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ giving $\alpha$ (1 to 2). $\Delta\alpha$ and 180° are applied to $\alpha$ (1 to 2) to obtain $\alpha'$ (2 to 1).

*b.* Figure 121 is a computation of lower-order accuracy made by following the instructions in the note at the bottom of the form.

*Note.* For log $s$ up to 4.0 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 3′, omit all terms below the heavy line except those printed (in whole or in part) in heavy type or those underscored, if using logarithms to seven decimal places.

*c.* Examples shown are for the Northern Hemisphere, west of Greenwich, and the following variations should be noted:

(1) For computations in the Southern Hemisphere, the sign of the cosine is reversed.

(2) For computations in the Eastern Hemisphere, the sign of the sine is reversed.

(3) $\Delta\alpha$ is applied according to the following rules:

(*a*) In the Northern Hemisphere, if $\alpha$ is less than 180°, $\Delta\alpha$ is minus; if $\alpha$ is greater than 180°, $\Delta\alpha$ is plus.

(*b*) In the Southern Hemisphere, if $\alpha$ is less than 180°, $\Delta\alpha$ is plus; if $\alpha$ is greater than 180°, $\Delta\alpha$ is minus.

## 71. Inverse Position Computation, Natural Function Solution

*a.* Figure 122 is an example of the inverse position computations performed with natural functions. The positions of the stations, arbitrarily numbered 1 and 2, are entered on the form, and $\Delta\phi$ and $\Delta\lambda$ are computed. $\phi$ is used to find $y_0$ (meridional arc in meters), and sin $\phi$ in USC&GS Sp. Pub. 241, and $\phi'$ is used to find $y_\phi$, sin $\phi'$, and H. $\Delta\lambda$ is changed to seconds, and the arc-sin correction factor is obtained from the formula; correction factor equals 0.39174 $\left(\frac{\Delta\lambda''}{100} \sin\phi'\right)^2$. This factor is in units of the seventh decimal place, and is added to 1.0 before being used. $\Delta\lambda$ is divided by the correction factor to find $Hx'$, which is divided by H to obtain $x'$. Now compute $a$ and $fa$. Use $y_2$ as the argument to interpolate K, and an approximate V, which is used to compute an approximate V$a$. The correction K $\left(\frac{Va}{1000}\right)^2$ is found and subtracted from $y_2$ to obtain the approximate $y_2$. V$a$ is added to the approximate $y_2$ to give an approximate $y_1$, which is used to interpolate a new V. A new V$a$ correction is computed and the correct $y_1$ is found. $y'$ is the difference between $y_0$ and $y_1$. $y'$ is divided by the $fa$ correction factor to find $y$. (Remember the correction factor is in units of the seventh decimal place and must be added to 1.0 before being used.) $b$ is found and the correction factor for $x$ is computed. $x'$ is divided by the factor to obtain $x$. (This factor must be subtracted from 1.0 before being used.) The distance ($s$) is computed by the Pythagorean theorem,

$$s=\sqrt{x^2+y^2}$$

Divide $x$ and $y$ by $s$ to find sin $\alpha$ and cos $\alpha$. Extract $\alpha$ from the tables of natural functions. $\Delta\alpha$ is computed by the same method used in the position computation, and the back azimuth is computed and entered on the form.

*b.* Figure 123 is an example of an inverse position computation for lines less than 8,000 meters. All correction factors are dropped and $x'$ is used as $x$, and $y'$ as $y$. This computation is sufficiently accurate for third-order surveys. DA Form 1923 is used for the computation.

| PROJECT | INVERSE POSITION COMPUTATION (TM 5-237) |
|---|---|
| 5-6886 | |
| LOCATION: Georgia | |
| ORGANIZATION: A&S, Inc. | |

$s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda_1 \cos\phi_m}{A_m}$ | $s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi_1 \cos\frac{\Delta\lambda}{2}}{B_m}$ | $-\Delta\alpha=\Delta\lambda \sin\phi_m \sec\frac{\Delta\phi}{2}+F(\Delta\lambda)^3$

in which $\log \Delta\lambda_1$=log ($\lambda'-\lambda$)—correction for arc to sin; $\log \Delta\phi_1$=log ($\phi'-\phi$)—correction for arc to sin; and $\log s$=$\log s_1$+correction for arc to sin.

| | | NAME OF STATIONS | | |
|---|---|---|---|---|
| 1. $\phi$ | 31° 27′ 35.461 | Atwood | $\lambda$ | 81° 20′ 09.455 |
| 2. $\phi'$ | 31 25 44.120 | Little Sapelo | $\lambda'$ | 81 18 19.536 |
| $\Delta\phi$ ($=\phi'-\phi$) | − 1 51.341 | | $\Delta\lambda$ ($=\lambda'-\lambda$) | − 1 49.919 |
| $\frac{\Delta\phi}{2}$ | − 55.671 | | $\frac{\Delta\lambda}{2}$ | − 54.960 |
| $\phi_m\left(=\phi+\frac{\Delta\phi}{2}\right)$ | 31 26 39.790 | | | |
| $\Delta\phi$ (secs.) | −111.341 | | $\Delta\lambda$ (secs.) | −109.919 |
| | | | | |
| log $\Delta\phi$ | 2.0466551 | | log $\Delta\lambda$ | −2.0410728 |
| cor. arc—sin | — | | cor. arc—sin | — |
| log $\Delta\phi_1$ | 2.0466551 | | log $\Delta\lambda_1$ | 2.0410728 |
| log $\cos\frac{\Delta\lambda}{2}$ | 10.0000000 | | log cos $\phi_m$ | 9.9310238 |
| colog $B_m$ | 1.4885250 | | colog $A_m$ | 1.4906737 |
| log $\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$ | +3.5351801 (opposite in sign to $\Delta\phi$) | | log $\left\{s_1 \sin\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$ | −3.4627703 |
| | | | log $\left\{s_1 \cos\left(\alpha+\frac{\Delta\alpha}{2}\right)\right\}$ | +3.5351801 |
| log $\Delta\lambda$ | −2.0410728 | 3 log $\Delta\lambda$ — | log tan $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ | −9.9275902 |
| log sin $\phi_m$ | 9.7173964 | log F — | $\alpha+\frac{\Delta\alpha}{2}$ | 319° 45′ 16.117 |
| log sec $\frac{\Delta\phi}{2}$ | — | log b — | log sin $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ | 9.8102758 |
| log a | −1.7584692 | | log cos $\left(\alpha+\frac{\Delta\alpha}{2}\right)$ | 9.8826856 |
| a | −57.342 | | log $s_1$ | 3.6524945 |
| b | — | | cor. arc—sin | + — |
| $-\Delta\alpha$ (secs.) | −57.342 | | log s | 3.6524945 |
| $-\frac{\Delta\alpha}{2}$ | −28.621 | | s | 4492.566 m. |
| | −28.621 | | | |
| $\alpha+\frac{\Delta\alpha}{2}$ | 319 45 16.117 | | | |
| $\alpha$ (1 to 2) | 319 44 47.496 | | | |
| $\Delta\alpha$ | + 57.342 | | | |
| | 180 | | | |
| $\alpha'$ (2 to 1) | 139° 45′ 44.″838 | | | |

NOTE.—For log s up to 4.0 and for $\Delta\phi$ or $\Delta\lambda$ (or both) up to 3′, omit all terms below the heavy line except those printed (in whole or in part) in heavy type or those underscored, if using logarithms to 7 decimal places.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| D.H. Smith | 27 May 56 | W. C. Aumen | 27 May 56 |

DA FORM 1 FEB 57 1924

*Figure 121. Inverse position computation, logarithmic, third-order (DA Form 1924).*

*c.* Examples shown are for the Northern Hemisphere, west of Greenwich, and the following variations should be noted:

(1) For computations in the Southern Hemisphere, the sign of the cosine is reversed.

(2) For computations in the Eastern Hemisphere, the sign of the sine is reversed.

(3) $\Delta\alpha$ is applied according to the following rules:

(*a*) In the Northern Hemisphere, if $\alpha$ is less than 180°, $\Delta\alpha$ is minus; if $\alpha$ is greater than 180°, $\Delta\alpha$ is plus.

(*b*) In the Southern Hemisphere, if $\alpha$ is less than 180°, $\Delta\alpha$ is plus; if $\alpha$ is greater than 180°, $\Delta\alpha$ is minus.

## 72. Inverse Position Computation (Long Line)

*a.* A noniterative solution of the inverse problem (fig. 124) based on Helmert's successive approximation method was developed at the Army Map Service by Emanuel Sodano. In this method it is assumed that $\Delta l$, the reduced difference of longitude, is equal to $\Delta\lambda+X$, where X is an unknown quantity. The computation procedure consists of computing the value of X directly, then determining $\Delta l$. Using $\Delta l$ in the Sodano equations, the azimuths ($\alpha$ and $\alpha'$) and the geodetic distance (S) are determined.

*b.* The procedure and formulas for computation are as follows:

(1) The known data are listed on DA Form 2858, Inverse Position Computation (Sodano Method).

$\phi_1$=Latitude of west point

$\phi_2$=Latitude of east point

$\Delta\lambda$=Difference of longitude, always positive

(2) The parametric latitudes, $\beta_1$ and $\beta_2$, are computed and the sines and cosines obtained (preferably from 10-place tables).

$$\tan\beta_1=\sqrt{1-e^2}\tan\phi_1$$

$$\tan\beta_2=\sqrt{1-e^2}\tan\phi_2$$

PROJECT 8-6886 — POSITION COMPUTATION, INVERSE — 1st ORDER TRIANGULATION (For calculating machine computation) (TM 5-237)

LOCATION Nevada — ORGANIZATION A&S, Inc. — DATE 18 May 56

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 2 | | 3 | | | | | $\alpha$ | 3 | To 2 | | | | | | | |
| 2d ∠ | | | & | | + | | | 3d ∠ | | & | | | − | | | | |
| $\alpha$ | 2 Pioche | | To 1 Hayford | | 34 | 59 | 03.63 | $\alpha$ | 3 | To 1 | | | | | | | |
| $\Delta\alpha$ | | | | | − | 41 | 47.82 | $\Delta\alpha$ | | | | | | | | | |
| | | | | | 180 | 00 | 00.00 | | | | | | | 180 | 00 | 00.00 | |
| $\alpha'$ | 1 Hayford | | To 2 Pioche | | 214 | 17 | 15.81 | $\alpha'$ | 1 | To 3 | | | | | | | |
| First Angle of Triangle | | | | | | | | | | | | | | | | | |
| $\phi$ | 37 | 59 | 10.102 | 2 Pioche | $\lambda$ | 114 | 03 | 03.921 | $\phi$ | | | | 3 | $\lambda$ | | | |
| | | | | s=179,171.949m | $\Delta\lambda$ | + 1 | 08 | 55.942 | | | | | s= | $\Delta\lambda$ | | | |
| $\phi'$ | 36 | 39 | 27.946 | 1 Hayford | $\lambda'$ | 115 | 11 | 59.863 | $\phi'$ | | | | 1 | $\lambda'$ | | | |

| | | | |
|---|---|---|---|
| $\Delta\phi$ | 1 19 42.2 | (log s= $b=(y/10,000)^2$ | 215.494 ) |
| sin $\alpha$ | + 0.57335255 | x cor.= $-\frac{1}{2}fb$ | 884.3 |
| cos $\alpha$ | + 0.81930876 | x′ | + 102,719.610 |
| x=s sin $\alpha$ | + 102,728.694 | H | 0.040263424 |
| y=−s cos $\alpha$ | − 146,797.148 | Hx′=(approx. $\Delta\lambda''$) | 4135.8432 |
| a=(x′/10,000)² | 105.5132 | Arc−sin cor = + V (Va) / 15 | 238.9 |
| y cor.=+fa | 866.0 | $\Delta\lambda''$ | + 4135.942 |
| $y_o$ | 4,205,753.823 | sin $\phi$ | 0.61547083 |
| y′ | − 146,809.861 | sin $\phi'$ | 0.59703393 |
| $y_1$ | 4,058,943.962 | 1 + cos $\Delta\phi$ | 1.99973125 |
| Va | − 614.839 963 | $\frac{\sin\phi+\sin\phi'}{1+\cos\Delta\phi}$ or sin $\phi_m$ | 0.60633386 |
| $y_2$ | 4,058,329.999 839 | $-\Delta\alpha''$ (approx.) | 2507.762 |
| V | 5.82 713 | + F $(\Delta\lambda'')^3$ | 0.053 |
| K $(Va/1,000)^2$+ | 0.035 | $-\Delta\alpha''$ | 2507.815 |
| COMPUTED BY RAS | DATE 23 May 56 | CHECKED BY W.C.A. | |

| | |
|---|---|
| $\Delta\phi$ | (log s= $b=(y/10,000)^2$ ) |
| sin $\alpha$ | x cor.= $-\frac{1}{2}fb$ |
| cos $\alpha$ | x′ |
| x=s sin $\alpha$ | H |
| y=−s cos $\alpha$ | Hx′=(approx. $\Delta\lambda''$) |
| a=(x′/10,000)² | Arc−sin cor = + V (Va) / 15 |
| y cor.=+fa | $\Delta\lambda''$ |
| $y_o$ | sin $\phi$ |
| y′ | sin $\phi'$ |
| $y_1$ | 1 + cos $\Delta\phi$ |
| Va − | $\frac{\sin\phi+\sin\phi'}{1+\cos\Delta\phi}$ or sin $\phi_m$ |
| $y_2$ | $-\Delta\alpha''$ (approx.) |
| V | + F $(\Delta\lambda'')^3$ |
| K $(Va/1,000)^2$+ | $-\Delta\alpha''$ |

DATE 25 May 56 — NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1923 FEB 57

*Figure 122. Inverse position computation using natural functions (DA Form 1923).*

$$e^2=(\text{eccentricity})^2=\frac{a_0^2-b_0^2}{a_0^2}$$

(3) The approximate spherical distance ($\theta$) between the two points is determined, and converted to radians (1 degree= 0.0174 5329 2520 radians).

$$\cos\theta=\sin\beta_1\sin\beta_2+\cos\beta_1\cos\beta_2\cos\Delta\lambda$$

$$\sin\frac{\theta}{2}=\sqrt{\cos\beta_1\cos\beta_2\sin^2\left(\frac{\Delta\lambda}{2}\right)+\sin^2\left(\frac{\beta_2-\beta_1}{2}\right)}$$

or

$$\sin\theta=1-\cos^2\theta$$

(4) The constant terms $k_1$ through $k_6$ used in the solution of X are computed, using the dimensions of the reference spheroid or taken from table XXI, appendix III.

$$k_1=\frac{16\mathrm{N}-e'^2}{16e^2\mathrm{N}^2+e'^2},\ \text{where}\ \mathrm{N}=\frac{f}{e^2}$$

$$k_2=\frac{16\sin 1''}{e^2(16e^2\mathrm{N}^2+e'^2)}$$

$$k_3=\frac{16e^2\mathrm{N}^2+e'^2}{e'^2}$$

$$k_4=\frac{2e'^2}{16e^2\mathrm{N}^2+e'^2}$$

$$k_5=\frac{16e^2\mathrm{N}^2}{16e^2\mathrm{N}^2+e'^2}$$

$$k_6=\frac{16e^2\mathrm{N}^2}{e'^2}$$

(5) The variable terms, A through G, used in the solution of X are computed and tabulated on the form:

PROJECT 5-6886 — POSITION COMPUTATION, 3RD ORDER TRIANGULATION (For calculating machine computation) — INVERSE (TM 5-237)

LOCATION Georgia — ORGANIZATION A&S, Inc. — DATE 18 May 56

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 2 | To 3 | | | | $\alpha$ | 3 | To 2 | | | | | | | |
| 2d ∠ | & | | + | | | 3d ∠ | & | | − | | | | | | |
| $\alpha$ | 2 Atwood | To 1 Little Sapelo | 319 | 44 | 47.5 | $\alpha$ | 3 | To 1 | | | | | | | |
| $\Delta\alpha$ | | | + | | 57.3 | $\Delta\alpha$ | | | | | | | | | |
| | | | 180 | 00 | 00.00 | | | | | | | 180 | 00 | 00.00 | |
| $\alpha'$ | 1 Little Sapelo | To 2 Atwood | 139 | 45 | 44.8 | $\alpha'$ | 1 | To 3 | | | | | | | |

First Angle of Triangle

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | 31 | 27 | 35.461 | 2. Atwood | $\lambda$ | 81 | 20 | 09.455 | $\phi$ | | | | 3 | $\lambda$ | | | | | |
| | | | | s= 4492.568 | $\Delta\lambda$ | − | 1 | 49.919 | | | | | s= | $\Delta\lambda$ | | | | | |
| $\phi'$ | 31 | 25 | 44.120 | 1 Little Sapelo | $\lambda'$ | 81 | 18 | 19.536 | $\phi'$ | | | | 1 | $\lambda'$ | | | | | |

| | | | |
|---|---|---|---|
| $\phi_m$ 31 26 39.790 | (log s = b=(y/10,000)$^2$ ) | $\Delta\phi$ | (log s = b=(y/10,000)$^2$ ) |
| sin $\alpha$ −0.64617030 | x cor. = $-\frac{1}{2}$fb | sin $\alpha$ | x cor. = $-\frac{1}{2}$fb |
| cos $\alpha$ +0.76319312 | x' −2902.964 | cos $\alpha$ | x' |
| x = s sin $\alpha$ (same as x') | H 0.037864399 | x = s sin $\alpha$ | H |
| y = −s cos $\alpha$ (same as y') | Hx' = (approx. $\Delta\lambda''$) | y = −s cos $\alpha$ | Hx' = (approx. $\Delta\lambda''$) |
| a = (x'/10,000)$^2$ .0843 | Arc−sin/cor = +V(Va)/15 | a = (x'/10,000)$^2$ | Arc−sin/cor = +V(Va)/15 |
| y cor. = +fa | $\Delta\lambda''$ 109.919 | y cor. = +fa | $\Delta\lambda''$ |
| $y_o$ 3,481,773.777 | sin $\phi_m$ 0.52167070 | $y_o$ | sin $\phi$ |
| y' 3,428.697 | sin $\phi'$ | y' | sin $\phi'$ |
| $y_1$ 3,478,345.080 | 1 + cos $\Delta\phi$ | $y_1$ | 1 + cos $\Delta\phi$ |
| Va − .403 | (sin $\phi$ + sin $\phi'$)/(1 + cos $\Delta\phi$) or sin $\phi_m$ | Va − | (sin $\phi$ + sin $\phi'$)/(1 + cos $\Delta\phi$) or sin $\phi_m$ |
| $y_2$ 3,478,344.677 | $-\Delta\alpha''$ (approx.) | $y_2$ | $-\Delta\alpha''$ (approx.) |
| V 4.78609 | + F ($\Delta\lambda''$)$^3$ | V | + F ($\Delta\lambda''$)$^3$ |
| K (Va/1,000)$^2$ + — | $-\Delta\alpha''$ −57.34 | K (Va/1,000)$^2$ + | $-\Delta\alpha''$ |

COMPUTED BY RHS — DATE 23 May 56 — CHECKED BY W.C.A. — DATE 25 May 56

NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1923 FEB 57

*Figure 123. Inverse position computation for short lines (DA Form 1923).*

**INVERSE POSITION COMPUTATION** *(SODANO METHOD)* *(TM 5-237)*

| PROJECT | LOCATION | DATUM |
|---|---|---|
| 64148 | AFA - ASA | WORLD 1955 |

| STATION (Westward) 1 | STATION (Eastward) 2 |
|---|---|
| AL (FFL, 1920) | MO (NIP, 1930) |

| | | | |
|---|---|---|---|
| $\phi_1$ 20° 00′ 00.000″ N | TAN $\phi_1$ .36397 02343 | TAN $\beta_1$ .36274 47453 | $\beta_1$ 19° 56′ 16.70600 |
| $\phi_2$ 45 00 00.000 N | TAN $\phi_2$ 1.00000 00000 | TAN $\beta_2$ .99663 29966 | $\beta_2$ 44 54 12.16752 |
| $\lambda_1$ 000 00 00.000 | $\Delta\lambda$ 106° | SIN $\beta_1$ .34100 26695 | COS $\beta_1$ .94006 23271 |
| $\lambda_2$ 106 00 00.000 E | SIN $\Delta\lambda$ .96126 16959 | SIN $\beta_2$ .70591 33541 | COS $\beta_2$ .70829 81972 |
| COS $\theta$ .05718 67342 1 | COS $\Delta\lambda$ -.27563 73558 | a .24071 83382 | b .66584 44515 |
| SIN $\theta$ .99836 34995 | $\theta$ 86 43 17.95062 ARC<br>$\theta$ 1.51357 8377 RADIANS | A .64109 99270 | E -.19244 45306 |
| | | B .41100 91164 | F .02692 77622 9 |
| C + D -.08979 29122 9 | $K_1$ + B 237.64990 09 | C .00675 47028 23 | E + F -.16551 67683 |
| | | D -.09654 76151 1 | G 2.29467 4739 |
| ① (C + D) SIN $\theta$ -.08964 60 | ② ($K_1$ + B) $\theta$ RADIANS 359.70175 13 | ③ (E + F) G -.37980 71 | X SEC. OF ARC 673.62305 |
| $\Delta l = \Delta\lambda + X$ | $\Delta l$ 106° 11′ 13.62305″ | COS $\Delta l$ -.27877 51848 | SIN $\Delta l$ .96035 63900 |
| $\cot\alpha = \frac{\tan\beta_2 \cos\beta_1 - \cos\Delta l \sin\beta_1}{\sin\Delta l}$ | | $\cot\alpha$ 1.07455 96393 | $\alpha$ 222° 56′ 30.0356″ |
| $\cot\alpha' = \frac{\cot\beta_2 \cos\Delta l - \cos\beta_2 \tan\beta_1}{\sin\Delta l}$ | | $\cot\alpha'$ -.47245 22892 | $\alpha'$ 115 17 18.5987 |
| COS $\theta_o$ .05509 74281 9 | $\theta_o$ 86° 50′ 29.58178 ARC | SIN $\theta_o$ .99848 09829 | H $\theta_o$ 96 44596.181 |
| SIN² $\beta_o$ .58986 12195 | $\theta_o$ 1.51567 0984 RADIANS | H 63 63251.842 | I SIN $\theta_o$ COS 2$\sigma$ 4816.692 |
| I 6338.31249 1 | J .79155 01899 | COS 2$\sigma$ .76108 89229 | J SIN 2 $\theta_o$ COS 4$\sigma$ .014 |
| SIN 2 $\theta_o$ .11002.74685 | COS² 2$\sigma$ .57925 63486 | COS 4$\sigma$ 15851 26972 | S METERS 96 49412.859 |

**CONSTANTS FROM TABLE**

| | |
|---|---|
| SPHEROID | INTERNATIONAL |
| $b_o$ (Semi-Minor Axis) | 63 56911.946 M. |
| $\sqrt{1-e^2}$ | 0.99663 29966 |
| $K_1$ | 237.23889 18 |
| $K_2$ | 0.34188 82393 |
| $K_3$ | 4.98652 0650 |
| $K_4$ | 0.40108 12630 |
| $K_5$ | 0.79945 93685 |
| $K_6$ | 3.98652 0650 |
| $K_7$ | 10756.16549 |
| $K_8$ | 2.27498 6211 |
| $K_9$ | 13.64991 726 |
| $K_{10}$ | 18.19988 965 |
| SIN 1″ | 0.00000 48481 36811 |
| 1° | 0.01745 32925 2 RADIANS |

$\Delta\lambda$ = DIFFERENCE IN LONGITUDE AND IS ALWAYS POSITIVE.

$J = K_8 \sin^4\beta_o$ $\quad \cos 4\sigma = 2\cos^2 2\sigma - 1$

$\tan\beta = \sqrt{1-e^2}\ \tan\phi$ $\quad A = \frac{b\sin\Delta\lambda}{\sin\theta}$ $\quad E = -aK_5$ $\quad \sin^2\beta_o = 1 - \left(\frac{b\sin\Delta l}{\sin\theta_o}\right)^2$ $\quad \cos 2\sigma = \frac{2a}{\sin^2\beta_o} - \cos\theta_o$ $\quad S = H\,\theta_o + I\sin\theta_o\cos 2\sigma - J\sin 2\theta_o\cos 4\sigma$

$a = \sin\beta_1 \sin\beta_2$ $\quad B = A^2$ $\quad F = CK_6$ $\quad H = b_o + \sin^2\beta_o(K_7 - K_9\sin^2\beta_o)$

$b = \cos\beta_1 \cos\beta_2$ $\quad C = \frac{\cos\theta - B\cos\theta}{K_3}$ $\quad G = \frac{\theta^2}{\sin\theta'}$ $\quad I = \sin^2\beta_o(K_7 - K_{10}\sin^2\beta_o)$

$\cos\theta = a + b\cos\Delta\lambda$ $\quad D = -aK_4$ $\quad X = \left(\frac{① + ② + ③}{K_2}\right)A$ $\quad \sin 2\theta_o = 2\sin\theta_o\cos\theta_o$

$\cos\theta_o = a + b\cos\Delta l$

$\alpha$ IN QUADRANT III OR IV FOR COT $\alpha$ + OR − CLOCKWISE FROM SOUTH RESPECTIVELY.

$\alpha'$ IN QUADRANT I OR II FOR COT $\alpha'$ + OR −

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R. A. Smith - AMS | JAN. '64 | G. Tennis - AMS | JAN. '64 |

DA FORM 2858, 1 OCT 64



*Figure 124. Inverse position computation (Sodano method) (DA Form 2858).*

$$A=\frac{\cos \beta_1 \cos \beta_2 \sin \Delta\lambda}{\sin \theta}$$

$$B=A^2$$

$$C=\frac{\cos \theta-B \cos \theta}{k_3}$$

$$D=-k_4 \sin \beta_1 \sin \beta_2$$

$$E=-k_5 \sin \beta_1 \sin \beta_2$$

$$F=Ck_6$$

$$G=\frac{\theta^2}{\sin \theta''},\ \theta \text{ in radians}$$

(6) X and $\Delta l$ are now computed.

$$X''=\left[\frac{(k_1+B)\theta+(C+D) \sin \theta+(E+F)G}{k_2}\right]A$$

$$\theta \text{ in radians } \Delta l=\Delta\lambda+X$$

(7) The azimuth ($\alpha$) and back azimuth ($\alpha'$) of the line are computed. The azimuth is referenced to south.

$$\cot \alpha=\frac{\tan \beta_2 \cos \beta_1-\cos \Delta l \sin \beta_1}{\sin \Delta l}$$

$$\cot \alpha'=\frac{\sin \beta_2 \cos \Delta l-\cos \beta_2 \tan \beta_1}{\sin \Delta l}$$

When: cot $\alpha$ is positive, $\alpha$ is between 180°–270°; cot $\alpha$ is negative, $\alpha$ is between 270°–360°; cot $\alpha'$ is positive, $\alpha'$ is between 0°–90°; cot $\alpha'$ is negative, $\alpha'$ is between 90°–180°.

(8) The true spherical distance ($\theta_o$) between the two points is computed, and converted to radians. The formulae in (3) above are used substituting $\Delta l$ for $\Delta\lambda$.

$$\cos \theta_0=\sin \beta_1 \sin \beta_2+\cos \beta_1 \cos \beta_2 \cos \Delta l$$

(9) The constants terms used in the solution of the geodetic distance (S) are computed or taken from table XXI, appendix III.

$$k_7=\frac{b_o e'^2}{4},\ b_o=\text{semi-minor axis (meters)}$$

$$k_8=\frac{b_o e'^4}{128}$$

$$k_9=\frac{3b_o e'^4}{64}=6k_8$$

$$k_{10}=\frac{b_o e'^4}{16}=8k_8$$

(10) The variable terms used in the solution of the geodetic distance (S) are computed.

$$\sin^2 \beta_o=1-\left(\frac{\cos \beta_1 \cos \beta_2 \sin \Delta l}{\sin \theta_o}\right)^2$$

$$\sin 2\theta_o=2 \sin \theta_o \cos \theta_o$$

$$\cos 2\theta=\frac{2 \sin \beta_1 \sin \beta_2}{\sin^2 \beta_o}-\cos \theta_o$$

$$\cos 4\sigma=2 \cos^2 2\sigma-1$$

$$I=k_7 \sin^2 \beta_o-k_{10} \sin^4 \beta_o$$

$$J=k_8 \sin^4 \beta_o$$

$$H=b_o+k_7 \sin^2 \beta_o-k_9 \sin^4 \beta_o$$

(11) The geodetic distance (S) is determined.

$$S_{\text{meters}}=H\theta_o+I \sin \theta_o \cos 2\sigma$$
$$-J \sin 2\theta_o \cos 4\sigma$$
$\theta_o$ is in radians.

*c.* For short lines or reduced accuracy on long lines, X becomes $A\theta \cdot F$ (Flattening), and all terms in $\sin^4 \beta_o$ are omitted. Use cofunction of tan $\beta$ or cot $\alpha$ when their values are too large.

*d.* Computations using the above formulas should yield results with a maximum distance error of one-half meter and azimuth error less than 0″.2. For shorter lines, the error is much less.

## Section IV. ADJUSTMENT OF A TRIANGULATION NET

### 73. Methods

Two methods of adjustment, the direction method and the angle method, are explained in this section. The azimuth, latitude, and longitude equations normally used in the direction method are explained in paragraph 75.

### 74. Direction Method

The most rigid method of triangulation adjustment used for all first- and second-order triangulation in which corrections to observed angles are used to make lengths, latitudes, longitudes, and azimuths agree with fixed data, is known as the

direction method. In this method each angle is considered to be made up of two directions, and corrections are made to each direction individually to correct the angle as a whole. The corrections to the directions are ordinarily designated by the letter $v$ and a subscript number such as $v_1$, $v_2$. The value for each $v$ is determined by a solution of certain conditions which must be satisfied to make all the elements of the net compatible with each other and with previously fixed data. The conditions are stated as equations and these equations are then solved by the method of least-squares. The Doolittle method of solution will be used throughout this section.

*a.* The conditions that may have to be satisfied are—

(1) *Triangle closure.* The three angles of a triangle must total 180° plus the spherical excess (unless the computations are made on a plane grid system where there is no spherical excess).

(2) *Side agreement.* The length of the common side of adjacent triangles must be the same, no matter which of two adjacent triangles is used to compute the length.

(3) *Length agreement.* The length of a line B as computed through a series of triangles from the fixed length of line A, must agree within certain limits with the fixed length of line B as determined by a base measurement or an adjustment.

(4) *Azimuth closure.* The azimuth of a line C–D as carried through a series of triangles from the fixed azimuth of a line A–B must agree with the fixed azimuth of line C–D as determined by an astronomic observation or a previous adjustment.

(5) *Latitude and longitude closure.* The latitude and longitude of a point B as computed through a series of triangles from a fixed position A must agree with the fixed latitude and longitude of point B as determined by an astronomic observation or a previous adjustment.

*b.* When stated in equation form, these conditions are known as angle, side, length, azimuth, latitude, and longitude equations. All of these conditions are not necessarily present in every triangulation problem. Figure 125 was chosen as a typical problem found in extending first- and second-order triangulation. This problem will

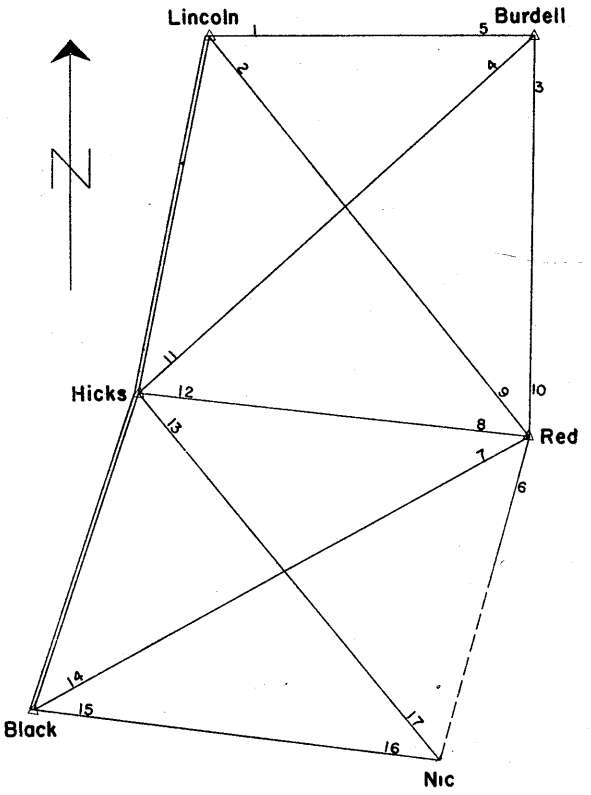



*Figure 125. Sketch of triangulation net for direction method.*

be treated in several different ways and the advantages and disadvantages of each method will be pointed out.

*c.* In the arc under consideration here, there are five angle equations, two side equations, and one length equation to be satisfied. The number of equations should be carefully checked to insure that all conditions are met and no identical equations are included. The formulas for determining the number of angle and side equations are as follows:

$$\text{Number of angle equations} = n' - s' + 1$$

$$\text{Number of side equations} = n - 2s + 3$$

in which $n$ is the total number of lines, $n'$ is the number of lines sighted over in both directions, $s$ is the total number of stations, and $s'$ is the number of occupied stations. Total numbers include fixed lines and stations. Any lines or triangles fixed by previous adjustments may require additional conditions. The number of angle and side equations may also be determined by building up the figure point by point. The rules are that at each point the number of angle equations is one less than the number of full lines (observed at both

ends) between the point and previously considered points, and the number of side equations is two less than the total number of lines from the point to previously considered points. Fixed points are previously considered at one end of the arc only. In our problem—

$n=11$, $n'=10$, $s=6$, $s'=6$

Number of angle equations $=10-6+1=5$

Number of side equations $=11-12+3=2$

Point-by-point determination starting from line Hicks-Lincoln—

| *Station* | *No. angle* | *No. side* |
|---|---|---|
| Burdell | 1 | 0 |
| Red | 2 | 1 |
| Nic | 0 | 0 |
| Black | 2 | 1 |
| | 5 | 2 |

This checks the number of equations found by using the formulas. On long or complicated nets, this check should always be made. Since there are two fixed lengths in this problem, a length equation is included. No azimuth equation is necessary in this case because the two fixed azimuths radiate from a common station, thus fixing an angle at that station. The fixed angle at Hicks from Lincoln to Black must be recognized and the list of directions altered to maintain the fixed angle. No latitude and longitude equation is necessary since the fixed positions are connected by the fixed lines.

*d.* To start an adjustment, a sketch of the net is drawn showing all lines observed, with lines observed in one direction only being shown solid halfway and dashed halfway (fig. 125). The line from Red to Nic illustrates a line observed only in the direction Red to Nic. Fixed lines and positions are distinguished. Position, lengths, and azimuths may be fixed by previous adjustments, astronomic observations, or measured bases. The directions are numbered clockwise around each station, starting with the first clockwise line in the figure. This system of numbering facilitates designation of the angles. No numbers are placed on the fixed lines. Theoretically, the observations made over the fixed lines should receive their share of any error to be distributed, but in practice, application of this theory can lead to complications. In this problem, for instance, applying a correction to the observations on the fixed lines from Hicks to Lincoln and from Hicks to Black would lead to the condition that the two corrections would have to add to zero to prevent any disturbance of the fixed angle Lincoln-Hicks-Black. Since the corrections to each direction are relatively small, it would be advisable to eliminate the corrections to the fixed directions altogether, and thereby eliminate the possibility of disturbing any fixed data. The numbers on the lines are used as subscripts to denote the correction ($v$) which is to be applied to the observed directions. Generally, the $v$ is dropped and the subscript alone is written in parentheses such as $v_1$ written as (1), $v_2$ as (2), and so on. If these subscripts are written on the list of directions (fig. 126), it greatly simplifies applying the final adopted correction ($v$) after the solution of the condition equations is made.

*e.* The triangles are written out on DA Form 1918 (fig. 127) and the observed angles (differences in directions), with the appropriate $v$ symbols in the left hand column, are entered. The directions used to obtain these angles are the directions reduced for eccentricity and sea level. In this problem, there is no correction necessary for either of these factors. Using the list of directions complete with its $v$'s, the correct angles and symbols are obtained directly. As an example, take the list of directions for station Burdell. The angle Hicks-Burdell-Lincoln is—

| *Station* | *Direction* | |
|---|---|---|
| Lincoln | 0° 00′ 00.″0 | $+v_5$ |
| Hicks | 337° 10′ 32.″9 | $+v_4$ |
| After subtracting | 22° 49′ 27.″1 | $-v_4+v_5$ |

The angle 22° 49′ 27.″1 is entered in the observed angle column of the triangle computation sheet and the symbol for the angle $[-(4)+(5)]$ is written in the left hand column. The symbols for the concluded angles at Nic in triangles Nic-Black-Red and Nic-Hicks-Red are obtained by changing the signs on all the other $v$'s in the triangle and applying them to the concluded angle. The concluded angle itself is inclosed in parentheses.

*f.* The total of the three angles in a triangle should be 180° plus the spherical excess ($\epsilon$). The computation of the spherical excess is explained in paragraph 57. The difference between the sum of the observed angles and $180°+\epsilon$ is the error of closure of the triangle.

*g.* Now the condition equations can be chosen for adjusting the net (fig. 128.). In this problem, three angle equations and one side equation are necessary to fix the quadrilateral Lincoln-Burdell-Red-Hicks, and two angle equations and one side equation will fix the quadrilateral Hicks-Red-

PROJECT 1-243 | ORGANIZATION 29th Engr. | LIST OF DIRECTIONS (TM 5-237)

LOCATION California | STATION BLACK (USC&GS)

OBSERVER Cpl. P.E. Smith | INST. (TYPE) (NO.) K&E #57460 | DATE 22 Aug 55

| | OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| | HICKS (USC&GS) | 0 00 00.00 | — | — | 0 00 00.00 | 00.0 |
| V14 | Red (29th Engr) | 51 57 43.3 | — | — | | 47.4 |
| V15 | Nic (29th Engr) | 90 39 09.9 | — | — | | 08.9 |

LOCATION California | STATION Burdell (29th Engrs)

OBSERVER Cpl. P.E. Smith | INST. (TYPE) (NO.) K&E #57460 | DATE 22 Aug 55

| | OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| V5 | LINCOLN (USC&GS) | 0 00 00.00 | — | — | 0 00 00.00 | 02.5 |
| V3 | Red (29th Engrs.) | 284 48 01.2 | — | — | | 58.9 |
| V4 | HICKS (USC&GS) | 337 10 32.9 | — | — | | 32.7 |

LOCATION California | STATION HICKS (USC+GS)

OBSERVER Cpl. P.E. Smith | INST. (TYPE) (NO.) K+E #57460 | DATE 22 Aug 55

| | OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| | LINCOLN (USC+GS) | 0 00 00.00 | | | 0 00 00.00 | 00.0 |
| V11 | Burdell (29th Engrs) | 49 00 55.2 | — | — | | 56.0 |
| V12 | Red (29th Engrs) | 91 29 06.2 | — | — | | 07.4 |
| V13 | Nic (29th Engrs) | 136 21 11.3 | — | — | | 13.8 |
| | BLACK (USC+GS) | 180 40 53.2 | — | — | Fixed by previous adjustment at | 54.5 |

LOCATION California | STATION LINCOLN (USC+GS)

OBSERVER Cpl. P.E. Smith | INST. (TYPE) (NO.) K+E #57460 | DATE 22 Aug 55

| | OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| | HICKS (USC+GS) | 0 00 00.00 | — | — | 0 00 00.00 | 00.0 |
| V1 | Burdell (29th Engrs) | 251 50 28.6 | — | — | | 25.7 |
| V2 | Red (29th Engrs) | 302 11 14.7 | — | — | | 11.2 |

LOCATION California | STATION Nic (29th Engrs)

OBSERVER Cpl. P.E. Smith | INST. (TYPE) (NO.) K+E #57460 | DATE 22 Aug 55

| | OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| V16 | BLACK (USC+GS) | 0 00 00.00 | — | — | 0 00 00.00 | 58.4 |
| V17 | HICKS (USC+GS) | 45 01 07.4 | — | — | | 08.9 |

LOCATION California | STATION Red (29th Engrs)

OBSERVER Cpl. P.E. Smith | INST. (TYPE) (NO.) K+E #57460 | DATE 22 Aug 55

| | OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|---|
| | | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| V8 | HICKS (USC+GS) | 0 00 00.00 | — | — | 0 00 00.00 | 58.1 |
| V9 | LINCOLN (USC+GS) | 30 41 59.7 | — | — | | 42 02.0 |
| V10 | Burdell (29th Engrs) | 95 09 11.0 | — | — | | 13.1 |
| V6 | Nic (29th Engrs) | 284 17 14.0 | — | — | | 11.8 |
| V7 | BLACK (USC+GS) | 321 09 32.8 | — | — | | 32.5 |

*Figure 126. List of directions for net adjustment (DA Form 1917).*

| PROJECT | DATE | COMPUTATION OF TRIANGLES (TM 5-237) |
|---|---|---|
| 1-243 | 19 Aug 55 | 1 of 2 |

LOCATION: California — ORGANIZATION: 29th Engrs

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM ~~OR NATURAL~~ |
|---|---|---|---|---|---|---|---|
| | | | | | | | m = 1.40471 |
| | 2-3 HICKS - LINCOLN | | | | | | 3.68755045 |
| -4 +5 | 1 Burdell | 22 49 27.1 | +2.7 | 29.8 | 0.0 | 29.8 | 9.58873869 |
| +11 | 2 HICKS | 49 00 55.2 | +0.8 | 56.0 | 0.0 | 56.0 | 9.87788233 |
| -1 | 3 LINCOLN | 108 09 31.4 | +2.9 | 34.3 | 0.1 | 34.2 | 9.97781164 |
| | 1-3 Burdell - LINCOLN | | | | | | 3.97669409 |
| | 1-2 Burdell - HICKS | | | | | | 4.07662340 |
| | | 53.7 | +6.4 | 00.1 | 0.1 | 0.11137 | ε = 9.04676 |
| | 2-3 HICKS - LINCOLN | | | | | | 3.68755045 |
| -8 +9 | 1 Red | 30 41 59.7 | +4.2 | 03.9 | 0.0 | 42' 03."9 | 9.70804608 |
| +12 | 2 HICKS | 91 29 06.2 | +1.2 | 07.4 | 0.1 | 07.3 | 9.99985404 |
| -2 | 3 LINCOLN | 57 48 45.3 | +3.5 | 48.8 | 0.0 | 48.8 | 9.92753420 |
| | 1-3 Red - LINCOLN | | | | | | 3.97935841 |
| | 1-2 Red - HICKS | | | | | | 3.90703857 |
| | | 51.2 | +8.9 | 00.1 | 0.1 | 0.09980 | ε = 8.99913 |
| | 2-3 HICKS - Burdell | | | | | | 4.07662340 |
| -8 +10 | 1 Red | 95 09 11.0 | +4.0 | 15.0 | 0.0 | 15.0 | 9.99824041 |
| -11 +12 | 2 HICKS | 42 28 11.0 | +0.4 | 11.4 | 0.1 | 11.3 | 9.82943345 |
| -3 +4 | 3 Burdell | 42 22 31.7 | +2.1 | 33.8 | 0.1 | 33.7 | 9.82865562 |
| | 1-3 Red - Burdell | | | | | | 3.90781644 |
| | 1-2 Red - HICKS | | | | | | 3.90703861 |
| | | 53.7 | +6.5 | 00.2 | 0.2 | 0.16513 | ε = 9.21783 |
| | 2-3 LINCOLN - Burdell | | | | | | 3.97669409 |
| -9 +10 | 1 Red | 64 27 11.3 | -0.2 | 11.1 | 0.1 | 11.0 | 9.95531835 |
| -1 +2 | 2 LINCOLN | 50 20 46.1 | -0.6 | 45.5 | 0.1 | 45.4 | 9.88644082 |
| -3 +5 | 3 Burdell | 65 11 58.8 | +4.8 | 03.6 | 0.0 | 12' 03."6 | 9.95798291 |
| | 1-3 Red - Burdell | | | | | | 3.90781656 |
| | 1-2 Red - LINCOLN | | | | | | 3.97935865 |
| | | 56.2 | +4.0 | 00.2 | 0.2 | 0.17669 | ε = 9.24721 |

COMPUTED BY: RMS — DATE: 22 Aug 55 — CHECKED BY: W. C. A. — DATE: 26 Aug 55

DA FORM 1 FEB 57 1918

*Figure 127. Computation of triangles for net adjustment (DA Form 1918).*

| PROJECT | DATE | COMPUTATION OF TRIANGLES (TM 5-237) |
|---|---|---|
| 1-243 | 19 Aug 55 | 2 of 2 |

| LOCATION | ORGANIZATION |
|---|---|
| California | 29th Engrs. |

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM ~~OR NATURAL~~ |
|---|---|---|---|---|---|---|---|
| | | | | | | | m = 1.40471 |
| | 2-3 BLACK - HICKS | | | | | | 3.80809910 |
| -7+8 | 1 Red | 38 50 27.2 | -1.6 | 25.6 | 0.0 | 25.6 | 9.79737405 |
| +14 | 2 BLACK | 51 57 43.3 | +4.1 | 47.4 | 0.0 | 47.4 | 9.89631387 |
| -12 | 3 HICKS | 89 11 48.3 | -1.2 | 47.1 | 0.1 | 47.0 | 9.99995728 |
| | 1-3 Red - HICKS | | | | | | 3.90703892 |
| | 1-2 Red - BLACK | | | | | | 4.01068233 |
| | | 58.8 | +1.3 | 00.1 | 0.1 | 0.13176 | ε = 9.11979 |
| | 2-3 BLACK - HICKS | | | | | | 3.80809910 |
| -16+17 | 1 Nic | 45 01 07.4 | +3.1 | 10.5 | 0.0 | 10.5 | 9.84963339 |
| +15 | 2 BLACK | 90 39 09.9 | -1.0 | 08.9 | 0.1 | 08.8 | 9.99997184 |
| -13 | 3 HICKS | 44 19 43.2 | -2.5 | 40.7 | 0.0 | 40.7 | 9.84433090 |
| | 1-3 Nic - HICKS | | | | | | 3.95843755 |
| | 1-2 Nic - BLACK | | | | | | 3.80279661 |
| | | 00.5 | -0.4 | 00.1 | 0.1 | 0.10365 | ε = 9.01559 |
| | 2-3 BLACK - Red | | | | | | 4.01068233 |
| +6-7 +14-15 | 1 Nic | (104 26 14.7) | +3.2 | 17.9 | 0.1 | 17.8 | 9.98606232 |
| -14+15 | 2 BLACK | 38 41 26.6 | -5.1 | 21.5 | 0.0 | 21.5 | 9.79594740 |
| -6+7 | 3 Red | 36 52 18.8 | +1.9 | 20.7 | 0.0 | 20.7 | 9.77817671 |
| | 1-3 Nic - Red | | | | | | 3.82056741 |
| | 1-2 Nic - BLACK | | | | | | 3.80279672 |
| | | 00.1 | 00.0 | 00.1 | 0.1 | 0.10331 | ε = 9.01414 |
| | 2-3 HICKS - Red | | | | | | 3.90703892 |
| +6-8 +12-13 | 1 Nic | (59 25 09.0) | -1.6 | 07.4 | 0.0 | 07.4 | 9.93495691 |
| -12+13 | 2 HICKS | 44 52 05.1 | +1.3 | 06.4 | 0.0 | 06.4 | 9.84848553 |
| -6+8 | 3 Red | 75 42 46.0 | +0.3 | 46.3 | 0.1 | 46.2 | 9.98635561 |
| | 1-3 Nic - Red | | | | | | 3.82056754 |
| | 1-2 Nic - HICKS | | | | | | 3.95843762 |
| | | 00.1 | — | 00.1 | 0.1 | 0.13142 | ε = 9.11865 |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RMS | 23 Aug 55 | W. C. A. | 25 Aug 55 |

DA FORM 1 FEB 57 1918

*Figure 127*—Continued.

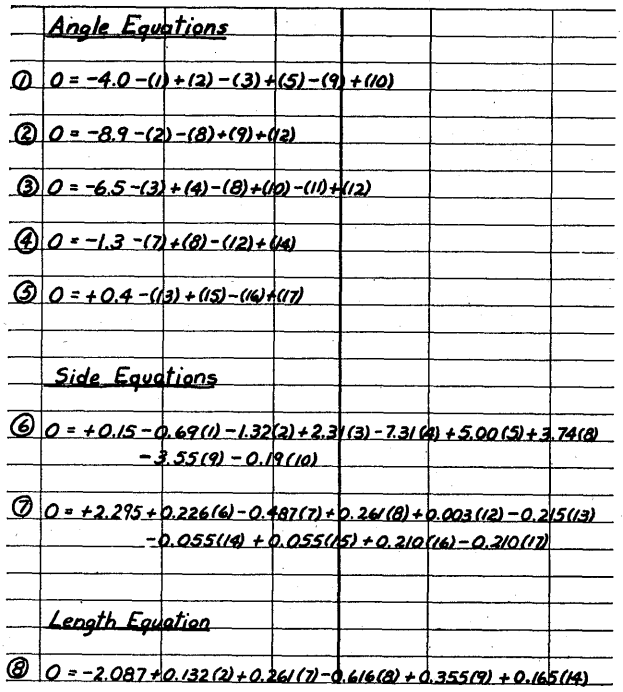



*Figure 128. Condition equations for net adjustment.*

Nic-Black. A length equation is needed to hold the fixed lengths Hicks-Lincoln and Hicks-Black constant.

(1) Generally, the triangles with the largest angles are used for the angle equations, and the triangles with the small angles are used for the side equations. Although four angle equations could be written from the four triangles, one of the four would be a combination of the other three and any solution which satisfies three of the equations automatically satisfies the fourth. The sum of the $v$'s designating the angles in a triangle must equal the correction needed to make the sum of the observed angles equal $180°+\epsilon$. For example, in the triangle Red-Lincoln-Burdell:

$$-(1)+(2)-(3)+(5)-(9)+(10)=4.0$$

or $0=-4.0-(1)+(2)-(3)+(5)-(9)+(10)$

which is equation number 1 in this problem. Number the equations in ascending order of $v$'s to keep the correlate equations from spreading too far apart.

(2) Side equations are now set up on DA Form 1926 (Side/Length Equations) to insure the condition of the side agreement previously stated (fig. 129). The simplest way to set up this condition is to select 1 station as a pole and write the product of the ratios of the lines running to that pole and equate the resulting expression to 1. By the law of sines, the sines of the angles opposite the sides can replace the sides. Replacing natural sines by logarithms reduces the expression to one which can be solved by addition and subtraction. In this problem, the side equation for the quadrilateral Lincoln-Burdell-Red-Hicks was written with the pole at Hicks. Therefore, the expression for the ratio of lines is—

$$\frac{\text{Hicks-Lincoln}}{\text{Hicks-Burdell}}\times\frac{\text{Hicks-Burdell}}{\text{Hicks-Red}}\times\frac{\text{Hicks-Red}}{\text{Hicks-Lincoln}}=1$$

Substituting the sines of the angles opposite these sides and using the symbol designation of the angles as they appear on the triangle sheet, the expression becomes:

$$\frac{\text{sine }(-4+5)}{\text{sine }(-1)}\times\frac{\text{sine }(-8+10)}{\text{sine }(-3+4)}\times\frac{\text{sine }(-2)}{\text{sine }(-8+9)}=1$$

Replacing sines by log sines:

$$\begin{aligned}&\log\text{ sine }(-4+5)+\log\text{ sine }(-8+10)\\&\quad+\log\text{ sine }(-2)-\log\text{ sine }(-1)\\&\quad-\log\text{ sine }(-3+4)\\&\qquad-\log\text{ sine }(-8+9)=0\end{aligned}$$

The items are entered on the side equations form under the appropriate heading. The column headed "Tab. Diff." contains the difference of logarithms per 1″ change in angle. The tabular difference is usually expressed in units of the sixth place of the logarithm. Notice that the sign of the tabular difference for angles over 90° is minus. Subtract the sum of the right hand log sine column from the sum of the left hand log sine column and express the difference in units of the sixth decimal.

PROJECT 1-243

SIDE/~~LENGTH~~ EQUATIONS (TM 5-237)

LOCATION California

ORGANIZATION 29th Engr.

DATE 19 Aug 55

| SYMBOL | ANGLE | LOG. SINE | TAB. DIFF. | SYMBOL | ANGLE | LOG. SINE | TAB. DIFF. |
|---|---|---|---|---|---|---|---|
| | Pole @ HICKS | | | | | | |
| -4 +5 | 22° 49' 27.1" | 9.58872519 | +5.00 | -1 | 108 09 31.4 | 9.97781357 | -0.69 |
| -8 +10 | 95 09 11.0 | 9.99824117 | -0.19 | -3 +4 | 42 22 31.7 | 9.82865101 | +2.31 |
| -2 | 57 48 45.3 | 9.92752957 | +1.32 | -8 +9 | 30 41 59.7 | 9.70803120 | +3.55 |
| | | 9.51449593 | | | | 9.51449578 | |
| | | 578 | | | | | |
| | | +0.15 | | | | | |
| | 0 = +0.15 -0.69(1) -1.32(2) +2.31(3) -7.31(4) +5.00(5) +3.74(8) | | | | | | |
| | -3.55(9) -0.19(10) | | | | | | |
| | Pole @ BLACK | | | | | | |
| -7 +8 | 38 50 27.2 | 9.79737823 | +2.61 | -12 | 89 11 48.3 | 9.99995732 | +0.03 |
| +6-7+14-15 | 104 26 14.7 | 9.98606400 | -0.55 | -6+7 | 36 52 18.8 | 9.77817138 | +2.81 |
| -13 | 44 19 43.2 | 9.84433629 | +2.15 | -16 +17 | 45 01 07.4 | 9.84962687 | +2.10 |
| | | 9.62777852 | | | | 9.62775557 | |
| | | 5557 | | | | | |
| | | +22.95 | | | | | |
| | 0 = +22.95 +2.26(6) -4.87(7) +2.61(8) +0.03(12) -2.15(13) -0.55(14) | | | | | | |
| | +0.55(15) +2.10(16) -2.10(17) | | | | | | |
| | ÷10 | | | | | | |
| | 0 = +2.295 +0.226(6) -0.487(7) +0.261(8) +0.003(12) -0.215(13) | | | | | | |
| | -0.055(14) +0.055(15) +0.210(16) -0.210(17) | | | | | | |

COMPUTED BY RAB — DATE 23 Aug. 55

CHECKED BY W.C.A. — DATE 25 Aug. 55

DA FORM 1 FEB 57 1926

*Figure 129. Computations of side equations (DA Form 1926).*

This difference will be the constant term of the side equation. Observe that if the sum of the right hand column is larger than the sum of the left hand column, the sign of the difference is minus. The tabular differences are the coefficients of the $v$'s shown in the symbol column. To form the equation, the coefficients of each $v$ are obtained by adding algebraically the coefficients for each $v$ from both sides of the equation. The signs of the tabular differences on the right hand side of the equation are reversed for this operation. For example, the first side equation in this problem is formed as follows:

$$0=+0.15+(+0.69)\ (-1)+(+1.32)\ (-2)+(-2.31)\ (-3)+[(+5.0)\ (-4)+(-2.31)\ (+4)]+(+5.00)\ (+5)+[-(0.19)\ (-8)+(-3.55)\ (-8)]+(-3.55)\ (+9)+(-0.19)\ (+10)$$

Remember that the numbers in the symbol column are only the subscripts of the corresponding $v$'s. A simpler method of writing the angles in their correct columns is to write the angles opposite the side *started from* in the left hand column, and the angle opposite the side *going to* in the right hand column. In this equation, the rule would work as follows with the pole at Hicks:

| | *Angle opposite* | *Column* |
|---|---|---|
| *Starting from side:* | | |
| Hicks-Lincoln | −4+5 | Left |
| *Going to side:* | | |
| Hicks-Burdell | −1 | Right |
| *Starting from side:* | | |
| Hicks-Burdell | −8+10 | Left |
| *Going to side:* | | |
| Hicks-Red | −3+4 | Right |
| *Starting from side:* | | |
| Hicks-Red | −2 | Left |
| *Going to side:* | | |
| Hicks-Lincoln | −8+9 | Right |

Use the smallest angles possible in the side equations, since a change in a small angle has more effect on its sine than the same change in a large angle would have on its sine. It may be necessary to divide a side equation by a constant, if the coefficients or constant term of the equation are large in comparison with those of the other equations. The second side equation in this problem was divided by 10. When dividing an equation by a constant, care must be taken not to forget to divide the constant term of the equation as well as the coefficients.

(3) Length equations are written in the same manner as side equations with the addition of the logarithms of the fixed lengths (fig. 130). The log of the length from which calculations are started is written on the right, and the log of the length on which calculations end is written on the left. In precise work, the logarithms should be corrected for the difference in arc and sine (known as the arc-sine correction). A table for this correction may be found in TM 5–236.

*h.* The explanation for the solution of condition equations by the Doolittle method may be found in paragraph 65, and in USC&GS Sp. Pubs. 28 and 138. By use of the accumulative features of the modern calculating machine, the forward solution of the normal equations can be fitted onto a prearranged sheet which is illustrated in the problem (fig. 131). The written-backward solution or solution of C's is also eliminated by use of the calculating machine. When the calculated C's are substituted in the normal equations, the equations should equal minus $\eta$. This check will prove the numerical value of the C's. The diagonal terms must be included when checking the C's by this method. After the $v$'s are computed from the correlate equations, they are applied to the angles and directions.

*i.* The corrected spherical angles on the triangle computation sheets are reduced to plane angles by subtracting ⅓ of the spherical excess from each angle. If the spherical excess is not evenly divisible by 3, apply the odd amount to the angle nearest 90°. The triangle sides are now computed by the sine law which, on DA Form 1918, means that log of the length 2–3 minus log sin angle 1 plus log sin angle 2 equals log length 1–3, and similarly log length 2–3 minus log sin angle 1 plus log sin angle 3 equals log length 1–2. Check the length of sides appearing in two or more triangles for agreement.

*j.* After computing the adjusted triangles and entering the final adjusted seconds on the list of directions, the geographic positions are computed (fig. 132). DA Forms 1922 and 1923 may be used for the computation of the geographic positions. These are triangulation position computa-

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| SYMBOL | ANGLE | LOG. SINE | TAB. DIFF. | SYMBOL | ANGLE | LOG. SINE | TAB. DIFF. |
|---|---|---|---|---|---|---|---|
| | Length Equation | | | | | | |
| | HICKS-BLACK | 3.8080991 -8 | | | HICKS-LINCOLN | 3.68755045 -4 | |
| -8+9 | 30 41 59.7 | 9.70803120 | +3.55 | -2 | 57 48 45.3 | 9.92752957 | +1.32 |
| +14 | 51 57 43.3 | 9.89630712 | +1.65 | -7+8 | 38 50 27.2 | 9.79737823 | +2.61 |
| | | 3.41243734 | | | | 3.41245821 | |
| | | 45821 | | | | | |
| | | − 20.87 | | | | | |
| | 0 = −20.87 + 1.32(2) + 2.61(7) − 6.16(8) + 3.55(9) + 1.65(14) | | | | | | |
| | ÷10 | | | | | | |
| | 0 = −2.087 + 0.132(2) + 0.261(7) − 0.616(8) + 0.355(9) + 0.165(14) | | | | | | |

*Figure 130. Computation of length equations (DA Form 1926).*

tion forms for logarithmic and machine solution, respectively. The forms can be modified for short lines by omitting certain correction factors, making them suitable for both first- and third-order triangulation. Paragraph 67 gives an example of both first- and third-order position computation by logarithms. USC&GS Sp. Pub. 200 is used for logarithmic computation on the International Ellipsoid. For machine computation of geographic positions, USC&GS Sp. Pub. 241 (Clarke Spheroid) and USC&GS G–58 (International Ellipsoid) are necessary.

*k.* Although azimuth, latitude, and longitude equations were not used in this example, they are necessities in some problems and their use should be mentioned. An azimuth equation is required whenever two or more fixed azimuths occur in an arc or net of triangulation, unless the fixed azimuths all radiate from a common point as in the example given in this manual. When two or more stations of fixed position occur in an arc of triangulation and are not connected by a line, it is necessary to relate the fixed positions by latitude and longitude equations.

## 75. Azimuth, Latitude, and Longitude Equations as Used in the Direction Method

Since neither azimuth nor latitude and longitude equations were required in the example as originally solved, the problem has been modified slightly, only for the sake of illustrating an azimuth and a latitude and longitude equation. The modified problem is shown in figure 133 and consists of fixing the positions of stations Black, Nic, Lincoln, and Burdell, and the azimuths of lines Black-Nic and Burdell-Lincoln. The fixed data is taken from the adjustment of the original figure by the direction method. The observed angles are from the original list of directions.

*a.* The purpose of an azimuth equation is to find a means of correcting observed directions or angles, so that the angles can be used to carry azimuths through a network of triangulation between fixed azimuths without a residual error.

Correlate Equations

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8' | Σc | v | Adopt. v | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | | | | | -0.69 | | | -1.69 | -2.86997 | -2.9 | 1 |
| 2 | +1 | -1 | | | | -1.32 | | +0.132 | -1.188 | -3.55022 | -3.5 | 2 |
| 3 | -1 | | -1 | | | +2.31 | | | +0.31 | -2.30591 | -2.3 | 3 |
| 4 | | | +1 | | | -7.31 | | | -6.31 | -0.20495 | -0.2 | 4 |
| 5 | +1 | | | | | +5.00 | | | +6.00 | +2.51086 | +2.5 | 5 |
| 6 | | | | | | | +0.226 | | +0.226 | -2.16980 | -2.2 | 6 |
| 7 | | | | -1 | | | -0.487 | +0.261 | -1.226 | -0.32922 | -0.3 | 7 |
| 8 | | -1 | -1 | +1 | | +3.74 | +0.261 | -0.616 | +2.385 | -1.89622 | -1.9 | 8 |
| 9 | -1 | +1 | | | | -3.55 | | +0.355 | -3.195 | +2.26597 | +2.3 | 9 |
| 10 | +1 | | +1 | | | -0.19 | | | +1.81 | +2.12927 | +2.1 | 10 |
| 11 | | | -1 | | | | | | -1 | +0.81402 | +0.8 | 11 |
| 12 | | +1 | +1 | -1 | | | +0.003 | | +1.003 | +1.18766 | +1.2 | 12 |
| 13 | | | | | -1 | | -0.215 | | -1.215 | +2.52422 | +2.5 | 13 |
| 14 | | | | +1 | | | -0.055 | +0.165 | +1.110 | +4.05456 | +4.1 | 14 |
| 15 | | | | | +1 | | +0.055 | | +1.055 | -0.98809 | -1.0 | 15 |
| 16 | | | | | -1 | | +0.210 | | -0.790 | -1.55616 | -1.6 | 16 |
| 17 | | | | | +1 | | -0.210 | | +0.790 | +1.55616 | +1.5 | 17 |

Normal Equations

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | n' | Σn | C | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | +6 | -2 | +2 | — | — | +5.42 | — | -0.223 | -4.0 | + 7.197 | +2.92746 | 1 |
| 2 | | +4 | +2 | -2 | — | -5.97 | -0.258 | +0.839 | -8.9 | - 12.289 | +6.12959 | 2 |
| 3 | | | +6 | -2 | — | -13.55 | -0.258 | +0.616 | -6.5 | - 11.692 | -0.81402 | 3 |
| 4 | | | | +4 | — | +3.74 | +0.690 | -0.712 | -1.3 | + 2.418 | +4.09911 | 4 |
| 5 | | | | | +4 | — | -0.150 | — | +0.4 | + 4.250 | -0.46003 | 5 |
| 6 | | | | | | +112.6169 | +0.97614 | -3.73833 | +0.15 | +99.64471 | -0.08332 | 6 |
| 7 | | | | | | | +0.49685 | -0.29696 | +2.295 | +3.49503 | -9.60089 | 7 |
| 8 | | | | | | | | +0.61825 | -2.087 | -4.98404 | -3.47028 | 8 |

Forward Solution of Normal Equations

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | n | -Σn |
|---|---|---|---|---|---|---|---|---|---|
| +6 | -2 | +2 | — | — | +5.42 | — | -0.223 | -4.0 | +7.197 |
| +6 | -2 | +2 | — | — | +5.42 | — | -0.223 | -4.0 | +7.197 |
| C1 | +0.33333 | -0.33333 | — | — | -0.90333 | — | +0.03717 | +0.66667 | -1.19950 |
| | +4 | +2 | -2 | — | -5.97 | -0.258 | +0.839 | -8.9 | -12.289 |
| | +3.33333 | +2.66667 | -2 | — | -4.16333 | -0.258 | +0.76467 | -10.23333 | -9.89000 |
| | C2 | -0.80000 | +0.60000 | — | +1.24900 | +0.07740 | -0.22940 | +3.07000 | +2.96700 |
| | | +6 | -2 | — | -13.55 | -0.258 | +0.616 | -6.5 | -11.692 |
| | | +3.20000 | -0.40000 | — | -12.02600 | -0.05160 | +0.07860 | +3.02000 | -6.17900 |
| | | C3 | +0.12500 | — | +3.75812 | +0.01612 | -0.02456 | -0.94375 | +1.93093 |
| | | | +4 | — | +3.74 | +0.690 | -0.712 | -1.3 | +2.418 |
| | | | +2.75000 | — | -0.26125 | +0.52875 | -0.24338 | -7.06250 | -4.28838 |
| | | | C4 | — | +0.09500 | -0.19227 | +0.08850 | +2.56818 | +1.55941 |
| | | | | +4 | — | -0.150 | — | +0.4 | +4.250 |
| | | | | +4 | — | -0.150 | — | +0.4 | +4.250 |
| | | | | C5 | — | +0.03750 | — | -0.10 | -1.06250 |
| | | | | | +112.6169 | +0.97614 | -3.73833 | +0.15 | +99.64471 |
| | | | | | +57.30088 | +0.51027 | -2.30956 | +1.66053 | +57.16212 |
| | | | | | C6 | -0.00891 | +0.04031 | -0.02898 | -0.99758 |
| | | | | | | +0.49685 | -0.29696 | +2.295 | +3.49503 |
| | | | | | | +0.36421 | -0.16914 | +2.90978 | +3.10485 |
| | | | | | | C7 | +0.46440 | -7.98929 | -8.52489 |
| | | | | | | | +0.61825 | -2.087 | -4.98404 |
| | | | | | | | +0.23943 | +0.83089 | +1.07032 |
| | | | | | | | C8 | -3.47028 | -4.47028 |

*Figure 131. Solution of condition equations.*

PROJECT 1-243 | POSITION COMPUTATION, 2nd ORDER TRIANGULATION (For calculating machine computation) (TM 5-237)

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| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 HICKS | To 3 LINCOLN | 212 | 00 | 51.34 | α | 3 LINCOLN | To 2 HICKS | 32 | 01 | 56.85 |
| 2nd ∠ | & | | +49 | 00 | 56.0 | 3rd ∠ | & | | −108 | 09 | 34.3 |
| α | 2 HICKS | To 1 Burdell | 261 | 01 | 47.34 | α | 3 LINCOLN | To 1 Burdell | 283 | 52 | 22.55 |
| Δα | | | + | 04 | 58.85 | Δα | | | + | 03 | 53.45 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| α' | 1 Burdell | To 2 HICKS | 81 | 06 | 46.19 | α' | 1 Burdell | To 3 LINCOLN | 103 | 56 | 16.00 |
| f = 8.3070 | First Angle of Triangle | | 22 | 49 | 29.8 | f = 8.3070 | | | | | |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 38 | 07 | 46.259 | 2 HICKS | λ | 122 | 43 | 29.366 | φ | 38 | 10 | 00.181 | 3 LINCOLN | λ | 122 | 41 | 43.305 |
| | | | | s = 11929.532 | Δλ | − | 08 | 03.927 | | | | | s = 9477.507 | Δλ | − | 06 | 17.866 |
| φ' | 38 | 08 | 46.310 | 1 Burdell | λ' | 122 | 35 | 25.439 | φ' | 38 | 08 | 46.310 | 1 Burdell | λ' | 122 | 35 | 25.439 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Δφ | 01′ 00″.05 | (log s = b = (y/10,000)² | 4.0766 2340 0.036 ) | Δφ | 01′ 13″.87 | (log s = b = (y/10,000)² | 3.9766 2340 0.052 ) |
| sin α | −0.98776962 | x cor. = −½fb | 0.144 | sin α | −0.97082987 | x cor. = −½fb | 0.213 |
| cos α | −0.15592045 | x′ | −11783.6291 | cos α | +0.23976940 | x′ | −9201.0467 |
| x = s sin α | −11783.6293 | H | 0.041067759 | x = s sin α | −9201.0469 | H | 0.041067759 |
| y = −s cos α | +1860.0580 | Hx′ = (approx. Δλ″) | −483.9272 | y = −s cos α | −2272.4162 | Hx′ = (approx. Δλ″) | −377.8664 |
| a = (x′/10,000)² | 1.3885 | Arc−sin cor = + V (Va) / 15 | 3.50 | a = (x′/10,000)² | 0.8466 | Arc−sin cor = + V (Va) / 15 | 2.13 |
| y cor. = +fa | 11.40 | Δλ″ | −483.927 | y cor. = +fa | 6.95 | Δλ″ | −377.866 |
| $y_o$ | 4 221 667.975 | sin φ | 0.61744118 | $y_o$ | 4 225 797.121 | sin φ | 0.61795179 |
| y′ | + 1 860.060 | sin φ′ | 0.61767017 | y′ | 2 272.418 | sin φ′ | 0.61767017 |
| $y_1$ | 4 223 528.035 | 1 + cos Δφ | 1.99999996 | $y_1$ | 4 223 524.703 | 1 + cos Δφ | 1.99999994 |
| Va − | 8.538 | (sin φ + sin φ′) / (1 + cos Δφ) or sin Va | 0.61755669 | Va − | 5.206 | (sin φ + sin φ′) / (1 + cos Δφ) or sin Va | 0.61781100 |
| $y_2$ | 4 223 519.497 | −Δα″ (approx.) | −298.852 | $y_2$ | 4223519.497 | −Δα″ (approx.) | −233.450 |
| V | 6.14898 | + F (Δλ″)³ | —— | V | 6.14898 | + F (Δλ″)³ | —— |
| K (Va/1,000)² + | —— | = Δα″ | −298.852 | K (Va/1,000)² + | —— | = Δα″ | −233.450 |

COMPUTED BY RAS | DATE 22 Aug 55 | CHECKED BY W.C.A. | DATE 25 Aug 55 | NOTE: For s under 5,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

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*Figure 132. Position computations (net adjustment) (DA Form 1923).*

*b.* The procedure in setting up the equation is to start with a fixed azimuth and, by successive use of observed angles, compute the azimuth of the next fixed azimuth. The difference between the computed and fixed azimuth thus obtained is the constant term of the azimuth equation. The sum of the corrections to all the observed angles used in obtaining the computed value of the fixed azimuth must numerically equal the constant term. When the adjustment is made by the direction method, the corrections are the *v*'s applied to the observed directions. When computations are in the geographic coordinate system, the forward and back azimuths of lines do not differ by exactly 180°, but by the amount known as geodetic convergence. It is to obtain the convergence that preliminary position computations are made for the lines through which the azimuth is carried. For this example, the convergence can be taken from the position computation made in the adjustment of of the original figure.

*c.* The numerical example is as follows:

| | | |
|---|---|---|
| Fixed azimuth Nic-Black | | 123°21′40″.99 |
| Observed angle at Nic | (+51) | +45 01 07 .4 |
| Azimuth Nic-Hicks | | 168 22 48 .39 |
| Convergence at Hicks | | − 00 46 .36 |
| | | 180 00 00 .00 |
| Azimuth Hicks-Nic | | 348 22 02 .03 |
| Observed angle at Hicks | (−56+57) | −44 52 05 .1 |
| Azimuth Hicks-Red | | 303 29 56 .93 |
| Convergence at Red | | + 2 50 .50 |
| | | 180 00 00 .00 |

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| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 HICKS | To 3 LINCOLN | 212 | 00 | 51.34 | α | 3 LINCOLN | To 2 HICKS | 32 | 01 | 56.85 |
| 24∠ | & | | +91 | 29 | 07.4 | 34∠ | & | | -57 | 48 | 48.8 |
| α | 2 HICKS | To 1 Red | 303 | 29 | 58.74 | α | 3 LINCOLN | To 1 Red | 334 | 13 | 08.05 |
| Δα | | | + | 02 | 50.50 | Δα | | | + | 01 | 45.08 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| α' | 1 Red | To 2 HICKS | 123 | 32 | 49.24 | α' | 1 Red | To 3 LINCOLN | 154 | 14 | 53.13 |
| f = 8.2070 | First Angle of Triangle | | 30 | 42 | 03.9 | f = 8.2070 | | | | | |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 38 | 07 | 46.259 | 2 HICKS | λ | 122 | 43 | 29.366 | φ | 38 | 10 | 00.181 | 3 LINCOLN | λ | 122 | 41 | 43.305 |
| | | | | s= 8073.067 | Δλ | - | 04 | 36.256 | | | | | s= 9535.828 | Δλ | - | 02 | 50.195 |
| φ' | 38 | 05 | 21.652 | 1 Red | λ' | 122 | 38 | 53.110 | φ' | 38 | 05 | 21.652 | 1 Red | λ' | 122 | 38 | 53.110 |

| | | | |
|---|---|---|---|
| Δφ | 02' 24.61 | (log s = 3.90703857 / b = (y/10,000)² 0.199) | |
| sin α | -0.83388919 | x cor. = -½fb | 0.817 |
| cos α | +0.55193189 | x' | -6732.0427 |
| x = s sin α | -6732.0433 | H | 0.041035936 |
| y = -s cos α | -4455.7831 | Hx' = (approx. Δλ") | -276.2557 |
| a = (x'/10,000)² | 0.4532 | Arc-sin cor = +V(Va)/15 | 1.14 |
| y cor. = +fa | 3.72 | Δλ" | -276.256 |
| $y_o$ | 4 221 667.975 | sin φ | 0.61744118 |
| y' | -4 455.785 | sin φ' | 0.61688956 |
| $y_1$ | 4 217 212.190 | 1 + cos Δφ | 1.99999975 |
| Va - | 2.781 | (sin φ + sin φ') / (1 + cos Δφ) or sin α | 0.61716545 |
| $y_2$ | 4 217 209.409 | -Δα" (approx.) | -170.496 |
| V | 6.13644 | + F (Δλ")³ | |
| K (Va/1,000)² + | | -Δα" | -170.496 |

| | | | |
|---|---|---|---|
| Δφ | 04' 38.53 | (log s = 3.97935841 / b = (y/10,000)² 0.737) | |
| sin α | -0.43493404 | x cor. = -½fb | 3.024 |
| cos α | +0.90046232 | x' | -4147.4549 |
| x = s sin α | -4147.4562 | H | 0.41035936 |
| y = -s cos α | -8586.6538 | Hx' = (approx. Δλ") | -170.1947 |
| a = (x'/10,000)² | 0.1720 | Arc-sin cor = +V(Va)/15 | 0.43 |
| y cor. = +fa | 1.41 | Δλ" | -170.195 |
| $y_o$ | 4 225 797.121 | sin φ | 0.61795119 |
| y' | -8 586.655 | sin φ' | 0.61688956 |
| $y_1$ | 4 217 210.466 | 1 + cos Δφ | 1.99999909 |
| Va - | 1.055 | (sin φ + sin φ') / (1 + cos Δφ) or sin α | 0.61742096 |
| $y_2$ | 4 217 209.411 | -Δα" (approx.) | -105.082 |
| V | 6.13644 | + F (Δλ")³ | |
| K (Va/1,000)² + | | -Δα" | -105.082 |

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NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

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*Figure 132*—Continued.

| | | |
|---|---|---|
| Azimuth Red-Hicks | | 123 32 47 .43 |
| Observed angle at Red | (−61+62) | +30 41 59 .7 |
| Azimuth Red-Lincoln | | 154 14 47 .13 |
| Convergence at Lincoln | | − 01 45 .08 |
| | | 180 00 00 .00 |
| Azimuth Lincoln-Red | | 334 13 02 .05 |
| Observed angle at Lincoln | (+66) | −50 20 46 .1 |
| Computed azimuth Lincoln-Burdell | | 283 52 15 .95 |
| Fixed azimuth Lincoln-Burdell | | 283 52 22 .55 |
| Constant term (computed minus fixed) | = | −6".60 |

Azimuth equation:

$$0 = -6.60 + (51) + (56) - (57) - (61) + (62) - (66)$$

(1) The azimuth equation is formed by setting *minus* the constant term equal to the sum of the corrections to the angles. Transferring all terms to one side of the equation and setting the equation equal to zero gives the form shown.

(2) A source of possible error occurs whenever an observed angle is *subtracted* to obtain an azimuth. The error frequently made is neglecting to change the signs of the corrections for that angle when the equation is written. In the example, the observed angle at Hicks is designated (−56+57), but the angle is subtracted to obtain the azimuth from Hicks to Red so the designation is written in the equation as

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| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 BLACK | To 3 HICKS | 212 | 40 | 17.91 | α | 3 HICKS | To 2 BLACK | 32 | 41 | 45.84 |
| 2d ∠ | & | | +90 | 39 | 08.9 | 3d ∠ | & | | -44 | 19 | 40.7 |
| α | 2 BLACK | To 1 Nic | 303 | 19 | 26.81 | α | 3 HICKS | To 1 Nic | 348 | 22 | 05.14 |
| Δα | | | + | 02 | 14.18 | Δα | | | + | 00 | 46.36 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| α' | 1 Nic | To 2 BLACK | 123 | 21 | 40.99 | α' | 1 Nic | To 3 HICKS | 168 | 22 | 51.50 |
| f = 8.2071 | First Angle of Triangle | | 45 | 01 | 10.5 | f = 8.2070 | | | | | |

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 38 | 04 | 50.777 | 2 BLACK | λ | 122 | 45 | 51.846 | φ | 38 | 07 | 46.259 | 3 HICKS | λ | 122 | 43 | 29.366 |
| | | | | s = 6350.335 | Δλ | - | 03 | 37.626 | | | | | s = 9087.356 | Δλ | - | 01 | 15.146 |
| φ' | 38 | 02 | 57.569 | 1 Nic | λ' | 122 | 42 | 14.220 | φ' | 38 | 02 | 57.569 | 1 Nic | λ' | 122 | 42 | 14.220 |

| | | | |
|---|---|---|---|
| Δφ | 01' 53".21 | Δφ | 04 48.69 |
| sin α | -0.83557623 | sin α | -0.20162338 |
| cos α | +0.54937453 | cos α | +0.97946313 |
| x = s sin α | -5306.1890 | x = s sin α | -1832.2234 |
| y = -s cos α | -3488.7123 | y = -s cos α | -8900.7302 |
| a = (x'/10,000)² | 0.2816 | a = (x'/10,000)² | 0.0336 |
| y cor. = +fa | 2.31 | y cor. = +fa | 0.28 |
| $y_o$ | 4 216 257.470 | $y_o$ | 4 221 667.975 |
| y' | - 3 488.713 | y' | -8 900.730 |
| $y_1$ | 4 212 768.757 | $y_1$ | 4 212 767.245 |
| Va - | 1.726 | Va - | 0.206 |
| $y_2$ | 4 212 767.03 5 | $y_2$ | 4 212 767.03 5 |
| V | 6.12763 | V | 6.12763 |
| K (Va/1,000)² + | ——— | K (Va/1,000)² + | ——— |
| COMPUTED BY RAS | DATE 23 Aug 55 | DATE 25 Aug 55 | |

| | | | |
|---|---|---|---|
| (log s = 3.80279661 / b = (y/10,000)² 0.122) | | (log s = 3.95843735 / b = (y/10,000)² 0.792) | |
| x cor. = -½fb | 0.501 | x cor. = -½fb | 3.250 |
| x' | -5306.1887 | x' | -1832.2228 |
| H | 0.041013585 | H | 0.041013585 |
| Hx' = (approx. Δλ") | -217.6258 | Hx' = (approx. Δλ") | -75.1460 |
| Arc-sin cor = +V(Va)/15 | 0.71 | Arc-sin cor = +V(Va)/15 | 0.08 |
| Δλ" | -217.626 | Δλ" | -75.146 |
| sin φ | 0.61677174 | sin φ | 0.61744118 |
| sin φ' | 0.61633963 | sin φ' | 0.61633963 |
| 1 + cos Δφ | 1.99999985 | 1 + cos Δφ | 1.99999902 |
| (sin φ + sin φ') / (1 + cos Δφ) ~~or sin +~~ | 0.61655573 | (sin φ + sin φ') / (1 + cos Δφ) ~~or sin +~~ | 0.6168907/ |
| -Δα" (approx.) | -134.179 | -Δα" (approx.) | -46.357 |
| + F (Δλ")³ | ——— | + F (Δλ")³ | ——— |
| - Δα" | -134.179 | - Δα" | -46.357 |
| CHECKED BY W.C.A. | | NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined. | |

DA FORM FEB 57 1923

*Figure 132*—Continued.

-(-56+57) or (+56-57). The same situation arises with the angle at Lincoln.

*d.* The discussion of latitude and longitude equations in this manual will be limited to a description and a short example problem. For a complete development of the formulas, see USC&GS Sp. Pub. 28.

*e.* The example is shown in figure 133. The fixed data is taken from the adjustment of the original figure by the direction method, as it was for the azimuth equation. The fixed data in this case is the positions of stations Black, Nic, Lincoln, and Burdell, and thereby the lengths and azimuths of the lines Black-Nic and Lincoln-Burdell.

*f.* The first step in forming the latitude and longitude equations is the setup and computation of preliminary triangles. A single chain of triangles is used for preliminary triangle and position computations, and it should be the $R_1$, or strongest chain (fig. 134). One angle in each triangle is concluded and the other two angles are observed angles. The observed angles are usually the distance angles, that is, the angles used to compute the unknown sides in the triangle. The concluded angle is known as the azimuth angle. The exception to the rule that the azimuth angle is always concluded occurs when one of the distance angles is not observed. In this case, the observed azimuth angle must be used and the unobserved distance angle concluded. The important point to remember when using concluded angles is that the designation of the correction to the angle is the sum of the corrections (with their signs changed) of the other two angles in the triangle.

PROJECT: I-243 | POSITION COMPUTATION, 2nd ORDER TRIANGULATION (For calculating machine computation) (TM 5-237)

LOCATION: California | ORGANIZATION: 29th Engrs | DATE: 19 Aug 55

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 HICKS | To 3 Red | 303 | 29 | 58.74 | α | 3 Red | To 2 HICKS | 123 | 32 | 49.24 |
| 2d ∠ | & | | +44 | 52 | 06.4 | 3d ∠ | & | | −75 | 42 | 46.3 |
| α | 2 HICKS | To 1 Nic | 348 | 22 | 05.14 | α | 3 Red | To 1 Nic | 47 | 50 | 02.94 |
| Δα | | | + | 0 | 46.36 | Δα | | | − | 02 | 04.01 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| α′ | 1 Nic | To 2 HICKS | 168 | 22 | 51.50 | α′ | 1 Nic | To 3 Red | 227 | 47 | 58.93 |
| f = 8.2070 | | First Angle of Triangle | 59 | 25 | 07.4 | f = 8.2071 | | | | | |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 38 | 07 | 46.259 | 2 HICKS | λ | 122 | 43 | 29.366 | φ | 38 | 05 | 21.652 | 3 Red | λ | 122 | 38 | 53.110 |
| | | | | s = 9087.358 | Δλ | − | 01 | 15.146 | | | | | s = 6615.574 | Δλ | + | 03 | 21.110 |
| φ′ | 38 | 02 | 57.569 | 1 Nic | λ′ | 122 | 42 | 14.220 | φ′ | 38 | 02 | 57.569 | 1 Nic | λ′ | 122 | 42 | 14.220 |

| | | | |
|---|---|---|---|
| Δφ | 04 48.69 | Δφ | 02 24.08 |
| sin α | −0.20162338 | sin α | +0.74120483 |
| cos α | +0.97946313 | cos α | +0.67127893 |
| x = s sin α | −1832.2238 | x = s sin α | +4903.4954 |
| y = −s cos α | −8900.7321 | y = −s cos α | −4440.8954 |
| a = (x′/10,000)² | −0.0336 | a = (x′/10,000)² | 0.2404 |
| y cor. = +fa | 0.28 | y cor. = +fa | 1.97 |
| $y_o$ | 4 221 667.975 | $y_o$ | 4 217 209.412 |
| y′ | −8 900.732 | y′ | −4 440.896 |
| $y_1$ | 4 212 767.243 | $y_1$ | 4 212 768.516 |
| Va | 0.206 | Va | −1.473 |
| $y_2$ | 4 212 767.037 | $y_2$ | 4 212 767.043 |
| V | 6.12763 | V | 6.12763 |
| K (Va/1,000)² + | ——— | K (Va/1,000)² + | ——— |

| | | | |
|---|---|---|---|
| (log s = b = (y/10,000)² | 3.95843762 0.792 | (log s = b = (y/10,000)² | 3.82056754 0.197 |
| x cor. = −½fb | 3.250 | x cor. = −½fb | 0.808 |
| x′ | −1832.2232 | x′ | +4903.4950 |
| H | 0.041013585 | H | 0.041013585 |
| Hx′ = (approx. Δλ″) | −75.1460 | Hx′ = (approx. Δλ″) | +201.1099 |
| Arc−sin cor = +V (Va)/15 | 0.08 | Arc−sin cor = +V (Va)/15 | 0.60 |
| Δλ″ | −75.146 | Δλ″ | +201.110 |
| sin φ | 0.61744118 | sin φ | 0.61688956 |
| sin φ′ | 0.61633963 | sin φ′ | 0.61633963 |
| 1 + cos Δφ | 1.99999902 | 1 + cos Δφ | 1.99999976 |
| (sin φ + sin φ′)/(1 + cos Δφ) or sin + | 0.61689071 | (sin φ + sin φ′)/(1 + cos Δφ) or sin + | 0.61661467 |
| −Δα″ (approx.) | −46.357 | −Δα″ (approx.) | +124.007 |
| + F (Δλ″)³ | ——— | + F (Δλ″)³ | ——— |
| − Δα″ | −46.357 | − Δα″ | +124.007 |

COMPUTED BY Adsmith | DATE 27 Aug. 55 | CHECKED BY W. C. Cumeo | DATE 28 Aug. 55

NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1923 FEB 57

*Figure 132*—Continued.

*g.* In addition to the correction symbols on the angles, each angle is also designated by A, B, or C. The azimuth angle is always called C, while the length angle adjacent to the known side in the triangle is called A, and the length angle opposite the known side is called B. The data from the preliminary triangles is used to compute the preliminary geographic positions. The combined data from the triangles and position computations is then used on DA Form 1927 (Latitude and Longitude Adjustment) to form the latitude and longitude equations.

*h.* The triangles used in the preliminary computations are shown in figure 134. The A, B, and C angles are labeled on the sketch for use in the latitude and longitude equations. The preliminary triangle computations are shown in figure 135. Notice that the C angle could not be concluded in triangle Red-Nic-Hicks, because the distance angle A was not observed. This is the case when the azimuth angle C must be used as observed. The triangle computation can be made using either natural or logarithmic functions. In this example, the computations were made using natural functions, as the position computations were also made using natural functions. Some argument could be raised that a complete logarithmic computation could be more efficient, since the tabular differences for 1″ for the log sines of the A and B angles are required for the latitude and longitude equations, but this is actually a minor point and the choice of logs or natural functions should be left to the individual computer.

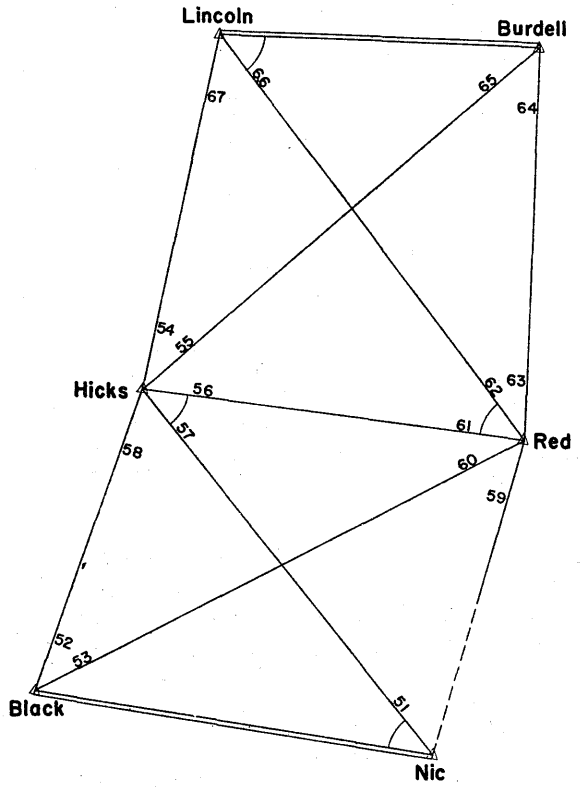



*Figure 133. Sketch of triangulation net (modified problem).*

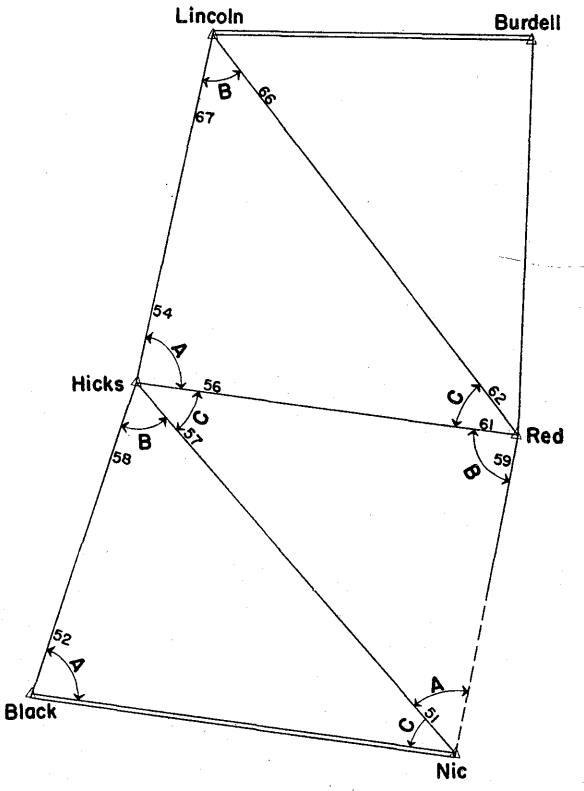



*Figure 134. Sketch of net ($R_l$ arc).*

*i.* Using the angles and distances from the preliminary triangle computations, the preliminary position computations are made (fig. 136). Notice that both sides of the position computation form check the position of station 1, even though no formal adjustment was made of the triangles. The check is possible because of the arbitrary adjustment made by concluding one angle in each triangle.

*j.* On completion of the preliminary triangle and position computations, all the necessary data is available for forming the latitude and longitude equations on DA Form 1927 (fig. 137). On the form, the following entries are made from the available data:

(1) In the right hand, upper corner of the form in the spaces $\phi_\eta$ and $\lambda_\eta$, the *computed* latitude and longitude (in degrees, minutes, and decimals of minutes) of the fixed end station.

(2) In the left hand column headed "Station," the names of the stations at which the C angles are recorded. It is convenient for later use, if a notation is made beside each station as to whether the C angle at the station is used in a clockwise or counterclockwise direction. A plus (+) sign indicates a clockwise and a minus (−) sign a counterclockwise C angle.

(3) The computed positions of the stations in the left-hand column in the columns headed $\phi_c$ (latitude) and $\lambda_c$ (longitude) in degrees, minutes, and decimals of minutes.

(4) In the columns headed A, B, and C, the correction symbols for the A, B, and C angles. The symbols for A, B, and C can be taken directly from the triangle computation sheet when C is plus, but when C is minus the symbols for C on the triangle sheet must be entered with opposite sign on DA Form 1927 by the following rules:

(*a*) When C is plus, the combination of the symbols for A, B, and C should be zero.

(*b*) When C is minus, the combination of the symbols for A and B should equal the symbols for C.

(5) The tabular differences for 1″ for the log sines of angles A and B in columns headed $+\delta_A$ and $-\delta_B$. The tabular difference for the log sine of angles over

| PROJECT | | DATE | COMPUTATION OF TRIANGLES (TM 5-237) |
|---|---|---|---|
| 1-243 | | 19 Aug 55 | Preliminary |
| LOCATION | California | ORGANIZATION | 29th Engrs. |

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: ~~Logarithm~~ or NATURAL |
|---|---|---|---|---|---|---|---|
| | 2–3 Nic - BLACK | | | | | | 6350.335 |
| B -57+58 | 1 HICKS | 44 19 41.9 | — | 41.9 | 0.0 | 41.9 | 0.69876877 |
| C (+52+57 -58) | 2 Nic | (45 01 08.3) | — | 08.3 | 0.0 | 08.3 | 0.70734089 |
| A -52 | 3 BLACK | 90 39 09.9 | — | 09.9 | 0.1 | 09.8 | 0.99993511 |
| | 1–3 HICKS - BLACK | | | | | | 6428.231 |
| | 1–2 HICKS - Nic | | | | | | 9087.302 |
| | | | | | 0.1 | | |
| | 2–3 Nic - HICKS | | | | | | 9087.302 |
| B -59+61 | 1 Red | 75 42 46.0 | — | 46.0 | 0.1 | 45.9 | 0.96907067 |
| A (+56-57 +59-61) | 2 Nic | (59 25 09.0) | — | 09.0 | 0.0 | 09.0 | 0.86091226 |
| C -56+57 | 3 HICKS | 44 52 05.1 | — | 05.1 | 0.0 | 05.1 | 0.70547688 |
| | 1–3 Red - HICKS | | | | | | 8073.064 |
| | 1–2 Red - Nic | | | | | | 6615.494 |
| | | | | | 0.1 | | |
| | 2–3 Red - HICKS | | | | | | 8073.064 |
| B -66+67 | 1 LINCOLN | 57 48 45.3 | — | 45.3 | 0.0 | 45.3 | 0.84631017 |
| C (+54-56 +66-67) | 2 Red | (30 42 08.6) | — | 08.6 | 0.0 | 08.6 | 0.51057877 |
| A -54+56 | 3 HICKS | 91 29 06.2 | — | 06.2 | 0.1 | 06.1 | 0.99966413 |
| | 1–3 LINCOLN-HICKS | | | | | | 4870.478 |
| | 1–2 LINCOLN - Red | | | | | | 9535.928 |
| | | | | | 0.1 | | |

*Figure 135. Preliminary computation of triangles (DA Form 1918).*

90° is minus. The tabular difference for the B angle is entered on DA Form 1927 with opposite algebraic sign than as given in the tables; therefore, if the tabular difference in the log tables is plus for the log sine of a B angle less than 90°, the tabular difference entered on DA Form 1927 in the $-\delta_B$ column will be minus.

(6) The computed value of the position of the fixed end station and the fixed value of the position of the fixed station. The computed value is designated $\phi_n$ and $\lambda_n$ and the fixed value is $\phi'_\eta$, $\lambda'_\eta$. Both values are entered in degrees, minutes, and seconds.

(7) The quantities $a_1$ and $a_2$ from the table at the right hand side of the form, using the

PROJECT 1-243 — ~~POSITION COMPUTATION~~, ~~ORDER TRIANGULATION~~ (For calculating machine computation)
LATITUDE AND LONGITUDE ADJUSTMENT (Preliminary TM 5-237)
LOCATION California — ORGANIZATION 29th Engrs. — DATE 19 Aug 55

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 Nic | To 3 BLACK | | 123 | 21 | 40.99 | α | 3 BLACK | To 2 Nic | | 303 | 19 | 26.81 |
| 3rd ∠ | & | | | +45 | 01 | 08.3 | 3rd ∠ | & | | | -90 | 39 | 09.9 |
| α | 2 Nic | To 1 HICKS | | 168 | 22 | 49.29 | α | 3 BLACK | To 1 HICKS | | 212 | 40 | 16.91 |
| Δα | | | | - | | 46.36 | Δα | | | | + | 01 | 27.92 |
| | | | | 180 | 00 | 00.00 | | | | | 180 | 00 | 00.00 |
| α' | 1 HICKS | To 2 Nic | | 348 | 22 | 02.93 | α' | 1 HICKS | To 3 BLACK | | 32 | 41 | 44.83 |
| f = 8.2072 | | First Angle of Triangle | | 44 | 19 | 41.9 | f = 8.2071 | | | | | | |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 38 | 02 | 57.569 | 2 Nic | λ | 122 | 42 | 14.220 | φ | 38 | 04 | 50.777 | 3 BLACK | λ | 122 | 45 | 51.846 |
| | | | | s = 9087.302 | Δλ | + | 01 | 15.150 | | | | | s = 6428.237 | Δλ | - | 02 | 22.476 |
| φ' | 38 | 07 | 46.257 | 1 HICKS | λ' | 122 | 43 | 29.370 | φ' | 38 | 07 | 46.257 | 1 HICKS | λ' | 122 | 43 | 29.370 |

| | | | |
|---|---|---|---|
| Δφ | 04 48.688 | (log s = b = (y/10,000)² | 0.792 ) |
| sin α | + 0.20141372 | x cor. = -½fb | 3.250 |
| cos α | - 0.97950626 | x' | + 1830.3067 |
| x = s sin α | + 1830.3073 | H | 0.04105841 |
| y = -s cos α | + 8901.0692 | Hx' = (approx. Δλ") | + 75.1495 |
| a = (x'/10,000)² | 0.0335 | Arc-sin cor = +V(Va)/15 | 0.08 |
| y cor. = +fa | 0.27 | Δλ" | + 75.150 |
| $y_0$ | 4 212 767.037 | sin φ | 0.61633963 |
| y' | + 8 901.069 | sin φ' | 0.61744117 |
| $y_1$ | 4 221 668.106 | 1 + cos Δφ | 1.99999902 |
| Va - | 0.206 | (sin φ + sin φ') / (1 + cos Δφ) ~~arc-sin~~ | 0.61689070 |
| $y_2$ | 4 221 667.900 | -Δα" (approx.) | + 46.359 |
| V | 6.14530 | + F (Δλ")³ | ——— |
| K (Va/1,000)² + | ——— | - Δα" | + 46.359 |

| | | | |
|---|---|---|---|
| Δφ | 02 55.480 | (log s = b = (y/10,000)² | 0.293 ) |
| sin α | -0.53981967 | x cor. = -½fb | 1.202 |
| cos α | -0.84178069 | x' | - 3470.0884 |
| x = s sin α | - 3470.0888 | H | 0.04105841 |
| y = -s cos α | + 5411.1658 | Hx' = (approx. Δλ") | - 142.4763 |
| a = (x'/10,000)² | 0.1204 | Arc-sin cor = +V(Va)/15 | 0.30 |
| y cor. = +fa | 0.99 | Δλ" | - 142.476 |
| $y_0$ | 4 216 257.470 | sin φ | 0.61677174 |
| y' | + 5 411.166 | sin φ' | 0.61744117 |
| $y_1$ | 4 221 668.636 | 1 + cos Δφ | 1.99999964 |
| Va - | 0.740 | (sin φ + sin φ') / (1 + cos Δφ) ~~arc-sin~~ | 0.61710657 |
| $y_2$ | 4 221 667.896 | -Δα" (approx.) | - 87.923 |
| V | 6.14530 | + F (Δλ")³ | ——— |
| K (Va/1,000)² + | ——— | - Δα" | - 87.923 |

COMPUTED BY RAS — DATE 23 Aug 55 — CHECKED BY W.C.A. — DATE 25 Aug 55

NOTE: For s under 6,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1 FEB 57 1923

*Figure 136. Preliminary position computations (DA Form 1923).*

computed value of $\phi_n$ from the top of the page as the argument.

*k.* Now that all the data has been entered from the preliminary triangle and position computations, the computation of the coefficients for the latitude and longitude equations can be completed on the form. The columns headed "$\phi_n - \phi_c$" and "$\lambda_n - \lambda_c$" need no explanation except the reminder to watch the algebraic signs of the quantities. The quantities in the columns headed "Latitude Equation" and "Longitude Equation" are simply the products of the quantities in the columns already filled out. Watch the algebraic signs.

*l.* In forming the latitude equation, the quantities in the column headed "$(\phi_n - \phi_c)\delta_A$" are the coefficients of the symbols in column A, the quantities in the "$(\phi_n - \phi_c)$ $(-\delta_B)$" column are the coefficients of the symbols in column B, the quantities in "$(\lambda_n - \lambda_c)a_1$" column are the coefficients of the symbols in column C. The constant term of the latitude equation is 7238.24 $(\phi_n - \phi_n')$.

*m.* The longitude equation is formed using the quantities in the column headed "$(\lambda_n - \lambda_c)\delta_A$" as the coefficients of the symbols in column A, the quantities in the column headed "$(\lambda_n - \lambda_c)$ $(-\delta_B)$" as the coefficients of the symbols in column B, and the quantities in the column headed "$(\phi_n - \phi_c)a_2$" as the coefficient of the symbols in column C. The constant term of the longitude equation is 7238.24 $(\lambda_n - \lambda_n')$. To help clarify the formation of the equations and the coefficients, the latitude equation in the example will be written out in full before any of the coefficients of the symbols are collected into the final form.

*n.* The latitude equation in full is—

PROJECT I-243 | ~~POSITION COMPUTATION~~, ~~ORDER TRIANGULATION~~ (For calculating machine computation) LATITUDE & LONGITUDE ADJUSTMENT (Prelim.) (TM 5-237)

LOCATION California | ORGANIZATION 29th Engrs. | DATE 19 Aug 55

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 Nic | To 3 HICKS | 168 | 22 | 49.29 | α | 3 HICKS | To 2 Nic | 348 | 22 | 02.93 |
| 2d∠ | & | | +59 | 25 | 09.0 | 3d∠ | & | | −44 | 52 | 05.1 |
| α | 2 Nic | To 1 Red | 227 | 47 | 58.29 | α | 3 HICKS | To 1 Red | 303 | 29 | 57.83 |
| Δα | | | + | 02 | 04.01 | Δα | | | + | 02 | 50.50 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| α′ | 1 Red | To 2 Nic | 47 | 50 | 02.30 | α′ | 1 Red | To 3 HICKS | 123 | 32 | 48.33 |
| f = 8.2072 | | First Angle of Triangle | 75 | 42 | 46.0 | f = 8.2070 | | | | | |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 38 | 02 | 57.569 | 2 Nic | λ | 122 | 42 | 14.220 | φ | 38 | 07 | 46.257 | 3 HICKS | λ | 122 | 43 | 29.370 |
| | | | | s = 6615.494 | Δλ | − | 03 | 21.106 | | | | | s = 8073.064 | Δλ | − | 04 | 36.256 |
| φ′ | 38 | 05 | 21.651 | 1 Red | λ′ | 122 | 38 | 53.114 | φ′ | 38 | 05 | 21.651 | 1 Red | λ′ | 122 | 38 | 53.114 |

| | | | |
|---|---|---|---|
| Δφ | 02 24.082 | (log s = b = (y/10,000)² | −0.197 ) |
| sin α | −0.74079903 | x cor. = −½fb | 0.808 |
| cos α | −0.67172673 | x′ | −4900.7511 |
| x = s sin α | −4900.7515 | H | 0.041035935 |
| y = −s cos α | +4443.8042 | Hx′ = (approx. Δλ″) | −201.1069 |
| a = (x′/10,000)² | 0.2402 | Arc−sin cor = +V(Va)/15 | 0.60 |
| y cor. = +fa | 1.97 | Δλ″ | −201.106 |
| $y_o$ | 4 212 767.037 | sin φ | 0.61633963 |
| y′ | + 4 443.805 | sin φ′ | 0.61688955 |
| $y_1$ | 4 217 210.842 | 1 + cos Δφ | 1.99999976 |
| Va | − 1.474 | (sin φ + sin φ′)/(1 + cos Δφ) ~~or sin t~~ | 0.6166466 |
| $y_2$ | 4 217 209.368 | −Δα″ (approx.) | −124.006 |
| V | 6.13644 | + F (Δλ″)³ | |
| K (Va/1,000)² + | | − Δα″ | −124.006 |
| COMPUTED BY RAdmitt | DATE 23 Aug 55 | CHECKED BY W. C. Aumen | |

| | | | |
|---|---|---|---|
| Δφ | 02 24.606 | (log s = b = (y/10,000)² | 0.199 ) |
| sin α | −0.83389163 | x cor. = −½fb | 0.817 |
| cos α | +0.55192821 | x′ | −6732.0599 |
| x = s sin α | −6732.0605 | H | 0.041035935 |
| y = −s cos α | −4455.7518 | Hx′ = (approx. Δλ″) | −276.2564 |
| a = (x′/10,000)² | 0.4532 | Arc−sin cor = +V(Va)/15 | 1.14 |
| y cor. = +fa | 3.72 | Δλ″ | −276.256 |
| $y_o$ | 4 221 667.913 | sin φ | 0.61744117 |
| y′ | − 4 455.753 | sin φ′ | 0.61688955 |
| $y_1$ | 4 217 212.160 | 1 + cos Δφ | 1.99999975 |
| Va | − 2.781 | (sin φ + sin φ′)/(1 + cos Δφ) ~~or sin t~~ | 0.6171654 |
| $y_2$ | 4 217 209.374 | −Δα″ (approx.) | −170.496 |
| V | 6.13644 | + F (Δλ″)³ | |
| K (Va/1,000)² + | | − Δα″ | −170.496 |
| DATE 23 Aug 55 | | NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined. | |

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*Figure 136*—Continued.

$$0=(7238.24)(+0.004)+(-0.21)(-52)$$
$$+(-15.14)(-57)+(-15.14)(58)$$
$$+(-0.86)(52)+(-0.86)(57)+(-0.88)(-58)$$
$$+(2.79)(56)+(2.79)(-57)+(2.79)(59)$$
$$+(2.79)(-61)+(-1.18)(-59)+(-1.18)(61)$$
$$+(-2.94)(56)+(-2.94)(-57)$$
$$+(-0.28)(-54)+(-0.28)(56)$$
$$+(-6.12)(-66)+(-6.12)(67)+(4.70)(54)$$
$$+(4.70)(-56)+(4.70)(66)+(4.70)(-67)$$

Combining coefficients of the same symbols with regard to algebraic sign, such as (+0.21−0.86) (52) and (+0.28+4.70) (54), and rearranging in ascending order of the symbols, the equation becomes—

$$0=+28.95296-0.65(52)+4.98(54)-5.13(56)$$
$$+14.43(57)-14.28(58)+3.97(59)-3.97(61)$$
$$+10.82(66)-10.82(67).$$

This equation would probably be divided by 10 when used in a least-squares solution. When dividing an equation by a constant, care must be taken not to lose any significant numbers by rounding off.

## 76. Adjustment of Triangulation by the Angle Method

*a.* The figure previously adjusted by the rigid direction method will now be adjusted by the

PROJECT 1-243 | ~~POSITION COMPUTATION~~, LATITUDE & LONGITUDE ADJUSTMENT | ~~ORDER TRIANGULATION~~ (For calculating machine computation) (TM 5-237) (Preliminary)

LOCATION California | ORGANIZATION 29th Engrs | DATE 19 Aug 55

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 2 Red | To 3 HICKS | 123 | 32 | 48.33 | $\alpha$ | 3 HICKS | To 2 Red | 303 | 29 | 57.83 |
| 2d ∠ | | & | +30 | 42 | 08.6 | 3d ∠ | | & | -91 | 29 | 06.2 |
| $\alpha$ | 2 Red | To 1 LINCOLN | 154 | 14 | 56.93 | $\alpha$ | 3 HICKS | To 1 LINCOLN | 212 | 00 | 51.63 |
| $\Delta\alpha$ | | | − | 01 | 45.08 | $\Delta\alpha$ | | | + | 01 | 05.52 |
| | | | 180 | 00 | 00.00 | | | | 180 | 00 | 00.00 |
| $\alpha'$ | 1 LINCOLN | To 2 Red | 334 | 13 | 11.85 | $\alpha'$ | 1 LINCOLN | To 3 HICKS | 32 | 01 | 57.15 |
| f = 8.2071 | | First Angle of Triangle | 57 | 48 | 48.3 | f = 8.2070 | | | | | |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | 38 | 05 | 21.651 | 2 Red | $\lambda$ | 122 | 38 | 53.114 | $\phi$ | 38 | 07 | 46.257 | 3 HICKS | $\lambda$ | 122 | 43 | 29.370 |
| | | | | s= 9535.928 | $\Delta\lambda$ | + | 02 | 50.190 | | | | | s= 4870.478 | $\Delta\lambda$ | − | 01 | 46.066 |
| $\phi'$ | 38 | 10 | 00.185 | 1 LINCOLN | $\lambda'$ | 122 | 41 | 43.304 | $\phi'$ | 38 | 10 | 00.185 | 1 LINCOLN | $\lambda'$ | 122 | 41 | 43.304 |

| | | | |
|---|---|---|---|
| $\Delta\phi$ | 04 38.534 | (log s = b = $(y/10{,}000)^2$ | .738 ) |
| sin $\alpha$ | +0.43445867 | x cor. = $-\frac{1}{2}fb$ | 3.028 |
| cos $\alpha$ | −0.90069177 | x′ | +4142.9653 |
| x = s sin $\alpha$ | +4142.9666 | H | 0.041079269 |
| y = −s cos $\alpha$ | +8588.9319 | Hx′ = (approx. $\Delta\lambda''$) | +170.1900 |
| a = $(x'/10{,}000)^2$ | 0.1716 | Arc−sin cor = $+\frac{V\,(Va)}{15}$ | 0.43 |
| y cor. = +fa | 1.41 | $\Delta\lambda''$ | +170.190 |
| $y_o$ | 4 217 209.382 | sin $\phi$ | 0.61688955 |
| y′ | + 8 588.933 | sin $\phi'$ | 0.61795181 |
| $y_1$ | 4 225 798.315 | 1 + cos $\Delta\phi$ | 1.99999909 |
| Va | − − 1.056 | $\frac{\sin\phi + \sin\phi'}{1 + \cos\Delta\phi}$ ~~or sin $\frac{1}{2}$~~ | 0.61742096 |
| $y_2$ | 4 225 797.258 | $-\Delta\alpha''$ (approx.) | +105.079 |
| V | 6.15350 | + F $(\Delta\lambda'')^3$ | ——— |
| K $(Va/1{,}000)^2$ + | ——— | $-\Delta\alpha''$ | +105.079 |

| | | | |
|---|---|---|---|
| $\Delta\phi$ | 02 13.928 | (log s = b = $(y/10{,}000)^2$ | 0.171 ) |
| sin $\alpha$ | −0.53013152 | x cor. = $-\frac{1}{2}fb$ | 0.702 |
| cos $\alpha$ | −0.84791542 | x′ | −2581.9937 |
| x = s sin $\alpha$ | −2581.9939 | H | 0.041079269 |
| y = −s cos $\alpha$ | +4129.7534 | Hx′ = (approx. $\Delta\lambda''$) | −106.0664 |
| a = $(x'/10{,}000)^2$ | 0.0667 | Arc−sin cor = $+\frac{V\,(Va)}{15}$ | 0.17 |
| y cor. = +fa | 0.55 | $\Delta\lambda''$ | −106.066 |
| $y_o$ | 4 221 667.913 | sin $\phi$ | 0.61744117 |
| y′ | + 4 129.754 | sin $\phi'$ | 0.61795181 |
| $y_1$ | 4 225 797.667 | 1 + cos $\Delta\phi$ | 1.99999979 |
| Va | − 0.410 | $\frac{\sin\phi + \sin\phi'}{1 + \cos\Delta\phi}$ ~~or sin $\frac{1}{2}$~~ | 0.61764655 |
| $y_2$ | 4 225 797.258 | $-\Delta\alpha''$ (approx.) | −65.517 |
| V | 6.15350 | + F $(\Delta\lambda'')^3$ | ——— |
| K $(Va/1{,}000)^2$ + | ——— | $-\Delta\alpha''$ | −65.517 |

COMPUTED BY RSmith | DATE 23 Aug 55 | CHECKED BY W. C. Cumen | DATE 25 Aug 55 | NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1 FEB 57 1923

*Figure 136*—Continued.

angle method (fig. 138). As the name implies, this method of adjustment solves the most probable angle, and individual directions are not corrected.

*b*. The angle method involves the solution of a series of triangles. Consequently, 1 diagonal in each quadrilateral must be omitted. Figure 139 illustrates the net with 2 diagonals omitted. The stations are numbered starting from a fixed line and each station added is given a consecutive number. Thus, building up the figure point by point and numbering each station as it is added, Burdell is number 1, Red number 2, Nic number 3, and Black number 4. Notice that Black is numbered, even though it is a fixed station. The angles in each triangle are lettered *a*, *b*, and *c*. Letter *b* is always assigned to the angle opposite the side of known length. Letter *a* is assigned to the angle adjacent to the known side and opposite the side through which the length is to be carried. Letter *c* is assigned to the remaining angle in the triangle which is also the angle used to carry the azimuth through the figure. Combining the letters with the number of the new point in the triangle completes the designation of the angles. For example, the angles in triangle Red-Hicks-Burdell will all carry the number 2 since Red is the new point in the triangle. The angle at Red will be letter *b* because it is opposite side Hicks-Burdell which is known from the computation of the first triangle. The length is to be carried through side Hicks-Red; therefore, the angle at Burdell is lettered *a*. The remaining angle is at Hicks and is lettered *c*.

*c*. The angles are taken from the list of directions (fig. 140) exactly as in the direction method of

PROJECT: 1-243 | LOCATION: CALIFORNIA | LATITUDE AND LONGITUDE ADJUSTMENT (TM 5-237)

ORGANIZATION: 29th ENGRS | FROM: BLACK - NIC BASE | TO: LINCOLN - BURDELL BASE

DATE: 22 AUG. 55 | STATION (End) LINCOLN | $\phi_n$ 38° 10′ 00 | $\lambda_n$ 122° 41′ 72

| Sign of "C" | Station | $\phi_c$ | $\lambda_c$ | $\phi_n - \phi_c$ | $\lambda_n - \lambda_c$ | $+\delta_A$ | $-\delta_B$ | Lat. equation $(\phi_n - \phi_c)\delta_A$ | $A$ | Long. equation $(\lambda_n - \lambda_c)\delta_A$ | Lat. equation $(\phi_n - \phi_c)(-\delta_B)$ | $B$ | Long. equation $(\lambda_n - \lambda_c)(-\delta_B)$ | Lat. equation $(\lambda_n - \lambda_c)a_1$ | $C$ $(+R$ or $-L)$ | Long. equation $(\phi_n - \phi_c)a_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| + | NIC | 38° 02′ 96 | 122° 42′ 24 | +7.04 | -0.52 | -0.03 | -2.15 | -0.21 | -52 | +0.02 | -15.14 | -57+58 | +1.12 | -0.86 | +52 +57-58 | -18.80 |
| − | HICKS | 38 07.77 | 122 43.49 | +2.23 | -1.77 | +1.25 | -0.53 | +2.79 | +56-57 +59-61 | -2.21 | -1.18 | -59+61 | +0.94 | -2.94 | +56-57 | -5.95 |
| + | RED | 38 05.36 | 122 38.89 | +4.64 | +2.83 | -0.06 | -1.32 | -0.28 | -54+56 | -0.17 | -6.12 | -66+67 | -3.74 | +4.70 | +54-56 +66-67 | -12.39 |

LINCOLN

$\phi_n$ = 38° 10′ 00.″185 | $\lambda_n$ = 122° 41′ 43.″304 | $a_1$ = +1.66

$\phi_n'$ = 38 10 00.181 | $\lambda_n'$ = 122 41 43.305

$(\phi_n - \phi_n')$ = +0.004 | $(\lambda_n - \lambda_n')$ = −0.001 | $a_2$ = −2.67

× 7238.24 | × 7238.24

+ 28.95296 | − 7.23824

Latitude Equation: $0 = +28.95296 - 0.65(52) + 4.98(54) - 5.13(56) + 14.43(57) - 14.28(58) + 3.97(59) - 3.97(61) + 10.82(66) - 10.82(67)$

Longitude Equation: $0 = -7.23824 - 18.82(52) - 12.22(54) + 4.06(56) - 11.76(57) + 19.92(58) - 3.15(59) + 3.15(61) - 8.65(66) + 8.65(67)$

TABLES

| $\phi_n$ | $a_1$ | $a_2$ |
|---|---|---|
| ° | + | − |
| 0 | 2.12 | 2.09 |
| 1 | 2.12 | 2.09 |
| 2 | 2.12 | 2.09 |
| 3 | 2.12 | 2.09 |
| 4 | 2.11 | 2.10 |
| 5 | 2.11 | 2.10 |
| 6 | 2.11 | 2.10 |
| 7 | 2.10 | 2.11 |
| 8 | 2.10 | 2.11 |
| 9 | 2.09 | 2.12 |
| 10 | 2.09 | 2.12 |
| 11 | 2.08 | 2.13 |
| 12 | 2.07 | 2.14 |
| 13 | 2.06 | 2.15 |
| 14 | 2.06 | 2.16 |
| 15 | 2.05 | 2.17 |
| 16 | 2.04 | 2.18 |
| 17 | 2.03 | 2.19 |
| 18 | 2.01 | 2.20 |
| 19 | 2.00 | 2.21 |
| 20 | 1.99 | 2.23 |
| 21 | 1.98 | 2.24 |
| 22 | 1.98 | 2.26 |
| 23 | 1.96 | 2.27 |
| 24 | 1.98 | 2.29 |
| 25 | 1.92 | 2.81 |
| 26 | 1.90 | 2.83 |
| 27 | 1.89 | 2.85 |
| 28 | 1.87 | 2.87 |
| 29 | 1.85 | 2.89 |
| 30 | 1.83 | 2.42 |
| 31 | 1.81 | 2.44 |
| 32 | 1.79 | 2.47 |
| 33 | 1.77 | 2.50 |
| 34 | 1.75 | 2.53 |
| 35 | 1.73 | 2.56 |
| 36 | 1.71 | 2.59 |
| 37 | 1.69 | 2.62 |
| 38 | 1.67 | 2.66 |
| 39 | 1.64 | 2.70 |
| 40 | 1.62 | 2.74 |
| 41 | 1.60 | 2.78 |
| 42 | 1.57 | 2.82 |
| 43 | 1.55 | 2.87 |
| 44 | 1.52 | 2.92 |
| 45 | 1.49 | 2.97 |
| 46 | 1.47 | 3.02 |
| 47 | 1.44 | 3.08 |
| 48 | 1.41 | 3.14 |
| 49 | 1.39 | 3.20 |
| 50 | 1.36 | 3.27 |
| 51 | 1.33 | 3.34 |
| 52 | 1.30 | 3.41 |
| 53 | 1.27 | 3.49 |
| 54 | 1.24 | 3.57 |
| 55 | 1.21 | 3.66 |
| 56 | 1.18 | 3.76 |
| 57 | 1.15 | 3.86 |
| 58 | 1.12 | 3.97 |
| 59 | 1.09 | 4.08 |
| 60 | 1.05 | 4.20 |
| 61 | 1.02 | 4.34 |
| 62 | .98 | 4.48 |
| 63 | .95 | 4.63 |
| 64 | .92 | 4.80 |
| 65 | .89 | 4.98 |
| 66 | .86 | 5.17 |
| 67 | .82 | 5.38 |
| 68 | .79 | 5.62 |
| 69 | .76 | 5.87 |
| 70 | .72 | 6.15 |
| 71 | .69 | 6.46 |
| 72 | .65 | 6.81 |

$\frac{M}{60} \times 10^6 = 7238.24$

COMPUTED BY: R.S. Smith | DATE: 27 Aug. 55 | CHECKED BY: W. C. Aumen | DATE: 27 Aug. 55

DA FORM 1 FEB 57 1927

*Figure 137. Computation of latitude and longitude equations (DA Form 1927).*

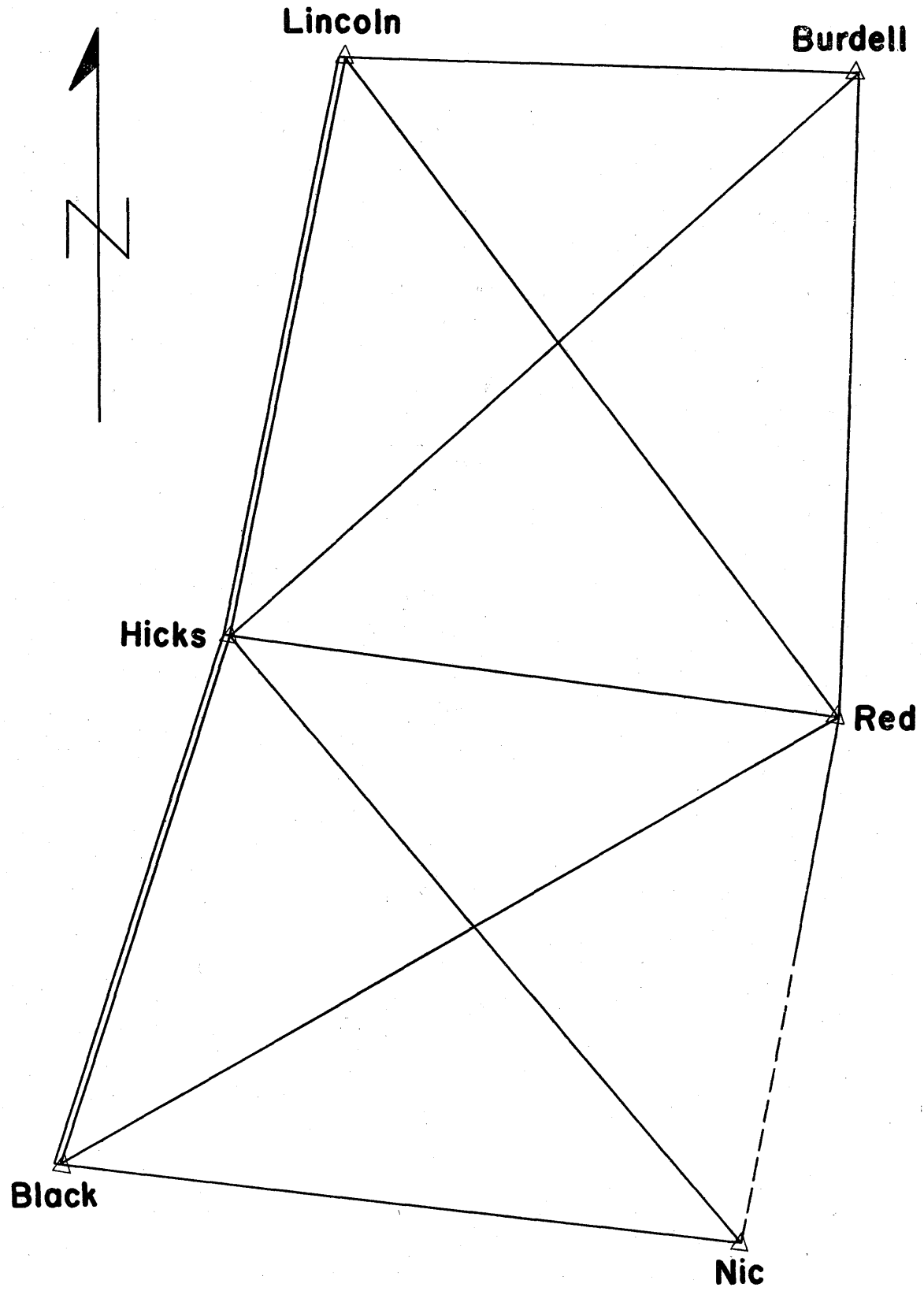



*Figure 138. Sketch of triangulation net.*

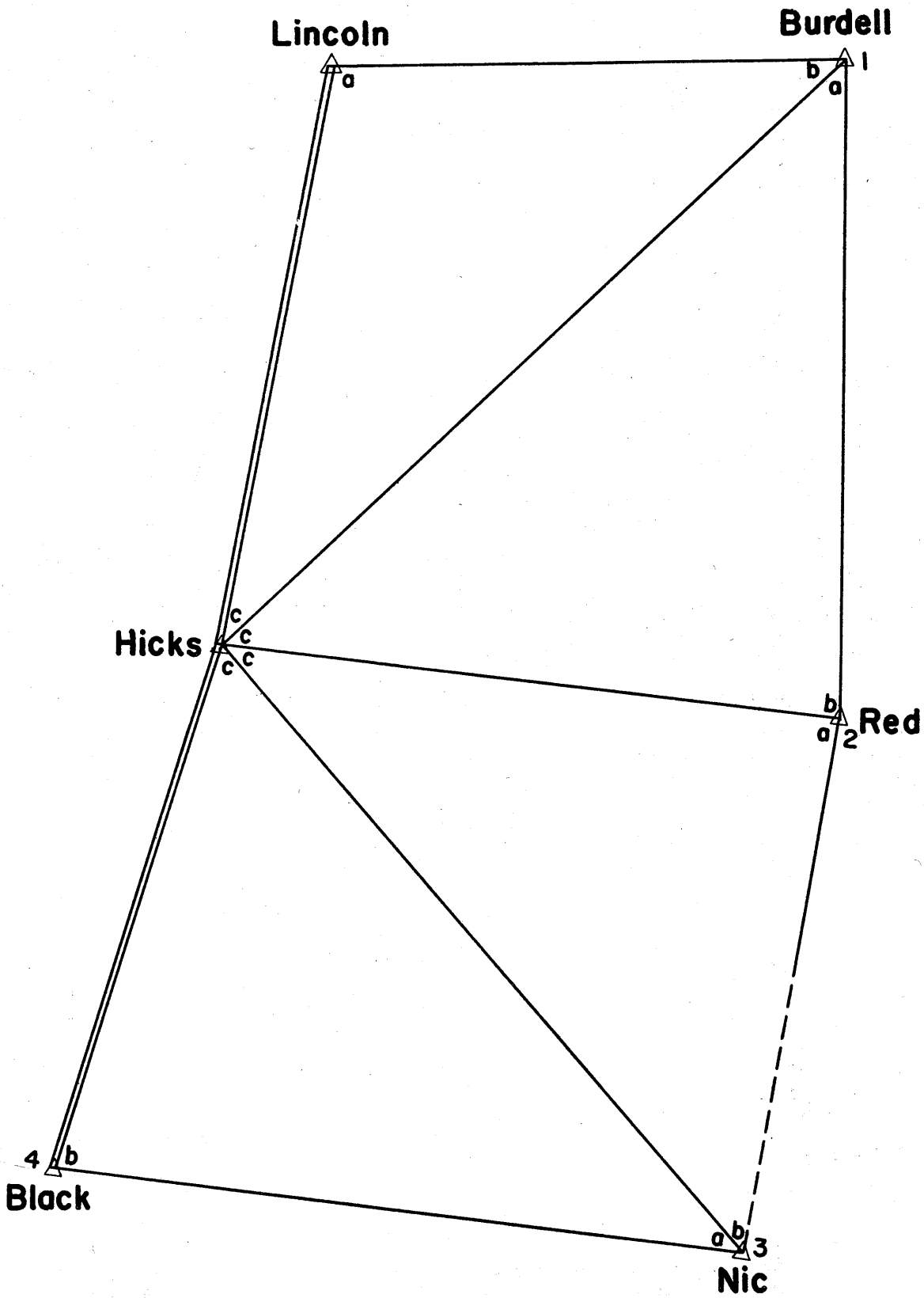



*Figure 139. Sketch of net (single triangles).*

PROJECT: I-243 | ORGANIZATION: 29th Engrs. | LIST OF DIRECTIONS (TM 5-237)

LOCATION: California | STATION: BLACK (USC&GS)

OBSERVER: Cpl. P.E. Smith | INST. (TYPE) (NO.): K&E 57460 | DATE: 22 Aug 55

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| HICKS (USC+GS) | 0 00 00.00 | —— | —— | 0 00 00.00 | 00.1 |
| Red (29th Engrs) | 51 57 43.3 | —— | —— | | 49.7 |
| Nic (29th Engrs) | 90 39 09.9 | —— | —— | | 10.2 |

LOCATION: California | STATION: Burdell (29th Engrs)

OBSERVER: Cpl. P.E. Smith | INST. (TYPE) (NO.): K&E 57460 | DATE: 22 Aug 55

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| LINCOLN (USC+GS) | 0 00 00.00 | —— | —— | 0 00 00.00 | 57.7 |
| Red (29th Engr) | 294 48 01.2 | —— | —— | | 53.9 |
| HICKS (USC+GS) | 337 10 32.9 | —— | —— | | 28.8 |

LOCATION: California | STATION: HICKS (USC+GS)

OBSERVER: Cpl. P.E. Smith | INST. (TYPE) (NO.): K&E 57460 | DATE: 22 Aug 55

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| LINCOLN (USC+GS) | 0 00 00.00 | —— | —— | 0 00 00.00 | 58.9 |
| Burdell (29th Engr) | 49 00 55.2 | —— | —— | | 55.7 |
| Red (29th Engr) | 91 29 06.2 | —— | —— | | 07.5 |
| Nic (29th Engr) | 136 21 11.3 | —— | —— | | 11.6 |
| BLACK (USC+GS) | 180 40 ~~54.5~~ 53.2 | —— | —— | Fixed by previous adjustment at 54.5 | 53.4 |

LOCATION: California | STATION: LINCOLN (USC+GS)

OBSERVER: Cpl. P.E. Smith | INST. (TYPE) (NO.): K&E 57460 | DATE: 22 Aug 55

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| HICKS (USC+GS) | 0 00 00.00 | —— | —— | 0 00 00.00 | 01.5 |
| Burdell (29th Engrs) | 251 50 28.6 | —— | —— | | 27.1 |
| Red (29th Engrs) | 302 11 14.7 | —— | —— | | 12.2 |

LOCATION: California | STATION: Nic (29th Engrs)

OBSERVER: Cpl. P.E. Smith | INST. (TYPE) (NO.): K&E 57460 | DATE: 22 Aug 55

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| BLACK (USC+GS) | 0 00 00.00 | —— | —— | 0 00 00.00 | 00.0 |
| HICKS (USC+GS) | 45 01 07.4 | —— | —— | | 08.2 |

LOCATION: California | STATION: Red (29th Engrs)

OBSERVER: Cpl. P.E. Smith | INST. (TYPE) (NO.): K&E 57460 | DATE: 22 Aug 55

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | SEA LEVEL REDUCTION* | CORRECTED DIRECTION WITH ZERO INITIAL | ADJUSTED DIRECTION* |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| HICKS (USC+GS) | 0 00 00.00 | —— | —— | 0 00 00.00 | 00.6 |
| LINCOLN (USC+GS) | 30 41 59.7 | —— | —— | | 42 02.8 |
| Burdell (29th Engrs) | 95 09 11.0 | —— | —— | | 14.1 |
| Nic (29th Engrs) | 284 17 14.0 | —— | —— | | 13.8 |
| BLACK (USC+GS) | 321 09 32.8 | —— | —— | | 36.0 |

*Figure 140. List of directions for angle method (DA Form 1917).*

adjustment. The triangles are entered on DA Form 1918 (fig. 141) as explained in the direction method. The symbols in the left-hand column are entered beside the corresponding angles as shown on the sketch. Notice that the angle at Hicks from Lincoln to Black is fixed, and the list of directions must be corrected to hold the fixed angle. Only four of the eight triangles in the complete net are written because the two diagonals were omitted. The missing triangles will be computed after the adjustment is completed. Since the net now consists of only four simple triangles, there will be only four angle equations to close these triangles, and one additional equation to preserve the fixed angle at Hicks (fig. 142). This latter equation is merely a statement that the sum of the four corrections to the four angles at Hicks must equal zero. There are no side equations, and only one length equation (fig. 143) as in the direction method.

*d.* The designation of the angle will also designate the correction to that angle. As in the direction method, the sum of the corrections to the angles in a triangle must equal the triangle closures. For example, in the triangle Burdell-Hicks-Lincoln, $(1a)+(1b)+(1c)=+6.4$, or written in the usual form, $0=-6.4+(1a)+(1b)+(1c)$. Equation 5 fixes the angle at Hicks. The length equation is formed as in the direction method. The six equations are now solved by the method of correlates (fig. 144) as were the equations in the direction method. The final adopted $v$'s in this case are the corrections to be applied to the angles. The angles are corrected on the triangle computation sheet, reduced to plane angles, and the lengths solved. Although the solution of the condition equations is much simpler in the angle method of adjustment than in the direction method, the application of the corrections to the list of directions is more complicated.

*e.* Examples of the method of correcting the directions are—

(1) *Station Lincoln.*

| *Station* | *Observed direction* | *Preliminary seconds* | *Final seconds* |
|---|---|---|---|
| Hicks | 0°00′00″.0 | 00″.0 | 01″.5 |
| Burdell | 251 50 28. 6 | 25. 6 | 27. 1 |
| Red | 302 11 14. 7 | ------- | ------- |

Triangle 1 angle Burdell to Hicks 108°09′34″.4
At Hicks correction is 0″.0
At Burdell correction is −3″.0

Total −3″.0
Average correction −1″.5

(2) *Station Burdell.*

| *Station* | *Observed direction* | *Preliminary seconds* | *Final seconds* |
|---|---|---|---|
| Lincoln | 0°00′00″.0 | 00″.0 | 57″.7 |
| Red | 294 48 01. 2 | 56. 2 | 53. 9 |
| Hicks | 337 10 32. 9 | 31. 1 | 28. 8 |

Triangle 1, angle Hicks to Lincoln 22° 49′ 28″.9 (A)
Triangle 2, angle Red to Hicks 42 22 34. 9
Angle Red to Lincoln 65 12 03. 8 (B)
At Lincoln correction is 0″.0
At Red correction is −5. 0
At Hicks correction is −1. 8

Total −6″.8
Average correction −2″.3

(3) *Station Hicks.*

| *Station* | *Observed direction* | *Preliminary seconds* | *Final seconds* |
|---|---|---|---|
| Lincoln | 0° 00′ 00″.0 | 00″.0 | 58″.9 |
| Burdell | 49 00 55. 2 | 56. 8 | 55. 7 |
| Red | 91 29 06. 2 | 08. 6 | 07. 5 |
| Nic | 136 21 11. 3 | 12. 7 | 11. 6 |
| Black | 180 40 54. 5 | 54. 5 | 53. 4 |

Triangle 1, angle Lincoln to Burdell 49° 00′ 56″.8 (A)
Triangle 2, angle Burdell to Red 42 28 11. 8
91 29 08. 6 (B)
Triangle 3, angle Red to Nic 44 52 04. 1
136 21 12. 7 (C)
Triangle 4, angle Nic to Black 44 19 41. 8
180 40 54. 5 (D)
At Lincoln correction is 0″.0
At Burdell correction is +1. 6
At Red correction is +2. 4
At Nic correction is +1. 4
At Black correction is 0. 0

Total +5″.4
Average correction +1″.1

(4) *Station Red.*

| *Station* | *Observed direction* | *Preliminary seconds* | *Final seconds* |
|---|---|---|---|
| Hicks | 0° 00 00″.0 | 00″.0 | 00″.6 |
| Lincoln | 30 41 59. 7 | ------- | ------- |
| Burdell | 95 09 11. 0 | 13. 5 | 14. 1 |
| Nic | 284 17 14. 0 | 13. 2 | 13. 8 |
| Black | 321 09 32. 8 | ------- | ------- |

Triangle 2, angle Hicks to Burdell 95° 09′ 13″.5 (A)
Triangle 3, angle Nic to Hicks 75 42 46. 8 (B)
At Hicks correction is 0″.0
At Burdell correction is +2. 5
At Nic correction is −0. 8

Total +1″.7
Average correction +0″.6

| PROJECT | | DATE | COMPUTATION OF TRIANGLES (TM 5-237) |
|---|---|---|---|
| | 1-243 | 20 Aug 55 | |
| LOCATION | California | ORGANIZATION | 29th Engrs |

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM ~~OR NATURAL~~ |
|---|---|---|---|---|---|---|---|
| | 2-3 | | | | | | 3.68755045 |
| 1b | 1 Burdell | 22 49 27.1 | +1.8 | 28.9 | 0.0 | 28.9 | 9.58873419 |
| c | 2 HICKS | 49 00 55.2 | +1.6 | 56.8 | 0.0 | 56.8 | 9.87788379 |
| a | 3 LINCOLN | 108 09 31.4 | +3.0 | 34.4 | 0.1 | 34.3 | 9.97781157 |
| | 1-3 Burdell-LINCOLN | | | | | | 3.97670005 |
| | 1-2 Burdell-HICKS | | | | | | 4.07662783 |
| | | $\overline{53.7}$ | +6.4 | | 0.1 | | |
| | 2-3 | | | | | | 4.07662783 |
| 2b | 1 Red | 95 09 11.0 | +2.5 | 13.5 | 0.0 | 13.5 | 9.99824070 |
| c | 2 HICKS | 42 28 11.0 | +0.8 | 11.8 | 0.1 | 11.7 | 9.82943437 |
| a | 3 Burdell | 42 22 31.7 | +3.2 | 34.9 | 0.1 | 34.8 | 9.82865816 |
| | 1-3 Red-Burdell | | | | | | 3.90782150 |
| | 1-2 Red-HICKS | | | | | | 3.90704529 |
| | | $\overline{53.7}$ | +6.5 | | 0.2 | | |
| | 2-3 | | | | | | 3.90704529 |
| 3b | 1 Nic | (59 25 09.0) | +0.2 | 09.2 | 0.0 | 09.2 | 9.93495914 |
| c | 2 HICKS | 44 52 05.1 | -1.0 | 04.1 | 0.0 | 04.1 | 9.84848067 |
| a | 3 Red | 75 42 46.0 | +0.8 | 46.8 | 01 | 46.7 | 9.98635588 |
| | 1-3 Nic-Red | | | | | | 3.82056682 |
| | 1-2 Nic-HICKS | | | | | | 3.95844203 |
| | | | — | | 0.1 | | |
| | 2-3 | | | | | | 3.95844203 |
| 4b | 1 BLACK | 90 39 09.9 | +0.2 | 10.1 | 0.1 | 10.0 | 9.99997181 |
| c | 2 HICKS | 44 19 43.2 | -1.4 | 41.8 | 0.0 | 41.8 | 9.84433327 |
| a | 3 Nic | 45 01 07.4 | +0.8 | 08.2 | 0.0 | 08.2 | 9.84962855 |
| | 1-3 BLACK-Nic | | | | | | 3.80280349 |
| | 1-2 BLACK-HICKS | | | | | | 3.80809877 |
| | | 00.5 | -0.4 | | -0.1 | | |

| COMPUTED BY | RAS | DATE | 23 Aug 55 | CHECKED BY | W.C.A. | DATE | 25 Aug 55 |
|---|---|---|---|---|---|---|---|

DA FORM 1 FEB 57 1918

*Figure 141. Computation of triangles for angle method (DA Form 1918).*

**Angle Equations**

① $0 = -6.4 + (1a) + (1b) + (1c)$

② $0 = -6.5 + (2a) + (2b) + (2c)$

③ $0 = 0 + (3a) + (3b) + (3c)$

④ $0 = +0.4 + (4a) + (4b) + (4c)$

⑤ $0 = (1c) + (2c) + (3c) + (4c)$

⑥ $0 = -1.28 + 0.69(1a) + 5.00(1b) - 2.31(2a) - 0.19(2b) - 0.54(3a) + 1.25(3b) - 2.10(4a) - 0.03(4b)$

*Figure 142. Condition equations for angle method.*

(5) *Station Black.*

| *Station* | *Observed direction* | *Preliminary seconds* | *Final seconds* |
|---|---|---|---|
| Hicks | 0° 00′ 00″.0 | 00″.0 | 00″.0 |
| Red | 51 57 43. 3 | | |
| Nic | 90 39 09. 9 | 10. 1 | 10. 2 |

Triangle 4, angle Hicks to Nic 90° 39′ 10″.1 (A)

At Hicks correction is 0″.0

At Nic correction is +0. 2

Total +0″.2

Average correction +0″.1

(6) *Station Nic.*

| *Station* | *Observed direction* | *Preliminary seconds* | *Final seconds* |
|---|---|---|---|
| Black | 0° 00′ 00″.0 | 00″.0 | 00″.0 |
| Hicks | 45 01 07. 4 | 08. 2 | 08. 2 |

*f.* The preliminary seconds are obtained by using the adjusted spherical angle from the triangle computation sheet. A direction which was not used to obtain an angle for a triangle used in the adjustment will not enter into this computation. These directions will be corrected later. The difference between the observed direction and the preliminary seconds is listed for each direction at which it occurs. The sum of these differences is divided by the number of directions involved (including the initial direction), giving the average correction per direction. The sign of the difference must be such as to change the observed direction to the preliminary seconds. The average correction is applied to the preliminary seconds to obtain the final seconds.

*g.* The final computation in the adjustment is the solution for the lengths of the diagonals omitted from the net when the adjustment was begun. This computation is best performed on DA Form 1919 (Triangle Computation Using Two Sides and Included Angle) (fig. 145). After

PROJECT 1-243 — ~~SIDE~~/LENGTH EQUATIONS (TM 5-237)

LOCATION California

ORGANIZATION 29th Engrs. — DATE 20 Aug 55

| Symbol | Angle | Log. Sine | Tab. Diff. | Symbol | Angle | Log. Sine | Tab. Diff. |
|---|---|---|---|---|---|---|---|
| | HICKS-BLACK | $3.8080991^{-8}$ | | | HICKS-LINCOLN | $3.68755045^{-4}$ | |
| 1b | 22 49 27.1 | 9.58872519 | +5.00 | 1a | 108 09 31.4 | 9.97781357 | -0.69 |
| 2b | 95 09 11.0 | 9.99824117 | -0.19 | 2a | 42 22 31.7 | 9.82865101 | +2.31 |
| 3b | 59 25 09.0 | 9.93495889 | +1.25 | 3a | 75 42 46.0 | 9.98635550 | +0.54 |
| 4b | 90 39 09.9 | 9.99997181 | -0.03 | 4a | 45 01 07.4 | 9.84962687 | +2.10 |
| | | 3.32999608 | | | | 3.32999736 | |
| | | 736 | | | | | |
| | | - 1.28 | | | | | |
| $0 = -1.28 + 0.69(1a) + 5.00(1b) - 2.31(2a) - 0.19(2b) - 0.54(3a) + 1.25(3b) - 2.10(4a) - 0.03(4b)$ | | | | | | | |

*Figure 143. Length equation for angle method (DA Form 1926).*

## Correlate Equations

| | 1 | 2 | 3 | 4 | 5 | $6^{\ell}$ | $\Sigma_c$ | $\nu$ | Adopt. $\nu$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1a | +1 | | | | | +0.69 | +1.69 | +3.03370 | +3.0 | 1a |
| b | +1 | | | | | +5.00 | +6.00 | +1.75613 | +1.8 | b |
| c | +1 | | | | +1 | | +2 | +1.61015 | +1.6 | c |
| 2a | | +1 | | | | -2.31 | -1.31 | +3.14708 | +3.2 | 2a |
| b | | +1 | | | | -0.19 | +0.81 | +2.51867 | +2.5 | b |
| c | | +1 | | | +1 | | +2 | +0.83427 | +0.8 | c |
| 3a | | | +1 | | | -0.54 | +0.46 | +0.77292 | +0.8 | 3a |
| b | | | +1 | | | +1.25 | +2.25 | +0.24232 | +0.2 | b |
| c | | | +1 | | +1 | | +2 | -1.01523 | -1.0 | c |
| 4a | | | | +1 | | -2.10 | -1.10 | +0.82138 | +0.8 | 4a |
| b | | | | +1 | | -0.03 | +0.97 | +0.20779 | +0.2 | b |
| c | | | | +1 | +1 | | +2 | -1.42918 | -1.4 | c |

## Normal Equations

| | 1 | 2 | 3 | 4 | 5 | 6 | n | $\Sigma_n$ | C | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | +3 | — | — | — | +1 | +5.69 | -6.4 | +3.29 | +3.23823 | 1 |
| 2 | | +3 | — | — | +1 | -2.50 | -6.5 | -5.00 | +2.46235 | 2 |
| 3 | | | +3 | — | +1 | +0.71 | 0 | +4.71 | +0.61285 | 3 |
| 4 | | | | +3 | +1 | -2.13 | +0.4 | +2.27 | +0.19890 | 4 |
| 5 | | | | | +4 | — | 0 | +8.0 | -1.62808 | 5 |
| 6 | | | | | | +37.1133 | -1.28 | +37.6033 | -0.29642 | 6 |

## Forward Solution

| 1 | 2 | 3 | 4 | 5 | 6 | n | $\Sigma_n$ |
|---|---|---|---|---|---|---|---|
| +3 | — | — | — | +1 | +5.69 | -6.4 | +3.29 |
| $C_1$ | — | — | — | -0.33333 | -1.89667 | +2.13333 | -1.09667 |
| | +3 | — | — | +1 | -2.50 | -6.5 | -5.00 |
| | $C_2$ | — | — | -0.33333 | +0.83333 | +2.16667 | +1.66667 |
| | | +3 | — | +1 | +0.71 | 0 | +4.71 |
| | | $C_3$ | — | -0.33333 | -0.23667 | — | -1.57000 |
| | | | +3 | +1 | -2.13 | +0.4 | +2.27 |
| | | | $C_4$ | -0.33333 | +0.71000 | -0.13333 | -0.75667 |
| | | | | +4 | — | 0 | +8.0 |
| | | | | +2.66667 | -0.59000 | +4.16667 | +6.24334 |
| | | | | $C_5$ | +0.22125 | -1.56250 | -2.34125 |
| | | | | | +37.1133 | -1.28 | +37.6033 |
| | | | | | +22.42704 | +6.64790 | +29.07494 |
| | | | | | $C_6$ | -0.29642 | -1.29642 |

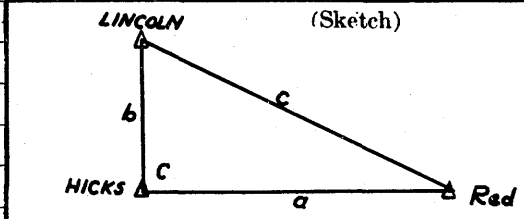
*Figure 144. Solution of equations.*

| PROJECT I-243 | TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE (TM 5-237) | |
|---|---|---|
| LOCATION California | ORGANIZATION 29th Engr. | DATE 19 Aug 55 |

$\left[\frac{a}{b}=\tan(45°+\phi)\right.$ (Call longer side $a$). $\tan\frac{1}{2}(A_p-B_p)=\tan\phi\tan\frac{1}{2}(A_p+B_p)$: $\left.c=\frac{a\sin C_p}{\sin A_p}\right]^*$

| | | | | | |
|---|---|---|---|---|---|
| $C_s$ | 91 29 08.6 | Log $a$ | 3.90704529 | Log $m$ | |
| Sph. excess / 3 | 0.1 | Log $b$ | 3.68755045 | Log sin $C_s$ | |
| $C_p$ | 91 29 08.5 | Log tan $(45°+\phi)$ | 0.21949484 | Log $a$ | |
| $\frac{1}{2}C_p$ | 45 44 34.25 | $(45°+\phi)$ | 58 53 56.626 | Log $b$ | |
| $90°-\frac{1}{2}C_p=\frac{1}{2}(A_p+B_p)$ | 44 15 25.75 | $\phi$ | 13 53 56.626 | Log sph. ex. | |
| $\frac{1}{2}(A_p-B_p)$ | 13 33 23.56 | Log tan $\phi$ | 9.39350083 | Sph. excess | 0.1 |
| Sum $=A_p$ | 57 48 49.31 | Log tan $\frac{1}{2}(A_p+B_p)$ | 9.98873737 | | |
| Diff $=B_p$ | 30 42 02.19 | Log tan $\frac{1}{2}(A_p-B_p)$ | 9.38223820 | | |

| | |
|---|---|
| $C_p$ | 91 29 08.5 |
| | |
| Log $a$ | 3.90704529 |
| Log sin $C_p$ | 9.99985398 |
| Colog sin $A_p$ | 0.07246512 |
| Log $c$ | 3.97936439 |




CHECK COMPUTATION

| No. | STATION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | LOGARITHM |
|---|---|---|---|---|---|
| | 2-3 HICKS - LINCOLN | | | | 3.68755045 |
| | 1 Red | 30 42 02.2 | | 02.2 | 9.70804006 |
| | 2 HICKS | 91 29 08.6 | 0.1 | 08.5 | 9.99985398 |
| | 3 LINCOLN | 57 48 49.3 | | 49.3 | 9.92753487 |
| | 1-3 Red - LINCOLN | | | | 3.97936437 |
| | 1-2 Red - HICKS | | | | 3.90704526 |
| | | | 0.1 | | |
| | 2-3 LINCOLN - Burdell | | | | 3.97670005 |
| | 1 Red | 64 27 11.3 | 0.1 | 11.2 | 9.95531855 |
| | 2 LINCOLN | 50 20 45.1 | | 45.1 | 9.88644030 |
| | 3 Burdell | 65 12 03.8 | 0.1 | 03.7 | 9.95798301 |
| | 1-3 Red - Burdell | | | | 3.90782180 |
| | 1-2 Red - LINCOLN | | | | 3.97936451 |
| | | | 0.2 | | |

*The subscripts $s$ and $p$ on this form refer to spherical and plane angles respectively.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 26 Aug 55 | W. C. A. | 28 Aug 55 |

DA FORM 1 FEB 57 1919

*Figure 145. Solution of triangles for angle method (DA Form 1919).*

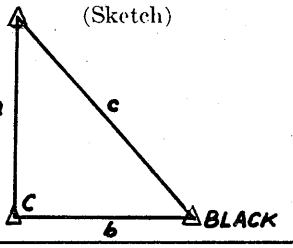
| PROJECT 1-243 | TRIANGLE COMPUTATION USING TWO SIDES AND INCLUDED ANGLE (TM 5-237) | |
|---|---|---|
| LOCATION California | ORGANIZATION 29th Engr. | DATE 19 Aug 55 |

$\left[\frac{a}{b}=\tan(45°+\phi)\right.$ (Call longer side $a$) $\tan\frac{1}{2}(A_p-B_p)=\tan\phi\tan\frac{1}{2}(A_p+B_p)$: $\left.c=\frac{a\sin C_p}{\sin A_p}\right]^*$

| | | | | | |
|---|---|---|---|---|---|
| $C_s$ | 89° 11' 45.9" | Log $a$ | 3.90704529 | Log $m$ | |
| Sph. excess / 3 | 0.1 | Log $b$ | 3.8080991 | Log sin $C_s$ | |
| $C_p$ | 89 11 45.8 | Log tan $(45°+\phi)$ | 0.09894619 | Log $a$ | |
| $\frac{1}{2}C_p$ | 44 35 52.9 | $(45°+\phi)$ | 51° 28' 16."185 | Log $b$ | |
| $90°-\frac{1}{2}C_p=\frac{1}{2}(A_p+B_p)$ | 45 24 07.1 | $\phi$ | 06 28 16.185 | Log sph. ex. | |
| $\frac{1}{2}(A_p-B_p)$ | 06 33 42.53 | Log tan $\phi$ | 9.05471183 | Sph. excess | 0.01 |
| Sum $=A_p$ | 51 57 49.63 | Log tan $\frac{1}{2}(A_p+B_p)$ | 0.00609399 | | |
| Diff $=B_p$ | 38 50 24.57 | Log tan $\frac{1}{2}(A_p-B_p)$ | 9.06080582 | | |
| $C_p$ | 89 11 45.8 | | | | |

| | |
|---|---|
| Log $a$ | 3.90704529 |
| Log sin $C_p$ | 9.99995724 |
| Colog sin $A_p$ | 0.10368246 |
| Log $c$ | 4.01068499 |

(Sketch)




CHECK COMPUTATION

| No. | STATION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | LOGARITHM |
|---|---|---|---|---|---|
| | 2-3 | | | | 3.8080991 |
| | 1 Red | 38 50 24.6 | 0.0 | 24.6 | 9.79737143 |
| | 2 BLACK | 51 57 49.6 | 0.0 | 49.6 | 9.89631749 |
| | 3 HICKS | 89 11 45.9 | 0.1 | 45.8 | 9.99995724 |
| | 1-3 Red - HICKS | | | | 3.90704516 |
| | 1-2 Red - BLACK | | | | 4.01068491 |
| | | | 0.1 | | |
| | 2-3 | | | | 3.80280349 |
| | 1 Red | 36 52 22.2 | 0.0 | 22.2 | 9.77818092 |
| | 2 Nic | 104 26 17.4 | 0.1 | 17.3 | 9.98606259 |
| | 3 BLACK | 38 41 20.5 | 0.0 | 20.5 | 9.79594476 |
| | 1-3 Red - BLACK | | | | 4.01068516 |
| | 1-2 Red - Nic | | | | 3.82056733 |
| | | | 0.1 | | |

*The subscripts $s$ and $p$ on this form refer to spherical and plane angles respectively.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 21 Aug 55 | W. C. A. | 25 Aug. 55 |

DA FORM 1 FEB 57 1919

*Figure 145*—Continued.

these computations are made, any missing angles can be obtained by combining angles fixed by the adjustment and computed angles. For example, computing the triangle Red-Hicks-Lincoln to obtain the diagonal Lincoln-Red, angle C at Hicks is found by adding the adjusted angles (1*c*) and (2*c*). The side Hicks-Lincoln is the original fixed length, and the side Hicks-Red is taken from triangle 2 of the adjustment. Solution of the triangle on DA Form 1919 gives the length of the diagonal Lincoln-Red plus the angles at Lincoln and Red. The check computation on DA Form 1919 is made by solving the triangle Red-Hicks-Lincoln by the sine law using the computed angles at Red and Lincoln. The second triangle in the check is Red-Lincoln-Burdell, the fourth triangle in the quadrilateral. In this triangle the length Lincoln-Burdell is taken from triangle 1 of the adjustment. The angle at Burdell is the sum of angles (1*b*) and (2*a*) of the adjustment. The angle at Red is found by subtracting the computed angle Lincoln-Red-Hicks from the adjusted angle (2*b*). Similarly, the angle at Lincoln is obtained by subtracting the computed angle Red-Lincoln-Hicks from the adjusted angle (1*a*).

*h.* The list of directions can now be completed by adding the angles from the check computations to the directions previously corrected.

*i.* Position computations are now made as in the direction method, using the adjusted angles and lengths (fig. 146).

PROJECT I-243 — POSITION COMPUTATION, 3rd ORDER TRIANGULATION (For calculating machine computation) (TM 5-237)

LOCATION California — ORGANIZATION 29th Engrs. — DATE 19 Aug 55

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 HICKS | To 3 LINCOLN | | | 212 | 00 | 51.34 | α | 3 LINCOLN | To 2 HICKS | | | 32 | 01 | 56.85 |
| 2d ∠ | & | | | | +49 | 00 | 56.8 | 3d ∠ | & | | | | 108 | 09 | 34.4 |
| α | 2 HICKS | To 1 Burdell | | | 261 | 01 | 48.14 | α | 3 LINCOLN | To 1 Burdell | | | 283 | 52 | 22.45 |
| Δα | | | | | + | 04 | 58.86 | Δα | | | | | + | 03 | 53.45 |
| | | | | | 180 | 00 | 00.00 | | | | | | 180 | 00 | 00.00 |
| α′ | 1 Burdell | To 2 HICKS | | | 81 | 06 | 47.00 | α′ | 1 Burdell | To 3 LINCOLN | | | 103 | 56 | 15.90 |
| f = 8.2070 | | First Angle of Triangle | | | 22 | 49 | 28.9 | f = 8.2070 | | | | | | | |
| | ° | ′ | ″ | | ° | ′ | ″ | | ° | ′ | ″ | | ° | ′ | ″ |
| φ | 38 | 07 | 46.259 | 2 HICKS | λ | 122 | 43 | 29.366 | φ | 38 | 10 | 00.181 | 3 LINCOLN | λ | 122 | 41 | 43.305 |
| | | | | s = 11929.654 | Δλ | − | 08 | 03.933 | | | | | s = 9477.636 | Δλ | − | 06 | 17.872 |
| φ′ | 38 | 08 | 46.309 | 1 Burdell | λ′ | 122 | 35 | 25.433 | φ′ | 38 | 08 | 46.309 | 1 Burdell | λ′ | 122 | 35 | 25.433 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Δφ | 01 00.05 | (log s = | 4.07662783 | Δφ | 01 13.87 | (log s = | 3.97670005 | |
| sin α | −0.98777022 | b = (y/10,000)² | 0.035 ) | sin α | −0.97082999 | b = (y/10,000)² | 0.052 ) | |
| cos α | −0.15591662 | x cor. = −½fb | 0.144 | cos α | +0.23976893 | x cor. = −½fb | 0.213 | |
| x = s sin α | −11783.7570 | x′ | −11783.7568 | x = s sin α | −9201.1733 | x′ | −9201.1731 | |
| y = −s cos α | +1860.0313 | H | 0.041067758 | y = −s cos α | −2272.4426 | H | 0.041067758 | |
| a = (x′/10,000)² | 1.3886 | Hx′ = (approx. Δλ″) | −483.9325 | a = (x′/10,000)² | 0.8466 | Hx′ = (approx. Δλ″) | −377.8716 | |
| y cor. = +fa | 11.40 | Arc−sin cor = +V(Va)/15 | 3.50 | y cor. = +fa | 6.95 | Arc−sin cor = +V(Va)/15 | 2.13 | |
| $y_o$ | 4 221 667.975 | Δλ″ | −483.933 | $y_o$ | 4 225 797.121 | Δλ″ | −377.872 | |
| y′ | +1860.033 | sin φ | 0.61744118 | y′ | −2272.444 | sin φ | 0.61795179 | |
| $y_1$ | 4 223 528.008 | sin φ′ | 0.61767017 | $y_1$ | 4 223 524.677 | sin φ′ | 0.61767017 | |
| Va − | 8.538 | 1 + cos Δφ | 1.99999996 | Va − | 5.206 | 1 + cos Δφ | 1.99999994 | |
| $y_2$ | 4 223 519.470 | (sin φ + sin φ′)/(1 + cos Δφ) or sin t = | 0.61755569 | $y_2$ | 4 223 519.471 | (sin φ + sin φ′)/(1 + cos Δφ) or sin t = | 0.61781100 | |
| V | 6.14898 | −Δα″ (approx.) | −298.856 | V | 6.14898 | −Δα″ (approx.) | −233.453 | |
| K (Va/1,000)² + | | + F(Δλ″)³ | | K (Va/1,000)² + | | + F(Δλ″)³ | | |
| | | −Δα″ | −298.856 | | | −Δα″ | −233.453 | |

COMPUTED BY RAS — DATE 31 Aug 55 — CHECKED BY W.C.A. — DATE 23 Aug 55

NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1 FEB 57 1923

*Figure 146. Position computations for angle method (DA Form 1923).*

PROJECT 1-243 | POSITION COMPUTATION, 3 Ch ORDER TRIANGULATION (For calculating machine computation) (TM 5-237)

LOCATION California | ORGANIZATION 29th Engrs. | DATE 19 Aug 55

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| α | 2 HICKS | To 3 LINCOLN | | | 212 | 00 | 51.34 | α | 3 LINCOLN | To 2 HICKS | | | 32 | 01 | 56.85 |
| 2d ∠ | & | | | | +91 | 29 | 08.6 | 3d ∠ | & | | | | -57 | 48 | 49.3 |
| α | 2 HICKS | To 1 Red | | | 303 | 29 | 59.94 | α | 3 LINCOLN | To 1 Red | | | 334 | 13 | 07.55 |
| Δα | | | | | + | 02 | 50.50 | Δα | | | | | + | 01 | 45.08 |
| | | | | | 180 | 00 | 00.00 | | | | | | 180 | 00 | 00.00 |
| α′ | 1 Red | To 2 HICKS | | | 123 | 32 | 50.44 | α′ | 1 Red | To 3 LINCOLN | | | 154 | 14 | 52.63 |
| f = 8.2070 | | First Angle of Triangle | | | 30 | 42 | 02.2 | f = 8.2070 | | | | | | | |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 38 | 07 | 46.259 | 2 HICKS | λ | 122 | 43 | 29.366 | φ | 38 | 10 | 00.181 | 3 LINCOLN | λ | 122 | 41 | 43.305 |
| | | | | s = 8073.192 | Δλ | — | 04 | 36.259 | | | | | s = 9535.959 | Δλ | — | 02 | 50.198 |
| φ′ | 38 | 05 | 21.648 | 1 Red | λ′ | 122 | 38 | 53.107 | φ′ | 38 | 05 | 21.648 | 1 Red | λ′ | 122 | 38 | 53.107 |

| | | | |
|---|---|---|---|
| Δφ | 02 24.61 | Δφ | 04 38.53 |
| sin α | -0.83388598 | sin α | -0.43493623 |
| cos α | +0.55193675 | cos α | +0.90046126 |
| x = s sin α | -6732.1216 | x = s sin α | -4147.5341 |
| y = -s cos α | -4455.8914 | y = -s cos α | -8586.7617 |
| a = (x′/10,000)² | 0.4532 | a = (x′/10,000)² | 0.1720 |
| y cor. = +fa | 3.72 | y cor. = +fa | 1.41 |
| $y_o$ | 4 221 667.975 | $y_o$ | 4 225 797.121 |
| y′ | - 4 455.893 | y′ | -8586.763 |
| $y_1$ | 4 217 212.082 | $y_1$ | 4 217 210.358 |
| Va - | 2.781 | Va - | 1.055 |
| $y_2$ | 4 217 209.301 | $y_2$ | 4 217 209.303 |
| V | 6.13644 | V | 6.13644 |
| K (Va/1,000)² + | | K (Va/1,000)² + | |

| | | | |
|---|---|---|---|
| (log s = b = (y/10,000)² | 3.90704526 0.199 ) | (log s = b = (y/10,000)² | 3.97936437 0.737 ) |
| x cor. = -½fb | 0.817 | x cor. = -½fb | 3.024 |
| x′ | -6732.1210 | x′ | -4147.5328 |
| H | 0.041035935 | H | 0.041035935 |
| Hx′ = (approx. Δλ″) | -276.2589 | Hx′ = (approx. Δλ″) | -170.1979 |
| Arc-sin cor = +V (Va)/15 | 1.14 | Arc-sin cor = +V (Va)/15 | 0.43 |
| Δλ″ | -276.259 | Δλ″ | -170.198 |
| sin φ | 0.61744118 | sin φ | 0.61795179 |
| sin φ′ | 0.61688954 | sin φ′ | 0.61688954 |
| 1 + cos Δφ | 1.99999975 | 1 + cos Δφ | 1.99999909 |
| (sin φ + sin φ′)/(1 + cos Δφ) or sin φ = | 0.61716544 | (sin φ + sin φ′)/(1 + cos Δφ) or sin φ = | 0.61742095 |
| -Δα″ (approx.) | -170.498 | -Δα″ (approx.) | -105.084 |
| + F (Δλ″)³ | | + F (Δλ″)³ | |
| = -Δα″ | -170.498 | = -Δα″ | -105.084 |

COMPUTED BY RAS | DATE 31 Aug 55 | CHECKED BY W.C.A. | DATE 23 Aug 55 | NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1 FEB 57 1923

*Figure 146*—Continued.

PROJECT 1-243

**POSITION COMPUTATION, 3 CS ORDER TRIANGULATION** (For calculating machine computation)
(TM 5-237)

LOCATION California

ORGANIZATION 29th Engrs.

DATE 19 Aug 55

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 2 BLACK | To 3 HICKS | | | 212 | 40 | 17.91 | $\alpha$ | 3 HICKS | To 2 BLACK | | | 32 | 41 | 45.84 |
| 2d ∠ | & | | | | +90 | 39 | 10.1 | 3d ∠ | & | | | | −44 | 19 | 41.8 |
| $\alpha$ | 2 BLACK | To 1 Nic | | | 303 | 19 | 28.01 | $\alpha$ | 3 HICKS | To 1 Nic | | | 348 | 22 | 04.04 |
| $\Delta\alpha$ | | | | | + | 02 | 14.18 | $\Delta\alpha$ | | | | | + | 00 | 46.36 |
| | | | | | 180 | 00 | 00.00 | | | | | | 180 | 00 | 00.00 |
| $\alpha'$ | 1 Nic | To 2 BLACK | | | 123 | 21 | 42.19 | $\alpha'$ | 1 Nic | To 3 HICKS | | | 168 | 22 | 50.40 |
| | f= 8.2071 | First Angle of Triangle | | | 45 | 01 | 08.2 | | f= 8.2070 | | | | | | |
| $\phi$ | 38 | 04 | 50.777 | 2 BLACK | $\lambda$ | 122 | 45 | 51.846 | $\phi$ | 38 | 07 | 46.259 | 3 HICKS | $\lambda$ | 122 | 43 | 29.366 |
| | | | | s= 6350.435 | $\Delta\lambda$ | − | 03 | 37.628 | | | | | s= 9087.450 | $\Delta\lambda$ | − | 01 | 15.149 |
| $\phi'$ | 38 | 02 | 57.566 | 1 Nic | $\lambda'$ | 122 | 42 | 14.218 | $\phi'$ | 38 | 02 | 57.566 | 1 Nic | $\lambda'$ | 122 | 42 | 14.217 |

| | | | |
|---|---|---|---|
| $\Delta\phi$ | 01 53.21 | $\Delta\phi$ | 04 48.69 |
| sin $\alpha$ | −0.83557302 | sin $\alpha$ | −0.20162860 |
| cos $\alpha$ | +0.54937939 | cos $\alpha$ | +0.97946205 |
| $x = s \sin \alpha$ | −5306.2522 | $x = s \sin \alpha$ | −1832.2898 |
| $y = -s \cos \alpha$ | −3488.7981 | $y = -s \cos \alpha$ | −8900.8124 |
| $a = (x'/10{,}000)^2$ | 0.2816 | $a = (x'/10{,}000)^2$ | 0.0336 |
| y cor. = +fa | 2.31 | y cor. = +fa | 0.28 |
| $y_o$ | 4 216 257.470 | $y_o$ | 4 221 667.975 |
| $y'$ | −3 488.799 | $y'$ | −8 900.813 |
| $y_1$ | 4 212 768.671 | $y_1$ | 4 212 767.162 |
| Va − | 1.726 | Va − | 0.206 |
| $y_2$ | 4 212 766.945 | $y_2$ | 4 212 766.956 |
| V | 6.12763 | V | 6.12763 |
| K (Va/1,000)$^2$ + | ——— | K (Va/1,000)$^2$ + | ——— |
| COMPUTED BY RAS | DATE 31 Aug 55 | DATE 31 Aug 55 | |

| | | | |
|---|---|---|---|
| (log s = b = (y/10,000)$^2$ 3.80280849) | 0.122 | (log s = b = (y/10,000)$^2$ 3.95844303) | 0.792 |
| x cor. = $-\frac{1}{2}$fb | 0.501 | x cor. = $-\frac{1}{2}$fb | 3.250 |
| $x'$ | −5306.2519 | $x'$ | −1832.2892 |
| H | 0.041013584 | H | 0.041013584 |
| Hx' = (approx. $\Delta\lambda''$) | −217.6284 | Hx' = (approx. $\Delta\lambda''$) | −75.1487 |
| Arc − sin cor = $+\frac{V\,(Va)}{15}$ | 0.71 | Arc − sin cor = $+\frac{V\,(Va)}{15}$ | 0.08 |
| $\frac{\Delta\lambda''}{}$ | −217.628 | $\frac{\Delta\lambda''}{}$ | −75.149 |
| sin $\phi$ | 0.61677174 | sin $\phi$ | 0.61744118 |
| sin $\phi'$ | 0.61633962 | sin $\phi'$ | 0.61633962 |
| 1 + cos $\Delta\phi$ | 1.99999985 | 1 + cos $\Delta\phi$ | 1.99999902 |
| $\frac{\sin\phi + \sin\phi'}{1 + \cos\Delta\phi}$ or sin $\phi$ | −0.61655573 | $\frac{\sin\phi + \sin\phi'}{1 + \cos\Delta\phi}$ or sin $\phi$ | −0.61689070 |
| $-\Delta\alpha''$ (approx.) | −134.180 | $-\Delta\alpha''$ (approx.) | −46.359 |
| + F $(\Delta\lambda'')^3$ | ——— | + F $(\Delta\lambda'')^3$ | ——— |
| $-\Delta\alpha''$ | −134.180 | $-\Delta\alpha''$ | −46.359 |
| CHECKED BY W. C. A. | | DATE 31 Aug 55 | |

NOTE: For s under 8,000 meters omit terms under the heavy black line not in heavy bold type or underlined.

DA FORM 1 FEB 57 1923

*Figure 146*—Continued.

# Section V. SPECIAL PROBLEMS

## 77. Description

*a.* Many of the special problems encountered in triangulation are connected with the location of additional points which are not normally considered to be part of the main scheme of triangulation. The methods most commonly used for locating additional points are intersection, three point (resection), inaccessible base, and special angle.

*b.* In these four methods, at least some of the directions have not been measured. At least one of the angles in some of the triangles must be concluded (deduced from known information) and the best results will be obtained if the concluded value approaches as closely as possible the probable measured value. In many cases, this may be accomplished by subtracting the sum of the two measured angles from 180° plus the spherical excess for the triangle. Each figure must be studied thoroughly before making a decision on this matter.

## 78. Intersection Station Problem

*a.* Intersection is the process of locating a point by observed directions from two or more stations of known position. This method is used in locating additional control points or prominent objects during the process of completing a survey.

*b.* In figure 147, stations Jones, Howard, and Kelley form the triangle for which the directions and lengths were previously determined. Station Spire is the intersected point. Directions have been measured from all three established stations to station Spire.

*c.* In this problem, there is one geometric condition to be met. The three adjusted directions must intersect at a common point, in this case, station Spire. A condition equation in the form of a length equation or a side equation with the pole at Spire may be used. In the sample adjustment, a length equation is used. The center line, which is the common side of two of the concluded triangles, is designated direction number 3; the line from the known station on the right is designated direction number 2; and, the line from the known station on the left is designated direction number 1. This numbering system is arbitrary.

*d.* On figure 148 is shown the length equation and its solution. The directions designated have been previously explained, the concluded angle carries the combined designation of the other two angles in the triangle, but with opposite sign. The length equation is set up from fixed line JONES-KELLEY to fixed line KELLEY-HOWARD. The equation is solved and the $v$'s are computed.

*e.* Figure 149 illustrates the computation of the triangles.

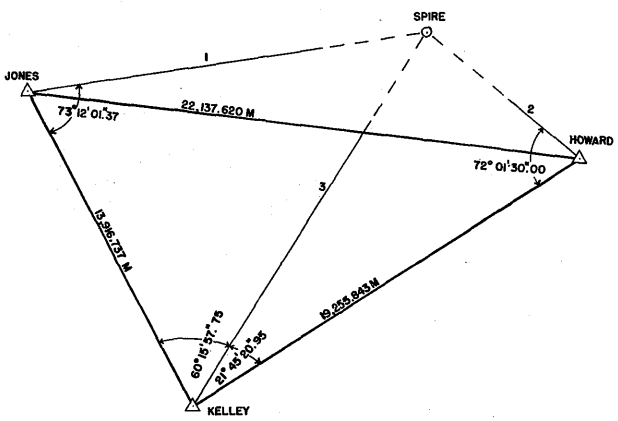



*Figure 147. Diagram of intersection station problem.*

PROJECT: TEST

~~SIDE~~/LENGTH EQUATIONS (TM 5-237)

LOCATION: C. A.

ORGANIZATION: T. O.

DATE: OCT. 1962

| SYMBOL | ANGLE | LOG. SINE | TAB. DIFF. | SYMBOL | ANGLE | LOG. SINE | TAB. DIFF. |
|---|---|---|---|---|---|---|---|
| JONES - KELLEY | | | | KELLEY - HOWARD | | | |
| | 13916.737 M | 4.14353742 | | | 19255.843 M | 4.28456254 | |
| -1 | 73 12 01.37 | 9.98105778 | +0.64 | +1 -3 | 46 32 01.45 | 9.86080473 | +2.00 |
| -2+3 | 86 13 09.38 | 9.99905381 | +0.14 | +2 | 72 01 30.00 | 9.97826785 | +0.68 |
| | ARC SINE | - 34 | | | ARC SINE | - 66 | |
| | | 4.12364867 | | | | 4.12363446 | |
| | | 3446 | | | | | |
| | | + 1421 | | | | | |

$O = +14.21 - 2.64(1) - 0.82(2) + 2.14(3)$

| | DIRECTION | COEFFICIENT | | COEFFICIENT$^2$ | V |
|---|---|---|---|---|---|
| | 1 | -2.64 | | 6.9696 | +3.07 |
| | 2 | -0.82 | | 0.6724 | +0.95 |
| | 3 | +2.14 | | 4.5796 | -2.49 |
| | | | | 12.2216 | |

SOLUTION OF NORMALS

$O = +14.21 + 12.2216c$

$12.2216c = -14.21$

$c = -1.16270$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| J.A. Andrews - AMS | FEB. '64 | G.T. Tennis - AMS | FEB. '64 |

DA FORM 1 FEB 57 1926

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420724

*Figure 148. Least squares adjustment of intersection station.*

| PROJECT | DATE | COMPUTATION OF TRIANGLES (TM 5-237) |
|---|---|---|
| TEST | OCT. 1962 | |

| LOCATION | ORGANIZATION |
|---|---|
| C. A. | T. O. |

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM OR NATURAL |
|---|---|---|---|---|---|---|---|
| | 2-3 | | | | | | 22137.620 |
| | 1 KELLEY | 82 01 18.70 | | | | | |
| | 2 JONES | 59 28 29.32 | FIXED | | | | |
| | 3 HOWARD | 38 30 12.66 | | | | | |
| | 1-3 | | | | | | 19255.843 |
| | 1-2 | | | | | | 13916.737 |
| | | 00.68 | | | 00.68 | | |
| | 2-3 | | | | | | 22137.620 |
| (+1-2) | 1 SPIRE | (132 45 10.83) | + 2.12 | 12.95 | -0.08 | 12.87 | .734 28016 |
| +2 | 2 HOWARD | 33 31 17.34 | + 0.95 | 18.29 | -0.07 | 18.22 | .552 25317 |
| -1 | 3 JONES | 13 43 32.05 | - 3.07 | 28.98 | -0.07 | 28.91 | .237 25691 |
| | 1-3 | | | | | | 16649.736 |
| | 1-2 | | | | | | 7152.996 |
| | | 00.22 | 00.00 | 60.22 | 00.22 | 00.00 | |
| | 2-3 | | | | | | 19255.843 |
| (-2+3) | 1 SPIRE | (86 13 09.38) | -3.44 | 05.94 | -0.11 | 05.83 | .997 82257 |
| +2 | 2 HOWARD | 72 01 30.00 | +0.95 | 30.95 | -0.11 | 30.84 | .951 19252 |
| -3 | 3 KELLEY | 21 45 20.95 | +2.49 | 23.44 | -0.11 | 23.33 | .370 66249 |
| | 1-3 | | | | | | 18355.983 |
| | 1-2 | | | | | | 7152.994 |
| | | 00.33 | 00.00 | 60.33 | 00.33 | 00.00 | |
| | 2-3 | | | | | | 13916.737 |
| (+1-3) | 1 SPIRE | (46 32 01.45) | +5.56 | 07.01 | -0.19 | 06.82 | .725 79746 |
| +3 | 2 KELLEY | 60 15 57.75 | -2.49 | 55.26 | -0.19 | 55.07 | .868 33127 |
| -1 | 3 JONES | 73 12 01.37 | -3.07 | 58.30 | -0.19 | 58.11 | .957 31685 |
| | 1-3 | | | | | | 16649.739 |
| | 1-2 | | | | | | 18355.984 |
| | | 00.57 | 00.00 | 60.57 | 00.57 | 00.00 | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| J.A. Andrews - AMS | FEB '64 | G.T. Tennis - AMS | FEB. '64 |

DA FORM 1 FEB 57 1918

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420469

*Figure 149. Computation of triangles for intersection station.*

(1) Observed angles extracted from the lists of directions are entered in the observed angle column. The corrections entered in the correction column, and the observed angles are added algebraically to determine the adjusted spherical angles. These angles are next reduced to plane angles by subtracting the spherical excess. Final lengths are computed using the adjusted plane angles. The double determination of all three lengths provides a valuable check on the adjustment. The lengths should agree within the limits of the computed precision of the angles.

(2) The fixed triangle (KELLEY-JONES-HOWARD) is shown for the starting lengths and also to provide a check on the spherical excess. No corrections are applied to the angles in the fixed triangle.

## 79. Three-Point Problem

*a.* A method sometimes used for supplemental or mapping control in an area where primary control exists is the three-point (resection) problem. In this method, the unknown station is occupied, and angles are turned to three or more known stations. Four known stations should be used if possible, as this will give a check on the work.

*b.* For purposes of illustration, a portion of the sample triangulation problem will be used for the sample three-point problem solution (fig. 150). The lengths and angles in the triangle Burdell-Hicks-Lincoln are those obtained from the adjustment by the direction method. Angles A and B (at Red) are the unadjusted angles as taken from the list of directions.

*c.* DA Form 1930, Special Angle Computation (fig. 151), provides a simple, rapid method for this computation as case 1. In this sample computation, angle A is 30°41′59″.7, angle B is 64°27′

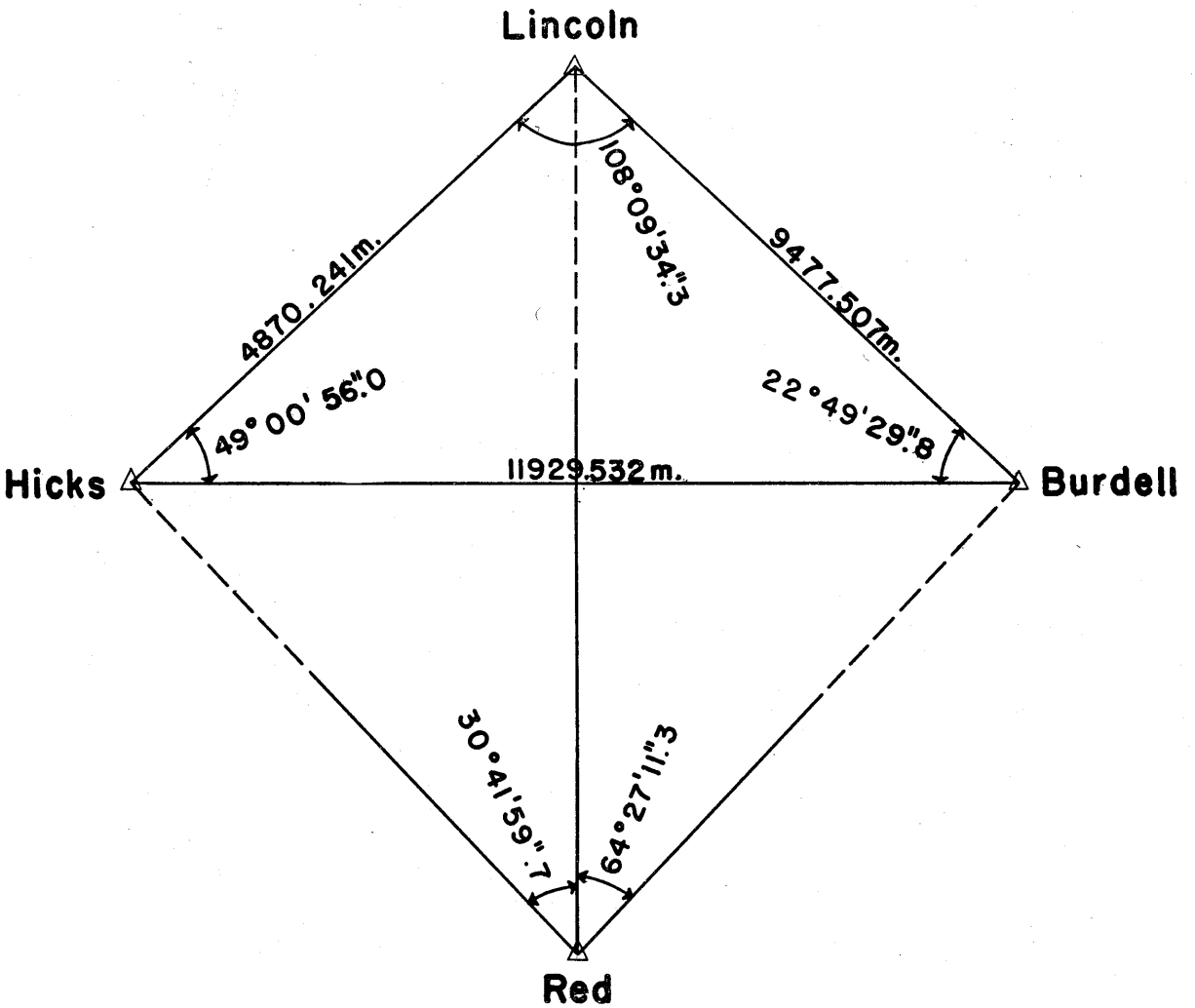
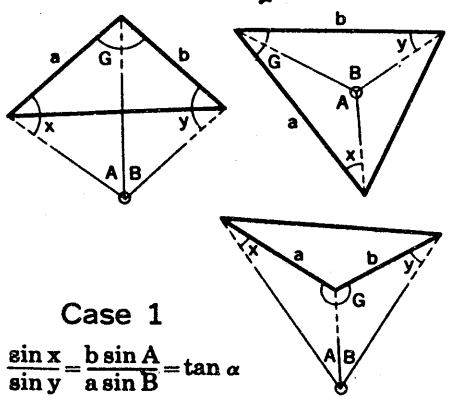



*Figure 150. Sketch, three-point problem.*

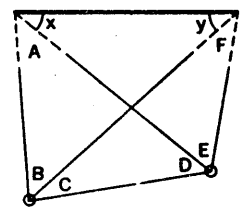
| PROJECT | SPECIAL ANGLE COMPUTATION (TM 5-237) |
|---|---|
| 1-243 | |
| LOCATION: California | DATE: 20 Aug 55 |
| ORGANIZATION: 29th Engrs. | CASE USED: 1. ☒ 2. ☐ 3. ☐ |




**Case 1**

$$\frac{\sin x}{\sin y} = \frac{b \sin A}{a \sin B} = \tan \alpha$$

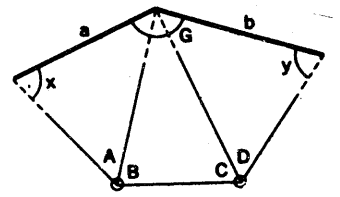
THREE-POINT PROBLEM




**Case 2**

$$\frac{\sin x}{\sin y} = \frac{\sin A \sin C \sin E}{\sin B \sin D \sin F} = \tan \alpha$$

INACCESSIBLE BASE PROBLEM




**Case 3**

$$\frac{\sin x}{\sin y} = \frac{b \sin A \sin C}{a \sin B \sin D} = \tan \alpha$$

SPECIAL ANGLE PROBLEM

$\frac{1}{2}(x+y) =$

| | |
|---|---|
| Case 1: $180° - \frac{1}{2}(A+B+G) =$ | 78° 20′ 37.35″ |
| Case 2: $\frac{1}{2}(C+D) =$ | |
| Case 3: $270° - \frac{1}{2}(A+B+C+D+G) =$ | |

Leave blanks below here for values not involved in the CASE used.

| | | | |
|---|---|---|---|
| log b | 3.97669409 | log a | 3.68755045 |
| log sin A | 9.70803120 | log sin B | 9.95531865 |
| log sin C | ——— | log sin D | ——— |
| log sin E | ——— | log sin F | ——— |
| *①Sum | 3.68472529 | *②Sum | 3.64286910 |
| —② — | 3.64286910 | —① — | |
| log tan α | 0.04185619 | log tan α | |
| α | 47° 45′ 24.285″ | α | ° ′ ″ |
| α-45° | 02 45 24.285 | α-45° | |
| log $\tan\frac{1}{2}(x+y)$ | 0.68551214 | log $\tan\frac{1}{2}(x+y)$ | |
| log tan (α-45°) | 8.68260940 | log tan (α-45°) | |
| Sum=log $\tan\frac{1}{2}(x-y)$ | 9.36812154 | Sum=log $\tan\frac{1}{2}(y-x)$ | |
| $\frac{1}{2}(x-y)$ | 13° 08′ 17.70″ | $\frac{1}{2}(y-x)$ | ° ′ ″ |
| $\frac{1}{2}(x+y)$ | 78 20 37.35 | $\frac{1}{2}(y+x)$ | |
| x | 91 28 55.05 | y | |
| y | 65 12 19.65 | x | |

α is an auxiliary angle needed only for the computation: it is always between 45° and 90°

* Where ① is greater than ② use only the left side of the form below here, and vice versa.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.A.Smith | 23 Aug 55 | W. C. Asmen | 25 Aug. 55 |

DA FORM 1 FEB 57 1930

*Figure 151. Computation of three-point problem (DA Form 1930).*

11.″3, angle G is 108°09′34.″3, side $a$ is 4870.241 meters, and side $b$ is 9477.507 meters. Subtract ½ of the sum of angles A, B, and G from 180°. (As the sum of angles A, B, and G nears 180°, the accuracy of the solution decreases. If the sum is 180° there is no solution.) This is ½$(x+y)$. Enter log $b$, log sin A, log $a$, and log sin B on the form where indicated. Log sin C, log sin D, log E, and log sin F do not enter the computation for case 1. Add log $b$ and log sin A to obtain sum ①. Add log $a$ and log sin B to obtain sum ②. Since ① is larger than ②, use the left side of the form for the rest of the computation. Subtract ② from ① to find log tan $\alpha$. From a table of logarithms of trigonometric functions, find $\alpha$. Subtract 45° from $\alpha$ and place the log tan of the resulting angle in the appropriate blank. Add the log tan of ½$(x+y)$ (if over 90° tan is negative) (sign is important) and the log tan of $(\alpha-45°)$ to find the log tan ½$(x-y)$. From tables, find the angle ½$(x-y)$. The desired angle $x$ is now obtained by adding the angle ½$(x+y)$ to the angle ½$(x-y)$. Angle $y$ is obtained by subtracting the angle ½$(x-y)$ from the angle ½$(x+y)$. All the angles in the figure are now known or can be obtained by subtracting the fixed angles at Hicks and Burdell from the computed angles $x$ and $y$ respectively. Note that an error in the computation of the angle ½$(x-y)$ [½$(y-x)$ on right side of form] will cause compensating errors in the angles $x$ and $y$ which will make the sums of the angles in the figure check, but the solution will be incorrect.

*d.* The same three-point problem previously solved by logarithms on DA Form 1930 is now solved on DA Form 1920. This solution utilizes natural trigonometric functions throughout. Referring to the schematic diagram (fig. 152), the formula used to solve the problem is:

$$\tan x = \frac{b \sin A \sin [360° - (A+B+C)]}{b \sin A \cos [360° - (A+B+C)] + a \sin B}$$

in which $a$ and $b$ are the known lengths, C is the known angle, and A and B are the observed angles. Angle $y$ equals $[360° - (A+B+C+x)]$. For use on this particular computation form, the formula for tan $x$ is modified to read as follows:

$$\tan x = \frac{\sin [360° - (A+B+C)]}{\cos [360° - (A+B+C)] + \frac{a/\sin A}{b/\sin B}}$$

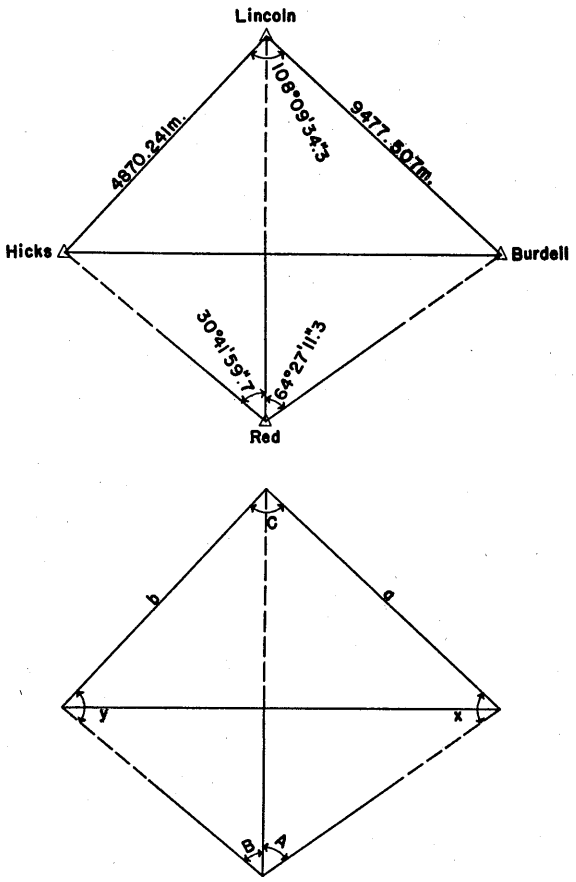



*Figure 152. Schematic diagram, three-point problem.*

The reason for this change is to obtain the ratios $a/\sin A$ and $b/\sin B$, which are used later to solve the triangles. In solving the formula, care must be taken to use the correct algebraic sign for the sine and cosine of $[360° - (A+B+C)]$. This is important. The algebraic sign of tan $x$ depends on the signs of these functions. The algebraic sign of tan $x$ determines the quadrant in which the angle $x$ lies. The cosine of angles between 90° and 270° is negative. Negative tangents indicate angles between 90° and 180° and between 270° and 360°.

*e.* In the numerical example, stations Hicks, Lincoln, and Burdell are fixed in position, thereby fixing the lengths Hicks-Lincoln and Lincoln-Burdell and the angle at Lincoln. The angles at Red of Hicks-Red-Lincoln and Lincoln-Red-Burdell are observed. The triangles to be solved are Red-Lincoln-Burdell and Red-Hicks-Lincoln. The triangles are written on the computation sheet (fig. 153), using the first two sections of the form. The remainder of the form is used for the additional computations required to solve the problem. The observed angles at Red are entered;

PROJECT 1-243

**TRIANGLE COMPUTATION (FOR CALCULATING MACHINE)** (TM 5-237)

LOCATION California | ORGANIZATION 29th Engrs | DATE 21 Aug 55

| SYMBOL | STATION | OBSERVED ANGLE | CORR'N | SPHER'L ANGLE | SPHER'L EXCESS | PLANE ANGLE | SINE | DISTANCE | | SIDE |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 Red | 64 27 11.3 | | | | | 0.90223288 | 9477.507 | 2–3 | LINCOLN-Burdell |
| | 2 LINCOLN | (50 20 29.1) | | | | | 0.76986109 | 8087.007 | 1–3 | Red - Burdell |
| | 3 Burdell | 65 12 19.6 | | | | | 0.90781733 | 9536.169 | 1–2 | Red - LINCOLN |
| | | | | | | | | D=Ratio, side/sine | | 10504.502 |
| | 1 Red | 30 41 59.7 | | | | | 0.51054167 | 4870.241 | 2–3 | HICKS-LINCOLN |
| | 2 HICKS | 91 28 55.1 | | | | | 0.99966551 | 9536.169 | 1–3 | Red-LINCOLN |
| | 3 LINCOLN | (57 49 05.2) | | | | | 0.84636157 | 8073.748 | 1–2 | Red-HICKS |
| | | | | | | | | D=Ratio, side/sine | | 9539.360 |
| | 1 LINCOLN | 108 09 34.3 | | | | | | | 2–3 | |
| | 2 Sum | 203 18 45.3 | | | | | | 1.10117471 | 1–3 | $D_1/D_2$ |
| | 3 360-Sum | 156 41 14.7 | | | | sin | 0.39574720 | -0.91835949 | 1–2 | cos. |
| | x = Burdell | 65 12 19.6 | | | | tan x = +2.16473880 | | D=Ratio, side/sine | | |
| | y = HICKS | 91 28 55.1 | | | | | | 0.18281522 | | |
| | 1 | | | | | | | | 2–3 | |
| | 2 | | | | | | | | 1–3 | |
| | 3 | | | | | | | | 1–2 | |
| | | | | | | | | D=Ratio, side/sine | | |

Case I a/sin A=b/sin B=c/sin C — Given: 3 angles, 1 side

Case II sin B=b sin A/a — Given: 2 sides and an angle opposite

Case III tan A=a sin B/c−a cos B — Given: 2 sides and included angle

Case IV cos A=[2s(s−a)/bc]−1 s=1/2(a+b+c) — Given: 3 sides

COMPUTED BY W. C. Aumen DATE 23 Aug. 55

CHECKED BY Rdsmith DATE 25 Aug. 55

DA FORM 1 FEB 57 1920

*Figure 153. Computation of three-point problem (DA Form 1920.)*

64°27′11″.3, angle A in the triangle Red-Lincoln-Burdell; 30°41′59″.7, angle B in the triangle Red-Hicks-Lincoln. The known lengths are Lincoln-Burdell, 9477.507 meters (side *a*) and Hicks-Lincoln, 4870.241 meters (side *b*). The sines of the known angles are entered on the form and the ratios, side/sine=D, are found for each triangle. For the triangle Red-Lincoln-Burdell, this ratio is 9477.507÷0.90223288 equals 10504.502. For the triangle Red-Hicks-Lincoln, the ratio is 4870.241 ÷0.51054167 equals 9539.360.

*f.* The computation now shifts to the third section of the form. The fixed angle at Lincoln, 108°09′34″.3 is entered and the sum of A+B+C is found to be 203°18′45″.3 (64°27′11″.3+30°41′59″.7+108°09′34″.3). This sum is subtracted from 360° and the answer 156°41′14″.7, entered on the form. The sine and cosine of this angle are entered on the form as shown in the example (sine equals +0.39574720, cosine equals −0.91835949). Notice the minus sign on the cosine because the angle 360°−(A+B+C) is between 90° and 270°. The ratio of triangle 1 (Red-Lincoln-Burdell) is divided by the ratio of triangle 2 (Red-Hicks-Lincoln), i.e., 10504.502÷9539.360, and the result 1.10117471 is added algebraically to the cosine of 360°−(A+B+C) which is −0.91835949. The result of this algebraic addition is +0.18281522. The sine of 360°−(A+B+C) which is +0.39574720, is divided by the result of the addition +0.18281522, to obtain the tangent of *x* which equals +2.16473880. The plus sign on the tangent indicates the angle is between 0° and 90° or 180° and 270°. Since angle *x* must be less than 360°−(A+B+C), which is 156°41′14″.7, the angle in this example must be between 0° and 90°. The angle corresponding to the tangent of +2.16473880 is found to be 65°12′19″.6. From the schematic diagram, the angle *x* is seen to be the angle at Burdell in the triangle Red-Lincoln-

Burdell. The angle $x$, 65°12′19″.6, can now be entered in the first triangle, and the third angle of the triangle concluded to be 50°20′29″.1. Angle $y$ is found by subtracting angle $x$ from the angle 360°−(A+B+C), i.e., 156°41′14″.7 minus 65°12′19″.6 equals angle $y$, 91°28′55″.1. Again from the schematic diagram, angle $y$ is seen to be the angle at Hicks in the triangle Red-Hicks-Lincoln, and can be entered in the second triangle. The concluding angle in the second triangle is found to be 57°49′05″.2. The two triangles can now be solved by the usual method. The common side Red-Lincoln in the two triangles should check, and the sum of the two concluded angles should equal the fixed angle C. In this example, the length of the common side as solved in the two triangles is 9536.169 and 9536.169, which checks within the accuracy of the computation. The sum of the concluded angles is 108°09′34″.3, which checks the fixed angle at Lincoln.

*g.* If the new point lies near the circle passing through the 3 fixed points, the solution is weak. If the new point lies on the circle or A+B+C= 180°, the solution is indefinite.

## 80. Inaccessible Base Computation

*a.* The same quadrilateral selected from the sample triangulation net for the three-point problem solution will again be used, but this time it will be solved as an inaccessible base problem (fig. 154). This problem deals with the solution of two unknown angles in a quadrilateral which are designated $x$ and $y$. This situation arises when two stations of unknown position, and unknown azimuth and distance between them, are mutually intervisible, and from which the stations at each end of a line of known length and azimuth can be observed. In the example, Red and Burdell are the unknown stations. Hicks and Lincoln designate the ends of the line of known length and azimuth. All the angles at Red and Burdell are known by observation. The angles Red-Hicks-Burdell and Red-Lincoln-Burdell are concluded in their respective triangles. Burdell-Hicks-Lincoln is the unknown angle $x$, and Hicks-Lincoln-Red is the unknown angle $y$.

*b.* The solution to this problem is best performed on DA Form 1930 as case 2 (fig. 155). Angles A and F on the form are the concluded angles at

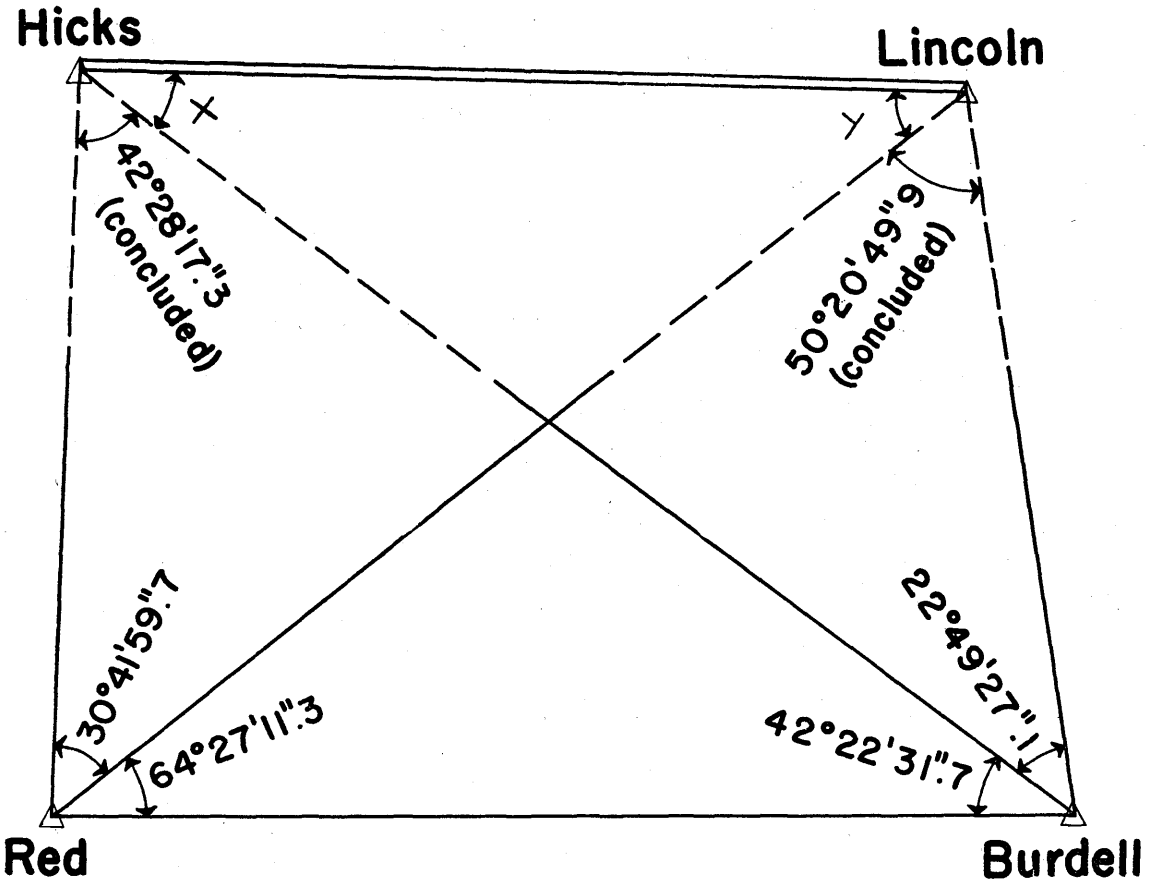
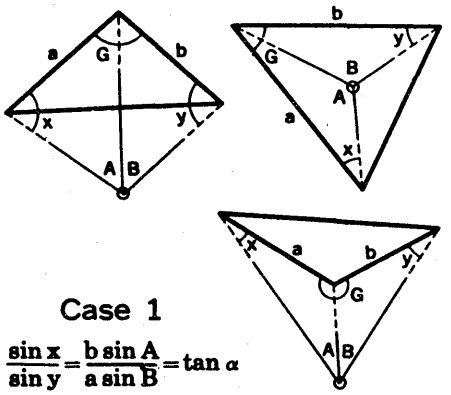



*Figure 154. Sketch, inaccessible-base problem.*

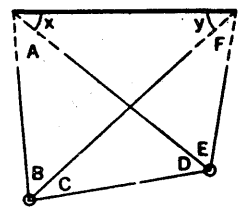
| PROJECT 1-243 | SPECIAL ANGLE COMPUTATION (TM 5-237) |
|---|---|
| LOCATION California | DATE 20 Aug 55 |
| ORGANIZATION 29th Engrs | CASE USED 1. ☐ 2. ☒ 3. ☐ |




**Case 1**

$$\frac{\sin x}{\sin y}=\frac{b \sin A}{a \sin B}=\tan \alpha$$

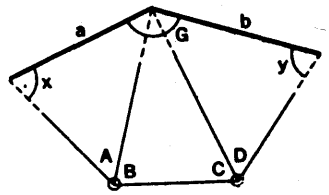
THREE-POINT PROBLEM




**Case 2**

$$\frac{\sin x}{\sin y}=\frac{\sin A \sin C \sin E}{\sin B \sin D \sin F}=\tan \alpha$$

INACCESSIBLE BASE PROBLEM




**Case 3**

$$\frac{\sin x}{\sin y}=\frac{b \sin A \sin C}{a \sin B \sin D}=\tan \alpha$$

SPECIAL ANGLE PROBLEM

| $\frac{1}{2}(x+y)=$ | | ° | ′ | ″ |
|---|---|---|---|---|
| | Case 1: $180°-\frac{1}{2}(A+B+G)=$ | | | |
| | Case 2: $\frac{1}{2}(C+D)=$ | 53 | 24 | 51.5 |
| | Case 3: $270°-\frac{1}{2}(A+B+C+D+G)=$ | | | |

Leave blanks below here for values not involved in the CASE used.

| log b | | log a | |
|---|---|---|---|
| log sin A | 9.82944725 | log sin B | 9.70803120 |
| log sin C | 9.95531865 | log sin D | 9.82865101 |
| log sin E | 9.58872519 | log sin F | 9.88644868 |
| *①Sum | 9.37349109 | *②Sum | 9.42313089 |
| —② — | | —① — | 9.37349109 |
| log tan α | | log tan α | 0.04963980 |
| α | ° ′ ″ | α | 48° 16′ 02.″436 |
| α-45° | | α-45° | 03 16 02.436 |
| log tan½(x+y) | | log tan½(x+y) | 0.12943328 |
| log tan (α-45°) | | log tan (α-45°) | 8.75654327 |
| Sum=log tan½(x−y) | | Sum=log tan½(y−x) | 8.88597655 |
| ½(x−y) | ° ′ ″ | ½(y−x) | 04° 23′ 52.″4 |
| ½(x+y) | | ½(y+x) | 53 24 51.5 |
| x | | y | 57 48 43.9 |
| y | | x | 49 00 59.1 |

α is an auxiliary angle needed only for the computation: it is always between 45° and 90°

* Where ① is greater than ② use only the left side of the form below here, and vice versa.

| COMPUTED BY RAS | DATE 23 Aug 55 | CHECKED BY W.C.A. | DATE 25 Aug 55 |
|---|---|---|---|

DA FORM 1 FEB 57 1930

*Figure 155. Computation of inaccessible-base problem (DA Form 1930).*

Hicks and Lincoln, respectively. Angles B and C are the observed angles at Red, and angles D and E are the observed angles at Burdell. Find ½ the sum of angles C+D and enter on the form as ½(x+y). From the table of logarithms of trigonometric functions, find the log sines of the angles and enter them in the designated spaces. Log *a* and log *b* do not enter this computation. Add log sin A, plus log sin C, plus log sin E, to obtain sum ①. Add log sin B, plus log sin D, plus log sin F, to obtain sum ②. Subtract the smaller sum from the larger, in this case ① from ②. The left side of the form is now ignored for the remainder of the computation. Subtracting sum ① from sum ② gives log tan $\alpha$. In the table of logarithms of trigonometric functions, find the angle ($\alpha$) corresponding to log tan $\alpha$. From this angle subtract 45°. From this table, find log tan ½(C+D) and log tan ($\alpha$−45°). Add these two logs to obtain log tan ½(*y*−*x*) [this sum is log tan ½(*x*−*y*) on left side of form]. Find the angle corresponding to this log tan which will be angle ½(*y*−*x*). To find angle *y*, add the angle ½(*y*−*x*), which was just obtained, to the angle ½(*y*+*x*), which was computed previously as ½(C+D). Angle *x* is found by subtracting the angle ½(*y*−*x*) from the angle ½(*y*+*x*). This completes the computation for the unknown angles. The triangles can now be computed by the usual method employing the law of sines.

*c.* It is emphasized that this solution may check within itself, as was the case with the solution of the three-point problem, but still be incorrect. An error in the angle ½(*y*—*x*) [or ½(*x*−*y*)] will make compensating errors in angles *x* and *y*, and the sum of the angles in the triangles or in the quadrilateral will still check.

## 81. Special Angle Computation

*a.* A sample figure (fig. 156) for solution by case 3, DA Form 1930 (Special Angle Computation) (fig. 157) was taken from the sample triangulation net. Again the problem is to solve two unknown angles, this time in a five-sided figure. This type of figure is not as common as the three-point or inaccessible base problems, but the occasion may arise where this figure can be observed and the others cannot.

*b.* As in the inaccessible base problem, two stations are intervisible, but no distance can be obtained between them. From each of these two stations, only two of the three available fixed stations can be seen. The sketch (fig. 156) illustrates the fixed lines Hicks-Black and Hicks-Lincoln and the fixed angle (G) at Hicks. The angles at Red and two angles at Burdell are

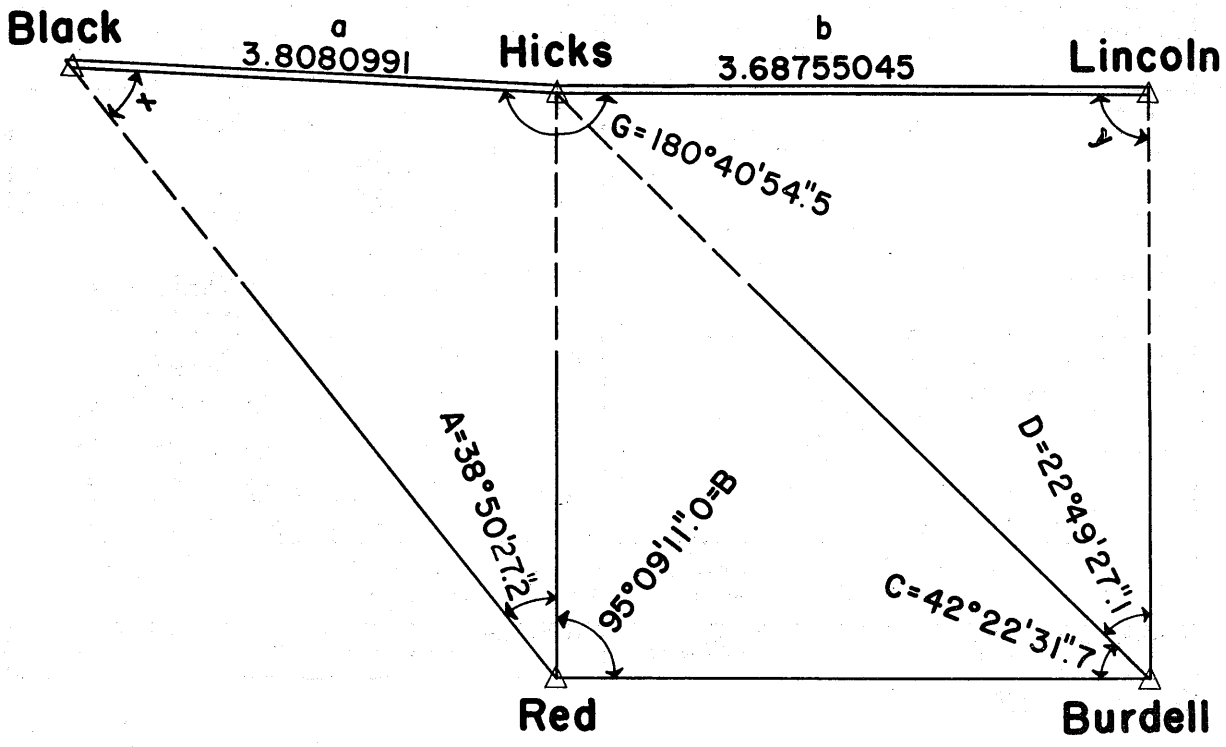
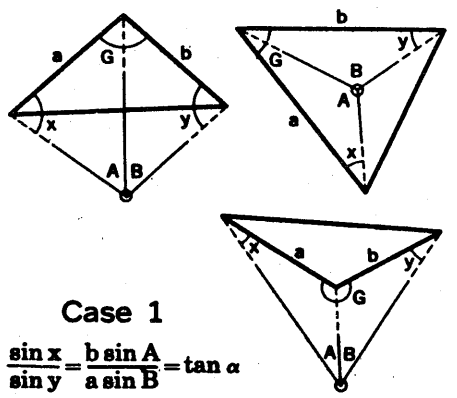



*Figure 156. Sketch, special angle problem.*

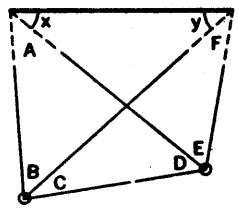
| PROJECT | I-243 | SPECIAL ANGLE COMPUTATION (TM 5-237) | |
|---|---|---|---|
| LOCATION | California | DATE | 20 Aug 55 |
| ORGANIZATION | 29th Engrs | CASE USED | 1. ☐ 2. ☐ 3. ☒ |




**Case 1**

$$\frac{\sin x}{\sin y}=\frac{b \sin A}{a \sin B}=\tan \alpha$$

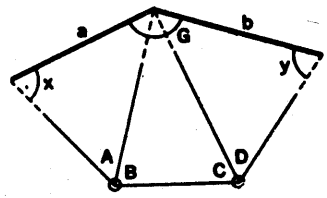
THREE-POINT PROBLEM




**Case 2**

$$\frac{\sin x}{\sin y}=\frac{\sin A \sin C \sin E}{\sin B \sin D \sin F}=\tan \alpha$$

INACCESSIBLE BASE PROBLEM




**Case 3**

$$\frac{\sin x}{\sin y}=\frac{b \sin A \sin C}{a \sin B \sin D}=\tan \alpha$$

SPECIAL ANGLE PROBLEM

$\frac{1}{2}(x+y)=$

| | |
|---|---|
| Case 1: $180°-\frac{1}{2}(A+B+G)=$ | ° ′ ″ |
| Case 2: $\frac{1}{2}(C+D)=$ | |
| Case 3: $270°-\frac{1}{2}(A+B+C+D+G)=$ | 80 03 44.25 |

Leave blanks below here for values not involved in the CASE used.

| | | | |
|---|---|---|---|
| log b | 3.68755045 | log a | 3.8080991 |
| log sin A | 9.79737823 | log sin B | 9.99824117 |
| log sin C | 9.82865101 | log sin D | 9.58872519 |
| log sin E | ——— | log sin F | ——— |
| *①Sum | 3.31357969 | *③Sum | 3.39506546 |
| —② — | | —① — | 3.31357969 |
| log tan α | | log tan α | 0.08148577 |
| α | ° ′ ″ | α | 50° 20′ 37.″97 |
| α-45° | | α-45° | 05 20 37.97 |
| log tan½(x+y) | | log tan½(x+y) | 0.75645054 |
| log tan (α-45°) | | log tan (α-45°) | 8.97099598 |
| Sum=log tan½(x−y) | | Sum=log tan½(y−x) | 9.72744652 |
| ½(x−y) | ° ′ ″ | ½(y−x) | 28° 05′ 49.″29 |
| ½(x+y) | | ½(y+x) | 80 03 44.25 |
| x | | y | 108 09 33.54 |
| y | | x | 51 57 54.96 |

α is an auxiliary angle needed only for the computation: it is always between 45° and 90°

* Where ① is greater than ③ use only the left side of the form below here, and vice versa.

| COMPUTED BY | RAG | DATE | 23 Aug 55 | CHECKED BY | W. C. A. | DATE | 25 Aug 55 |
|---|---|---|---|---|---|---|---|

DA FORM 1 FEB 57 1930

*Figure 157. Computation of special angle problem (DA Form 1930).*

observed. The lines Red-Lincoln or Burdell-Black cannot be observed, so the inaccessible base solution is not applicable. Angle Hicks-Black-Red is the unknown angle $x$, and angle Hicks-Lincoln-Burdell is the unknown angle $y$. The angles are lettered A, B, C, D, and G as shown on the sketch. The fixed length Hicks-Black is designated by the letter $a$, and the fixed length Hicks-Lincoln by the letter $b$. Solve for the angle $\frac{1}{2}(x+y)$ for case 3 on DA Form 1930 as $270°-\frac{1}{2}(A+B+C+D+G)$. Enter the logarithms for the lengths $a$ and $b$ and the log sines of angles A, B, C, and D in their designated places on the form. Log sin E and log sin F do not enter this computation, and these spaces are left blank. Log $b$ plus log sin A plus log sin C equals sum ①. Log $a$ plus log sin B plus log sin D equals sum ②. Subtract the smaller sum from the larger sum, in this case sum ① is subtracted from sum ②. From this point on, the left side of the form is not used. The result of this subtraction is log tan $\alpha$. $\alpha$ is an auxiliary angle needed only for the computation. Using log tan $\alpha$, find angle $\alpha$ from a table of logarithms of trigonometric functions. Subtract 45° from $\alpha$ and find the log tan of this new angle. Find the log tan of the angle $\frac{1}{2}(x+y)$. The sum of log tan $\frac{1}{2}(x+y)$ plus log tan $(\alpha-45°)$ equals log tan $\frac{1}{2}(y-x)$. From the tables obtain the angle $\frac{1}{2}(y-x)$. Add the angles $\frac{1}{2}(y-x)$ and $\frac{1}{2}(y+x)$ to obtain angle $y$. Subtract angle $\frac{1}{2}(y-x)$ from angle $\frac{1}{2}(y+x)$ to obtain angle $x$.

*c.* This completes the computation on this form. Now two angles in each triangle are known and all of the triangles can be computed by the sine law, starting with the triangles containing the fixed lengths $a$ and $b$.

## Section VI. SHORE-SHIP TRIANGULATION

### 82. Description

The shore-ship method of triangulation establishes distances between shore-traverse stations that cannot be measured directly. The formula used is—

$$b' = \frac{b \sin A \sin (A'+C')}{\sin A' \sin (A+C)}$$

Where $b$ is a known distance, $b'$ is the unknown distance, and angles A, C, A′, and C′ are the observed angles at the shore stations. Figure 158 illustrates the problem.

### 83. Computation

The computation is performed in four steps. The procedure is: first, abstract the directions from the field books; second, perform the length computation to find the values for log X, where—

$$X = \frac{\sin A \sin (A'+C')}{\sin A' \sin (A+C)};$$

third, compute the probable error to establish the rejection limit for values of log X, and final log X; and fourth, using the final log X, compute log $b'$, and interpolate the length of $b'$ from the tables. A correction will be applied to this length when the error in the traverse distances is determined by a check on another known length (base).

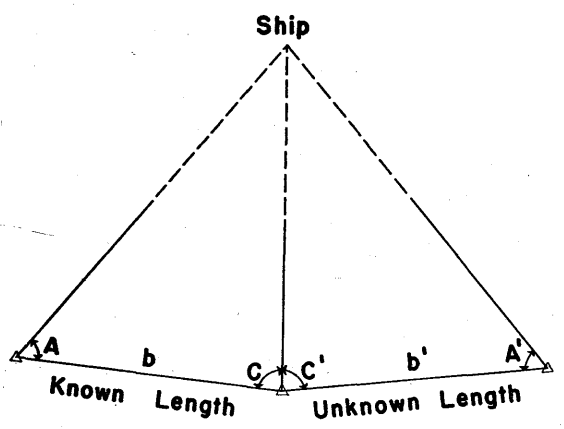



*Figure 158. Sketch, shore-ship triangulation.*

*a.* Certain specifications must be followed to insure third-order results in this method. These are—

(1) Angles will be abstracted and computed to the nearest 1″.

(2) Logarithms for the individual computations will be taken to five decimals, the computations using meaned logarithms to six decimals. The angular argument for these logarithms can be rounded off to the nearest 10″ before entering the tables.

(3) The rejection limit for log X will be determined by the rule: *Reject any log X whose residual exceeds three times the probable error of a single measurement*, when the probable error is found by the follow-

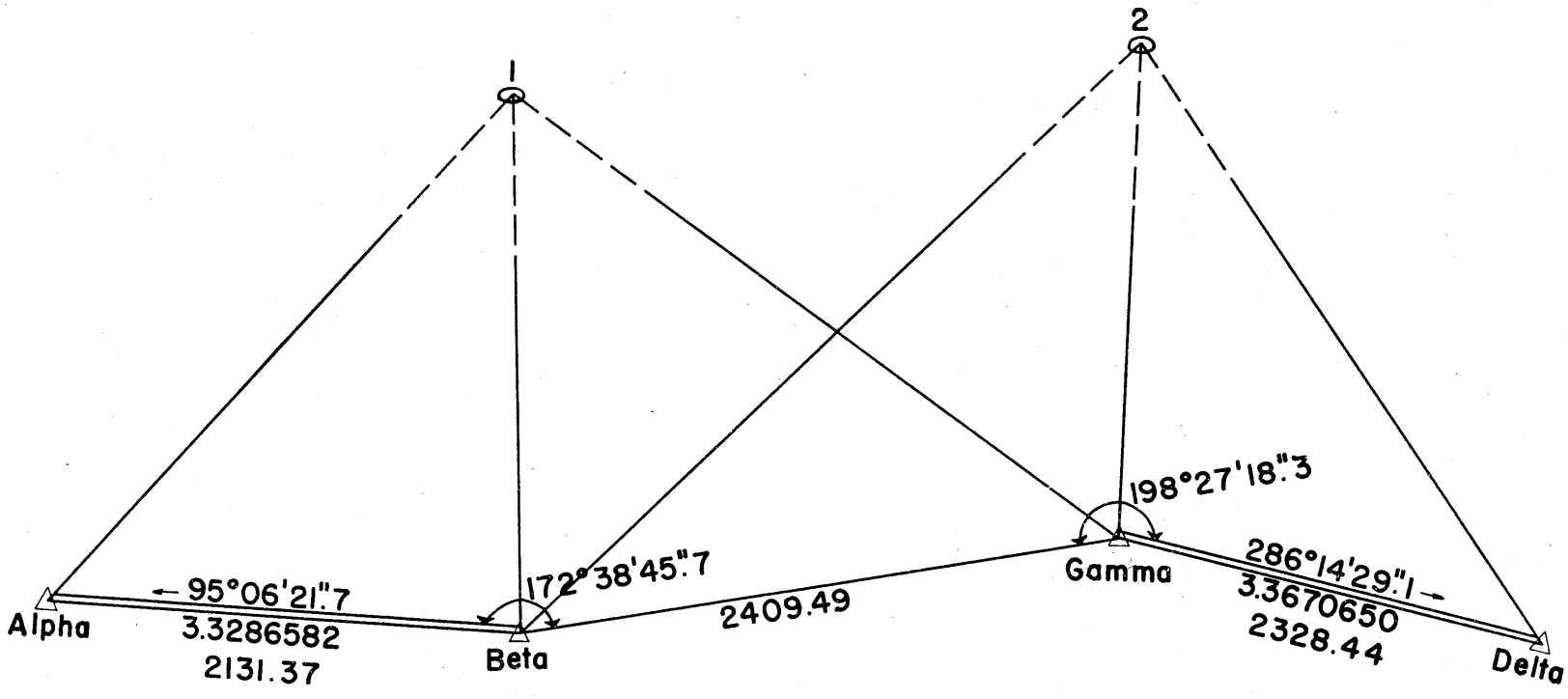



*Figure 159. Sketch, complete traverse (shore-ship).*

ing method: Find the mean of *all* log X's, and find the residual for each. Compute the probable error of a single measurement by the following formula:

$$E=0.6745\sqrt{\frac{\Sigma v^2}{n-1}}$$

Where: E=probable error of a single observation.
$v$=residual or algebraic difference between the mean of a set of log X's and the log X for each pointing.
$\Sigma v^2$=sum of the squares of the residuals.
$n$=number of observations.

(4) The lengths between traverse stations are adjusted by—

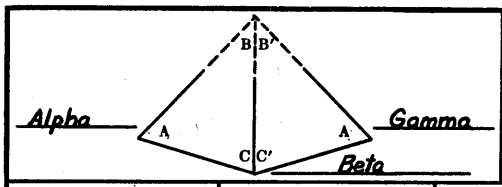
(*a*) Determining the discrepancy between the computed and the fixed length of the closing base.

(*b*) Dividing the sum of the lengths of the traverse into the discrepancy to furnish a correction factor.

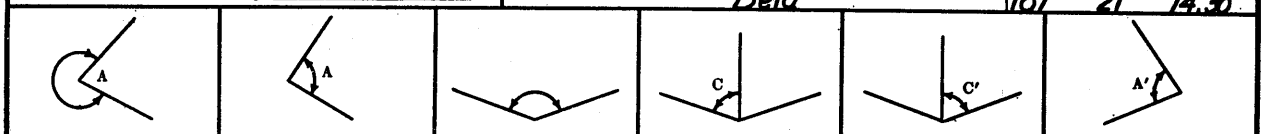
(*c*) Multiplying the correction factor by each length accumulatively to determine the correction for each course, the final course having the whole correction.

*b.* Figures 159, 160 (DA Form 1928, Abstract of Angles, Shore-Ship), 161 (DA Form 1929, Shore-Ship Length Computation), and 162 illustrate the computations performed to determine the length of the course Beta-Gamma from the fixed length Alpha-Beta. The corrections applied to the length was computed from the complete traverse (fig. 159) using the rule stated previously.

*c.* After the lengths have been computed, they are used with the traverse angles to perform a regular traverse computation.




PROJECT: Navel
ABSTRACT OF ANGLES, SHORE-SHIP (TM 5-237)
LOCATION: Washington, D.C.
SHIP (NAME AND/OR NUMBER): Endeavor II
ORGANIZATION: A&S. Inc.
SHIP POSITION NUMBER: 7
DATE: 15 April 54
SHORE TRAVERSE ANGLE AT: Beta 187° 21' 14.30"

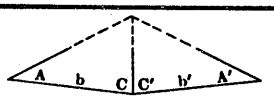



| RECORDED ANGLE | SHORE-SHIP ANGLE | SHORE ANGLE | ANGLE C | ANGLE C' | SHORE-SHIP ANGLE |
|---|---|---|---|---|---|
| 323° 35' 08" | 36° 24' 52" | 172° 38' 46" | 85° 15' 49" | 87° 22' 57" | 32° 23' 39" |
| 38 55 | 21 05 | " | 19 47 | 18 59 | 22 10 |
| 41 43 | 18 17 | " | 22 58 | 15 48 | 20 54 |
| 43 14 | 16 46 | " | 24 41 | 14 05 | 20 37 |
| 44 39 | 15 21 | " | 26 38 | 12 08 | 20 35 |
| 46 04 | 13 56 | " | 28 12 | 10 34 | 19 44 |
| 48 58 | 11 02 | " | 31 42 | 07 04 | 19 15 |
| 49 35 | 10 25 | " | 32 46 | 06 00 | 19 21 |
| 50 56 | 09 04 | " | 34 20 | 04 26 | 19 11 |
| 52 29 | 07 31 | " | 37 52 | 00 54 | 18 52 |
| 55 39 | 04 21 | " | 38 03 | 00 43 | 16 28 |
| 58 04 | 01 56 | " | 39 47 | 86 58 59 | 15 40 |
| 59 43 | 00 17 | " | 42 24 | 56 22 | 15 28 |
| 324 01 50 | 35 58 10 | " | 45 09 | 53 37 | 14 51 |
| 05 34 | 54 26 | " | 48 38 | 50 08 | 12 57 |
| 07 58 | 52 02 | " | 51 27 | 47 19 | 12 17 |

COMPUTED BY: RSchmidt — DATE: 25 Aug 55
CHECKED BY: W. C. Aumen — DATE: 25 Aug. 55

DA FORM 1 FEB 57 1928

*Figure 160. Abstract of directions for shore-ship triangulation (DA Form 1928).*




PROJECT: Navel | LOCATION: Washington, D.C. | ORGANIZATION: A&S, Inc.

FROM: Alpha — Beta | TO: Beta — Gamma

SECT. NR. 1 | FROM (STARTING POINT OF TRAVERSE): Alpha | TO (ENDING POINT OF TRAVERSE): Delta | DATE: 15 April 54

| POSITION (SHIP) | ( / ) a | ( / ) b. | ( / ) c | ( / ) d | ( / ) e | ( / ) f | ( / ) g | ( / ) h |
|---|---|---|---|---|---|---|---|---|
| A angle at ( Alpha ) | 36 24 52 | 36 21 05 | 36 18 17 | 36 16 46 | 36 15 21 | 36 13 56 | 36 11 02 | 36 10 25 |
| C angle at ( Beta ) | 85 15 49 | 85 19 47 | 85 22 58 | 85 24 41 | 85 26 38 | 85 28 12 | 85 31 42 | 85 32 46 |
| A+C | 121 40 41 | 121 40 52 | 121 41 15 | 121 41 27 | 121 41 59 | 121 42 08 | 121 42 44 | 121 43 11 |
| A′ angle at ( Gamma ) | 32 23 39 | 32 22 10 | 32 20 54 | 32 20 37 | 32 20 35 | 32 19 44 | 32 19 15 | 32 19 21 |
| C′ angle at ( Beta ) | 87 22 57 | 87 18 59 | 87 15 48 | 87 14 05 | 87 12 08 | 87 10 34 | 87 07 04 | 87 06 00 |
| A′+C′ | 119 46 36 | 119 41 09 | 119 36 42 | 119 34 42 | 119 32 43 | 119 30 18 | 119 26 19 | 119 25 21 |
| log sin A | 9.77351 | 9.77286 | 9.77238 | 9.77212 | 9.77188 | 9.77163 | 9.77113 | 9.77102 |
| log sin (A′+C′) | 9.93850 | 9.93890 | 9.93922 | 9.93936 | 9.93950 | 9.93968 | 9.93996 | 9.94003 |
| colog sin (A+C) | 0.07006 | 0.07008 | 0.07011 | 0.07012 | 0.07017 | 0.07018 | 0.07022 | 0.07026 |
| colog sin A′ | 0.27105 | 0.27134 | 0.27159 | 0.27165 | 0.27166 | 0.27183 | 0.27192 | 0.27190 |
| Log X | 0.05312 | 0.05318 | 0.05330 | 0.05325 | 0.05321 | 0.05332 | 0.05323 | 0.05321 |
| **POSITION (SHIP)** | ( / ) i | ( / ) j | ( / ) k | ( / ) l | ( / ) m | ( / ) n | ( / ) o | ( / ) p |
| A angle at ( Alpha ) | 36 09 04 | 36 07 31 | 36 04 21 | 36 01 56 | 36 00 17 | 35 58 10 | 35 54 26 | 35 52 02 |
| C angle at ( Beta ) | 85 34 20 | 85 37 52 | 85 38 03 | 85 39 47 | 85 42 24 | 85 45 09 | 85 48 38 | 85 51 27 |
| A+C | 121 43 24 | 121 45 23 | 121 42 24 | 121 41 43 | 121 42 41 | 121 43 19 | 121 43 04 | 121 43 29 |
| A′ angle at ( Gamma ) | 32 19 11 | 32 18 52 | 32 16 28 | 32 15 40 | 32 15 28 | 32 14 51 | 32 12 57 | 32 12 17 |
| C′ angle at ( Beta ) | 87 04 26 | 87 00 54 | 87 00 43 | 86 58 59 | 86 56 22 | 86 53 37 | 86 50 08 | 86 47 19 |
| A′+C′ | 119 23 37 | 119 19 46 | 119 17 11 | 119 14 39 | 119 11 50 | 119 08 28 | 119 03 05 | 118 59 36 |
| log sin A | 9.77079 | 9.77052 | 9.76997 | 9.76955 | 9.76927 | 9.76890 | 9.76825 | 9.76783 |
| log sin (A′+C′) | 9.94015 | 9.94043 | 9.94061 | 9.94079 | 9.94099 | 9.94122 | 9.94160 | 9.94185 |
| colog sin (A+C) | 0.07028 | 0.07043 | 0.07020 | 0.07014 | 0.07022 | 0.07027 | 0.07025 | 0.07028 |
| colog sin A′ | 0.27194 | 0.27200 | 0.27248 | 0.27264 | 0.27268 | 0.27280 | 0.27318 | 0.27332 |
| Log X | 0.05316 | 0.05338 | 0.05326 | 0.05312 | 0.05316 | 0.05319 | 0.05328 | 0.05328 |

| | |
|---|---|
| Mean Log X | 0.053228 |
| log b | 3.3286582 |
| log b | 3.381886 |

| | |
|---|---|
| Unadjusted length Beta to Gamma | 2409.27 meters |
| Correction | + 0.22 |
| Adjusted length Beta to Gamma | 2409.49 |
| Adjusted Shore Traverse Angle at Beta | 172° 38′ 45″.7 |

COMPUTED BY: W. C. Cannon | DATE: 16 April 54 | CHECKED BY: Kidmith

DA FORM 1 FEB 57 1929 — SHORE-SHIP LENGTH COMPUTATION (TM 5-237)

*Figure 161. Length computation (DA Form 1929).*

| | | Ship | Position | No. 1 | | |
|---|---|---|---|---|---|---|
| | Log x | V | $V^2$ | | | |
| 1 | 0.05312 | -11 | 121 | | | |
| 2 | 318 | -05 | 25 | | | |
| 3 | 330 | +07 | 49 | | | |
| 4 | 325 | +02 | 4 | | | |
| 5 | 321 | -02 | 4 | | | |
| 6 | 332 | +09 | 81 | | | |
| 7 | 323 | 0 | 0 | | | |
| 8 | 321 | -02 | 4 | | | |
| 9 | 316 | -07 | 49 | | | |
| 10 | 338 | +15 | 225 | | | |
| 11 | 326 | +03 | 9 | | | |
| 12 | 312 | -11 | 121 | | | |
| 13 | 316 | -07 | 49 | | | |
| 14 | 319 | -04 | 16 | | | |
| 15 | 328 | +05 | 25 | | | |
| 16 | 328 | +05 | 25 | | | |
| Sum | 0.85165 | $\Sigma V^2 = 8.07 \times 10^{-8}$ | | | | |
| Mean | 0.05323 | | | | | |
| | | $\sqrt{\Sigma V^2 / n-1}$ | | $\sqrt{8.07\times10^{-8}}$ | | |
| E= | 3 x 0.6745 | V | E= 2V | 15 | $E = 2\times10^{-4}$ | 0.05380 |
| E= | 2 x 0.000073 | | E= 0.000015 | | Log X = | 0.05323 |

*Figure 162. Computation of rejection limit of log X.*

# CHAPTER 6

# TRILATERATION

## Section I. PREPARING DATA FOR ADJUSTMENT

### 84. Introduction

*a.* Trilateration is the term used to describe the establishment of horizontal geodetic control by determining the lengths of the sides of triangles. Although trilateration may be employed in all cases when triangulation is possible, it is especially adaptable under conditions which preclude the measurement of angles. Smoke, haze, instability of instruments at one or more stations, and wartime conditions are among the factors which may necessitate the use of trilateration.

*b.* The most desirable scheme of trilateration would be one consisting of one or more braced hexagons. If this is not possible, a redundancy of measured lines in the net will materially improve the accuracy of the results.

*c.* The U.S. Air Force is involved in accomplishing a special type of trilateration over long distances by methods identified as HIRAN, SHORAN, and, more recently, SHIRAN. These latter methods are especially adaptable to the establishment of control in areas where stations are not intervisible.

*d.* This chapter is only concerned with trilateration wherein the lengths have been determined from observations obtained by electronic measuring instruments over lines which could otherwise be observed by triangulation.

### 85. Preliminary Computations

*a.* The observational data must be reduced to geodetic distances in accordance with procedures outlined in chapter 4.

*b.* Triangle computations, including the determination of spherical excess, will be performed using DA Form 1920, in accordance with the methods prescribed in paragraphs 54 through 61.

*c.* Preliminary geographic positions must be computed for all new stations in accordance with the procedures set forth in paragraphs 66 through 72. This can be accomplished through a single chain of triangles. Although approximate scaled positions may be used if accurate large scale maps are available, it is not considered desirable since large differences between the actual and approximate positions will make the adjustment much more difficult.

*d.* For all measured lines, it will be necessary to have a computed azimuth and length. These are determined by inverse position computations using geographic positions. Where the lengths and azimuths have previously been determined from the computation of positions, the inverse will provide a check on the position computation.

*e.* A tabulation should be prepared, listing the azimuths of all lines with their sines and cosines to four decimal places. The difference between lengths is listed as the inverse length minus the measured length.

## Section II. TRILATERATION ADJUSTMENT BY THE METHOD OF LEAST SQUARES

### 86. Observation Equations

*a.* An observation equation in the following form must be derived for each measured side in the trilateration net:

$$V_{1-2}=X_1 \sin \alpha_{1-2}+Y_1 \cos \alpha_{1-2}+X_2 \sin \alpha_{2-1} +Y_2 \cos \alpha_{2-1}+(I-S)_{1-2}$$

Additional terms may be required if it is necessary to determine constant and calibration errors in cases where only one set of measuring instruments has been used. In the equation, East longitude and North latitude are considered positive and—

$V_{1-2}$=the residual or correction to the measured length

$X_1$=the correction in meters to the preliminary longitude for station 1

$Y_1$=the correction in meters to the preliminary latitude for station 1

$\alpha_{1-2}$=the geodetic azimuth (from south) from station 1 to station 2

I=the inversed distance in meters

S=the fixed or measured distance in meters

*b.* Having derived equations for each of the lines, it is necessary to eliminate the known conditions. Thus, if the longitude and latitude of a station is fixed, the appropriate X (sin $\alpha$) and Y (cos $\alpha$) terms would be eliminated. Also, if I and S are identical, these terms would be dropped.

## 87. Conditions Equations

The condition equations which enter into the adjustment consist of those for fixed lengths and for fixed azimuths. An equation must be derived for each fixed condition in the net.

*a.* The fixed length equation is in the following form:

$$0=X_1 \sin \alpha_{1-2}+Y_1 \cos \alpha_{1-2}+X_2 \sin \alpha_{2-1} +Y_2 \cos \alpha_{2-1}+(I-S)_{1-2}$$

*b.* The fixed azimuth equation is in the following form:

$$0=X_1 \frac{\sin \alpha_{1-2}}{\tan \alpha_{2-1}}-Y_1 \sin \alpha_{1-2}+X_2 \cos \alpha_{2-1} -Y_2 \sin \alpha_{2-1}-\Delta\alpha_{1-2} S \sin 1''$$

Where $\Delta\alpha_{1-2}$=the difference in seconds between the fixed azimuth and the inversed azimuth

S=the fixed or measured length of the line

*c.* If the azimuth equation involves an astronomic azimuth and hence a LaPlace condition, the formula in *b* above becomes:

$$X_1\left(\frac{\sin \alpha_{1-2}}{\tan \alpha_{2-1}}-\frac{S \tan \phi_1}{N}\right)-Y_1 \sin \alpha_{1-2} +X_2 \cos \alpha_{2-1}-Y_2 \sin \alpha_{2-1}=\Delta\alpha$$

Where: $\Delta\alpha=[\alpha_a-\alpha_p+\sin \phi_1(\lambda_a-\lambda_p)]I \sin 1''$

*a*=astronomic

*p*=preliminary

I=preliminary inversed length

N=radius of prime vertical

## 88. Probable Error

The probable error of a single observation is obtained by the following formula:

$$p.e.=\pm.6745\sqrt{\frac{\Sigma V^2}{n-c}}$$

Where: V=residual of an observation equation

*n*=number of observation

*c*=number of unknowns

## 89. Solution of Equations

*a.* The two methods that are generally used for solving the observation and condition equations are outlined below.

(1) *Vertical insertion method.* In this method, the normal equations are formed from the observation equations and then the condition equations are written in vertical columns following the normal equations so that the coefficient of each unknown in the condition equation is on the horizontal line corresponding to the normal equation for that unknown.

(2) *Substitution method.* In this method, one unknown in each condition equation is solved for in terms of one of the unknowns in the corresponding observation equation. These revised observation equations are used to form the normal equations. Values determined from the solution of the normal equations are used to solve the condition equations.

*b.* The forward and back solutions of the normal equations are performed in the same manner as in the triangulation adjustment which is fully covered in paragraphs 62 through 65.

## 90. Application of Adjustment

*a.* The values of X and Y for each station as obtained from the solution are the corrections in the longitude and latitude respectively. As these values are in meters, they must be converted to seconds of arc before being applied to the preliminary positions to obtain the final positions. Using the preliminary latitude as an argument, a shift in latitude is obtained by dividing Y by "meters per second of arc", and a shift in longitude is obtained by multiplying X by H. "Meters per second of arc" and H can be obtained from the following USC&GS publications: for the Clarke 1866 spheroid, use Sp. Pub. 241; for the International spheroid, use Sp. Pub. G–58.

*b.* The final lengths are obtained by applying the V's, obtained from the solution, to the preliminary lengths. As a check of the solution, these lengths should agree very closely with the lengths obtained by inversing the final positions.

# CHAPTER 7

# TRIANGULATION–TRILATERATION COMBINATION

## Section I. PREPARING DATA FOR ADJUSTMENT

### 91. Introduction

The use of electronic distance measuring equipment in conjunction with triangulation instruments has produced geodetic figures in which both the directions and the lengths are measured. In the adjustment of these figures, corrections for both the directions and the lengths are obtained simultaneously.

### 92. Observed Directions

The observed directions to be used in the adjustment are abstracted and listed as explained in paragraphs 54 through 61.

### 93. Measured Lengths

The measured lengths to be used in the adjustment are computed as explained in paragraphs 47 through 53.

### 94. Relative Weights for Lengths and Directions

*a.* The weights ($p$) for directions and lengths are inversely proportional to the squares of the mean errors (M), also known as standard deviations and standard errors. Appropriate mean errors, assumed in accordance with *b* and *c* below, are used in the least squares adjustment. Unless there is a wide disparity between the assumed and true weights of the observed quantities, the correction obtained from the adjustment will not be appreciably altered by a different weighting system.

*b.* The relative mean errors assumed for directions are as follows:

(1) For first-order and modified second-order triangulation observations (normally 16 positions with precise instruments), assume a mean error of 0.6 second of arc.

(2) For second-order triangulation observations (normally 8 or more positions with precise instruments), assume a mean error of 1.0 seconds of arc.

(3) For third-order triangulation observations, depending upon the number of observations and instruments used, assume a mean error of 2.0 to 5.0 seconds of arc.

(4) The weights for directions based on the above mean errors would be as follows:

$$p=\left(\frac{1}{0.6}\right)^2=2.78 \text{ for first-order.}$$

$$p=\left(\frac{1}{1}\right)^2=1.00 \text{ for second order.}$$

$$p=\left(\frac{1}{2}\right)^2 \text{ to } \left(\frac{1}{5}\right)^2=0.25 \text{ to } 0.04 \text{ for third order.}$$

*c.* The relative mean errors for lengths will vary extensively depending upon the type of equipment used, the length of line measured, the number of observations, the interval between measurements, and the spread of the measurements. Since all manufacturers specify an inherent error in their instruments regardless of line length, short lines (under 1000 meters) should be assigned a low relative accuracy. However, when the lines exceed 8 kilometers, with at least four separate determinations of the length having residuals that seldom exceed $\pm 10$ ppm., an equal relative accuracy for all lengths may be assumed. For the example problem, a relative accuracy of 1/200,000 has been assumed.

(1) If natural length values (not logarithms) are to be used in the adjustment, individual weights must be determined by the following formula:

$$\frac{1}{p}=(\text{S}\times\text{relative accuracy})^2$$

S must be in units that will limit $p$ to values compatible with the weight values used for the directions. That is, for a 10 kilometer length with the correction to be determined in centimeters (S=10 km=1,000,000 cm) and a relative accuracy of 1/200,000;

$$\frac{1}{p}=\left[\frac{10^6}{2(10^5)}\right]^2=25$$

(2) If logarithm values for the lengths are to be used with the corrections to these logarithms determined in the adjustment, then equal weights can be used for all lengths of equal relative accuracy. The mean error for the logarithm lengths are computed by multiplying the modulus of common logarithms by the relative accuracy.

mean error=relative accuracy $\times$ 0.4343, i.e. for a 1/200,000 relative accuracy, the mean error=$\frac{0.4343}{0.2(10^6)}$ =2.171 in units of the sixth decimal place of logs, and $p=\left(\frac{1}{2.17}\right)^2=0.2124$.

## Section II. ADJUSTMENT USING COMBINED MEASUREMENTS

### 95. Angle Equations

The angle equations for a combination adjustment are formed in the same manner as for an adjustment of triangulation (ch. 5). In a triangle, the sum of the three adjusted angles must equal 180° plus the spherical excess ($\epsilon$). Since the sum of the observed angles minus (180°+$\epsilon$) equals the triangle error ($n$), the sum of the corrections ($v's$) to the directions plus $n$ must equal zero to satisfy the angle condition.

$$\sum v's+n=0$$

### 96. Length Equations

All length equations are derived from the sine formula:

$$a/\sin A=b/\sin B=c/\sin C$$

A, B, C, are the angles of a triangle and $a$, $b$, $c$, the opposite sides.

*a.* Considering only a single triangle, if the adjustment is to be made using natural functions, the length equation would be derived as follows:

$$(a+v_a)\sin(B+v_B-\tfrac{1}{3}\epsilon)-(b+v_b)\sin(A+v_A-\tfrac{1}{3}\epsilon)=0$$

substituting: $B'=B-\frac{1}{3}\epsilon$, $A'=A-\frac{1}{3}\epsilon$

$$(a+v_a)\sin(B'+v_B)-(b+v_b)\sin(A'+v_A)=0$$

expanding, and neglecting the small terms, the length equation becomes:

$$a\sin B'+a\delta_B v_B+\sin B'v_a-b\sin A'-b\delta_A v_A-\sin A'v_b=0$$

where $\delta_A$ and $\delta_B$ are the differences in the natural sine for an increase of 1″ in angles A′ and B′ respectively.

*b.* Where the adjustment is to be made using logarithmic functions, the observed spherical angles can be used directly by including the arc-sine corrections ($C_s$) for the lengths in the length equation. The length equation then becomes:

$$C_{s(a)}+\log a+\Delta a+\log\sin B+\delta_B v_B-C_{s(b)}-\log b-\Delta b-\log\sin A-\delta_A v_A=0.$$

Where: $C_{s(a)}$ and $C_{s(b)}$ are the log arc-sine corrections, $\Delta a$ and $\Delta b$ are the corrections to the log lengths, and $\delta_A$ and $\delta_B$ are the differences in the log sines for an increase of 1″ in angles A and B respectively.

### 97. Quadrilateral Adjustment (Combined Measurements)

*a.* Figure 163 is a diagram of a quadrilateral in which all directions and lengths are measured. The directions, 1 through 12, are numbered clockwise around each station, and the lengths are lettered $a$ through $f$.

*b.* The observed directions, corrected for eccentricity and elevation, and the measured spheroidal lengths are tabulated in figure 164. These values are normally obtained from the Lists of Directions (DA Form 1917) and Electronic Distance Measurement Summary sheets (DA Form 2854). In this solution, the directions are considered to have a mean error of unity, and the lengths are considered to have a relative accuracy of 1/200,000.

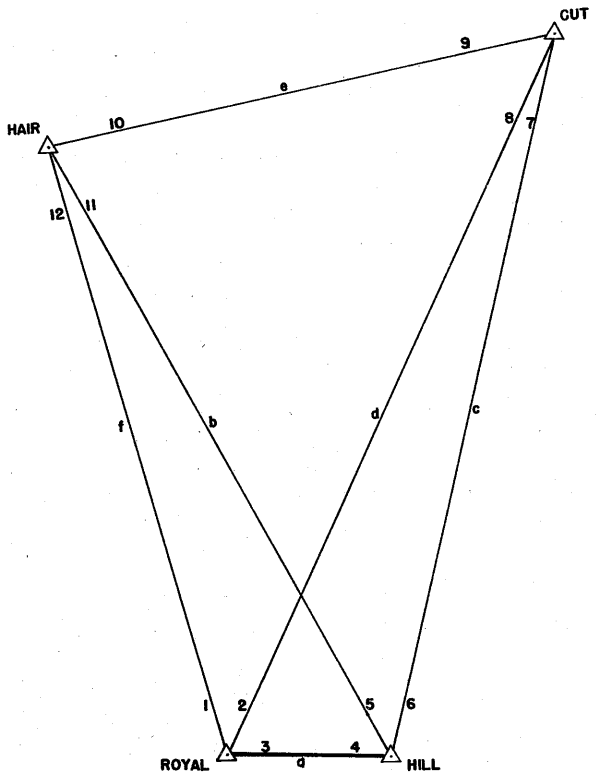



*Figure 163. Quadrilateral sketch (combination adjustment).*

*c.* A preliminary computation of triangles is made to obtain the spherical excess and the triangle closures. The angle equations are formed using any three of the four triangles in the quadrilateral. The angle equations chosen for this adjustment are as follows:

$$0 = -0.47 - (2) + (3) - (4) + (6) - (7) + (8)$$
$$0 = +4.04 - (1) + (2) - (8) + (9) - (10) + (12)$$
$$0 = +2.00 - (1) + (3) - (4) + (5) - (11) + (12)$$

*d.* Since all the lengths of the quadrilateral have been observed, the side equation is replaced with a length equation and five additional length equations are required. There are 15 possible length equations from which the computer may select any six which include all lengths. In the example (fig. 165), logarithmic equations were set up for $a$ to $c$, $c$ to $d$, $d$ to $e$, $e$ to $f$, $f$ to $b$, and $b$ to $a$. The length of $a$ is fixed since it has been considered to be a taped base and, therefore, no correction for this length can be inserted into the equations.

*e.* The correlate equations (fig. 166) are formed from the nine condition equations (three angle and six length). The values entered in the weight column of the correlate sheet are the reciprocals of the weights ($1/p$). As previously discussed (par. 94), $1/p$ is equal to the square of the mean error ($M^2$). In this solution, the mean errors of the directions are unity. However, for convenience in other solutions, where the mean error of the directions is not unity, a constant (K) can be applied to the weight column to change the direction weight to unity. This will cause a proportionate change in the length weights. Thus, if the mean error of the directions is 0.″ 6, $M^2=0.36$, and a K value of 2.77778 multiplied by the $M^2$ column will change the direction weights to 1.00.

*f.* The normal equations (fig. 167) are obtained by taking the algebraic sums of $KM^2$ times the products of the various columns in the correlates.

*g.* The solution of the normal equations is completed as explained in chapter 5. Apply the C's to the correlate equations to obtain the $v$'s, remembering to multiply the products of the coefficients by the C values, then by $KM^2$.

*h.* The $v$'s are applied to the directions and a final computation of triangles made (fig. 168). The adjusted lengths obtained from the triangles should agree closely with the lengths from the adjustment.

*i.* When the corrections to either the lengths or the directions appear unreasonable, it may be necessary to re-evaluate the field observations and assign new weights to obtain a more compatible solution.

| PROJECT: BETTY | | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|---|
| LOCATION: JASPER | | | ORGANIZATION: USAMS | |
| STATION | OBSERVED DIRECTION | MEAN ERROR | | |
| ROYAL | | | | |
| 1. HAIR | 00° 00' 00.00" | ± 1.00" | | |
| 2. CUT | 38 49 49.28 | ± 1.00 | | |
| 3. HILL | 118 11 13.65 | ± 1.00 | | |
| | | | | |
| HILL | | | | |
| 4. ROYAL | 00° 00' 00.00" | ± 1.00 | | |
| 5. HAIR | 50 56 47.47 | ± 1.00 | | |
| 6. CUT | 87 32 55.86 | ± 1.00 | | |
| | | | | |
| CUT | | | | |
| 7. HILL | 00° 00' 00.00" | ± 1.00 | | |
| 8. ROYAL | 13 05 40.69 | ± 1.00 | | |
| 9. HAIR | 78 09 17.57 | ± 1.00 | | |
| | | | | |
| HAIR | | | | |
| 10. CUT | 00° 00' 00.00" | ± 1.00 | | |
| 11. HILL | 65 14 39.49 | ± 1.00 | | |
| 12. ROYAL | 76 06 41.53 | ± 1.00 | | |
| | | | | |
| | | | | |
| | | | | |
| | MEASURED LENGTH (M.) | | LOG. LENGTH (M.) | |
| ROYAL-HILL = a | 11260.189 | (BASE) | 4.051 54568 | |
| ROYAL-CUT = d | 49655.842 | ± 0.248 | 4.695 97035 | ± 2.17 × 10⁻⁶ |
| ROYAL-HAIR = f | 46381.616 | ± 0.232 | 4.666 34588 | " |
| HILL-HAIR = b | 52644.175 | ± 0.263 | 4.721 35032 | " |
| HILL-CUT = c | 48846.311 | ± 0.244 | 4.688 83177 | " |
| CUT-HAIR = e | 32072.371 | ± 0.160 | 4.506 13106 | " |
| | | | | |
| | | | | |
| | | | | |

| TABULATED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R. A. Smith - AMS | FEB. '64 | L. E. Smith - AMS | FEB. '64 |

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*Figure 164. Observed directions and measured lengths (combination adjustment).*

PROJECT: BETTY

~~SIDE~~/LENGTH EQUATIONS (TM 5-237)

LOCATION: JASPER

ORGANIZATION: USAMS

DATE

| Symbol | Angle | Log. Sine | Tab. Diff. | Symbol | Angle | Log. Sine | Tab. Diff. |
|---|---|---|---|---|---|---|---|
| | | - 23 | | | | - 424 | |
| a | 11260.189 | 4.05154568 | — | c | 48846.311 | 4.68883177 | $\Delta_c$ |
| -2+3 | 79° 21' 24.37" | 9.99246350 | +0.39 | -7+8 | 13° 05' 40.69" | 9.35518348 | +9.05 |
| | | 14.04400895 | | | | 14.04401100 | |

④ $0 = -2.05 - 0.39(2) + 0.39(3) + 9.05(7) - 9.05(8) - \Delta_c$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | - 424 | | | | - 438 | |
| c | 48846.311 | 4.68883177 | $\Delta_c$ | d | 49655.842 | 4.69597035 | $\Delta_d$ |
| -4+6 | 87 32 55.86 | 9.99960246 | +0.09 | -2+3 | 79 21 24.37 | 9.99246350 | +0.39 |
| | | 14.68842999 | | | | 14.68842947 | |

⑤ $0 = +0.52 + 0.39(2) - 0.39(3) - 0.09(4) + 0.09(6) + \Delta_c - \Delta_d$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | - 438 | | | | - 183 | |
| d | 49655.842 | 4.69597035 | $\Delta_d$ | e | 32072.371 | 4.50613106 | $\Delta_e$ |
| -1+2 | 38 49 49.28 | 9.79727905 | +2.62 | -10+12 | 76 06 41.53 | 9.98711404 | +0.52 |
| | | 14.49324502 | | | | 14.49324327 | |

⑥ $0 = +1.75 - 2.62(1) + 2.62(2) + 0.52(10) - 0.52(12) + \Delta_d - \Delta_e$

ANGLE EQUATIONS

① $0 = -0.47 - (2) + (3) - (4) + (6) - (7) + (8)$

② $0 = +4.04 - (1) + (2) - (8) + (9) - (10) + (12)$

③ $0 = +2.00 - (1) + (3) - (4) + (5) - (11) + (12)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | - 183 | | | | - 382 | |
| e | 32072.371 | 4.50613106 | $\Delta_e$ | f | 46381.616 | 4.66634588 | $\Delta_f$ |
| -8+9 | 65 03 36.88 | 9.95748835 | +0.98 | -1+2 | 38 49 49.28 | 9.79727905 | +2.62 |
| | | 14.46361758 | | | | 14.46362111 | |

⑦ $0 = -3.53 + 2.62(1) - 2.62(2) - 0.98(8) + 0.98(9) + \Delta_e - \Delta_f$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | - 382 | | | | - 492 | |
| f | 46381.616 | 4.66634588 | $\Delta_f$ | b | 52644.175 | 4.72135032 | $\Delta_b$ |
| -1+3 | 118 11 13.65 | 9.94517779 | -1.13 | -4+5 | 50 56 47.47 | 9.89017401 | +1.71 |
| | | 14.61151985 | | | | 14.61151941 | |

⑧ $0 = +0.44 + 1.13(1) - 1.13(3) + 1.71(4) - 1.71(5) - \Delta_b + \Delta_f$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | - 492 | | | | - 23 | |
| b | 52644.175 | 4.72135032 | $\Delta_b$ | a | 11260.189 | 4.05154568 | — |
| -11+12 | 10 52 02.04 | 9.27538932 | +10.97 | -1+3 | 118 11 13.65 | 9.94517779 | -1.13 |
| | | 13.99673472 | | | | 13.99672324 | |

⑨ $0 = +11.48 - 1.13(1) + 1.13(3) - 10.97(11) + 10.97(12) + \Delta_b$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
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*Figure 165. Condition equations (combination adjustment).*

| V \ EQ. | $(1/p) = m^2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | $\Sigma c$ | V | ADOPTED V | LOG LENGTH CORR. $\times 10^6$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.00 | | -1 | -1 | · | | -2.62 | +2.62 | +1.13 | -1.13 | -2.00 | +.7314 | +0.73 | | 1 |
| 2 | 1.00 | -1 | +1 | | -0.39 | +0.39 | +2.62 | -2.62 | | | 0.00 | -.6545 | -0.66 | | 2 |
| 3 | 1.00 | +1 | | +1 | +0.39 | -0.39 | | | -1.13 | +1.13 | +2.00 | -.0769 | -0.08 | | 3 |
| 4 | 1.00 | -1 | | -1 | | -0.09 | | | +1.71 | | -0.38 | +.0193 | +0.02 | | 4 |
| 5 | 1.00 | | | +1 | | | | | -1.71 | | -0.71 | -.1586 | -0.16 | | 5 |
| 6 | 1.00 | +1 | | | | +0.09 | | | | | +1.09 | +.1393 | #0.14 | | 6 |
| 7 | 1.00 | -1 | | | +9.05 | | | | | | +8.05 | +.4272 | +0.43 | | 7 |
| 8 | 1.00 | +1 | -1 | | -9.05 | | | -0.98 | | | -10.03 | +.1995 | +0.20 | | 8 |
| 9 | 1.00 | | +1 | | | | | +0.98 | | | +1.98 | -.6267 | -0.63 | | 9 |
| 10 | 1.00 | | -1 | | | | +0.52 | | | | -0.48 | +.8805 | +0.88 | | 10 |
| 11 | 1.00 | | | -1 | | | | | | -10.97 | -11.97 | +.0666 | +0.07. | | 11 |
| 12 | 1.00 | | +1 | +1 | | | -0.52 | | | +10.97 | +12.45 | -.9471 | -0.94 | | 12 |
| $\Delta b$ | 4.7089 | | | | | | | | -1 | +1 | 0.00 | +.5533 | | +0.55 | $\Delta b$ |
| $\Delta c$ | 4.7089 | | | | -1 | +1 | | | | | 0.00 | +.2298 | | +0.23 | $\Delta c$ |
| $\Delta d$ | 4.7089 | | | | | -1 | +1 | | | | 0.00 | +.5354 | | +0.54 | $\Delta d$ |
| $\Delta e$ | 4.7089 | | | | | | -1 | +1 | | | 0.00 | -.3951 | | -0.40 | $\Delta e$ |
| $\Delta f$ | 4.7089 | | | | | | | -1 | +1 | | 0.00 | -1.1042 | | -1.10 | $\Delta f$ |

*Figure 166. Correlate equations (combined adjustment).*

| V \ EQ. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | n | $\Sigma n$ | $c_w$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | +6 | -2 | +2 | -17.32 | -0.60 | -2.62 | +1.64 | -2.84 | +1.13 | -0.47 | -15.08 | +0.1294 |
| 2 | | +6 | +2 | +8.66 | +0.39 | +4.20 | -3.28 | -1.13 | +12.10 | +4.04 | +30.98 | -0.7640 |
| 3 | | | +6 | +0.39 | -0.30 | +2.10 | -2.62 | -5.68 | +24.20 | +2.00 | +30.09 | -0.3200 |
| 4 | | | | +168.8181 | -5.0131 | -1.0218 | +9.8908 | -0.4407 | +0.4407 | -2.05 | +162.3540 | +0.0615 |
| 5 | | | | | +9.7382 | -3.6871 | -1.0218 | +0.2868 | -0.4407 | +0.52 | -0.1277 | +0.1103 |
| 6 | | | | | | +23.6874 | -18.4377 | -2.9606 | -2.7438 | +1.75 | +0.2664 | +0.2240 |
| 7 | | | | | | | +25.0674 | -1.7483 | -2.9606 | -3.53 | +2.9998 | +0.1401 |
| 8 | | | | | | | | +17.8198 | -7.2627 | +0.44 | -3.5157 | -0.0944 |
| 9 | | | | | | | | | +247.9445 | +11.48 | +283.8874 | +0.0231 |

*Figure 167. Normal equations (combination adjustment).*

| PROJECT | | DATE | COMPUTATION OF TRIANGLES (TM 5-237) |
|---|---|---|---|
| BETTY | | | |
| LOCATION | JASPER | ORGANIZATION | USAMS |

| | STATION | OBSERVED ANGLE | CORRECTION | SPHERICAL ANGLE | SPHERICAL EXCESS | PLANE ANGLE | FUNCTION: LOGARITHM OR NATURAL |
|---|---|---|---|---|---|---|---|
| | | | | $\phi = 51°$ | 40′ | | |
| | 2-3 HILL - ROYAL | | | 11260 | .189M | | 4.05154568 |
| -7+8 | 1 CUT | 13 05 40.69 | -0.23 | 40.46 | 0.46 | 40.00 | 9.35517723 |
| -4+6 | 2 HILL | 87 32 55.86 | +0.12 | 55.98 | 0.47 | 55.51 | 9.99960243 |
| -2+3 | 3 ROYAL | 79 21 24.37 | +0.58 | 24.95 | 0.46 | 24.49 | 9.99246355 |
| | 1-3 CUT - ROYAL | (4.6959 | 7089 = | 4965 | 5.904 | M.)* | 4.69597088 |
| | 1-2 CUT - HILL | (4.6888 | 3200 = | 4884 | 6.336 | M.) | 4.68883200 |
| | | 00.92 | +0.47 | | 1.39 | | |
| | 2-3 CUT - ROYAL | | | | | | 4.69597088 |
| -10+12 | 1 HAIR | 76 06 41.53 | -1.82 | 39.71 | 1.21 | 38.50 | 9.98711246 |
| -8+9 | 2 CUT | 65 03 36.88 | -0.83 | 36.05 | 1.22 | 34.83 | 9.95748635 |
| -1+2 | 3 ROYAL | 38 49 49.28 | -1.39 | 47.89 | 1.22 | 46.67 | 9.79727223 |
| | 1-3 HAIR - ROYAL | (4.6663 | 4478 = | 4638 | 1.499 | M.)* | 4.66634477 |
| | 1-2 HAIR - CUT | (4.5061 | 3066 = | 3207 | 2.341 | M.) | 4.50613065 |
| | | 07.69 | -4.04 | | 3.65 | | |
| | 2-3 HILL - ROYAL | | | | | | 4.05154568 |
| -11+12 | 1 HAIR | 10 52 02.04 | -1.01 | 01.03 | 0.39 | 00.64 | 9.27537396 |
| -4+5 | 2 HILL | 50 56 47.47 | -0.18 | 47.29 | 0.39 | 46.90 | 9.89017304 |
| -1+3 | 3 ROYAL | 118 11 13.65 | -0.81 | 12.84 | 0.38 | 12.46 | 9.94517913 |
| | 1-3 HAIR - ROYAL | | | | | | 4.66634476 |
| | 1-2 HAIR - HILL | (4.7213 | 5087 = | 5264 | 4.241 | M.)* | 4.72135085 |
| | | 03.16 | -2.00 | | 1.16 | | |
| | 2-3 HILL - HAIR | | | | | | 4.72135085 |
| -7+9 | 1 CUT | 78 09 17.57 | -1.06 | 16.51 | 1.30 | 15.21 | 9.99065123 |
| -5+6 | 2 HILL | 36 36 08.39 | +0.30 | 08.69 | 1.29 | 07.40 | 9.77543107 |
| -10+11 | 3 HAIR | 65 14 39.49 | -0.81 | 38.68 | 1.29 | 37.39 | 9.95813237 |
| | 1-3 CUT - HAIR | | | | | | 4.50613069 |
| | 1-2 CUT - HILL | | | | | | 4.68883199 |
| | | 05.45 | -1.57 | | 3.88 | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
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*Figure 168. Triangle computation (combination adjustment).*

# CHAPTER 8

# GEOGRAPHIC TRAVERSE

## Section I. INTRODUCTION

### 98. Definition of Traverse

A geographic traverse is a sequence of lengths and directions of lengths between points on the earth. Each length is accurately measured and an angle is observed at the beginning and end of each length to determine the direction of each length. Geodetic traverses are normally closed traverses starting at a station of known position with a fixed azimuth to a mark (or other station) and ending on a station of known position with a fixed azimuth to a mark.

### 99. Types of Traverse

*a. Electronic Traverse.* Traverses whose lengths are measured by electronic distance measuring equipment are known as electronic traverses. This equipment is of two types; one uses light waves to measure distances, the other type uses radio microwaves to measure distances. As the lines measured by electronic methods are usually long and of high order, this type of traverse should be adjusted by the least squares method.

*b. Taped Traverse.* Traverses whose lengths are usually measured by steel or invar tapes although other types of tapes may be used are referred to as taped traverses. These types of traverses may be of a high or low order accuracy.

*c. Tachymetry Traverse.* These are traverses whose lengths are measured directly by tacheometer, or using tachymetry methods such as stadia or subtense measurements. These types of traverses are of a low order accuracy and the approximate method of adjustment should be used.

### 100. Traverse Specifications

Appendix III of TM 5–441 contains the general specifications for traverses. There will occasionally be changes from these general specifications required on surveying projects. The computer should check the project specifications for these changes.

## Section II. ADJUSTMENT OF TRAVERSE (LEAST SQUARES METHOD)

### 101. Purpose

The least squares adjustment of a traverse is used to simultaneously eliminate closing errors in azimuths and geographic positions. These closing errors are caused by normal observation errors in either the observed angles or in the measured lengths, or a combination of both. Some of the error is due to previous adjustments of the control surveys establishing the geographic positions of the known stations. This error, however, must be considered zero. In a traverse, there is no mathematical relation between the lengths of courses and the angles between them. However, the adjustment procedure used in the following example assumes the weight of any angular correction is unity, while the weight of a linear correction is inversely proportional to the square of the length of the course. There will probably be cases when it is necessary to use other systems of weighting. These can be determined using the methods explained in chapter 7.

### 102. Preliminary Positions

*a.* The traverse chosen for an example contains a total of 22 angle stations and 21 measured

lengths (fig. 169). The total length of the traverse is 248 kilometers and the preliminary geographic positions showed a linear closure of 1:31,950. The system of weighting given above was used in this example, as the discrepancies in the angles and lengths appeared to be distributed proportionally.

*b.* The starting and ending positions of the traverse, three fixed azimuths, one at each end and one in the center of the traverse, are—

| STATION | LATITUDE (North) | LONGITUDE (West) | AZIMUTH (From south) | TO STATION |
|---|---|---|---|---|
| Roy | 25°47′53.784″ | 64°36′18.478″ | 75°31′25.3″ | Roy Azimuth Mk. |
| 10 | | | 105 04 24.3 | 10 Azimuth Mk. |
| Dallas | 26 10 23.922 | 65 57 18.338 | 345 38 49.7 | Dallas Azimuth Mk. |

The observed angles (fig. 170) and the electronically measured lengths (fig. 171) were used to compute the preliminary geographic positions using the method given in paragraphs 66 through 72. The fixed position and azimuth at station ROY was used as a starting point for the computation of the preliminary geographic positions. The closures in position and azimuth were made at station DALLAS. The azimuth at station number 10 was not held fixed in the preliminary geographic position computations. A tabulation of these positions, along with $\Delta\alpha$ (Geodetic convergence), is shown in figure 172.

## 103. Condition Equations

*a.* Two azimuth, one latitude, and one longitude equations were required in this adjustment. Numerals were used to designate the angles, and letters were used to designate the lengths. Due to the two observed angles at station number 10, 10A was introduced to designate the angle turned to the azimuth mark.

*b.* The first azimuth equation was formed by starting with the azimuth of the line, ROY to azimuth mark, and was closed on the azimuth of line DALLAS to azimuth mark. The absolute term in this equation consists of two errors; the error in the closing line (computed minus fixed azimuth), and the convergence error due to the change in the final longitude of the closing point from the computed to the fixed value. The formula for this error is as follows:

$$E_c=(\lambda_n-\lambda_f)\sin\phi_n$$

Where:

$E_c$=the convergence error.
$\lambda_f$=fixed longitude of the closing point.
$\lambda_n$=computed longitude of the closing point.
$\phi_n$=computed latitude of the closing point.
East longitude being positive.
North latitude being positive.

In this example, $E_c=(-18''.239+18''.338)\sin 26°10'24''=+0''.044$.

Since the angles were only used to the nearest tenth of a second, this error became negligible.

*c.* The second azimuth equation was formed by starting with the azimuth ROY to azimuth mark and was closed on the azimuth, station 10 to azimuth mark. As there was no fixed position at station 10, no convergence correction was determined.

*d.* The first azimuth equation showed an error of 11″.0, and the second azimuth equation showed an error of 4″.2. As the azimuth closure was small (less than 1 second per station), no azimuth adjustment was made prior to the least squares adjustment. In cases, however, where the azimuth discrepancies are quite large, it might be better to adjust the azimuth first, by dividing the azimuth error equally among the angles so that the azimuth closure becomes zero. In this case each angle carries the same weight. It would then be necessary to recompute the preliminary geographic positions and a new azimuth equation or equations would be formed for the traverse.

*e.* The latitude and longitude equations are expressed as follows:

Latitude:

$$0=7238.24(\phi_n-\phi_f)''+[(\phi_2-\phi_1)'(V_{s1})+(\phi_3-\phi_2)'(V_{s2})+\ldots+(\phi_n-\phi_{n-1})'(V_{sn})]+a_1[(\lambda_n-\lambda_1)'(V_{c1})+(\lambda_n-\lambda_2)'(V_{c2})+\ldots+(\lambda_n-\lambda_{n-1})'(V_{cn})].$$




*Figure 169. Diagram of traverse.*

| PROJECT | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|
| LOCATION | | ORGANIZATION | |

| STATION | OBSERVED | | ADJUSTED | |
|---|---|---|---|---|
| | FROM - TO | ANGLE | CORRECTION | ANGLE |
| ROY | Az. Mk. - 1 | 30° 41' 44."9 | - 2."5 | 30° 41' 42."4 |
| 1 | ROY - 2 | 220 08 07.9 | - 0.6 | 220 08 07.3 |
| 2 | 1 - 3 | 179 55 13.4 | - 0.6 | 179 55 12.8 |
| 3 | 2 - 4 | 179 58 20.0 | - 0.6 | 179 58 19.4 |
| 4 | 3 - 5 | 179 57 03.2 | - 0.5 | 179 57 02.7 |
| 5 | 4 - 6 | 179 59 19.6 | - 0.5 | 179 59 19.1 |
| 6 | 5 - 7 | 180 02 13.4 | - 0.5 | 180 02 12.9 |
| 7 | 6 - 8 | 179 57 03.5 | - 0.5 | 179 57 03.0 |
| 8 | 7 - 9 | 180 31 20.6 | - 0.5 | 180 31 20.1 |
| 9 | 8 - 10 | 134 05 17.1 | - 0.5 | 134 05 16.6 |
| 10 | 9 - Az. Mk. | 184 32 17.2 | + 3.1 | 184 32 20.3 |
| 10 | 9 - 11 | 263 14 26.5 | - 1.6 | 263 14 24.9 |
| 11 | 10 - 12 | 110 49 11.1 | - 3.5 | 110 49 07.6 |
| 12 | 11 - 13 | 166 09 46.6 | - 1.9 | 166 09 44.7 |
| 13 | 12 - 14 | 150 14 40.7 | - 0.2 | 150 14 40.5 |
| 14 | 13 - 15 | 241 53 36.5 | 0.0 | 241 53 36.5 |
| 15 | 14 - 16 | 160 18 51.2 | + 0.5 | 160 18 51.7 |
| 16 | 15 - 17 | 168 24 05.4 | + 1.3 | 168 24 06.7 |
| 17 | 16 - 18 | 233 14 00.9 | + 2.1 | 233 14 03.0 |
| 18 | 17 - 19 | 55 19 58.4 | + 1.8 | 55 20 00.2 |
| 19 | 18 - 20 | 139 05 24.1 | + 6.3 | 139 05 30.4 |
| 20 | 19 - DALLAS | 151 47 52.9 | + 6.9 | 151 47 59.8 |
| DALLAS | 20 - Az. Mk. | 204 55 26.6 | + 6.6 | 204 55 33.2 |

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*Figure 170. Traverse angles.*

| PROJECT | | TABULATION OF GEODETIC DATA (TM 5-237) | | |
|---|---|---|---|---|
| LOCATION. | | ORGANIZATION | | |
| STATION FROM - TO | | MEASURED LENGTH METERS | CORRECTION METERS | FINAL LENGTH METERS |
| ROY - 1 | a | 21942.5400 | -.0535 | 21942.4865 |
| 1 - 2 | b | 5170.6788 | -.0039 | 5170.6749 |
| 2 - 3 | c | 5893.1010 | -.0051 | 5893.0959 |
| 3 - 4 | d | 2455.5300 | -.0009 | 2455.5291 |
| 4 - 5 | e | 4346.9958 | -.0027 | 4346.9931 |
| 5 - 6 | f | 3652.0437 | -.0020 | 3652.0417 |
| 6 - 7 | g | 1573.9597 | -.0004 | 1573.9593 |
| 7 - 8 | h | 2154.6322 | -.0007 | 2154.6315 |
| 8 - 9 | i | 2670.9276 | -.0011 | 2670.9265 |
| 9 - 10 | j | 20799.8896 | -.0438 | 20799.8458 |
| 10 - 11 | k | 23773.1425 | -.0667 | 23773.0758 |
| 11 - 12 | l | 22451.3322 | -.0627 | 22451.2695 |
| 12 - 13 | m | 17161.9140 | -.0299 | 17161.8841 |
| 13 - 14 | n | 1744.3828 | -.0001 | 1744.3827 |
| 14 - 15 | o | 13257.1596 | -.0251 | 13257.1345 |
| 15 - 16 | p | 10698.9385 | -.0140 | 10698.9245 |
| 16 - 17 | q | 8125.8182 | -.0068 | 8125.8114 |
| 17 - 18 | r | 15481.9380 | -.0350 | 15481.9030 |
| 18 - 19 | s | 37427.3103 | +.0944 | 37427.4047 |
| 19 - 20 | t | 11164.3251 | +.0170 | 11164.3421 |
| 20 - DALLAS | u | 16199.0849 | +.0384 | 16199.1233 |
| | | | | |
| | | | | |
| | TOTAL LENGTH | 248105.6445 | | |

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|---|---|---|---|
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*Figure 171. Traverse lengths.*

PRELIMINARY

| PROJECT | TABULATION OF GEODETIC DATA (TM 5-237) |
|---|---|
| LOCATION, | ORGANIZATION |

| STATION FROM - TO | LATITUDE (N) | LONGITUDE (W) | CONVERGENCE Δα | |
|---|---|---|---|---|
| ROY - 1 | 25° 47' 53."784 | 64° 36' 18."478 | - 05' 29."6 | |
| 1 - 2 | 25 51 12.407 | 64 48 55.127 | - 00 45.0 | |
| 2 - 3 | 25 53 32.129 | 64 50 38.290 | - 00 51.6 | |
| 3 - 4 | 25 56 11.199 | 64 52 36.193 | - 00 21.5 | |
| 4 - 5 | 25 57 17.451 | 64 53 25.382 | - 00 38.2 | |
| 5 - 6 | 25 59 14.656 | 64 54 52.609 | - 00 32.1 | |
| 6 - 7 | 26 00 53.098 | 64 56 05.950 | - 00 13.9 | |
| 7 - 8 | 26 01 35.540 | 64 56 37.539 | - 00 19.0 | |
| 8 - 9 | 26 02 33.603 | 64 57 20.847 | - 00 23.3 | |
| 9 - 10 | 26 03 46.014 | 64 58 13.820 | - 05 23.4 | |
| 10 - 11 | 26 05 50.132 | 65 10 29.503 | + 00 24.9 | |
| 11 - 12 | 26 18 40.957 | 65 09 33.042 | - 05 26.9 | |
| 12 - 13 | 26 23 44.170 | 65 21 49.514 | - 04 30.7 | |
| 13 - 14 | 26 25 27.119 | 65 31 58.127 | - 00 26.5 | |
| 14 - 15 | 26 25 08.517 | 65 32 57.590 | - 02 36.7 | |
| 15 - 16 | 26 30 00.706 | 65 38 49.239 | - 02 38.8 | |
| 16 - 17 | 26 32 16.431 | 65 44 44.989 | - 02 08.6 | |
| 17 - 18 | 26 33 08.357 | 65 49 32.805 | - 01 47.8 | |
| 18 - 19 | 26 40 42.568 | 65 53 33.358 | - 05 00.3 | |
| 19 - 20 | 26 23 07.438 | 66 04 45.610 | + 00 34.5 | |
| 20 - DALLAS | 26 17 11.482 | 66 03 27.909 | + 02 43.4 | |
| DALLAS | 26 10 24.158 | 65 57 18.239 | | |

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*Figure 172. Preliminary positions for traverse adjustment.*

Longitude:

$$0=7238.24(\lambda_n-\lambda_f)''+[(\lambda_2-\lambda_1)'(V_{s1})$$
$$+(\lambda_3-\lambda_2)'(V_{s2})+\ldots.+(\lambda_n-\lambda_{n-1})'(V_{sn})]$$
$$+a_2[(\phi_n-\phi_1)'(V_{c1})+(\phi_n-\phi_2)'(V_{c2})+\ldots.$$
$$+(\phi_n-\phi_{n-1})'(V_{cn})].$$

Where: $\phi_n$=computed latitude of the closing point.
$\lambda_n$=computed longitude of the closing point.
$\phi_f$=fixed latitude of the closing point.
$\lambda_f$=fixed longitude of the closing point.
$V_s$ is the change in the logarithm of the distance S.
$V_c$ is the small change in seconds in the observed angles.

The minute and second symbols after the parentheses show whether the difference indicated within the parentheses is expressed in minutes or seconds.

$$a_1=10^6\mathrm{M}\frac{\mathrm{B\ arc\ 1''}}{\mathrm{A}_n\sec\phi_n}$$

$$a_2=10^6\mathrm{M}\frac{\mathrm{A}_n\sec\phi_n\ \mathrm{arc\ 1''}}{\mathrm{B}}$$

A and B are factors depending on the latitude and the dimensions of the reference spheroid. M is the modulus of the common system of logarithms. $10^6$ is used, as the sixth place of the logarithm was considered unity.

$a_1$ is (+) in the Western Hemisphere.
$a_1$ is (−) in the Eastern Hemisphere.
$a_2$ is (−) in the Northern Hemisphere.
$a_2$ is (+) in the Southern Hemisphere.

All clockwise angles are considered positive. All counterclockwise angles are considered negative. Azimuths are from south. 7238.24 is obtained from the formula $\frac{10^6\mathrm{M}}{60}$. 60 is used to change seconds to minutes.

*f.* The development of the latitude and longitude equations is shown in figure 173. A table for the values of $a_1$ and $a_2$ is given on DA Form 1927, which could be used in forming these equations. The columns that are not needed on the form can be left blank. The four condition equations completely developed are as follows:

(1) 0=− 11.0 + (R) + (1) +(2) + (3) + (4) + (5) + (6) + (7) + (8) + (9) + (10) + (11) + (12) + (13) + (14) + (15) + (16) + (17) + (18) + (19) + (20) + (21).

(2) 0=+ 4.2 + (R) + (1) + (2) + (3) + (4) + (5) + (6) + (7) + (8) + (9) + (10A).

(3) 0=+ 1708.22464 + 3.31 (*a*) + 2.33 (*b*) + 2.65 (*c*) + 1.10 (*d*) + 1.95 (*e*) + 1.64 (*f*) + 0.71 (*g*) + 0.97 (*h*) + 1.21 (*i*) + 2.07 (*j*) + 12.84 (*k*) + 5.06 (*l*) + 1.71 (*m*) − 0.31 (*n*) + 4.87 (*o*) + 2.26 (*p*) + 0.87 (*q*) + 7.57 (*r*) − 17.59 (*s*) − 5.93 (*t*) − 6.79 (*u*) + 153.88 (R) + 129.92 (1) + 126.65 (2) + 122.93 (3) + 121.37 (4) + 118.60 (5) +116.28 (6) +115.27 (7)+113.91 (8)+ 112.23 (9) +88.94 (10)+90.73 (11)+67.39 (12) + 48.13 (13) + 46.25 (14) + 35.11 (15) + 23.84 (16) + 14.73 (17) + 7.11 (18) − 14.17 (19) − 11.72 (20).

(4) 0=−716.58576 + 12.61 (*a*) + 1.72 (*b*) +1.96 (*c*) + 0.82 (*d*) + 1.46 (*e*) + 1.22 (*f*) + 0.53 (*g*) + 0.72 (*h*) + 0.88 (*i*) + 12.26 (*j*) −0.94 (*k*) + 12.28 (*l*) + 10.14 (*m*) + 0.99 (*n*) + 5.86 (*o*) + 5.93 (*p*) + 4.80 (*q*) + 4.01 (*r*) + 11.20 (*s*) − 1.29 (*t*) − 6.17 (*u*) − 52.42 (R) − 44.71 (1) − 39.28 (2) − 33.11 (3) − 30.55 (4) − 26.00 (5) − 22.18 (6) − 20.53 (7) − 18.27 (8) − 15.45 (9) − 10.62 (10) + 19.29 (11) + 31.08 (12) + 35.07 (13) + 34.34 (14) + 45.69 (15) + 50.96 (16) + 52.98 (17) + 70.62 (18) + 29.64 (19) + 15.82 (20).

## 104. Solution

The solution of the azimuths, latitude, and longitude equations is shown on figure 174. The Doolittle method was used in this solution. The correction to the observed angles ($V_c$) is in seconds of arc and is applied directly to the observed angles (fig. 170). The correction to the measured lengths ($V_s$) is in logarithms and is converted to meters before being applied to the measured

## LATITUDE AND LONGITUDE ADJUSTMENT

| STATION | ANGLE DESIGNATION | SIDE | $\phi_c$ | $\lambda_c$ | $\phi_n - \phi_c$ | $\lambda_n - \lambda_c$ | $\phi_2 - \phi_1 - \phi_n - \phi_{n-1}$ | $\lambda_2 - \lambda_1 - \lambda_n - \lambda_{n-1}$ | LAT. EQ. $(\lambda_n - \lambda_c)a_1$ | LONG. EQ. $(\phi_n - \phi_c)a_2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| ROY | R | a | 25° 47.90 | 64° 36.31 | + 22.50 | + 80.99 | + 3.31 | + 12.61 | + 153.88 | - 52.42 |
| 1 | 1 | b | 25 51.21 | 64 48.92 | + 19.19 | + 68.38 | + 2.33 | + 1.72 | + 129.92 | - 44.71 |
| 2 | 2 | c | 25 53.54 | 64 50.64 | + 16.86 | + 66.66 | + 2.65 | + 1.96 | + 126.65 | - 39.28 |
| 3 | 3 | d | 25 56.19 | 64 52.60 | + 14.21 | + 64.70 | + 1.10 | + 0.82 | + 122.93 | - 33.11 |
| 4 | 4 | e | 25 57.29 | 64 53.42 | + 13.11 | + 63.88 | + 1.95 | + 1.46 | + 121.37 | - 30.55 |
| 5 | 5 | f | 25 59.24 | 64 54.88 | + 11.16 | + 62.42 | + 1.64 | + 1.22 | + 118.60 | - 26.00 |
| 6 | 6 | g | 26 00.88 | 64 56.10 | + 9.52 | + 61.20 | + 0.71 | + 0.53 | + 116.28 | - 22.18 |
| 7 | 7 | h | 26 01.59 | 64 56.63 | + 8.81 | + 60.67 | + 0.97 | + 0.72 | + 115.27 | - 20.53 |
| 8 | 8 | i | 26 02.56 | 64 57.35 | + 7.84 | + 59.95 | + 1.21 | + 0.88 | + 113.91 | - 18.27 |
| 9 | 9 | j | 26 03.77 | 64 58.23 | + 6.63 | + 59.07 | + 2.07 | + 12.26 | + 112.23 | - 15.45 |
| 10 | 10 | k | 26 05.84 | 65 10.49 | + 4.56 | + 46.81 | + 12.84 | - 0.94 | + 88.94 | - 10.62 |
| 11 | 11 | l | 26 18.68 | 65 09.55 | - 8.28 | + 47.75 | + 5.06 | + 12.28 | + 90.73 | + 19.29 |
| 12 | 12 | m | 26 23.74 | 65 21.83 | - 13.34 | + 35.47 | + 1.71 | + 10.14 | + 67.39 | + 31.08 |
| 13 | 13 | n | 26 25.45 | 65 31.97 | - 15.05 | + 25.33 | - 0.31 | + 0.99 | + 48.13 | + 35.07 |
| 14 | 14 | o | 26 25.14 | 65 32.96 | - 14.74 | + 24.34 | + 4.87 | + 5.86 | + 46.25 | + 34.34 |
| 15 | 15 | p | 26 30.01 | 65 38.82 | - 19.61 | + 18.48 | + 2.26 | + 5.93 | + 35.11 | + 45.69 |
| 16 | 16 | q | 26 32.27 | 65 44.75 | - 21.87 | + 12.55 | + 0.87 | + 4.80 | + 23.84 | + 50.96 |
| 17 | 17 | r | 26 33.14 | 65 49.55 | - 22.74 | + 7.75 | + 7.57 | + 4.01 | + 14.73 | + 52.98 |
| 18 | 18 | s | 26 40.71 | 65 53.56 | -30.31 | + 3.74 | - 17.59 | + 11.20 | + 7.11 | + 70.62 |
| 19 | 19 | t | 26 23.12 | 66 04.76 | - 12.72 | - 7.46 | - 5.93 | - 1.29 | - 14.17 | + 29.64 |
| 20 | 20 | u | 26 17.19 | 66 03.47 | - 6.79 | -6.17 | -6.79 | - 6.17 | - 11.72 | + 15.82 |
| DALLAS | 21 | | | | | | | | | |
| | | | | | | | $\phi$ | | | $\lambda$ |
| | $\phi_n$ = | 26° 10.40 | | 21 (FIXED) | | 26° 10' | 23.922 | | 65° 57' | 18.338 |
| | $\lambda_n$ = | 65 57.30 | | 21 (COMPUTED) | | 26 10 | 24.158 | | 65 57 | 18.239 |
| | $a_1$ = | 1.90 | | (COMPUTED) | - (FIXED) | + | 00.236 | | - | 00.099 |
| | $a_2$ = | - 2.33 | | | | | | | | |
| | | | | $\phi$ = | 7238.24 | (+.236) | = +1708 | .22464 | | |
| | | | | $\lambda$ = | 7238.24 | (-.099) | = - 716 | .58576 | | |

**TABLES**

| $\phi_n$ | $a_1$ | $a_2$ |
|---|---|---|
| ° | + | − |
| 0 | 2.12 | 2.09 |
| 1 | 2.12 | 2.09 |
| 2 | 2.12 | 2.09 |
| 3 | 2.12 | 2.09 |
| 4 | 2.11 | 2.10 |
| 5 | 2.11 | 2.10 |
| 6 | 2.11 | 2.10 |
| 7 | 2.10 | 2.11 |
| 8 | 2.10 | 2.11 |
| 9 | 2.09 | 2.12 |
| 10 | 2.09 | 2.12 |
| 11 | 2.08 | 2.13 |
| 12 | 2.07 | 2.14 |
| 13 | 2.06 | 2.15 |
| 14 | 2.06 | 2.16 |
| 15 | 2.05 | 2.17 |
| 16 | 2.04 | 2.18 |
| 17 | 2.03 | 2.19 |
| 18 | 2.01 | 2.20 |
| 19 | 2.00 | 2.21 |
| 20 | 1.99 | 2.23 |
| 21 | 1.98 | 2.24 |
| 22 | 1.96 | 2.26 |
| 23 | 1.95 | 2.27 |
| 24 | 1.93 | 2.29 |
| 25 | 1.92 | 2.31 |
| 26 | 1.90 | 2.33 |
| 27 | 1.89 | 2.35 |
| 28 | 1.87 | 2.37 |
| 29 | 1.85 | 2.39 |
| 30 | 1.83 | 2.42 |
| 31 | 1.81 | 2.44 |
| 32 | 1.79 | 2.47 |
| 33 | 1.77 | 2.50 |
| 34 | 1.75 | 2.53 |
| 35 | 1.73 | 2.56 |
| 36 | 1.71 | 2.59 |
| 37 | 1.69 | 2.62 |
| 38 | 1.67 | 2.66 |
| 39 | 1.64 | 2.70 |
| 40 | 1.62 | 2.74 |
| 41 | 1.60 | 2.78 |
| 42 | 1.57 | 2.82 |
| 43 | 1.55 | 2.87 |
| 44 | 1.52 | 2.92 |
| 45 | 1.49 | 2.97 |
| 46 | 1.47 | 3.02 |
| 47 | 1.44 | 3.08 |
| 48 | 1.41 | 3.14 |
| 49 | 1.39 | 3.20 |
| 50 | 1.36 | 3.27 |
| 51 | 1.33 | 3.34 |
| 52 | 1.30 | 3.41 |
| 53 | 1.27 | 3.49 |
| 54 | 1.24 | 3.57 |
| 55 | 1.21 | 3.66 |
| 56 | 1.18 | 3.76 |
| 57 | 1.15 | 3.86 |
| 58 | 1.12 | 3.97 |
| 59 | 1.09 | 4.08 |
| 60 | 1.05 | 4.20 |
| 61 | 1.02 | 4.34 |
| 62 | .99 | 4.48 |
| 63 | .96 | 4.63 |
| 64 | .92 | 4.80 |
| 65 | .89 | 4.98 |
| 66 | .86 | 5.17 |
| 67 | .82 | 5.38 |
| 68 | .79 | 5.62 |
| 69 | .76 | 5.87 |
| 70 | .72 | 6.15 |
| 71 | .69 | 6.46 |
| 72 | .65 | 6.81 |

$\frac{M}{60} \times 10^6 = 7238.24$

*Figure 173. Development of latitude and longitude equations for traverse adjustment.*

## CORRELATE EQUATIONS

| V / EQ. | Az. | Az. | LAT. | LONG. | Σc | V | ADOPT. V | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | | | | |
| R | +1 | +1 | +15.388 | - 5.242 | +12.146 | -2.49907 | -2.5 | |
| 1 | +1 | +1 | +12.992 | - 4.471 | +10.521 | - .56988 | -0.6 | |
| 2 | +1 | +1 | +12.665 | - 3.928 | +10.737 | - .56004 | -0.6 | |
| 3 | +1 | +1 | +12.293 | - 3.311 | +10.982 | - .54843 | -0.6 | |
| 4 | +1 | +1 | +12.137 | - 3.055 | +11.082 | - .54197 | -0.5 | |
| 5 | +1 | +1 | +11.860 | - 2.600 | +11.260 | - .53075 | -0.5 | |
| 6 | +1 | +1 | +11.628 | - 2.218 | +11.410 | - .52189 | -0.5 | |
| 7 | +1 | +1 | +11.527 | - 2.053 | +11.474 | - .51728 | -0.5 | |
| 8 | +1 | +1 | +11.391 | - 1.827 | +11.564 | - .51328 | -0.5 | |
| 9 | +1 | +1 | +11.223 | - 1.545 | +11.678 | - .50998 | -0.5 | |
| 10 A | | +1 | | | + 1.000 | +3.11349 | +3.1 | |
| 10 | +1 | | + 8.894 | - 1.062 | + 8.832 | - 1.59397 | - 1.6 | |
| 11 | +1 | | + 9.073 | + 1.929 | +12.002 | - 3.50308 | - 3.5 | |
| 12 | +1 | | + 6.739 | + 3.108 | +10.847 | - 1.87157 | - 1.9 | |
| 13 | +1 | | + 4.813 | + 3.507 | + 9.320 | - .19300 | -0.2 | |
| 14 | +1 | | + 4.625 | + 3.434 | + 9.059 | + .03566 | 0.0 | |
| 15 | +1 | | + 3.511 | + 4.569 | + 9.080 | + .48305 | +0.5 | |
| 16 | +1 | | + 2.384 | + 5.096 | + 8.480 | +1.29533 | + 1.3 | |
| 17 | +1 | | + 1.473 | + 5.298 | + 7.771 | +2.08161 | + 2.1 | |
| 18 | +1 | | + 0.711 | + 7.062 | +8.773 | +1.81584 | + 1.8 | |
| 19 | +1 | | - 1.417 | + 2.964 | + 2.547 | +6.29823 | + 6.3 | |
| 20 | +1 | | - 1.172 | + 1.582 | + 1.410 | +6.85542 | + 6.9 | |
| 21 | +1 | | | | + 1.000 | +6.60932 | + 6.6 | |
| a | | | + 0.331 | + 1.261 | + 1.592 | - 1.05823 | | |
| b | | | + 0.233 | + 0.172 | + 0.405 | - .33059 | | |
| c | | | + 0.265 | + 0.196 | + 0.461 | - .37621 | | |
| d | | | + 0.110 | + 0.082 | + 0.192 | - .15654 | | |
| e | | | + 0.195 | + 0.146 | + 0.341 | - .27786 | | |
| f | | | + 0.164 | + 0.122 | + 0.286 | - .23323 | | |
| g | | | + 0.071 | + 0.053 | + 0.124 | - .10108 | | |
| h | | | + 0.097 | + 0.072 | + 0.169 | - .13786 | | |
| i | | | + 0.121 | + 0.088 | + 0.209 | - .17092 | | |
| j | | | + 0.207 | + 1.226 | + 1.433 | - .91503 | | |
| k | | | + 1.284 | - 0.094 | + 1.190 | - 1.21863 | | |
| l | | | + 0.506 | + 1.228 | + 1.734 | - 1.21263 | | |
| m | | | + 0.171 | + 1.014 | + 1.185 | - .75660 | | |
| n | | | - 0.031 | + 0.099 | + 0.068 | - .02658 | | |
| o | | | + 0.487 | + 0.586 | + 1.073 | - .82211 | | |
| p | | | + 0.226 | + 0.593 | + 0.819 | - .56739 | | |
| q | | | + 0.087 | + 0.480 | + 0.567 | - .36415 | | |
| r | | | + 0.757 | + 0.401 | + 1.158 | - .98270 | | |
| s | | | - 1.759 | + 1.120 | - 0.639 | +1.09557 | | |
| t | | | - 0.593 | - 0.129 | - 0.722 | + .66263 | | |
| u | | | - 0.679 | - 0.617 | - 1.296 | +1.03042 | | |

## NORMAL EQUATIONS

| V / EQ. | 1 | 2 | 3 | 4 | n | Σ n | |
|---|---|---|---|---|---|---|---|
| 1 | +22. | +10. | +162.73800 | + 7.23700 | - 11.00000 | +190.97500 | |
| 2 | | +11. | +123.10400 | -30.25000 | + 4.20000 | +118.05400 | |
| 3 | | | +811.41492 | -287.19527 | +170.82246 | +980.88411 | |
| 4 | | | | +287.99887 | - 71.65858 | - 93.86798 | |

## SOLUTION OF NORMALS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | +22. | +10. | +162.73800 | + 7.23700 | - 11.00000 | +190.97500 | | |
| | | - .45454 | - 7.39718 | - .32895 | + .50000 | - 8.68068 | +6.60932 | $c_1$ |
| | | +11. | +123.10400 | -30.25000 | + 4.20000 | +118.05400 | | |
| | | + 6.45460 | +49.13307 | -33.53951 | + 9.19994 | + 31.24810 | | |
| | | | - 7.61210 | + 5.19622 | - 1.42533 | - 4.84121 | +3.11349 | $c_2$ |
| | | | +811.41492 | -287.19527 | +170.82246 | +980.88411 | | |
| | | | +233.60680 | - 85.42256 | +182.16058 | +330.34482 | | |
| | | | | + .36567 | - .77977 | - 1.41410 | - .99147 | $c_3$ |
| | | | | +287.99887 | - 71.65858 | - 93.86798 | | |
| | | | | + 80.10312 | + 46.37544 | +126.47856 | | |
| | | | | | - .57895 | - 1.57895 | - .57895 | $c_4$ |

*Figure 174. Solution of traverse adjustment.*

| PROJECT | | TABULATION OF GEODETIC DATA (TM 5-237) | |
|---|---|---|---|
| LOCATION | | ORGANIZATION | |

| STATION | LATITUDE (N) | | LONGITUDE (W) | |
|---|---|---|---|---|
| | FINAL ADJUSTED | FINAL MINUS PRELIMINARY | FINAL ADJUSTED | FINAL MINUS PRELIMINARY |
| ROY | 25° 47' 53."784 | FIXED | 64° 36' 18."478 | FIXED |
| 1 | 25 51 12.398 | -0."009 | 64 48 55.128 | +0."001 |
| 2 | 25 53 32.119 | -0.010 | 64 50 38.293 | +0.003 |
| 3 | 25 56 11.187 | -0.012 | 64 52 36.199 | +0.006 |
| 4 | 25 57 17.438 | -0.013 | 64 53 25.389 | +0.007 |
| 5 | 25 59 14.640 | -0.016 | 64 54 52.620 | +0.011 |
| 6 | 26 00 53.081 | -0.017 | 64 56 05.964 | +0.014 |
| 7 | 26 01 35.522 | -0.018 | 64 56 37.554 | +0.015 |
| 8 | 26 02 33.584 | -0.019 | 64 57 20.863 | +0.016 |
| 9 | 26 03 45.993 | -0.021 | 64 58 13.839 | +0.019 |
| 10 | 26 05 50.087 | -0.045 | 65 10 29.526 | +0.023 |
| 11 | 26 18 40.913 | -0.044 | 65 09 33.101 | +0.059 |
| 12 | 26 23 44.085 | -0.085 | 65 21 49.592 | +0.078 |
| 13 | 26 25 26.996 | -0.123 | 65 31 58.211 | +0.084 |
| 14 | 26 25 08.390 | -0.127 | 65 32 57.673 | +0.083 |
| 15 | 26 30 00.556 | -0.150 | 65 38 49.344 | +0.105 |
| 16 | 26 32 16.259 | -0.172 | 65 44 45.104 | +0.115 |
| 17 | 26 33 08.168 | -0.189 | 65 49 32.923 | +0.118 |
| 18 | 26 40 42.367 | -0.201 | 65 53 33.501 | +0.143 |
| 19 | 26 23 07.209 | -0.229 | 66 04 45.704 | +0.094 |
| 20 | 26 17 11.253 | -0.229 | 66 03 27.998 | +0.089 |
| DALLAS | 26 10 23.922 | -0.236 | 65 57 18.338 | +0.099 |

| TABULATED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| J. A. Andrews - AMS | FEB. '64 | L. E. Smith - AMS | FEB. '64 |

DA FORM 1 FEB 57 1962

GPO 921961

U. S. GOVERNMENT PRINTING OFFICE : 1957 O - 421182

*Figure 175. Final geographic positions of traverse stations.*

lengths (fig. 171). As the $V_s$ is small, it can be converted to meters by the following formula:

$$V=\frac{S \cdot V_s}{M}$$

Where S is the measured length and M is the modulus of the common system of logarithms.

## 105. Final Positions

The final geographic positions are computed using the adjusted angles and lengths. Figure 175 is a tabulation of the final adjusted geographic positions and a comparison of the final positions with the preliminary positions.

# Section III. ADJUSTMENT OF TRAVERSE (APPROXIMATE METHOD)

## 106. Azimuth Adjustment

*a.* DA Form 1931 (Traverse Computation) (Geographic) will be used for this illustration. The corrections to the field measurement of distances are explained in chapter 4. Distances can be used in feet or in meters (fig. 176). In traverse solutions on DA Form 1931, the lengths should not exceed one thousand (1,000) meters without consideration of the second term in the position determination (pars. 66 through 72).

*b.* The clockwise angles are abstracted from the field books and are entered in the first column of the form in the same block and directly below the station name at which the angle was turned. The fixed latitude and longitude of Lenox are entered at the right-hand side of the form under the headings "$\phi$ or Northing" and "$\lambda$ or Easting," respectively. The fixed azimuth from Lenox to the azimuth mark is entered on the first line in the column headed "Adjusted Azimuth." This azimuth must be the azimuth from the starting station of the traverse to the azimuth mark. Since this traverse is being computed on the geographic system, the azimuths and back-azimuths do not differ by exactly 180°.

*c.* The next step is to obtain a set of preliminary or field azimuths by adding each angle to the back-azimuth of the preceding station, i.e., azimuth 2—3=azimuth 1—2±180°+ angle at 2. Since the azimuth at the known starting station is already a back-azimuth, 180° need not be added. In this case, there is an error in each azimuth except the first, because the back-azimuths were taken as 180° different from the forward azimuths, and the effect of convergence was neglected. Even though these preliminary azimuths are in error, they are sufficiently accurate for the purpose of finding the convergence for each course. This step is necessary before the closing error of the azimuths can be found and the final adjusted azimuths obtained. The preliminary azimuths are written in the column headed "Field Azimuths" and beside the station from which they emanate. For example, the preliminary azimuth from Lenox to station 1 is 101°22′45″.6 and is written beside Lenox. The preliminary azimuths are now used in conjunction with the distances to obtain the convergence for each course.

*d.* Geodetic convergence is calculated from the formula: $\Delta\alpha=-\Delta\lambda$ (sin $\phi_m$) where $\Delta\lambda$=difference in longitude between the stations concerned, and sin $\phi_m$=sine of the mean latitude of the stations concerned. Several different graphs have been developed to solve this convergence by use of the azimuth and distance of the course for which the convergence is desired. Two such graphs are included in TM 5–236; one for distances in feet, and one for distances in meters. As the convergence for each course is found, it is written in the column headed "Conv." in the space beside the forward azimuth to which it is to be applied. For example, the convergence for the course Lenox to station 2 (station 1 is on line) is obtained from the graph using the field azimuth of 101°22′45″.6 and the distance from Lenox to station 2, which is Lenox to station 1 plus station 1 to station 2 (877.388+750.595), a total distance of 1,627.983 feet. Notice that the convergence is found only *between angle stations*. No angle corrections of any kind can be applied at a through station. The convergence found above was —12″.0. The algebraic sign is required and the method of determining this sign is: When the course is west of the preceding station, the convergence is negative (—); and positive (+) when east of the preceding station (in the Southern Hemisphere, the rule is reversed). The value —12″.0 is entered beside the field azimuth 101°22′45″.6 in the convergence column. This shows that the azimuth from Lenox to station 2 is 101°22′45″.6 (before angle correction) and the azimuth from station 2 to Lenox is—

$$101°22'45''.6-12''.0+180°=281°22'33''.6$$

| PROJECT | LENGTH OF TRAVERSE (MILES) | FROM | TO | TRAVERSE COMPUTATION (GEOGRAPHIC) |
|---|---|---|---|---|
| Test Traverse | 5.09 | LENOX 1947 USC+GS | ANUTT 1947 USC+GS | PAGE 1 OF 2 |
| LOCATION | LINEAR CLOSURE | ERROR OF CLOSURE | ORDER | |
| Ft. Leonard Wood, Missouri | 0.775 ft | 1/34700 | 3rd | |
| ORGANIZATION | AZIMUTH CLOSURE | NUMBER OF STATIONS | DATE | FIELD BOOK PAGE |
| AMS | -45".2 | 27 (24 angle sta.) | 31 Oct 52 | Vols 30+50 |

| STATION AND OBSERVED ANGLE | FIELD AZIMUTH | ADJUSTMENT CONV. | ADJUSTMENT ∠ | ADJUSTMENT TOT. | ADJUSTED (GEOD.) AZIMUTH | MEAN AZIMUTH | DISTANCE (FEET) | COSINE | SINE | Δ NORTHING | Δ EASTING | FACTOR M | FACTOR P | Δφ | Δλ | φ OR NORTHING | λ OR EASTING | STATION |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LENOX Az Mk | | | | | LENOX to Az Mk 101 30 46.1 | | | | | | | | | | | | | |
| LENOX 359-51-59.5 | 101 22 45.6 | -12.0 | +1.9 | +1.9 | 101 22 47.5 | 41.5 | 877.388 | 0.197284 | 0.980346 | +173.095 | +860.144 | | 0.0124332 | +1.711 | +10.694 | 37 38 47.807 | 91 44 17.976 | LENOX |
| Sta. #1 (thru) | " | — | — | — | " | " | 750.595 | " | " | +148.080 | +735.843 | | 333 | +1.464 | +9.149 | 49.518 | 28.670 | Sta. #1 HPT 315Z |
| Sta. #2 188-34-34.8 | 109 57 20.4 | -4.9 | +1.9 | -8.2 | 109 57 12.2 | 09.8 | 688.747 | 0.341245 | 0.939975 | +235.031 | +647.405 | | 334 | +2.324 | +8.049 | 50.982 | 37.819 | Sta. #2 |
| Sta. #3 160-15-25.5 | 98 12 45.9 | -13.4 | +1.9 | -11.2 | 98 12 34.7 | 28.0 | 1189.605 | 0.142763 | 0.989757 | +169.832 | +1177.420 | | 335 | +1.679 | +14.639 | 53.305 | 45.867 | Sta. #3 |
| Sta. #4 (thru) | " | — | — | — | " | " | 601.053 | " | " | +85.808 | +594.896 | | 335 | +0.848 | +7.397 | 54.984 | 45 00.506 | Sta. #4 HPT 315Y |
| Sta. #5 189-43-02.8 | 107 55 48.7 | -4.6 | +1.9 | -22.7 | 107 55 26.0 | 23.7 | 642.547 | 0.307743 | 0.951470 | +197.739 | +611.364 | | 336 | +1.955 | +7.601 | 55.832 | 07.903 | Sta. #5 |
| Sta. #6 181-10-48.8 | 109 06 37.5 | -8.7 | +1.8 | -25.5 | 109 06 12.0 | 07.6 | 1223.829 | 0.327253 | 0.944937 | +400.502 | +1156.441 | | 337 | +3.960 | +14.379 | 57.787 | 15.504 | Sta. #6 |
| Sta. #7 196-36-34.0 | 125 43 11.5 | -4.9 | +1.9 | -32.3 | 125 42 39.2 | 36.8 | 803.823 | 0.583686 | 0.811979 | +469.180 | +652.687 | | 339 | +4.639 | +8.115 | 39 01.747 | 29.883 | Sta. #7 |
| Sta. #8 216-56-06.5 | 162 39 18.0 | -1.4 | +1.9 | -35.3 | 162 38 42.7 | 42.0 | 597.844 | 0.954475 | 0.298291 | +570.627 | +178.331 | 0.009885 | 342 | +5.642 | +2.217 | 06.385 | 37.997 | Sta. #8 |
| Sta. #9 201-49-55.5 | 184 29 13.5 | +0.5 | +1.9 | -34.8 | 184 28 38.7 | 39.0 | 857.955 | 0.996948 | 0.078068 | +855.337 | -66.979 | | 345 | +8.456 | -0.833 | 12.027 | 40.214 | Sta. #9 HPT 315W |
| Sta. #10 192-04-36.8 | 196 33 50.3 | +2.6 | +1.9 | -32.4 | 196 33 17.9 | 19.2 | 1200.755 | 0.958545 | 0.284941 | +1150.978 | -342.144 | | 349 | +11.379 | -4.255 | 20.483 | 39.381 | Sta. #10 |
| Sta. #11 185-55-52.8 | 202 29 43.1 | +2.5 | +1.9 | -27.9 | 202 29 15.2 | 16.4 | 858.094 | 0.923960 | 0.382488 | +792.845 | -328.211 | | 354 | +7.838 | -4.081 | 31.862 | 35.126 | Sta. #11 |
| Sta. #12 185-32-48.0 | 208 02 31.1 | +5.6 | +1.8 | -23.6 | 208 02 07.5 | 10.3 | 1579.470 | 0.882651 | 0.470029 | +1394.121 | -742.397 | | 359 | +13.783 | -9.232 | 39.700 | 31.045 | Sta. #12 |
| Sta. #13 157-38-42.5 | 185 41 13.6 | +1.9 | +1.9 | -16.1 | 185 40 57.5 | 58.4 | 1885.228 | 0.995085 | 0.099023 | +1875.962 | -186.681 | | 366 | +18.547 | -2.322 | 53.482 | 21.812 | Sta. #13 CWA 339 |
| Sta. #14 200-41-43.5 | 206 22 57.1 | +6.0 | +1.9 | -12.3 | 206 22 44.8 | 47.8 | 1163.558 | 0.895867 | 0.444322 | +1042.393 | -516.994 | | 373 | +10.306 | -6.430 | 40 12.029 | 19.490 | Sta. #14 CWA 338 |
| Sta. #15 (thru) | " | — | — | — | " | " | 609.845 | " | " | +546.340 | -270.968 | | 377 | +5.401 | -3.370 | 22.335 | 13.060 | Sta. #15 HPT 315U |
| Sta. #16 187-42-20.0 | 214 05 17.1 | +3.1 | +1.9 | -4.4 | 214 05 12.7 | 14.2 | 746.928 | 0.828185 | 0.560455 | +618.595 | -418.620 | | 379 | +6.116 | -5.207 | 27.736 | 09.690 | Sta. #16 |
| Sta. #17 173-05-04.8 | 207 10 21.9 | +6.1 | +1.9 | +0.6 | 207 10 22.5 | 25.6 | 1765.029 | 0.889625 | 0.456691 | +1570.214 | -806.073 | | 384 | +15.524 | -10.026 | 33.851 | 04.482 | Sta. #17 |
| Sta. #18 176-57-53.5 | 204 08 15.4 | +5.3 | +1.8 | +8.5 | 204 08 23.9 | 26.6 | 1716.955 | 0.912544 | 0.408979 | +1566.797 | -702.199 | | 392 | +15.490 | -8.735 | 49.325 | 44 54.456 | Sta. #18 |
| Sta. #19 173-43-45.5 | 197 52 00.9 | +2.2 | +1.9 | +15.7 | 197 52 16.6 | 17.7 | 948.602 | 0.951747 | 0.306885 | +902.829 | -291.112 | | 397 | +8.926 | -3.621 | 41 04.865 | 45.721 | Sta. #19 |
| Sta. #20 178-21-38.2 | 196 13 39.1 | +2.9 | +1.9 | +19.8 | 196 13 58.9 | 60.4 | 1401.006 | 0.960131 | 0.279552 | +1345.149 | -391.654 | | 402 | +13.299 | -4.872 | 13.790 | 42.099 | Sta. #20 |
| Sta. #21 186-38-19.2 | 202 51 58.3 | +3.2 | +1.9 | +24.6 | 202 52 22.9 | 24.5 | 1104.506 | 0.921365 | 0.388697 | +1017.653 | -429.318 | | 408 | +10.061 | -5.341 | 27.089 | 37.227 | Sta. #21 |
| Sta. #22 209-55-33.0 | 232 47 31.3 | +10.9 | +1.9 | +29.7 | 232 48 01.0 | 06.4 | 1821.413 | 0.604574 | 0.796549 | +1101.179 | -1450.845 | | 413 | +10.887 | -18.050 | 37.150 | 31.886 | Sta. #22 |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RSSmith | 6 June 54 | W. C. Aumen | 8 June 54 |

DA FORM 1 FEB 57 1931

① Using factors M and P

*Figure 176. Computation of geographic traverse (DA Form 1931).*

| PROJECT | LENGTH OF TRAVERSE (MILES)(KM.) | FROM | TO | TRAVERSE COMPUTATION (GEOGRAPHIC) |
|---|---|---|---|---|
| Test Traverse | | LENOX 1947 USC&GS | ANUTT 1947 USC&GS | PAGE 2 OF 2 |
| LOCATION: Ft Leonard Wood, Missouri | LINEAR CLOSURE | ERROR OF CLOSURE | ORDER | |
| ORGANIZATION: AMS | AZIMUTH CLOSURE | NUMBER OF STATIONS | DATE | FIELD BOOK Vols. 30 & 50 PAGE |

| STATION AND OBSERVED ANGLE | FIELD AZIMUTH | ADJUSTMENT CONV. | ADJUSTMENT ∠ | ADJUSTMENT TOT. | ADJUSTED (GEOD.) AZIMUTH | MEAN AZIMUTH | DISTANCE (FEET) (M) | COSINE | SINE | Δ NORTHING | Δ EASTING | FACTOR | Δφ | Δλ | φ OR NORTHING | λ OR EASTING | STATION |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sta. #22 (See 1st page) | | | | | (see 1st page) | | (see 1st page) | | | | | | | | (see 1st page) | see 1st page | Sta. #22 |
| Sta. #23 172-25-55.8 | 225 13 27.1 | +4.3 | +1.9 | +42.5 | 225 14 09.6 | 11.8 | 797.058 | 0.704181 | 0.710021 | +561.273 | -565.928 | 0.0124416 | +5.549 | -7.041 | 37 41 48.036 | 91 44 13.835 | Sta. #23 |
| Sta. #24 166-08-29.8 | 211 21 56.9 | +2.2 | +1.8 | +48.6 | 211 22 45.5 | 46.6 | 561.528 | 0.853736 | 0.520706 | +479.397 | -292.391 | 419 | +4.740 | -3.638 | 53.585 | 06.794 | Sta. #24 |
| Sta. #25 166-19-20.8 | 197 41 17.7 | +1.1 | +1.9 | +52.7 | 197 42 10.4 | 11.0 | 492.574 | 0.952645 | 0.304084 | +469.248 | -149.784 | 420 | +4.639 | -1.864 | 58.325 | 03.156 | Sta. #25 |
| ANUTT 103-49-38.5 | 121 30 56.2 | — | +1.9 | +55.7 | 121 31 51.9 (Fixed) | | | | | | | | | | 42 02.964 | 01.292 | ANUTT |
| ANUTT Az. Mk. | | | | | | | | | | | | Fixed = | +03 | 15."157 | -16."684 | | |
| | Σ conv. = +10.5 | | | | 121 31 06.7 (Computed) | | 26885.935 | | | | | Computed = | +03 | 15.163 | -16 678 | | |
| | | | | | | | | | | | | Error = | +0 | ."006 or 607ft | +0 ."006 or +0 .482ft | | |
| | | | Azimuth closure = -45."2 | | | | | | | | | | | | | | |
| | | | No. of angles = 24 | | | | | | | | | Linear Error of Closure = 0.775ft | | | Error of Closure Ratio = 1/34700 | | |
| | | | Error per angle = -1."88 | | | | | | | | | Corr. per foot = | | -0."2232x10⁻⁴ | 0."2232x10⁻⁴ | | |
| | | | Corr. per angle = 20@+1.9 4@+1.8 | | | | | | | | | | | | | | |
| | | | | | | | | SIDE | SHOTS | | | | | | | | |
| Sta. #7 | | -4.9 | | | 125 42 39 | | Substance dist | | | | | | | | | | |
| Sta. #8 185 13 31 | | | | | 130 56 05 | | 250.14 | 0.655199 | 0.755457 | +163.891 | +188.970 | 0.0124342 | +1.620 | +2.350 | 37 39 06.385 | 91 45 37.997 | Sta. #8 |
| Sta. #8-A HPT 315-X | | | | | | | | | | | | | | | 37 39 08.005 | 91 45 40.347 | Sta. #8-A HPT 315-X |
| Sta. #12 | | +5.6 | | | 208 02 08 | | | | | | | 0.0098865 | | | | | |
| Sta. #13 11 27 02 | | | | | 39 29 16 | | 141.772 | 0.771760 | 0.635914 | -109.414 | +90.155 | 0.0124366 | -1.082 | +1.121 | 37 39 53.482 | 91 45 21.812 | Sta. #13 |
| Sta. #13-A HPT 315-V | | | | | | | | | | | | | | | 37 39 52.400 | 91 45 22.933 | Sta. #13-A HPT 315-V |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RDSmith | 6 June 54 | W. C. Aumen | 6 June 54 |

DA FORM 1 FEB 57 1931

② Using factors M and P—Continued

*Figure 176*—Continued.



757-381 O - 65 - 17

| PROJECT | LENGTH OF TRAVERSE (KM.) | FROM | TO | TRAVERSE COMPUTATION (GEOGRAPHIC) |
|---|---|---|---|---|
| Test Traverse | 8.195 | LENOX 1947 USC+GS | ANUTT 1947 USC+GS | PAGE 1 OF 2 |
| LOCATION | LINEAR CLOSURE | ERROR OF CLOSURE | ORDER | |
| Ft. Leonard Wood, Missouri | 0.213 m. | 1/38,500 | 3rd | |
| ORGANIZATION | AZIMUTH CLOSURE | NUMBER OF STATIONS | DATE | FIELD BOOK PAGE |
| AMS | -45"2 | 27 (24 angle stations) | 31 Oct. 52 | Vols. 30 & 50 |

| STATION AND OBSERVED ANGLE | FIELD AZIMUTH | ADJUSTMENT CONV. | ADJUSTMENT ∠ | ADJUSTMENT TOT. | ADJUSTED (GEOD.) AZIMUTH | MEAN AZIMUTH | DISTANCE (M.) | COSINE | SINE | Δ NORTHING | Δ EASTING | FACTOR Diff. per Sec. | FACTOR H | Δφ | Δλ | φ OR NORTHING | λ OR EASTING | STATION |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LENOX Az. Mk. | | | | | LENOX To As. Mk. 101 30 46.1 | | | | | | | | (from S.P. 24) | | | | | |
| LENOX | | | | | | | | | | | | | | | | | | |
| 359 - 51 - 59.5 | 101 22 45.6 | -12.0 | +1.9 | +1.9 | 101 22 47.5 | 41.5 | 267 428 | 0.197284 | 0.980346 | + 52 759 | + 262 172 | | 0.0407913 | + 1 711 | + 10 694 | 37 38 47.807 | 91 44 17.976 | LENOX |
| Sta. 1 (thru) | " | — | — | — | " | " | 228 782 | " | " | + 45 135 | + 224 286 | | 7915 | + 1 464 | + 9 149 | 49.518 | 28.670 | Sta. 1 HPT 315Z |
| Sta. 2 | | | | | | | | | | | | | | | -1 | | | |
| 188 - 34 34.8 | 109 57 20.4 | -4.9 | +1.9 | -8.2 | 109 57 12.2 | 09.8 | 209 931 | 0.341245 | 0.939975 | + 71 638 | + 197 330 | | 7918 | + 2 324 | + 8 049 | 50.982 | 37.819 | 2 |
| Sta. 3 | | | | | | | | | | | | | | -1 | | | | |
| 168 15 25.5 | 98 12 45.9 | -13.4 | +1.9 | -11.2 | 98 12 34.7 | 28.0 | 362 592 | 0.142763 | 0.989757 | + 51 765 | + 358 878 | | 7921 | + 1 679 | +14 639 | 53.306 | 45.867 | 3 |
| Sta. 4 (thru) | " | — | — | — | " | " | 183 201 | " | " | + 26 154 | + 181 324 | | 7923 | + 0 848 | + 7 397 | 54.984 | 45 00.506 | Sta. 4 HPT 315Y |
| Sta. 5 | | | | | | | | | | | | | | | | | | |
| 189 43 02.8 | 107 55 48.7 | -4.6 | +1.9 | -22.7 | 107 55 26.0 | 23.7 | 195 849 | 0.307743 | 0.951470 | + 60 271 | + 186 344 | | 7925 | + 1 955 | + 7 601 | 55.832 | 07.903 | 5 |
| Sta. 6 | | | | | | | | | | | | | | | | | | |
| 181 10 48.8 | 109 06 37.5 | -8.7 | +1.8 | -25.5 | 109 06 12.0 | 07.6 | 373 024 | 0.327253 | 0.944937 | + 122 073 | + 352 484 | | 7930 | + 3 960 | +14 379 | 57.787 | 15.504 | 6 |
| Sta. 7 | | | | | | | | | | | | | | | -1 | | | |
| 196 36 34.0 | 125 43 11.5 | -4.9 | +1.9 | -32.3 | 125 42 39.2 | 36.8 | 245 006 | 0.583686 | 0.811979 | + 143 007 | + 198 940 | | 7936 | + 4 639 | + 8 115 | 39 01.747 | 29.883 | 7 |
| Sta. 8 | | | | | | | | | | | | | | | | | | |
| 216 56 06.5 | 162 39 18.0 | -1.4 | +1.9 | -35.3 | 162 38 42.7 | 42.0 | 182 223 | 0.954475 | 0.298291 | + 173 927 | + 54 355 | | 7944 | + 5 641 | + 2 217 | 06.386 | 37.997 | 8 |
| Sta. 9 | | | | | | | | | | | | | | -1 | | | | Sta. 9 |
| 201 49 55.5 | 184 29 13.5 | +0.5 | +1.9 | -34.8 | 184 28 38.7 | 39.0 | 261 505 | 0.996948 | 0.078068 | + 260 707 | - 20 415 | | 7955 | + 8 456 | - 0 833 | 12.027 | 40.214 | HPT 315W |
| Sta. 10 | | | | | | | | | | | | 0.0824359 | | | | | | |
| 192 04 36.8 | 196 33 50.3 | +2.6 | +1.9 | -32.4 | 196 33 17.9 | 19.2 | 365 991 | 0.958545 | 0.284941 | + 350 819 | - 104 286 | | 7970 | +11 379 | - 4 255 | 20.482 | 39.381 | 10 |
| Sta. 11 | | | | | | | | | | | | | | | | | | |
| 185 55 52.8 | 202 29 43.1 | +2.5 | +1.9 | -27.9 | 202 29 15.2 | 16.4 | 261 548 | 0.923960 | 0.382488 | + 241 660 | - 100 039 | | 7984 | + 7 838 | - 4 081 | 31.861 | 35.126 | 11 |
| Sta. 12 | | | | | | | | | | | | | | | -1 | | | |
| 185 32 48.0 | 208 02 31.1 | +5.6 | +1.8 | -23.6 | 208 02 07.5 | 10.3 | 481 423 | 0.882651 | 0.470029 | + 424 928 | - 226 283 | | 8001 | +13 783 | - 9 232 | 39.699 | 31.045 | 12 |
| Sta. 13 | | | | | | | | | | | | | | -1 | | | | Sta. 13 |
| 157 38 42.5 | 185 41 13.6 | +1.9 | +1.9 | -16.1 | 185 40 57.5 | 58.4 | 574 619 | 0.995085 | 0.099023 | + 571 795 | - 56 900 | | 8025 | +18 547 | - 2 322 | 53.482 | 21.812 | CWA 339 |
| Sta 14 | | | | | | | | | | | | | | | | | | Sta. 14 |
| 200 41 43.5 | 206 22 57.1 | +6.0 | +1.9 | -12.3 | 206 22 44.8 | 47.8 | 354 653 | 0.895867 | 0.444322 | + 317 722 | - 157 580 | | 8047 | +10 306 | - 6 430 | 40 12.028 | 19.490 | CWA 338 |
| Sta. 15 (thru) | " | — | — | — | " | " | 185 881 | " | " | + 166 525 | - 82 591 | | 8059 | + 5 401 | - 3 370 | 22.334 | 13.060 | Sta. 15 HPT 315U |
| Sta. 16 | | | | | | | | | | | | | | | -1 | | | |
| 187 42 20.0 | 214 05 17.1 | +3.1 | +1.9 | -4.4 | 214 05 12.7 | 14.2 | 227 664 | 0.828185 | 0.560455 | + 188 548 | - 127 595 | | 8068 | + 6 116 | - 5 207 | 27.735 | 09.680 | 16 |
| Sta. 17 | | | | | | | | | | | | | | | | | | |
| 173 05 04.8 | 207 10 21.9 | +6.1 | +1.9 | +0.6 | 207 10 22.5 | 25.6 | 537 982 | 0.889625 | 0.456691 | + 478 602 | - 245 692 | | 8084 | +15 524 | -10 026 | 33.851 | 04.482 | 17 |
| Sta. 18 | | | | | | | | | | | | | | | -1 | | | |
| 176 57 53.5 | 204 08 15.4 | +5.3 | +1.8 | +8.5 | 204 08 23.9 | 26.6 | 523 329 | 0.912544 | 0.408979 | + 477 561 | - 214 031 | | 8108 | +15 490 | - 8 735 | 49.375 | 44 54.456 | 18 |
| Sta. 19 | | | | | | | | | | | | | | | -1 | | | |
| 173 43 45.5 | 197 52 00.9 | +2.2 | +1.9 | +15.7 | 197 52 16.6 | 17.7 | 289 134 | 0.951747 | 0.306885 | + 275 182 | - 88 731 | | 8127 | + 8 926 | - 3 621 | 41 04.864 | 45.721 | 19 |
| Sta. 20 | | | | | | | | | | | | | | | | | | |
| 178 21 38.2 | 196 13 39.1 | +2.9 | +1.9 | +19.8 | 196 13 58.9 | 60.9 | 427 027 | 0.960131 | 0.279552 | + 410 002 | - 119 376 | | 8144 | +13 299 | - 4 872 | 13.790 | 42.099 | 20 |
| Sta. 21 | | | | | | | | | | | | | | | | | | |
| 186 38 19.2 | 202 51 58.3 | +3.2 | +1.9 | +24.6 | 202 52 22.9 | 24.5 | 336 654 | 0.921365 | 0.388697 | + 310 181 | - 130 856 | | 8161 | +10 061 | - 5 341 | 27.089 | 37.227 | 21 |
| Sta. 22 | | | | | | | | | | | | | | -1 | -1 | | | |
| 209 55 33.0 | 232 47 31.3 | +10.9 | +1.9 | +29.7 | 232 48 01.0 | 06.9 | 555 168 | 0.604574 | 0.796549 | + 335 640 | - 442 219 | | 8177 | +10 887 | -18 050 | 37.150 | 31.886 | 22 |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RDsmith | 6 June 54 | W. C. Aumen | 8 June 54 |

DA FORM 1 FEB 57 1931

③ Using factors "Diff. per sec." and H

*Figure 176*—Continued.

| PROJECT | LENGTH OF TRAVERSE (~~MILES~~)(KM.) | FROM | TO | TRAVERSE COMPUTATION (GEOGRAPHIC) |
|---|---|---|---|---|
| Test Traverse | | LENOX 1947 USC+GS | ANUTT 1947 USC+GS | |
| LOCATION | LINEAR CLOSURE | ERROR OF CLOSURE | ORDER | PAGE 2 OF 2 |
| Ft Leonard Wood, Missouri | | | | |
| ORGANIZATION | AZIMUTH CLOSURE | NUMBER OF STATIONS | DATE | FIELD BOOK PAGE |
| AMS | | | 31 Oct. 52 | Vols. 30 & 50 |

| STATION AND OBSERVED ANGLE | FIELD AZIMUTH | ADJUSTMENT CONV. | ADJUSTMENT ∠ | ADJUSTMENT TOT. | ADJUSTED (GEOD.) AZIMUTH | MEAN AZIMUTH | DISTANCE (~~FEET~~) (M.) | COSINE | SINE | Δ NORTHING | Δ EASTING | FACTOR | Δφ | Δλ | φ OR NORTHING | λ OR EASTING | STATION |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sta. 22 (see 1st page) | | | | | (See 1st page) | | (See 1st page) | | | | | | | | (see 1st page) | (see 1st page) | |
| Sta. 23 172 25 55.8 | 225 13 27.1 | +4.3 | +1.9 | +42.5 | 225 14 09.6 | 11.8 | 242 944 | 0.704181 | 0.710021 | +171 077 | -172 495 | 0.0408190 | +5 549 | -7 041 | 37 41 48.036 | 91 44 13.835 | Sta. 23 |
| Sta. 24 166 08 29.8 | 211 21 56.9 | +2.2 | +1.8 | +48.6 | 211 22 45.5 | 46.6 | 171 154 | 0.853736 | 0.520706 | +146 120 | - 89 121 | 198 | +4 740 | -3 638 | 53.585 | 06.794 | 24 |
| Sta. 25 166 19 20.8 | 197 41 17.7 | +1.1 | +1.9 | +52.7 | 197 42 10.4 | 11.0 | 150 137 | 0.952645 | 0.304084 | +143 027 | - 45 654 | 205 | +4 639 | -1 864 | 58.325 | 03.156 | 25 |
| ANUTT 103 49 38.5 | 121 30 56.2 | — | +1.9 | +55.7 | 121 31 51.9 (Fixed) | | | | | | | | | | 42 02.964 | 01.292 | ANUTT |
| ANUTT Az. Mk. | | | | | | | | | | | | Fixed = | +03 15."157 | -16 "684 | | | |
| | Σ conv= | +10.5 | | | 121 31 06.7 (Computed) | | 8194 849 | | | | | Computed = | +03 15."162 | -16 678 | | | |
| | | | | | | | | | | | | Error = | 0."005 or 0.154m. | 0."006 or 0.147m. | | | |
| Number of angles = 24 | Azimuth closure = - 45."2 | | | | | | | | | | | | | | | | |
| | Error per angle = - 1."88 | | | | | | | | | | | Linear Error of Closure = 0.213m. | | | | | |
| | Corr. per angle = 20 @ +1."9 = 4 @ +1."8 | | | | | | | | | | | Corr. per m. | -0.6101x10$^{-6}$ | -0.7322x10$^{-6}$ | | | |
| | | | | | | | | SIDE | SHOTS | | | | | | | | |
| Sta. 7 | | -4.9 | | | 125 42 39 | | | | | | | | | | | | |
| Sta. 8 185 13 31 | | | | | 130 56 05 | | 76 243 | 0.655199 | 0.755457 | + 49 954 | + 57 598 | 0.0407941 | +1 620 | +2 350 | 37 39 06.386 | 91 45 37.997 | Sta. 8 |
| Sta. 8-A HPT 315-X | | | | | | | | | | | | | | | 37 39 08.006 | 91 45 40.347 | Sta. 8-A HPT 315-X |
| Sta. 12 | | +5.6 | | | 208 02 08 | | | | | | | 0.0324360 | | | | | |
| Sta. 13 11 27 02 | | | | | 39 29 16 | | 43 212 | 0.771760 | 0.635914 | - 33 349 | + 27 479 | 0.0408010 | -1 082 | +1 121 | 37 39 53.482 | 91 45 21.812 | Sta. 13 |
| Sta. 13-A HPT 315-V | | | | | | | | | | | | | | | 37 39 52.400 | 91 45 22.933 | Sta. 13-A HPT 315-V |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RSSmith | 6 June 54 | W. C. Aumen | 8 June 54 |

DA FORM 1 FEB 57 1931

④ Using factors "Diff. per sec." and H—Continued

*Figure 176*—Continued.

*e.* After the convergence for each course is found and entered in the Conv. column, the summation of the convergence (Σ convergence) is obtained. Remember that this is an algebraic summation of the convergences for each course. The summation for this traverse is +10″.5. The total effect of applying the convergences to each course would be to increase the final field azimuth +10″.5. Adding +10″.5 to the field azimuth Anutt to Az. Mk., 121°30′56″.2, the unadjusted azimuth Anutt to Az. Mk. is found to be 121°31′06″.7. The fixed value for the azimuth Anutt to Az. Mk., 121°31′51″.9, is subtracted from the unadjusted azimuth 121°31′06″.7, and the angular error of closure or azimuth closure is obtained as −45″.2. The maximum azimuth error allowed in a third-order traverse is $15'' \sqrt{\text{Number of angles}}$.

*f.* Divide this closure of −45″.2 by the number of angles turned, 24 (through stations are not angle stations), to find the error per angle. It is −1″.88. The correction per angle is numerically the same as the error per angle, but has the opposite algebraic sign and is, in this example, +1″.88 per angle. Since the angles used in this traverse are only given to tenths of a second, the corrections to the angles can be applied only to tenths. Therefore, if 20 angles are corrected +1″.9 and 4 angles are corrected+1.8, the correction for the 24 angles is still +45″.2, which is required. The corrections are now entered in the adjustment angle (∠) column for each angle. The four odd corrections of +1″.8 are distributed through the traverse at approximately equal intervals. No attempt should be made to find the most probable angles which should have the odd corrections, as the final coordinates will not be affected at all.

*g.* The final subheading in the adjustment column is now filled in. This is the total adjustment to be applied to the *field azimuths. The total adjustment at each angle station is the total at the preceding angle station plus the convergence at the preceding plus the angle correction at the station itself.* For example, at Lenox there is no preceding angle station, so the total correction is the angle correction at Lenox, +1″.9. At station 2, the next angle station, the total correction is the total at Lenox, +1″.9, plus the convergence at Lenox, −12″.0, plus the angle correction at station 2, +1″.9, which equals −8″.2. At station 3, the total is −8″.2 (total at 2) plus −4″.9 (convergence at 2) plus +1″.9 (angle correction at 3) which equals −11″.2. This process is continued until a total correction is found at each angle station. The total correction at the last angle station, when added to the field azimuth, should equal the fixed ending azimuth, i.e., +55″.7 (total at Anutt) added to 121°30′56″.2 (field azimuth Anutt to Az. Mk.) should equal 121°31′51″.9 (fixed azimuth Anutt to Az. Mk.) and it does exactly which indicates that the total corrections throughout the traverse are correct, discounting the possibility of compensating errors.

*h.* The total corrections are now applied to the field azimuths and the resulting adjusted geodetic azimuths are entered on the form. The mean azimuth column contains the seconds of the adjusted forward azimuths to which one-half of the convergences for the courses has been applied. For example, the forward azimuth from Lenox to station 1 is 101°22′47″.5 and the convergence for the course is −12″.0. The value in the mean azimuth column is then 47″.5−6″.0=41″.5. In the same manner, the mean azimuth is found for each course.

*i.* The sines and cosines of the mean azimuths are entered in the appropriate columns. Do not attempt to apply an algebraic sign to these trigonometric functions. Instead, apply the signs to the Δ Northings and Δ Eastings by inspection of the azimuths producing them, i.e., when referred to south as 0°, in the Western Hemisphere and north of the Equator, azimuths between 0° and 180° indicate plus (+) Δ Eastings, and azimuths between 90° and 270° indicate plus (+) Δ Northings.

## 107. Latitude and Longitude Difference

*a.* The distance (in feet or meters) is multiplied by the cosines and sines to obtain the Δ Northings and Δ Eastings, respectively: ΔNorthing=distance×cosine; ΔEasting=distance×sine.

*b.* The Δ Northings and Δ Eastings must now be converted from feet or meters to seconds of arc. The factors used to make this conversion when using feet are known as the M and P factors. M is the factors used to convert Δ Northings in feet to $\Delta\phi$ in seconds. M=1/R arc 1″, and R=radius of curvature in the meridian. If R is used in units of feet, the factor M must be used to convert Δ Northings in feet. Most tables are made up on this basis. P=1/N cos $\phi$ arc 1″, where N=radius of curvature in the prime vertical and cos $\phi$=cosine of the latitude at which the factor P is computed. The U.S. Geological Survey has published two sets of tables containing M and P

factors. One set is a reprint of the State of Texas Reclamation Department Bulletin 8 and can be used only between latitudes 25° and 50°. A second set of tables published by the U.S. Geological Survey contains the M and P factors (along with several other factors) from latitudes 0° to 72° and is much more useful than the first tables mentioned. This latter publication is titled: Factors for Computation of Geodetic Positions. Both tables mentioned are based on the Clarke Spheroid of 1866. The factors for converting Δ Northings and Δ Eastings in meters to seconds of arc are "Diff. per sec." and H, which are taken from USC&GS Sp. Pub. 241. "Diff. per sec." is found by interpolating the difference per second of meridional arc for the mean latitude of the traverse, and making a reciprocal of this value.

$$\text{Factor "Diff. per sec."} = \frac{1}{\text{Difference per second of meridional arc}}$$

H is taken directly from the tables using the mean latitude of each course. Both "Diff. per sec." and H are used to seven decimal places. The following parts of the computation are explained in feet, but the procedure is the same for meters. A sample problem in both feet and meters (fig. 176) is included. Since the M factor changes slowly, for ordinary traverse computation the factor for the mean latitude of the ends of the traverse will suffice for converting all the Δ Northings.

## 108. Adjustment and Final Positions

*a.* The first step in finding the positions of the traverse stations is to convert all the Δ Northings to $\Delta\phi$'s by using the M factor. Each $\Delta\phi$ has the same algebraic sign as the corresponding Δ Northing. The summation of the $\Delta\phi$'s is then obtained by adding, algebraically, the $\Delta\phi$'s for the courses. The difference in latitude between the starting and ending positions is subtracted from the summation of $\Delta\phi$'s (which could also be called the computed difference in latitude between fixed positions). The result of the subtraction is the error of closure in $\Delta\phi$ in seconds of arc. Dividing this closure by M will give the error of closure in feet. In this traverse, the difference in latitude between the fixed positions is +03′15″.157; the summation of the $\Delta\phi$'s equals +03′15″.163. Subtraction of the fixed difference, +03′15″.157, from the computed difference, +03′15″.163, gives an error of closure in $\Delta\phi$ of +0.607 feet.

*b.* The correction factor for the $\Delta\phi$'s is computed by dividing the error of closure (in seconds) by the total length of the traverse (in feet) and changing the sign, which makes the correction factor units of seconds per foot. The correction factor for $\Delta\phi$ in this traverse is $-(+0''.006 \div 26885.935 \text{ feet}) = -0.2232 \times 10^{-6}$ seconds per foot.

*c.* The correction for each course is found by multiplying the length of the course by the correction factor. Round off the length to the nearest foot for this operation. Because the courses in a traverse usually are not long enough to show any correction per foot, it is necessary to accumulate the corrections for each course until the accumulation will change the last place of decimal carried. In this traverse, the correction to $\Delta\phi$ accumulates to −0″.001 after multiplying by the first three courses. The correction of −0″.001 is written above the third $\Delta\phi\begin{pmatrix}-1\\+2''.324\end{pmatrix}$ in units of the third place and the accumulation is continued. After the eighth course, the correction has accumulated to −0″.002, which means that the $\Delta\phi$ for this course can be corrected by −0″.001, and the correction is again written above the $\Delta\phi\begin{pmatrix}-1\\+4.639\end{pmatrix}$. This process continues until six of the $\Delta\phi$'s have been corrected by −0″.001. The total accumulation for the complete length of the traverse should be −0″.006, which checks the error of closure. The corrections are applied algebraically to the $\Delta\phi$'s, i.e., a minus correction would make a plus $\Delta\phi$ smaller numerically.

*d.* The adjusted latitudes of the traverse stations can now be computed by successively adding the adjusted $\Delta\phi$'s. Starting with the fixed latitude of Lenox, add algebraically the $\Delta\phi$ for the course Lenox to station 1(+1″.711) to obtain the latitude of station 1; to this latitude, add the $\Delta\phi$ for the course station 1 to station 2(+1″.464) to obtain the latitude of station 2. Continue this procedure to the end of the traverse. The closure on the fixed position should now be exact.

*e.* The longitude computation is now begun by finding the factor P for each course. This factor changes quite rapidly and a new value must be found for each course. A mean value of the latitudes of the ends of the course will suffice for the interpolation of P. Seven decimal places are enough for the P factor. After all the factors are

obtained, the factor of each course is multiplied by the Δ Easting of the course, which gives the $\Delta\lambda$ of the course in seconds. The sign of $\Delta\lambda$ is the same as that of Δ Easting. The same procedure is used to find the error of closure for $\Delta\lambda$ as was used for $\Delta\phi$. The error in seconds is divided by the P factor of the last course to find the error in feet. The correction factor for $\Delta\lambda$ is found as for $\Delta\phi$ and applied in the same manner.

*f.* The linear error of closure is the square root of the sum of the squares of the errors in $\Delta\phi$ and $\Delta\lambda$ expressed in feet. The error of closure ratio is an indication of the accuracy of the survey and is expressed as a fraction with a numerator of 1. The ratio is the fraction, linear closure÷total length, with the numerator 1. The fraction is 1/(total length÷linear closure). For this traverse, the error in $\Delta\phi$ is 0.607 feet, and for $\Delta\lambda$ it is 0.482 feet. The linear error is $\sqrt{0.607^2+0.482^2}=0.775$ feet. The error of closure ratio is 0.775/26885.935= 1/34700.

*g.* The computation is now completed by computing the side shots, using the adjusted positions and azimuths of the traverse and the M and P factors at the station from which the side shot was made.

*h.* At this point, it should be mentioned that the form used for the computation on the geographic system can easily be utilized for computations on a grid system. To use DA Form 1931 for grid computations, simply ignore the convergence column in the azimuth adjustment, and the total column will then become the preceding total plus the angle adjustment at the station. The factors, M and P or "Diff. per sec." and H, also drop out on the grid, as do the $\Delta\phi$ and $\Delta\lambda$. The adjustment is performed as explained for the UTM grid traverse.

# CHAPTER 9

# RESULTS OF HORIZONTAL CONTROL SURVEYS

## Section I. TABULATION OF RESULTS

### 109. Geodetic Data Cards

After the completion of computations, the results should be tabulated to make the information readily available for future use. One method is by the use of geodetic data cards. DA Form 1961 (Geodetic Data) (fig. 177) is a form designed to tabulate the station name, establishing agency, numerical results and datum references, for monumented or marked stations. The spaces provided are self-explanatory.

### 110. Control Data Sheets

DA Form 1962 (Tabulation of Geodetic Data) (fig. 178) is used when partial or specialized listing of results is required.

### 111. Statistics on Triangulation and Baseline Results

After a net of triangulation has been adjusted, statistics for the results are recorded on DA Form 1964 (fig. 179). The form is self-explanatory.

| COUNTRY | TYPE OF MARK | STATION | |
|---|---|---|---|
| UNITED STATES | DISK | COTTON, AMS 1962 | |
| LOCALITY | STAMPING ON MARK COTTON | AGENCY (CAST IN MARK) | ELEVATION 1179.2 (FT) |
| MARICOPA, ARIZONA | 1962 ARMY MAP SERVICE | C. of E. | 359.42 (M) |
| LATITUDE | LONGITUDE | DATUM | DATUM |
| N 33° 03' 15."771 | W 112° 01' 32."659 | 1927 NAD | SLD of 1929 |
| (NORTHING) ~~(EASTING)~~ ~~(FT)~~ | (EASTING) ~~(NORTHING)~~ ~~(FT)~~ | GRID AND ZONE | ESTABLISHED BY (AGENCY) |
| 3657 591.90 (M) | 404 234.41 (M) | UTM 12 | AMS |
| (NORTHING) ~~(EASTING)~~ (FT) ~~(M)~~ | (EASTING) ~~(NORTHING)~~ (FT) ~~(M)~~ | GRID AND ZONE | DATE / ORDER |
| 747 307.98 | 466 591.19 | ARIZONA (CENTRAL) | 1962 / 4th |

TO OBTAIN UTM — GRID AZIMUTH, ADD 180 °33 ′34.26″ TO THE GEODETIC AZIMUTH

TO OBTAIN ARIZONA (CENTRAL) — GRID AZ. (ADD) ~~(SUB.)~~ 0 ° 03 ′ 34.17″ TO THE GEODETIC AZIMUTH

| OBJECT | AZIMUTH ~~OR DIRECTION~~ (GEODETIC) ~~(GRID)~~ ~~(MAGNETIC)~~ | BACK AZIMUTH | GEOD. DISTANCE (METERS) ~~(FEET)~~ | GRID DISTANCE (METERS) ~~(FEET)~~ |
|---|---|---|---|---|
| R. M. No. 2 | 90° 46′ 50″ | ° ′ ″ | 11.412 | |
| TT N-31 (AZ. MK.) | 136 46 25.1 | | 553.1705 | |
| R. M. No. 1 | 180 56 25 | | 6.828 | |
| PIMA BUTTE | 183 44 24.77 | 003 44 38.95 | 10315.1376 | 10312.1469 |
| GILA BUTTE | 233 45 03.73 | 53 50 27.08 | 19023.6945 | 19017.8585 |
| BASE STATION "A" | 263 54 08.51 | 83 59 53.69 | 16505.6605 | 16500.6268 |

DA FORM 1 OCT 64 1961 — REPLACES DA FORM 1961, 1 FEB 57, WHICH IS OBSOLETE. — GEODETIC DATA (TM 5-237)

*Figure 177. Geodetic data card (DA Form 1961).*

PROJECT 1-243

TABULATION OF GEODETIC DATA (TM 5-237)

LOCATION California

ORGANIZATION 29th Engrs.

| STATION | Ø | λ | UTM - Clarke 1866 - Zone 10 N (Meters) | E |
|---|---|---|---|---|
| Burdell | 38° 08' 46."310 | 122° 35' 25."439 | 4 221 909.33 | 535 891.28 |
| Nic | 38 02 57.569 | 122 42 14.220 | 4 211 123.29 | 525 975.66 |
| Red | 38 05 21.652 | 122 38 53.110 | 4 215 580.99 | 530 860.39 |

TABULATED BY R.H. Smith | DATE 27 Aug. 55 | CHECKED BY W. C. Aumen | DATE 27 Aug. 55

DA FORM 1 FEB 57 1962

*Figure 178. Coordinate position tabulation sheet (DA Form 1962).*

## 112. Army Map Service Trigonometric Lists

*a.* Horizontal and vertical survey control data for many areas are compiled and published by the Army Map Service. These trig lists contain a compilation of the descriptions, geographic positions, Universal Transverse Mercator grid coordinates, and elevations of the stations in an area. In some cases, the lists also contain the azimuths and back azimuths between observed stations.

*b.* The data is generally arranged by quadrangle on map sheet areas. An explanation of the contents provides the arrangement and area covered.

| PROJECT | STATISTICS ON COMPUTATIONS (TM 5-237) |
|---|---|
| 1-243 | |
| LOCATION: California | SPHEROID: Clarke 1866 |
| ORGANIZATION: 29th Engrs. | NAME OF ☐ QUAD. ☒ NET ☐ BASE EXP.: Typical |

**TRIANGULATION**

| | | | |
|---|---|---|---|
| TOTAL NUMBER OF TRIANGLES. | 8 | AVERAGE CLOSURE OF TRIANGLES WITHOUT REGARD TO SIGN. | 4."6 |
| NUMBER OF TRIANGLES WITH PLUS CLOSURES. | 5 | MAXIMUM CLOSURE OF TRIANGLE. | +8."9 |
| NUMBER OF TRIANGLES WITH MINUS CLOSURES. | 1 | MEAN ERROR OF AN ANGLE ($\alpha$). | ±3."2 |
| NUMBER OF TRIANGLES WITH ZERO CLOSURES. | 0 | PROBABLE ERROR OF AN OBSERVED DIRECTION ($d$). | ±2."2 |
| NUMBER OF CONCLUDED TRIANGLES. | 2 | | |

**BASE LINES**

| BASE LINE | CLOSURE | BASE LINE | CLOSURE |
|---|---|---|---|
| | | | |

**REMARKS**

Line LINCOLN-Burdell will be used to extend control to the North (Log length 3.97669409).

Probable error of an observed direction, $d=0.6745\sqrt{\frac{\Sigma v^2}{c}}$, where $\Sigma v^2$=sum of squares of corrections to directions, and $c$ is the number of conditions.

Mean error of an angle $\alpha=\sqrt{\frac{\Sigma\Delta^2}{3n}}$, where $\Sigma\Delta^2$ is the sum of the squares of the closing errors of the triangles, and $n$ is the number of triangles.

| COMPUTED BY | DATE | APPROVED BY | DATE |
|---|---|---|---|
| R.A. Smith | 28 Aug. 55 | W. C. Aumen | 28 Aug. 55 |

DA FORM 1 FEB 57 1964

*Figure 179. Computation statistics tabulation sheet (DA Form 1964).*

## Section II. DESCRIPTION OF HORIZONTAL CONTROL STATION

### 113. Requirements

The description, sketch, and numerical information for marked or monumented stations are the final product of the expenditure of man-hours and money. The Chief of Party is responsible for the accuracy and completeness of this information. If the record does not provide sufficient information to locate the mark by measurements at the site, or in case of destruction to reestablish it, and does not provide azimuths to objects from which later surveys can be initiated, most of the effort of the survey has been wasted. Every reasonable effort should be made to insure that—

*a.* The type of mark and the exact point at which an instrument was centered are described.

*b.* Sufficient measurements (within .01 foot accuracy) are made which would enable the station or point to be replaced.

*c.* Two or more directions or angles, related to the survey are observed to objects for later use as starting azimuths.

*d.* All reference marks, azimuth marks, and objects are described for recovery purposes.

*e.* The station description is written at the site.

*f.* Where steeples, tanks, towers, or other objects are observed as azimuth marks or intersections, the exact point sighted upon, and the general appearance of the object are described.

*g.* Inconspicuous types of marks, rather than disks, are used in areas where superstition and vandalism are probable.

### 114. Control Card

*a.* DA Form 1959 is to be used for all marked or monumented stations that can be recovered and used. On the reverse side of this form, brief "NOTES ON COMPLETION OF FORM" are provided as a guide to the information required. Good judgment should be used in selecting the general location of a mark. The expression "TO REACH THE STATION, ETC." should not be used in settled or definite locations. This is intended for use in describing the route to stations in isolated locations. Where applicable, expressions such as, "IN N.W. WASH., D.C., NEAR THE NORTH END OF KEY BRIDGE" will be sufficient.

*b.* The date and letters of the organization establishing the station are an integral part of the station designation such as, "COTTON, AMS, 1962." If a station is replaced in exactly the same location, its designation remains the same with the exception of a new date. If it is replaced by a new mark not in the same precise location, the same station name should be retained with the figure 2, and a new date, as part of the station designation. Recovery notes should identify whether or not the reference marks, azimuth marks, and ties were checked. A station should never be reported as destroyed unless positive evidence of destruction is present. If the exact location where an instrument was centered can be defined from local measurements of the original description, the station is not destroyed. A statement should be included as to the thoroughness of the search which was made for a station that could not be located. Samples of properly written descriptions are provided in figures 180 and 181.

| COUNTRY | TYPE OF MARK | STATION | |
|---|---|---|---|
| UNITED STATES | DISK | COTTON, AMS 1962 | |
| LOCALITY | STAMPING ON MARK COTTON | AGENCY (CAST IN MARK) | ELEVATION 1179.2 (FT) 359.42 (M) |
| MARICOPA, ARIZONA | 1962 ARMY MAP SERVICE | C. OF E. | |
| LATITUDE | LONGITUDE | DATUM | DATUM SEA LEVEL DATUM OF 1929 |
| 33° 03' 15."771 | 112° 01' 32."659 | 1927 NAD | |
| (NORTHING) ~~(EASTING)~~ ~~(FT)~~ (M) | (EASTING) ~~(NORTHING)~~ ~~(FT)~~ (M) | GRID AND ZONE | ESTABLISHED BY (AGENCY) |
| 3,657,591.90 | 404,234.41 | UTM 12 | AMS |
| (NORTHING) ~~(EASTING)~~ (FT) ~~(M)~~ | (EASTING) ~~(NORTHING)~~ (FT) ~~(M)~~ | GRID AND ZONE | DATE / ORDER |
| 747,307.98 | 466,591.19 | ARIZ. S.P.C. (CENTRAL) | 1962 / FOURTH |

TO OBTAIN Universal Transverse Mercator GRID AZIMUTH, ADD 180° 33' 34."26 TO THE GEODETIC AZIMUTH

TO OBTAIN ARIZONA S.P.C. (CENTRAL) GRID AZ. (ADD) ~~(SUB.)~~ 0° 03' 34."17 TO THE GEODETIC AZIMUTH

| OBJECT | AZIMUTH ~~OR DIRECTION~~ (GEODETIC) ~~(GRID)~~ ~~(MAGNETIC)~~ | BACK AZIMUTH | GEOD. DISTANCE (METERS) ~~(FEET)~~ | GRID DISTANCE (METERS) ~~(FEET)~~ |
|---|---|---|---|---|
| R.M. NO. 2 | 90° 46' 50" | ° — " | 11.412 | — |
| TT N-31 (AZ. MK.) | 136 46 25.1 | — | 553.1705 | — |
| R.M. NO. 1 | 180 56 25 | — | 6.828 | — |
| PIMA BUTTE | 183 44 24.77 | 003 44 38.95 | 10315.1376 | 10312.1469 |
| GILA BUTTE | 233 45 03.73 | 53 50 27.08 | 19023.6945 | 19017.8585 |
| BASE STATION "A" | 263 54 08.51 | 83 59 53.69 | 16505.6605 | 16500.6268 |

The station is marked by a Corps of Engineers, U. S. Army disk stamped: "COTTON 1962 ARMY MAP SERVICE" set in the top of a round concrete post 10 inches in diameter the top of which projects 0.1 foot above ground.

The station is located 7.5 feet south of the centerline of a road west; 25.0 feet west of the centerline of a road north; 5.61 feet north of the center of the north edge of a concrete irrigation headwall; 3.50 feet east of a 4 x 4 inch white witness post.

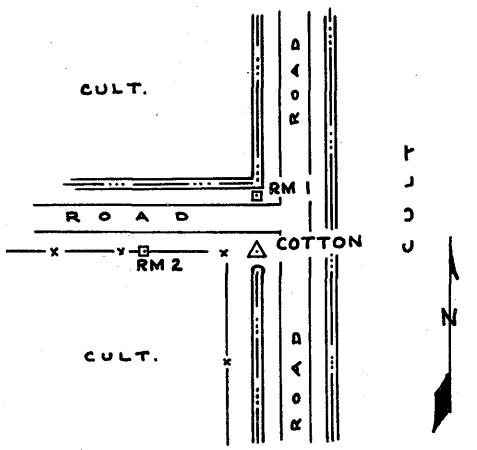
To reach from the Southern Pacific Railroad grade crossing in Maricopa, Arizona, go north on the Maricopa-Phoenix Highway for 0.1 mile to a crossroad; turn right and go east for 1.3 miles to a road to the right; turn right and go south for 0.25 mile to the station site near the center of the northwest 1/4 of section 26; T-4-S; R-3-E.

R.M. No.1: A Corps of Engineers disk stamped: "COTTON R.M. NO.1 1962 ARMY MAP SERVICE", set in the top of a round concrete post the top of which is flush with the ground, located 22.40 feet north of the station mark.

R.M. NO.2: A Corps of Engineers disk stamped: "COTTON R.M. NO.2 ARMY MAP SERVICE", set in the top of a round concrete post the top of which projects 0.2 foot above ground, located 37.44 feet west of the station mark and 43.68 feet west-southwest of R.M. NO. 1.

AZIMUTH MARK: A Corps of Engineers disk stamped "TT N-31 ARMY MAP SERVICE 1961" set in the top of a concrete monument 10 inches in diameter the top of which is flush with the ground, located 10.5 feet north of the north edge of the road bank; 14.1 feet west northwest of a telephone pole; and 3.2 feet northeast of a fencepost.

Visibility is clear in all directions from tripod height.




SKETCH

DA FORM 1 OCT 64 1959 REPLACES DA FORMS 1959 AND 1960, 1 FEB 57, WHICH ARE OBSOLETE. DESCRIPTION OR RECOVERY OF HORIZONTAL CONTROL STATION (TM 5-237)

① Front of DA Form 1959

*Figure 180. Description of horizontal control station (occupied).*

## NOTES ON COMPLETION OF FORM

1. GENERAL: This form may be used in the field or, as an office form to record and publish positions, descriptions, and related data.

2. FIELD USE OF FORM: The information required should be obtained and recorded *AT THE STATION SITE.* The field engineer should fill in only the information available and applicable to field use. In general, the geographic and grid positions, azimuths, distances, and elevations should not be filled in at field level except when the information is required for an immediate specific purpose.

a. ORIGINAL DESCRIPTION OF NEW STATION: The type of mark used for the station, reference marks, and azimuth marks, and a description of each must be given in the text of the description. If a disk is used, the identity of the agency whose name is cast in the disk and all of the letters and numbers stamped on the mark which identify the organization establishing or setting the mark should be given. In many areas the use of disks is not desirable because of their loss, due to vandalism or superstition. Less conspicuous marks should be used under these conditions. This requires exact statements of the character of the marks. Information for all marks as to the elevation above or below ground and approximate elevation above or below nearby prominent features is important. At least three measurements within .01 foot should be made from the station to any permanent marks, features, or structures that would permit relocating the spot where an instrument was centered. Good judgment should be exercised as to how far these measurements should be made. It is recommended that they be made to items which are not in the immediate vicinity of the station. Angles should also be turned to these items, particularly where no azimuth mark or marks have been established.

b. VIEW: Provide information on height of tower or stand used in occupying or establishing the station and information on view from a normal tripod, i.e., a 50-foot tower was used at the station; view from a tripod height is clear to the south and east but is obstructed by rise in ground *(by 50 foot trees)* to the north and west.

c. PHOTOGRAPHIC IDENTIFICATION: Provide when possible, two measurements from the station to natural or cultural features which might be visible on aerial photography and a description of the terrain. If photographs are available identify the station thereon and note estimated accuracy of the identification.

d. NOTES ON RECOVERED STATIONS: A diligent search should be made for *ALL* previously established stations in the vicinity and no station should be reported as destroyed unless conclusive evidence of destruction is present. A statement of the diligence of the search and reason for the nonrecovery of a previously established mark is required. If the spot where a station mark was located can be reproduced by measurement given in the description, the station is not destroyed. The reproduced spot should be tied in by azimuth and distance and the estimated accuracy of the reproduced location given. If a new mark is set in the exact location of a previously established but destroyed mark, the designation of the station should be identical with the original with only a new date added to its designation. If a new disk is set in the approximate location of the old station, the name should be preserved but the number "2" and a new date should be added.

| (DESCRIBED) ([redacted]) BY | |
|---|---|
| Wm. Kuronen. | |
| PROJECT | |
| 61821-003 | |
| DATE | FIELD BOOK |
| 1962 | 2 |

② Reverse of DA Form 1959

*Figure 180*—Continued.

| COUNTRY | TYPE OF MARK | STATION | |
|---|---|---|---|
| UNITED STATES | — | SPIRE, 1st METH. CHURCH, AMS 1960 | |
| LOCALITY | STAMPING ON MARK | AGENCY (CAST IN MARK) | ELEVATION (FT) (M) |
| MESA, ARIZONA | — | — | |
| LATITUDE | LONGITUDE | DATUM | DATUM |
| 33° 24' 45."131 | 111° 49' 49."773 | 1927 NAD | |
| (NORTHING) ~~(EASTING)~~ ~~(FT)~~ (M) | (EASTING) ~~(NORTHING)~~ ~~(FT)~~ (M) | GRID AND ZONE | ESTABLISHED BY (AGENCY) |
| 3,697,137.95 | 422,779.12 | UTM 12 | |
| (NORTHING) ~~(EASTING)~~ (FT) ~~(M)~~ | (EASTING) ~~(NORTHING)~~ (FT) ~~(M)~~ | GRID AND ZONE | DATE / ORDER |
| 877,607.56 | 526,295.70 | STATE PLANE CENTRAL | |

TO OBTAIN U.T.M. GRID AZIMUTH, ADD 180° 27' 26.44" TO THE GEODETIC AZIMUTH

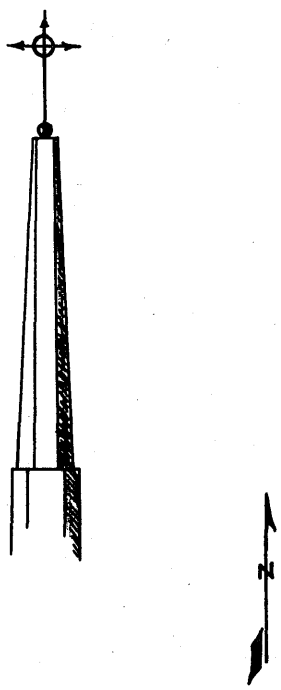
TO OBTAIN S.P.C. GRID AZ. ~~(ADD)~~(SUB.) 0° 02' 50.83" TO THE GEODETIC AZIMUTH

| OBJECT | AZIMUTH ~~OR DIRECTION~~ (GEODETIC) ~~(GRID)~~ ~~(MAGNETIC)~~ | BACK AZIMUTH | GEOD. DISTANCE (METERS) ~~(FEET)~~ | GRID DISTANCE (METERS) (FEET) |
|---|---|---|---|---|
| AA-5 | 196° 33' 26."14 | 16° 33' 43."15 | 2799.9048 | |
| AA-8 | 339 54 00.49 | 159 54 17.20 | 2282.6339 | |
| AA-11 | 354 52 23.82 | 174 52 37.12 | 6994.4263 | |
| | | | | |
| | | | | |

Intersection Station

The point observed was the center of the vertical shaft of the cross at the top center of the octagon shaped, tapered copper spire on the First Methodist Church. Estimated height 100 feet.

The church is located southeast of the intersection of South Center and East 1st Avenue in Mesa, Arizona.




SKETCH

DA FORM 1 OCT 64 1959 REPLACES DA FORMS 1959 AND 1960, 1 FEB 57, WHICH ARE OBSOLETE. DESCRIPTION OR RECOVERY OF HORIZONTAL CONTROL STATION (TM 5-237)

*Figure 181. Description of intersection station.*

# CHAPTER 10

# DIFFERENTIAL LEVELING

## Section I. DIFFERENTIAL LEVEL LINE

### 115. Scope

Leveling is the operation of measuring vertical distances with respect to a reference or datum. This section will deal with a single line of levels, from the abstracting of the data from field books, to the computation and adjustment of the data. See TM 5–441 for details on field procedures and notekeeping.

### 116. Abstract of Data

*a.* Before the level data is abstracted, the field book must be completely checked. Normally, this is done by the chief of the field party. The computer should make a review of the field book to see that all of the necessary information has been recorded.

*b.* The information necessary for the adjustment is abstracted directly from the field book on DA Form 1942 (Computation of Levels) (fig. 182).

### 117. Computations

*a.* Fill in the headings, crossing out the units of measurement that are not applicable—miles, kilometers, feet, or meters.

LINE: BM #33 to BM #34 — ORDER: 3rd — COMPUTATION OF LEVELS (TM 5-237)

PROJECT: 7-2438 — ORGANIZATION: A & S, Inc. — CLOSURE (~~MISTAKE~~) (FEET): 0.01

LOCATION: Oregon — LENGTH OF LINE (~~KMS.~~) (MILES): 4.0 — CORRECTION PER (~~KM.~~) (MILE): −0.0025

| SECTION | LENGTH mi. ~~km.~~ | DIST. FROM B.M. #33 mi. ~~km.~~ | F B | DIFFERENCE feet ~~meters~~ | F–B DIFF. | MEAN DIFFERENCE | B. M. | OBS. ELEV. feet ~~meters~~ | CORR. | ADJ. ELEV. feet ~~meters~~ | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | #33 | | | 1262.22 | |
| #33-A | 1.0 | 1.0 | F | +46.70 | | | A | 1308.92 | — | 1308.92 | |
| A-B | 1.0 | 2.0 | F | +73.31 | | | B | 1382.23 | — | 1382.23 | |
| B-C | 1.0 | 3.0 | F | +55.20 | | | C | 1437.43 | -0.01 | 1437.42 | |
| C-#34 | 1.0 | 4.0 | F | +42.97 | | | #34 | 1480.40 | -0.01 | 1480.39 | |
| Observed Diff. of Elev. | | | | +218.18 | | | | | | | |
| | | | | | | | | Error = | +0.01 | | |
| | | | | | | | | Allowable Error = | 0.10 | = 0.05′ √4 miles | |

COMPUTED BY: RAS — DATE: 29 Dec 54 — CHECKED BY: WCA — DATE: 30 Dec 54

DA FORM 1 FEB 57 1942

*Figure 182. Adjustment of a level line (DA Form 1942).*

*b.* The starting bench mark with its adjusted elevation is entered in the adjusted elevation column. From the field book is abstracted the length, direction of measurement—forward (F) or backward (B)—and difference of elevation for the section. In the case of a section being leveled both in a forward and backward direction, enter the F–B differences in the F–B column and then enter the mean of the runnings in the mean difference column. The sign of the mean difference is the sign of the forward running. The observed elevation for each bench mark is computed and the difference between the observed elevation and the fixed elevation of the ending bench mark is the error of closure.

### 118. Adjustment

*a.* The single line of levels is adjusted by distributing the closure error, in proportion to the distance from the starting bench mark.

*b.* The correction factor is obtained by dividing the closure error by the total distance of the line. The correction is then computed by multiplying the correction factor by the total distance from the starting bench mark to each point to be adjusted.

## Section II. ADJUSTMENT OF A LEVEL NET

### 119. Methods

There are many methods of adjusting a level net; some give a rigorous adjustment, and some give an approximate adjustment. To adjust a level net of third- or lower-order control points, which are established for a specific project (either mapping or construction), and which are not to be permanent (monumented) control points, any approximate adjustment that meets the accuracy requirements of the project can be used. Where the control points established are third- or higher-order, and are to be permanent (monumented), a more rigorous method of adjustment should be used.

### 120. Approximate Adjustment

*a.* Figure 183 is a small level net consisting of four permanent bench marks and seven vertical control points. Figure 184 demonstrates one of the approximate solutions that could be applied in this net to meet the accuracy requirements of third-order work. In the example, the net is broken into four lines, and each is computed separately, the last three utilizing one or more elevations computed from the preceding line or lines. The corrections are applied in the same manner as a single line computation.

*b.* Another adjustment sometimes necessary for vertical control points is the datum adjustment. This occurs when the elevations are referenced to a datum other than the fixed datum for the country or area where the points are located.

### 121. Adjustment By Least Squares Method

*a.* An adjustment of a net by the least squares method will be discussed here. The level net (fig. 185) consists of nine lines, and five junction points, which form five circuits and contain four fixed elevations.

*b.* On a diagram of the level net, indicate fixed elevations, junction points, circuit numbers, line numbers, observed differences of elevation between junction points, and between fixed elevations, also distance, error, and allowable error for each circuit.

*c.* Prepare a table of lines (fig. 186) listing the number assigned to each line, the designations of the bench marks at the ends of each line, the length of each line, the difference of elevation between the terminal ends of each line, and the sign and number of each "*v*" or symbol representing the correction to be applied to the corresponding observed difference in elevation. The entries for correction, adjusted difference of elevations, and rate are filled in after the adjustment computations have been completed.

*d.* Prepare a computation of circuit closures (fig. 187). Whenever a fixed difference of elevation enters into the computation of a circuit closure, the line number, distance, *v*, and adjustment correction are deleted, and the observed difference and adjusted difference are identical. Note that while circuit 5 is computed for check purposes, it is not used in the adjustment. Its inclusion in the solution would create an identity,

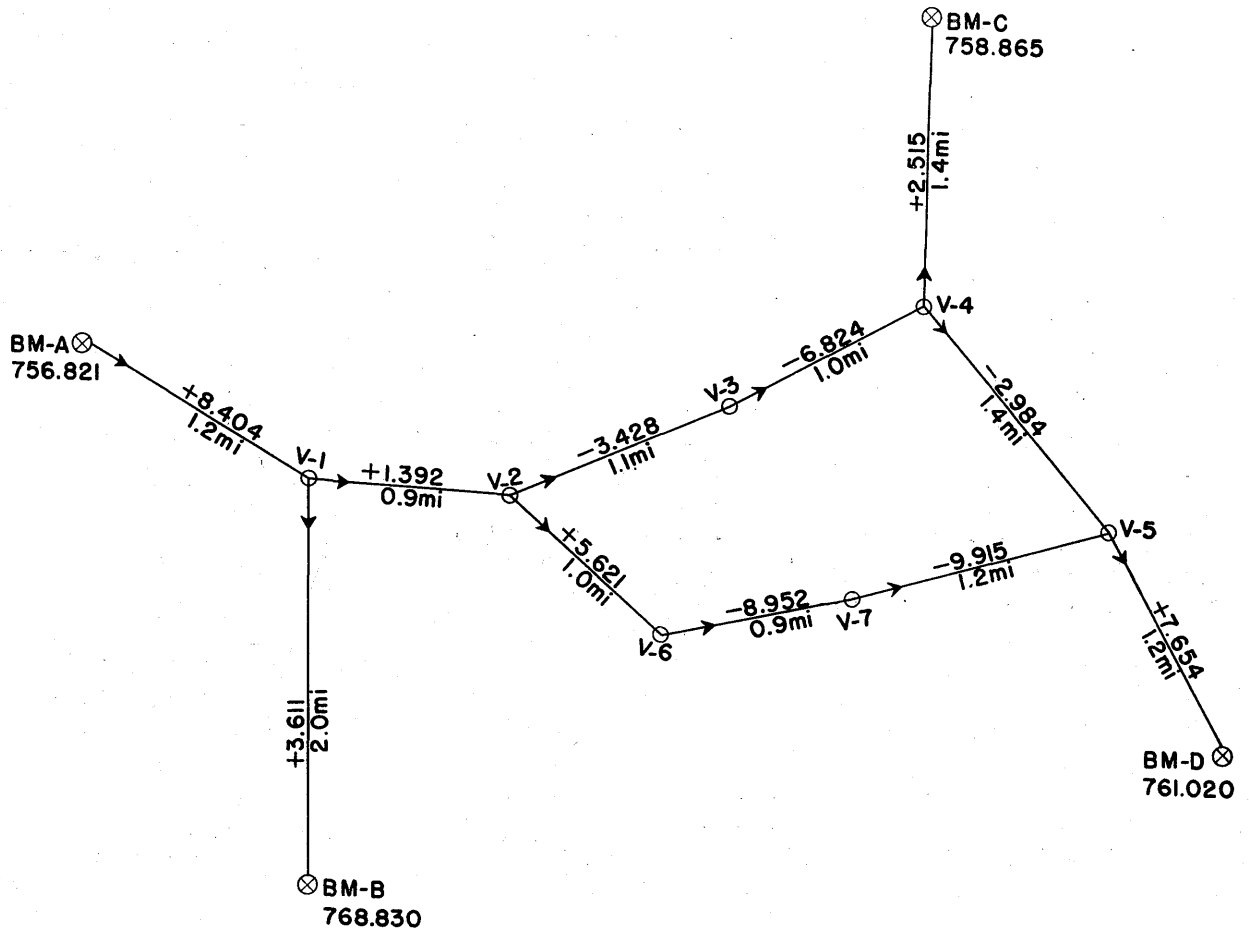



*Figure 183. Diagram of a level net.*

in that all terms have already been used in the first four circuits.

*e.* When computing the circuit closures using the observed differences of elevation from BM to BM, proceed around the circuits in a clockwise direction. When the BM to BM column in the computation of circuit closures shows the designations of the bench marks in the order opposite to that in which they appear in the table of lines, the algebraic signs of both the difference of elevation and the correction symbol must be changed. Otherwise the signs are as shown in the table of lines.

*f.* Prepare a table of correlates (fig. 188). This table has as many horizontal lines of entries as there are $v$'s to be computed, and three more columns than the number of circuits in the adjustment.

(1) The entries in the $1/p$ column are the reciprocals of the weights assigned to the lines. The reciprocal of the weight of the line is computed by dividing the length of each line by some constant to produce the values of $1/p$. In this adjustment, the length of the longest line in the adjustment was used as the constant and each line in the adjustment was divided by this constant to secure the individual entries in column 2 of the Table of Correlates.

(2) In the table of lines (fig. 186), the sum of the line lengths is 13.3 kilometers. This figure is used to check the $1/p$ entries of the table of correlates. The sum of the $1/p$ entries is 6.34. This is equal to the sum of the lengths (13.3) divided by 2.1 and is, except for the possibility of compensating errors, a check on the computed values of $1/p$.

(3) The entries in the numbered columns of the table of correlates are the coefficients of the $v$'s of the four circuits. Since the numerical coefficient of each $v$ is unity,

LINE NET: BM-A, BM-B, BM-C, BM-D — ORDER: 3d

COMPUTATION OF LEVELS (TM 5-237)

PROJECT: 7-2438 — ORGANIZATION: A & S, Inc. — CLOSURE (METERS) (FEET):

LOCATION: Oregon — LENGTH OF LINE NET (KMS.) (MILES): 13.3 — CORRECTION PER (KM.) (MILE): See Remarks

| SECTION | LENGTH mi. km | DIST. FROM B.M. A mi. km | F B | DIFFERENCE feet meters | F–B DIFF | MEAN DIFFERENCE | B. M. | OBS. ELEV. feet meters | CORR. | ADJ. ELEV. feet meters | Correction REMARKS Factor |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | A | | | 756.821 | |
| A – V-1 | 1.2 | 1.2 | F | +8.404 | | | V-1 | 765.225 | -.002 | 765.223 | .0018750 |
| V-1 – B | 2.0 | 3.2 | F | +3.611 | | | B | 768.836 | -.006 | 768.830 | |
| | | DIST. FROM V-1 | | | | | V-1 | | | 765.223 | |
| V-1 – V-2 | 0.9 | 0.9 | F | +1.392 | | | V-2 | 766.615 | -.003 | 766.612 | |
| V-2 – V-3 | 1.1 | 2.0 | F | -3.428 | | | V-3 | 763.187 | -.006 | 763.181 | .0028545 |
| V-3 – V-4 | 1.0 | 3.0 | F | -6.824 | | | V-4 | 756.363 | -.009 | 756.354 | |
| V-4 – C | 1.4 | 4.4 | F | +2.515 | | | C | 758.878 | -.013 | 758.865 | |
| | | DIST. FROM V-4 | | | | | V-4 | | | 756.354 | |
| V-4 – V-5 | 1.4 | 1.4 | F | -2.984 | | | V-5 | 753.370 | -.002 | 753.368 | .0015385 |
| V-5 – D | 1.2 | 2.6 | F | +7.654 | | | D | 761.024 | -.004 | 761.020 | |
| | | DIST. FROM V-2 | | | | | V-2 | | | 766.612 | |
| V-2 – V-6 | 1.0 | 1.0 | F | +5.621 | | | V-6 | 772.233 | +.001 | 772.234 | .0006452 |
| V-6 – V-7 | 0.9 | 1.9 | F | -8.952 | | | V-7 | 763.281 | +.001 | 763.282 | |
| V-7 – V-5 | 1.2 | 3.1 | F | -9.915 | | | V-5 | 753.366 | +.002 | 753.368 | |

COMPUTED BY: R. A. Smith — DATE: 4 Jan. 55 — CHECKED BY: W. C. Aumen — DATE: 6 Jan. 55

DA FORM 1 FEB 57 1942.

*Figure 184. Adjustment of a level net (DA Form 1942).*

each entry in the numbered columns must be either +1 or −1.

(*a*) For circuit 1 in column 6 of the tabulation of the circuit-closure computations was entered $+v_1+v_2$. Therefore, in column 1 of the table of correlates, enter +1 on line 1 and +1 on line 2.

(*b*) The *v*'s for circuit 2 are $-v_7$, $-v_5$, $-v_3$, $-v_1$. Enter in column 2 of the table of correlates, −1 on line 7, −1 on line 5, −1 on line 3, and −1 on line 1.

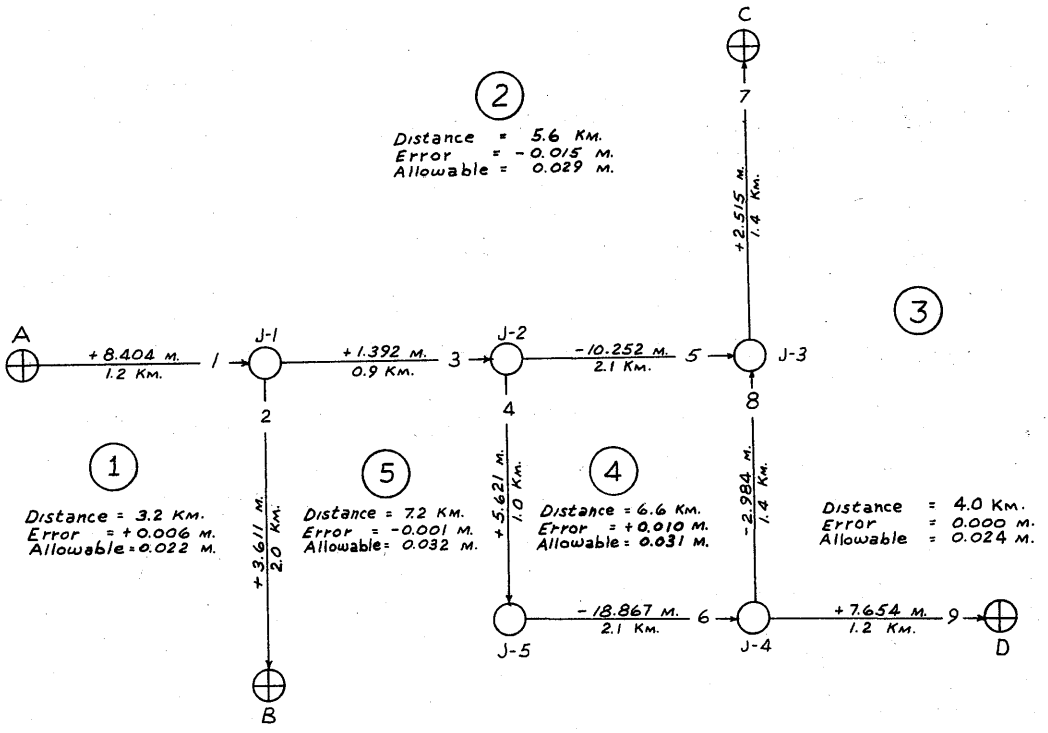
(*c*) This same procedure is followed until the coefficients of all the *v*'s in the four circuit closures have been entered in the proper places in the table of correlates.

(4) Each entry in the Σ column is the algebraic sum of the entries on the same horizontal line in the four numbered columns.

(5) Notice that all of the entries in the Σ column are zero except on lines 2, 3, 4, 6, and 9 and that these are the lines in circuit 5 which are not used in the adjustment computation. Except in cases where overlapping circuits occur, this peculiarity of the Σ column entries will be the regular thing and will serve as a partial check on the table of correlates. However, in any extensive adjustment, the table of correlates should be checked thoroughly before proceeding with the next step, the formation of the normal equations.

*g*. Form the normal equations (fig. 189) using the data in the table of correlates.

(1) In column 1 of the table of correlates are two entries, +1 on line 1 and +1 on line 2. Multiply each of these entries by itself and then multiply each product by




| LIST OF FIXED ELEVATIONS | | | |
|---|---|---|---|
| BM | ELEVATION METERS | DIRECTION | DIFFERENCE IN ELEVATION METERS |
| A | 756.821 | C → A | -2.044 |
| B | 768.830 | B → A | -12.009 |
| C | 758.865 | C → D | +2.155 |
| D | 761.020 | D → B | +7.810 |

LEGEND

⊕ FIXED ELEV.

○ JUNCTION PT.

(1) CIRCUIT NO.

*Figure 185. Level net for least squares adjustment.*

the corresponding entry in the $1/p$ column, respectively. The sum of the products is the entry for line 1 in column 1 of the normal equations. In mathematical form, this is $(+1)(+1)(+0.57)+(+1)(+1)(+0.95)=+1.52$.

(2) The entry for line 1, column 2 of the normal equations is the product of the $+1$ on line 1 of column 1 of the correlates, multiplied by the $-1$ on line 1 of column 2 and then multiplied by the 0.57 of line 1 of the $1/p$ column. Thus: $(+1)(-1)(+0.57)=-0.57$, which is entered on line 1 of column 2 of the normal equations.

(3) Columns 3 and 4 of the correlates have no entries on lines 1 or 2, so no entries are made on line 1 of the normal equations in columns 3 and 4.

(4) The entry on line 1 of the N column of the normal equations is the closure of circuit 1 from the circuit-closure computations. Multiply entries in column 1 by the corresponding entries in the $\Sigma$ column of the correlates, and multiply these products by the corresponding entries in the $1/p$ column. The sum of these products, plus the closure of circuit, is entered for line 1 of the $\Sigma$ column of the normal equations.

$$(+1)(0)(+0.57)= \quad 0.00$$

$$(+1)(+1)(+0.95)=+0.95$$

$$+0.95+0.006=+0.956$$

(5) This completes the formation of the first normal equation. The equation is

TABLE OF LINES

| 1 LINE | 2 BM TO BM | 3 LENGTH KM. | 4 OBSERVED DIFFERENCE METERS | 5 "V" | 6 ADJUSTMENT CORRECTION METERS | 7 ADJUSTED DIFFERENCE METERS | 8 RATE M/KM |
|---|---|---|---|---|---|---|---|
| 1 | "A" TO J-1 | 1.2 | + 8.404 | + $V_1$ | - .004 | + 8.400 | -.003333 |
| 2 | J-1 TO "B" | 2.0 | + 3.611 | + $V_2$ | - .002 | + 3.609 | -.001000 |
| 3 | J-1 TO J-2 | 0.9 | + 1.392 | + $V_3$ | - .002 | + 1.390 | -.002222 |
| 4 | J-2 TO J-5 | 1.0 | + 5.621 | + $V_4$ | + .001 | + 5.622 | +.001000 |
| 5 | J-2 TO J-3 | 2.1 | - 10.252 | + $V_5$ | - .006 | - 10.258 | -.002857 |
| 6 | J-5 TO J-4 | 2.1 | - 18.867 | + $V_6$ | + .001 | - 18.866 | +.000476 |
| 7 | J-3 TO "C" | 1.4 | + 2.515 | + $V_7$ | - .003 | + 2.512 | -.002143 |
| 8 | J-3 TO J-4 | 1.4 | - 2.984 | + $V_8$ | - .002 | - 2.986 | -.001429 |
| 9 | J-4 TO "D" | 1.2 | + 7.654 | + $V_9$ | - .001 | + 7.653 | -.000833 |
| | | 13.3 | | | | | |

*Figure 186. Table of lines.*

checked by adding the values on line 1 of the normal equations and comparing the total with the entry on line 1 of the $\Sigma$ column.

$$+1.52-0.57+.006=+0.956 \text{ (check)}$$

(6) For the second normal equation, the product of the correlate entries in column 2 by column 1 by column $1/p$ was previously computed and entered on line 1 of column 2 of the table of normal equations.

(7) Multiply the correlate entries in column 2 by themselves and by the corresponding entries in the $1/p$ column, and enter on line 2 of column 2, normal equations.

$$\begin{aligned}(-1)(-1)(+0.57)&=+0.57\\(-1)(-1)(+0.43)&=+0.43\\(-1)(-1)(+1.00)&=+1.00\\(-1)(-1)(+0.67)&=+0.67\\&\overline{+2.67}\end{aligned}$$

(8) $(-1)(+1)(+0.67)=-0.67$ is the entry for line 2, column 3 of the normal equations.

(9) $(-1)(+1)(+1.00)=-1.00$ is the entry on line 2, column 4 of the normal equations.

(10) Enter on line 2 of column N of the normal equations, the circuit closure of circuit 2.

(11) The product of columns 2, $\Sigma$, and $1/p$ of the correlates is:

$$\begin{aligned}(-1)(0)(+0.57)&=\ 0.00\\(-1)(-1)(+0.43)&=+0.43\\(-1)(0)(+1.00)&=\ 0.00\\(-1)(0)(+0.67)&=\ \underline{0.00}\\&+0.43-0.015=+0.415\end{aligned}$$

(12) The check on the second normal equation is obtained by adding down column 2, to and including line 2 and then along line 2 to the right, to and including the N column, thus $-0.57+2.67-0.67-1.00-0.015=+0.415$ (check). This

COMPUTATION OF CIRCUIT CLOSURES

| 1 CIRCUIT | 2 LINE | 3 BM TO BM | 4 DIST. (KM.) | 5 OBS. DIFF. (M) | 6 V | 7 ADJ. CORR'N (M) | 8 ADJ. DIFF. (M) |
|---|---|---|---|---|---|---|---|
| | - | "A" TO "B" | - | - 12.009 | - | - | - 12.009 |
| 1 | 1 | "A" TO J-1 | 1.2 | + 8.404 | + $V_1$ | - .004 | + 8.400 |
| | 2 | J-1 TO "B" | 2.0 | + 3.611 | + $V_2$ | - .002 | + 3.609 |
| | | | 3.2 | + .006 | | - .006 | 0.000 |
| | - | "C" TO "A" | - | + 2.044 | - | - | + 2.044 |
| 2 | 7 | "C" TO J-3 | 1.4 | - 2.515 | - $V_7$ | + .003 | - 2.512 |
| | 5 | J-3 TO J-2 | 2.1 | + 10.252 | - $V_5$ | + .006 | + 10.258 |
| | 3 | J-2 TO J-1 | 0.9 | - 1.392 | - $V_3$ | + .002 | - 1.390 |
| | 1 | J-1 TO "A" | 1.2 | - 8.404 | - $V_1$ | + .004 | - 8.400 |
| | | | 5.6 | - .015 | | + .015 | 0.000 |
| | - | "D" TO "C" | - | + 2.155 | - | - | + 2.155 |
| 3 | 9 | "D" TO J-4 | 1.2 | - 7.654 | - $V_9$ | + .001 | - 7.653 |
| | 8 | J-4 TO J-3 | 1.4 | + 2.984 | - $V_8$ | + .002 | + 2.986 |
| | 7 | J-3 TO "C" | 1.4 | + 2.515 | + $V_7$ | - .003 | + 2.512 |
| | | | 4.0 | 0.000 | | .000 | 0.000 |
| | 5 | J-2 TO J-3 | 2.1 | - 10.252 | + $V_5$ | - .006 | - 10.258 |
| 4 | 8 | J-3 TO J-4 | 1.4 | - 2.984 | + $V_8$ | - .002 | - 2.986 |
| | 6 | J-4 TO J-5 | 2.1 | + 18.867 | - $V_6$ | - .001 | + 18.866 |
| | 4 | J-5 TO J-2 | 1.0 | - 5.621 | - $V_4$ | - .001 | - 5.622 |
| | | | 6.6 | + .010 | | - .010 | 0.000 |
| | - | "B" TO "D" | - | + 7.810 | - | - | + 7.810 |
| 5 | 2 | "B" TO J-1 | 2.0 | - 3.611 | - $V_2$ | + .002 | - 3.609 |
| | 3 | J-1 TO J-2 | 0.9 | + 1.392 | + $V_3$ | - .002 | + 1.390 |
| | 4 | J-2 TO J-5 | 1.0 | + 5.621 | + $V_4$ | + .001 | + 5.622 |
| | 6 | J-5 TO J-4 | 2.1 | - 18.867 | + $V_6$ | + .001 | - 18.866 |
| | 9 | J-4 TO "D" | 1.2 | + 7.654 | + $V_9$ | - .001 | + 7.653 |
| | | | 7.2 | - .001 | | + .001 | 0.000 |

*Figure 187. Circuit closures.*

same routine is continued for the remainder of the normal equations.

*h.* The four simultaneous linear equations in the normal equations are solved (figs. 190 and 191) by the Doolittle method (para. 65).

*i.* The computation of the $v$'s (fig. 192) is made using a table quite similar in appearance to that of the table of correlates. The lines of the table are numbered from 1 to 9 inclusive. The columns are headed, No. of "$v$," 1, 2, 3, 4, $\Sigma$, $1/p$, $(\Sigma)(1/p)$, and Adopted "$v$."

(1) Use the total of column 1 at the right of the back solution as a constant multiplier for the entries in column 1 of the table of correlates, and enter the products in the corresponding spots in column 1 of the computation of $v$'s. Use the total for column 2 of the back solution as a constant multiplier for all entries in column 2 of the table of correlates, and enter these products in column 2 of the computation of $v$'s. Continue this operation until

TABLE OF CORRELATES

| V | 1/p | 1 | 2 | 3 | 4 | Σ |
|---|---|---|---|---|---|---|
| 1 | 0.57 | + 1 | - 1 | | | 0 |
| 2 | 0.95 | + 1 | | | | + 1 |
| 3 | 0.43 | | - 1 | | | - 1 |
| 4 | 0.48 | | | | - 1 | - 1 |
| 5 | 1.00 | | - 1 | | + 1 | 0 |
| 6 | 1.00 | | | | - 1 | - 1 |
| 7 | 0.67 | | - 1 | + 1 | | 0 |
| 8 | 0.67 | | | - 1 | + 1 | 0 |
| 9 | 0.57 | | | - 1 | | - 1 |
| | 6.34 | | | | | |
| 13.3 ÷ 2.1 = 6.33 (CHECK) | | | | | | |

*Figure 188. Table of correlates.*

NORMAL EQUATIONS

| | 1 | 2 | 3 | 4 | N | Σ |
|---|---|---|---|---|---|---|
| 1 | + 1.52 | - 0.57 | | | + 0.006 | + 0.956 |
| 2 | | + 2.67 | - 0.67 | - 1.00 | - 0.015 | + 0.415 |
| 3 | | | + 1.91 | - 0.67 | 0.000 | + 0.570 |
| 4 | | | | + 3.15 | + 0.010 | + 1.490 |

*Figure 189. Normal equations.*

all four numbered columns of the computation of $v$'s have been filled in. At this point, the numbered columns of the computation of $v$'s should contain the same number of entries as the four numbered columns of the table of correlates, and the arrangement of these entries in the various columns and lines should be the same as for the corresponding entries in the table of correlates.

(2) Enter in the Σ column, computation of $v$'s, the sum of the entries on the same line in the four numbered columns of the table. Complete the column headed $1/p$. Each entry in the column headed $(\Sigma)(1/p)$ is the product of the corresponding entries in the two columns to the left.

(3) The adopted $v$ column is completed by rounding the entries in the $(\Sigma)(1/p)$ column.

*j*. Enter the adopted $v$'s in the Adj. Corr'n. column of the circuit-closure computations. The $v$'s for circuits 2 and 3 failed to close by 0.001. It was noted that $v_7$, common to circuits 2 and 3, had a computed $v$ of 0.0025 *m*. Changing $v$ to 0.003 closed all circuits.

FORWARD SOLUTION

| 1 | 2 | 3 | 4 | N | Σ | C |
|---|---|---|---|---|---|---|
| + 1.52 | - 0.57 | - | - | + 0.006 | + 0.956 | |
| $C_1$ | + 0.37500 | - | - | - 0.00395 | - 0.62895 | - .00207 |
| | + 2.67000 | - 0.67000 | - 1.00000 | - 0.015 | + 0.415 | |
| | - 0.21375 | - | - | + 0.00225 | + 0.35850 | |
| | + 2.45625 | - 0.67000 | - 1.00000 | - 0.01275 | + 0.77350 | |
| | $C_2$ | + 0.27277 | + 0.40712 | + 0.00519 | - 0.31491 | + .00501 |
| | | + 1.91 | - 0.67 | 0.00000 | + 0.57 | |
| | | - | - | - | - | |
| | | - 0.18276 | - 0.27277 | - 0.00348 | + 0.21099 | |
| | | + 1.72724 | - 0.94277 | - 0.00348 | + 0.78099 | |
| | | $C_3$ | + 0.54582 | + 0.00201 | - 0.45217 | + .00129 |
| | | | + 3.15000 | + 0.01000 | + 1.49000 | |
| | | | - | - | - | |
| | | | - 0.40712 | - 0.00519 | + 0.31491 | |
| | | | - 0.51458 | - 0.00190 | + 0.42628 | |
| | | | + 2.22830 | + 0.00291 | + 2.23121 | |
| | | | $C_4$ | - 0.00131 | - 1.00131 | - .00131 |

*Figure 190. Forward solution.*

BACK SOLUTION

| 4 | 3 | 2 | 1 |
|---|---|---|---|
| - 0.00131 | + 0.00201 | + 0.00519 | - 0.00395 |
| | - 0.00072 | - 0.00053 | - |
| | + 0.00129 | + 0.00035 | - |
| | | + 0.00501 | + 0.00188 |
| | | | - 0.00207 |

*Figure 191. Back solution.*

*k.* Enter adjustment corrections in the table of lines. Compute the adjusted differences of elevation, and the rates of correction in *mm/km.*

*l.* Adjust the elevations of the bench marks, between the junction points, of each line in the adjustment. The correction to each elevation is obtained by multiplying the correction factor for each individual line by the distance to each mark from the starting bench mark (fig. 193).

COMPUTATION OF "V's"

| NO. OF "V" | 1 | 2 | 3 | 4 | Σ | 1/p | (Σ)(1/p) | ADOPTED "V" (M) |
|---|---|---|---|---|---|---|---|---|
| 1 | - .00207 | -. 00501 | | | - .00708 | 0.57 | - .0040 | -. 004 |
| 2 | - .00207 | | | | - .00207 | 0.95 | - .0020 | - .002 |
| 3 | | -. 00501 | | | - .00501 | 0.43 | - .0022 | - .002 |
| 4 | | | | + .00131 | + .00131 | 0.48 | + .0006 | + .001 |
| 5 | | -. 00501 | | - .00131 | - .00632 | 1.00 | - .0063 | - . 006 |
| 6 | | | | + .00131 | + .00131 | 1.00 | + .0013 | + . 001 |
| 7 | | -. 00501 | + .00129 | | - .00372 | 0.67 | - .0025 | - . 003 |
| 8 | | | - .00129 | - .00131 | - .00260 | 0.67 | - .0017 | - . 002 |
| 9 | | | - .00129 | | - .00129 | 0.57 | - .0007 | - . 001 |

*Figure 192. Computation of corrections.*

LINE | ORDER | COMPUTATION OF LEVELS (TM 5-237)

PROJECT | ORGANIZATION | CLOSURE (METERS) (FEET)

LOCATION | LENGTH OF LINE (KMS.) (MILES) | CORRECTION PER (KM.) ~~(MILE)~~ -.003333 METERS BM "A" TO BM J-1

| SECTION | LENGTH ~~mi.~~ km. | DIST. FROM B.M. "A" ~~mi.~~ km. | F B | OBSERVED DIFFERENCE ~~feet~~ meters | F-B DIFF. | MEAN DIFFERENCE | B. M. | OBS. ELEV. ~~feet~~ meters | CORR. (M.) | ADJ. ELEV. ~~feet~~ meters | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| BM "A" | | | | | | | A | | | 756.821 | |
| BM H-1 | 0.5 | 0.5 | | + 4.000 | | | H-1 | 760.821 | -.002 | 760.819 | |
| BM J-1 | 0.7 | 1.2 | | + 4.404 | | | J-1 | 765.225 | -.004 | 765.221 | |
| | | | | | | | | | | | |
| | | J-1 | | | | | | -.001000 M/KM. | | BM H-1 TO BM "B" | |
| BM J-1 | | | | | | | J-1 | | | 765.221 | |
| BM K-1 | 0.8 | 0.8 | | - 2.100 | | | K-1 | 763.121 | -.001 | 763.120 | |
| BM "B" | 1.2 | 2.0 | | + 5.711 | | | "B" | 768.832 | -.002 | 768.830 | |
| | | | | | | | | | | | |
| | | J-1 | | | | | | -.022222 M/KM. | | BM J-1 TO BM J-2 | |
| BM J-1 | | | | | | | J-1 | | | 765.221 | |
| BM J-2 | 0.9 | 0.9 | | + 1.392 | | | J-2 | 766.613 | -.002 | 766.611 | |
| | | | | | | | | | | | |
| | | J-2 | | | | | | +.00100 M/KM. | | BM J-2 TO BM J-5 | |
| BM J-2 | | | | | | | J-2 | | | 766.611 | |
| BM J-5 | 1.0 | 1.0 | | + 5.621 | | | J-5 | 772.232 | +.001 | 772.233 | |
| | | | | | | | | | | | |
| | | | | | | | | | | | |
| | | | | | | | | | | | |
| | | | | | | | | | | | |

COMPUTED BY G. Tennis - AMS | DATE FEB. '64 | CHECKED BY L. E. Smith - AMS | DATE FEB. '64

DA FORM 1 FEB 57 1942 GPO 919185

*Figure 193. Final elevations, level net adjustment.*

## Section III. DESCRIPTION OF VERTICAL CONTROL STATION

### 122. Requirements

The description, sketch, and numerical information for marked or monumented stations are the final product of the expenditure of man-hours and money. The Chief of Party is responsible for the accuracy and completeness of this information. If the record does not provide sufficient information to locate the mark by measurements at the site and does not provide reference elevations from which later surveys can be initiated, most of the effort of the survey has been wasted. Every reasonable effort should be made to insure that—

*a.* The station description is written at the site.

*b.* All reference marks are described for recovery purposes.

*c.* Inconspicuous types of marks, rather than disks, should be used in areas where superstition and vandalism are probable.

*d.* Turning points and semipermanent or temporary bench marks (TBM's) at the ends of sections are described if their character is such that they might be recoverable for a reasonable time.

### 123. Control Card

*a.* DA Form 1958 (Description or Recovery of Bench Mark) is to be used for all marked or monumented bench marks that can be recovered and used. On the reverse side of this form, brief "NOTES ON COMPLETION OF FORM" are provided as a guide to the information required. Good judgment should be used in selecting the general location of a mark.

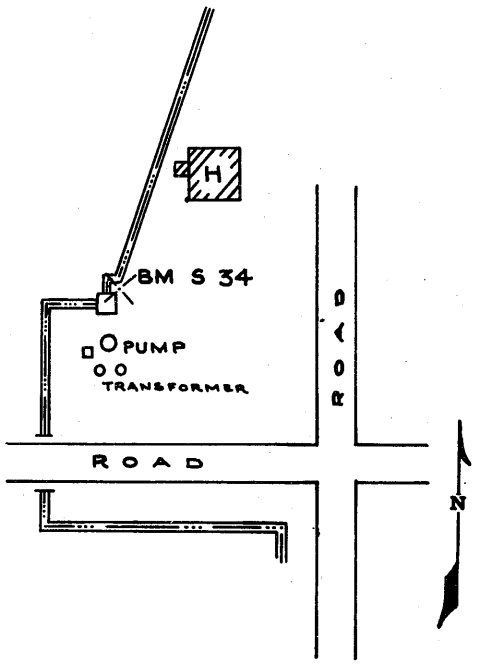
*b.* The date and letters of the organization establishing the station are an integral part of the station designation such as, "S–34, AMS, 1959". A bench mark should never be reported as destroyed unless positive evidence of destruction is present. A statement should be included as to the thoroughness of the search which was made for a bench mark that could not be located.

| COUNTRY | TYPE OF MARK | STATION | |
|---|---|---|---|
| UNITED STATES | DISK | S-34, 1952 AMS 1959 | |
| PROVINCE OR STATE | STAMPING ON MARK | ELEVATION | (FT) (M) |
| ARIZONA | SEE DESCRIPTION | 1225.736<br>373.605 | |
| MUNICIPALITY | AGENCY (Cast in Mark) | DATUM | |
| SE OF MARICOPA | CORPS OF ENGINEERS | SEA LEVEL DATUM OF 1929 | |
| LINE | ESTABLISHED BY (Agency) | EST. BY | ORDER |
| | ARMY MAP SERVICE | AMS 1959 | SECOND |
| LATITUDE | LONGITUDE | DATUM | |
| 33° 00.8 N | 111° 57.7 W | Scaled from map | |
| NORTHING (FT) (M) | EASTING | GRID AND ZONE | |
| NORTHING (FT) (M) | EASTING | GRID AND ZONE | |

The station is marked by a Corps of Engineers Bench Mark disk set in the top of an eight inch diameter concrete post the top of which projects 0.4 foot above the ground. The disk is stamped "BM S-34 1948 RESET AMS 1952."

The station mark is located 38.5 feet southwest of the southwest corner of a house; 20 feet north-northeast of an electric water pump; 2 feet northeast of the northeast corner of a concrete water box.

To reach from the Southern Pacific RR crossing in Maricopa, Arizona; go southeast on main gravel road along the Railroad for 6.25 miles to cotton gin and a farm road north; turn left and go north for 0.7 mile along the farm road to a farmhouse and the station site.




SKETCH

DA FORM 1 OCT 64 1958 REPLACES DA FORM 1958, 1 FEB 57, WHICH IS OBSOLETE. DESCRIPTION OR RECOVERY OF BENCH MARK (TM 5-237)

① Front of DA Form 1958

*Figure 194. Description or recovery of bench mark.*

INSTRUCTIONS

1. GENERAL. This form is used in the field or, as an office form to record and publish elevations, descriptions and related data.

2. FIELD USE OF FORM. The information required should be obtained and recorded at the Station site. The field engineer should fill in only the information available and applicable to field use. In general, the elevations, datum, and order should not be filled in at the field level except when the information is required for an immediate specific purpose.

*a.* Original Description of New Station. The station and reference marks must be described for recovery purposes. The following information is required for each mark; Identify exactly what the mark is, letters or numbers stamped on each mark, the object in which the mark is set, its elevation above or below ground, and its approximate elevation above or below some permanent feature such as the highest point of hill, top of rail, surface of highway, etc. If a disk is used, identify the agency whose name is cast in the disk. The description should contain reference measurements, immediate area description and the route to the area from the nearest town or city.

*b.* Photographic Identification. Provide measured distances to and descriptions of at least two natural or cultural features in the immediate vicinity which might be identifiable on an aerial photograph *(preferably at right angles to the mark)*. Photos taken by hand-held cameras or identification of the station on aerial photography should be furnished when available.

*c.* Notes on Recovered Stations. A diligent search should be made for previously established marks in the vicinity and no mark should be reported as destroyed unless conclusive evidence of destruction is present. A statement of the diligence of the search and reason for non-recovery of a previously established mark is required. If a new disk is set in the approximate location of the old station, the name should be preserved, but the number "2" and a new date should be added. When the station mark is recovered as previously described and additional reference measurements are not necessary, a note to that effect is necessary with the headings completed as in paragraph *a* above.

| 3. COMPLETE THE FOLLOWING | |
|---|---|
| PROJECT NUMBER AND NAME<br>61821-003 | |
| (DESCRIBED) (~~RECOVERED~~) BY<br>Wm. Kuronen | |
| DATE<br>1962 | FIELD BOOK NO<br>2 |
| PHOTO NO<br>560 | STRIP NO<br>8 |

② Reverse of DA Form 1958

*Figure 194*—Continued.

*c.* Space has been provided to enter the horizontal position of the bench mark for the reason that gravity surveys often follow level lines. The horizontal positions are required to an accuracy of one-tenth of a minute of latitude and longitude. These positions are sometimes scaled from maps. In addition, bench marks are often incorporated in traverse lines at a later date.

*d.* A sample of a properly written description is provided in figure 194.

## Section IV. COMPUTATION OF TIDE OBSERVATIONS

### 124. Tide Stations

*a.* Tides are caused by the gravitational pull of celestial bodies, principally the moon, and to a lesser degree the sun, acting on a pliable earth and the earth's fluid oceans. A detailed description of the causes of tides, the types of tides, and the methods of recording tides is given in TM 5–235.

*b.* Tidal observation stations are located at inland bays and deep inlets along the coast line, which are directly affected by the rise and fall of the tide and at the same time protected from storms and high winds. Tide stations are classified as primary and secondary. Primary stations are those at which observations have been continued for a complete tide cycle which is approximately 19 years. Secondary stations are those for which observations are generally limited to a period of a few months. In extending vertical control for mapping, secondary stations are of sufficient accuracy to be used as the origin as well as a terminating station.

### 125. Tidal Definitions

Tidal observations and recording are referred to some plane of reference called a datum or a water level. The more commonly used references are defined below.

*a.* Mean sea level (MSL) (fig. 195) is the mean level of the sea at any point. It is derived from the mean of a large number of observations taken at hourly intervals along the tidal curve.

*b.* Mean tide level (MTL) or half tide level (HTL) is the plane that lies exactly midway between the planes of mean high water and mean low water. It agrees very closely with MSL.

*c.* Mean high water (MHW) is the mean level of all high waters during a period of observation.

*d.* Mean low water (MLW) is the mean level of all low waters during a period of observation.

*e.* Mean higher high water (MHHW) is the mean level of all higher high water observations.

*f.* Mean lower low water (MLLW) is the mean level of all lower low water observations.

*g.* Spring low water (SpLW) is the average of low waters that come at the time of spring tides.

*h.* Mean range of tide ($M_N$) is the difference between mean high water and mean low water.

*i.* Diurnal inequality (DHQ for high water, DLQ for low water) occurs when the high tide is greater than average and the low tide is less than average, and is caused by changes in the moon's orbit to the equator and by changes in the declination of the sun.

### 126. Tabulations and Computation

*a.* At primary stations, three tabulations are made from the tide graph.

(1) Comparative readings to obtain the relation between the datum line of a tide-gage record and the datum adopted for the tabulations.

(2) Hourly heights scaled from the tide curve at hourly intervals. They are taken on the exact hour and are tabulated to tenths of a foot. The mean of hourly heights over a 19 year period is the MSL.

(3) High and low water are also tabulated from the tide graph. These values determine the MHW and MLW, and are used in the computation of simultaneous comparisons of tides with secondary stations.

*b.* Since the observations at secondary stations cover a much shorter period of time than primary stations, corrections must be applied to the secondary values. There are two methods available.

(1) *Comparison by simultaneous observations.* The comparison method, which is more satisfactory, requires two separate corrections; the first to derive the mean value of high water and low water above and below HTL, and the second to derive the value of HTL. An example of the computation of comparison of simultaneous observations for 7 days is given in figure 196. In the column of heights, the higher high and lower low waters are marked with a check from which the appropriate sums and means

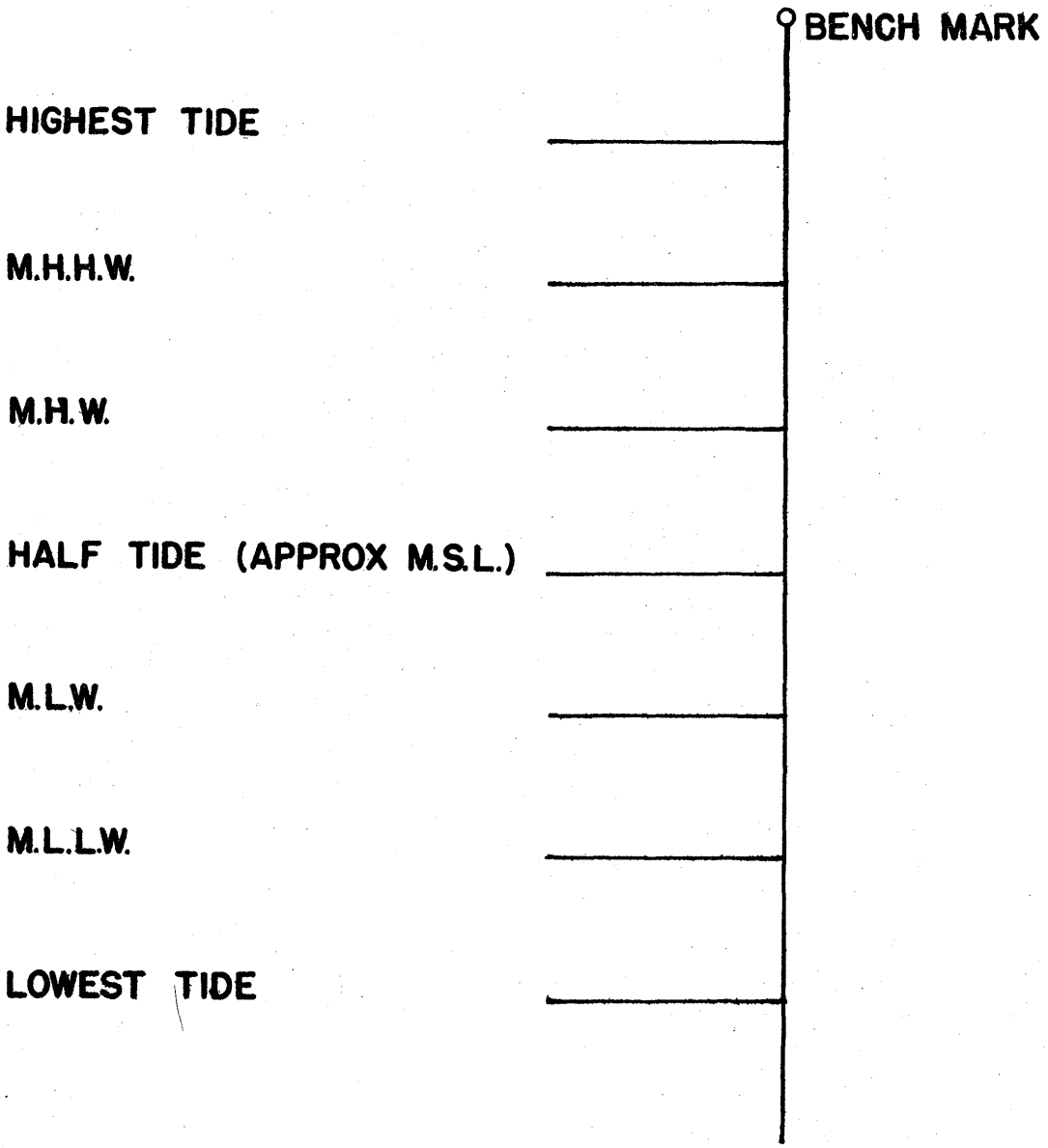



*Figure 195. Tidal datums.*

are derived. If the two high or two low waters on the same day are equal, either may be selected as the higher high or the lower low water. When only one high or one low water occurs on a calendar day, because one of the tides occurs after midnight and therefore on the next calendar day, the single tide is to be checked if the tide just above it is unchecked; otherwise, it should not be checked. If, however, the tide has become diurnal and only one high and one low water occurs during the tidal day, these should both be checked. The back of the form carries instructions and explanations.

(2) *Correction by tabular values.* This method uses the ratio of $K_1+O_1$ to $M_2+S_2$ to compute factors for correcting tides at secondary stations. $K_1$ and $O_1$ represent the amplitudes of the principal daily constituents of the tide and $M_2$ and $S_2$ represent the principal semidaily constituents. When harmonic constants of

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 248
Ed. May, 1928

# TIDES : Comparison of Simultaneous Observations

(A) Subordinate station DUKE Lat. 32°22′ N Long. 75°12′ W
(B) Standard station TIP Lat. 32 54 N Long. 73 15 W
Chief of party ____ Time Meridian: (A) 30 W (B) 30 W

| DATE. Year. | (A) STATION. Time of— | | (B) STATION. Time of— | | (A)—(B) Time difference. | | (A) STATION. Height of— | | (B) STATION. Height of— | | (A)—(B) Height difference. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1957 | HW. | LW. | HW. | LW. | HW. | LW. | HW. | LW. | HW. | LW. | HW. | LW. |
| Mo. D. | Hours. | Hours. | Hours. | Hours. | Hours. | Hours. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. |
| 12 6 | +03.4 | +09.4 | -03.7 | -10.3 | -0.3 | -0.9 | 6.0 | 3.4 | 6.1 | 3.0 | -0.1 | +0.4 |
| | 15.1 | 21.8 | 16.0 | 22.7 | 0.9 | 0.9 | 5.9 | 2.3 | 5.0 | 1.9 | 0.1 | 0.4 |
| 7 | 03.6 | 10.5 | 04.2 | 10.7 | 0.6 | 0.2 | 6.0 | 3.3 | 6.0 | 2.9 | 0.0 | 0.4 |
| | 15.8 | 22.2 | 16.5 | 22.9 | 0.7 | 0.7 | 5.9 | 2.2 | 5.9 | 1.8 | 0.0 | 0.4 |
| 8 | 04.5 | 10.5 | 04.8 | 11.2 | 0.3 | 0.7 | 6.0 | 3.2 | 6.0 | 2.8 | 0.0 | 0.4 |
| | 16.4 | 22.6 | 17.2 | 23.2 | 0.8 | 0.6 | 5.8 | 2.1 | 5.8 | 1.8 | 0.0 | 0.3 |
| 9 | 05.0 | 11.2 | 05.5 | 11.7 | 0.5 | 0.5 | 5.9 | 3.2 | 5.9 | 2.8 | 0.0 | 0.4 |
| | 17.1 | 23.1 | 17.9 | 23.7 | 0.8 | 0.6 | 5.9 | 2.2 | 5.9 | 1.9 | 0.0 | 0.3 |
| 10 | 05.5 | — | 06.5 | — | 0.9 | — | 5.9 | — | 5.9 | — | 0.0 | — |
| | 18.0 | 12.2 | 18.6 | 12.5 | 0.6 | 0.3 | 6.0 | 3.3 | 6.0 | 3.0 | 0.0 | 0.3 |
| 11 | 06.6 | 00.2 | 06.7 | 01.0 | 0.1 | 0.8 | 5.9 | 3.4 | 6.0 | 3.1 | 0.1 | 0.3 |
| | 18.9 | 12.8 | 19.5 | 13.3 | 0.6 | 0.5 | 5.8 | 2.3 | 6.0 | 1.9 | 0.2 | 0.4 |
| 12 | 07.5 | 00.8 | 07.7 | 01.3 | 0.2 | 0.5 | 5.9 | 2.3 | 6.0 | 2.2 | 0.1 | 0.1 |
| | 19.5 | 13.8 | 20.5 | 14.1 | 1.0 | 0.3 | 6.0 | 3.4 | 6.0 | 3.4 | 0.0 | 0.0 |
| Sums | | | | | | | 7/ HHW. 41.8 | 6/ HLW. 19.9 | 7/ HHW. 42.0 | 6/ HLW. 18.0 | 7/ HHW. -0.2 | 6/ HLW. +1.9 |
| Means | | | | | | | 5.97 | 3.12 | 6.00 | 3.0 | -.03 | +.32 |
| Sums | | | | | -8.3 | -7.5 | 7/ LHW. 41.1 | 7/ LLW. 16.7 | 7/ LHW. 40.5 | 7/ LLW. 14.5 | 7/ LHW. -0.4 | 7/ LLW. +2.2 |
| Means | | | | | -0.59 | -0.58 | 5.87 | 2.39 | 5.79 | 2.07 | -.06 | +.31 |

| | HW. | LW. | |
|---|---|---|---|
| (1)= | 0.59 | 0.58 | =Mean difference in time of high and low water respectively. |
| (2)= | 0.14 | 0.14 | =Correction for difference in longitude. (Table on back of form.) |
| (3)= | 0.45 | 0.44 | =(1)+(2)=Mean difference in high and low water intervals, respectively. |

(4)= 5.97 Feet. =Mean HHW height at (A).
(5)= 3.12 Feet. =Mean HLW height at (A).
(6)= 5.87 =Mean LHW height at (A).
(7)= 2.39 =Mean LLW height at (A).
(8)= 0.10 =(4)−(6)=2DHQ at (A).
(9)= 0.73 =(5)−(7)=2DLQ at (A).
(10)= 5.92 =½[(4)+(6)]=Mean HW height at (A).
(11)= 2.76 =½[(5)+(7)]=Mean LW height at (A).
(12)= 3.16 =(10)−(11)=Mn at (A).
(13)= 4.34 =½[(10)+(11)]=MTL at (A).

(14)= -0.03 =Mean HHW difference.
(15)= 0.32 =Mean HLW difference.
(16)= -0.06 =Mean LHW difference.
(17)= 0.31 =Mean LLW difference.
(18)= +0.03 =(14)−(16)=2DHQ difference.
(19)= +0.01 =(15)−(17)=2DLQ difference.
(20)= -0.045 =½[(14)+(16)]=Mean HW difference.
(21)= +0.315 =½[(15)+(17)]=Mean LW difference.
(22)= -0.360 =(20)−(21)=Mn difference.
(23)= +0.135 =½[(20)+(21)]=MTL difference.

(24)= 0.898 =(12)÷[(12)−(22)]=Mn ratio.
(25)= 1.429 =(8)÷[(8)−(18)]=DHQ ratio.
(26)= 1.014 =(9)÷[(9)−(19)]=DLQ ratio.

| Results from comparison of Stations A and B. | HWI. | LWI. | MTL. | Mn. | DHQ. | DLQ. |
|---|---|---|---|---|---|---|
| Length of Series. | Hours. | Hours. | Feet. | Feet. | Feet. | Feet. |
| Accepted values for standard station, from 19 YEARS | 5.530 | 1.162 | 4.500 | 0.932 | 3.000 | 0.500 |
| Differences and ratios: (3), (23), (24), (25), (26) | -.450 | -.440 | +.135 | ×.898 | ×1.429 | ×1.014 |
| Corrected values for subordinate station | 5.080 | 0.722 | 4.635 | 0.837 | 4.287 | 0.507 |

Mean LW on staff at subordinate station=MTL−½Mn = 4.217 feet.
Mean LLW on staff at subordinate station=MTL−½Mn−DLQ= 3.710 feet.

Computed by P.E. Hoffman (AMS) Dec. '63 (Date.) Verified by J.A. Andrews (AMS), Dec. '63 (Date.)

① Front of USCGS Form 248

*Figure 196. Comparison of simultaneous tide observations.*

# EXPLANATION OF FORM 248.

This form is designed for the comparison of tides at a subordinate station for which tidal results are sought, with the tides observed simultaneously at a standard station for which tidal constants are known.

For short series of observations the high and low waters observed at the subordinate station may be tabulated immediately in this form, in which case it will be unnecessary to tabulate them also in Form 138.

The time and height differences are to be obtained by subtracting the values at the standard station from the values at the subordinate station, and the results entered with proper signs in the columns indicated.

Find the sums and means of columns of time difference, height of tide at both stations, and height difference. For stations on the Pacific coast, where the plane of reference is mean lower low water, the heights of the higher high, lower high, higher low, and lower low waters are to be summed separately, the higher highs and lower lows being indicated by pencil check marks. For stations on the Atlantic coast, where the plane of reference is mean low water, the heights of the high waters may be all combined into a single sum, and similarly the low water heights; the headings of their sums being made to read HW and LW, respectively, by striking out the extra letters. Mean results for time and height should be carried to two decimals places, and ratios to three decimal places. If any individual difference varies greatly from the apparent average, and an examination of the original record fails to show an error, that difference should not be included in the sum; and such a value should be encircled to show that it has been rejected.

For stations on the Atlantic coast omit (4) to (9), (14) to (19), (25), (26), and the computation of DHQ and DLQ at the bottom of the form. Take (10)=mean high water height at the subordinate station, (11)=mean low water height at subordinate station, (20)=mean high water difference, and (21)=mean low water difference.

For stations on the Pacific coast, the lower part of the form should be filled out completely as indicated.

The correction for difference in longitude (2) may be obtained from the following table. Find the difference in longitude by subtracting the longitude of the subordinate station from the longitude of the standard station, considering west as positive and east as negative. The correction has the same sign as the resulting difference of longitudes.

If the kind of time used at the two stations is different, apply this difference, expressed in hours, to the difference in the time of tide as directly obtained, adding if the time meridian of the subordinate station is west of the time meridian of the standard station, and subtracting if the time meridian of the subordinate station is east of that of the standard station.

*Correction for difference in longitude.*

| Difference. | Correction. | Difference. | Correction. | Difference. | Correction. | Difference. | Correction. | Difference. | Correction. | Difference. | Correction. | Difference. | Correction. | Difference. | Correction. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | *Hour.* | ′ | *Hour.* | ° | *Hour.* | ° | *Hour.* | ° | *Hour.* | ° | *Hour.* | ° | *Hour.* | ° | *Hour.* |
| 1 | 0.001 | 31 | 0.036 | 1 | 0.069 | 31 | 2.139 | 61 | 4.209 | 91 | 6.279 | 121 | 8.349 | 151 | 10.420 |
| 2 | 0.002 | 32 | 0.037 | 2 | 0.138 | 32 | 2.208 | 62 | 4.278 | 92 | 6.348 | 122 | 8.418 | 152 | 10.489 |
| 3 | 0.003 | 33 | 0.038 | 3 | 0.207 | 33 | 2.277 | 63 | 4.347 | 93 | 6.417 | 123 | 8.487 | 153 | 10.558 |
| 4 | 0.005 | 34 | 0.039 | 4 | 0.276 | 34 | 2.346 | 64 | 4.416 | 94 | 6.486 | 124 | 8.556 | 154 | 10.627 |
| 5 | 0.006 | 35 | 0.040 | 5 | 0.345 | 35 | 2.415 | 65 | 4.485 | 95 | 6.555 | 125 | 8.625 | 155 | 10.696 |
| 6 | 0.007 | 36 | 0.041 | 6 | 0.414 | 36 | 2.484 | 66 | 4.554 | 96 | 6.624 | 126 | 8.694 | 156 | 10.765 |
| 7 | 0.008 | 37 | 0.043 | 7 | 0.483 | 37 | 2.553 | 67 | 4.623 | 97 | 6.693 | 127 | 8.763 | 157 | 10.834 |
| 8 | 0.009 | 38 | 0.044 | 8 | 0.552 | 38 | 2.622 | 68 | 4.692 | 98 | 6.762 | 128 | 8.832 | 158 | 10.903 |
| 9 | 0.010 | 39 | 0.045 | 9 | 0.621 | 39 | 2.691 | 69 | 4.761 | 99 | 6.831 | 129 | 8.901 | 159 | 10.972 |
| 10 | 0.012 | 40 | 0.046 | 10 | 0.690 | 40 | 2.760 | 70 | 4.830 | 100 | 6.900 | 130 | 8.970 | 160 | 11.041 |
| 11 | 0.013 | 41 | 0.047 | 11 | 0.759 | 41 | 2.829 | 71 | 4.899 | 101 | 6.969 | 131 | 9.039 | 161 | 11.110 |
| 12 | 0.014 | 42 | 0.048 | 12 | 0.828 | 42 | 2.898 | 72 | 4.968 | 102 | 7.038 | 132 | 9.108 | 162 | 11.179 |
| 13 | 0.015 | 43 | 0.049 | 13 | 0.897 | 43 | 2.967 | 73 | 5.037 | 103 | 7.107 | 133 | 9.177 | 163 | 11.248 |
| 14 | 0.016 | 44 | 0.051 | 14 | 0.966 | 44 | 3.036 | 74 | 5.106 | 104 | 7.176 | 134 | 9.246 | 164 | 11.317 |
| 15 | 0.017 | 45 | 0.052 | 15 | 1.035 | 45 | 3.105 | 75 | 5.175 | 105 | 7.245 | 135 | 9.315 | 165 | 11.386 |
| 16 | 0.018 | 46 | 0.053 | 16 | 1.104 | 46 | 3.174 | 76 | 5.244 | 106 | 7.314 | 136 | 9.384 | 166 | 11.455 |
| 17 | 0.020 | 47 | 0.054 | 17 | 1.173 | 47 | 3.243 | 77 | 5.313 | 107 | 7.383 | 137 | 9.453 | 167 | 11.524 |
| 18 | 0.021 | 48 | 0.055 | 18 | 1.242 | 48 | 3.312 | 78 | 5.382 | 108 | 7.452 | 138 | 9.522 | 168 | 11.593 |
| 19 | 0.022 | 49 | 0.056 | 19 | 1.311 | 49 | 3.381 | 79 | 5.451 | 109 | 7.521 | 139 | 9.591 | 169 | 11.662 |
| 20 | 0.023 | 50 | 0.058 | 20 | 1.380 | 50 | 3.450 | 80 | 5.520 | 110 | 7.590 | 140 | 9.660 | 170 | 11.731 |
| 21 | 0.024 | 51 | 0.059 | 21 | 1.449 | 51 | 3.519 | 81 | 5.589 | 111 | 7.659 | 141 | 9.729 | 171 | 11.800 |
| 22 | 0.025 | 52 | 0.060 | 22 | 1.518 | 52 | 3.588 | 82 | 5.658 | 112 | 7.728 | 142 | 9.798 | 172 | 11.869 |
| 23 | 0.026 | 53 | 0.061 | 23 | 1.587 | 53 | 3.657 | 83 | 5.727 | 113 | 7.797 | 143 | 9.867 | 173 | 11.938 |
| 24 | 0.028 | 54 | 0.062 | 24 | 1.656 | 54 | 3.726 | 84 | 5.796 | 114 | 7.866 | 144 | 9.936 | 174 | 12.007 |
| 25 | 0.029 | 55 | 0.063 | 25 | 1.725 | 55 | 3.795 | 85 | 5.865 | 115 | 7.935 | 145 | 10.005 | 175 | 12.076 |
| 26 | 0.030 | 56 | 0.064 | 26 | 1.794 | 56 | 3.864 | 86 | 5.934 | 116 | 8.004 | 146 | 10.074 | 176 | 12.145 |
| 27 | 0.031 | 57 | 0.066 | 27 | 1.863 | 57 | 3.933 | 87 | 6.003 | 117 | 8.073 | 147 | 10.143 | 177 | 12.214 |
| 28 | 0.032 | 58 | 0.067 | 28 | 1.932 | 58 | 4.002 | 88 | 6.072 | 118 | 8.142 | 148 | 10.212 | 178 | 12.283 |
| 29 | 0.033 | 59 | 0.068 | 29 | 2.001 | 59 | 4.071 | 89 | 6.141 | 119 | 8.211 | 149 | 10.281 | 179 | 12.352 |
| 30 | 0.035 | 60 | 0.069 | 30 | 2.070 | 60 | 4.140 | 90 | 6.210 | 120 | 8.280 | 150 | 10.351 | 180 | 12.421 |

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② Reverse of USC&GS Form 248

*Figure 196*—Continued.

the tide are not available, a rough approximation of the ratio can be derived, where,

$$(K_1+O_1)-M_2+S_2$$

$$=1.4(DHQ+DLQ)\div M_N$$

The constants $K_1$, $O_1$, and $M_2$ were used by the USC&GS in developing factors for correcting tides at secondary stations. They are shown in tabular forms in USC&GS Sp. Pubs. 135 and 196. Additional information on tides may also be obtained from TM 5–235.

# CHAPTER 11

# TRIGONOMETRIC LEVELING

## Section I. ABSTRACT OF ZENITH DISTANCES

### 127. Preliminary Data

The first step in the computation of zenith distances is the abstracting of the zenith-distance observations from the field records. The form used in this illustration is DA Form 1943 (Abstract of Zenith Distances) (fig. 197). This form provides space at the bottom to list all signals shown at the station. It is extremely important to provide complete information as to the heights of the signals shown to other stations and the dates on which the signal was shown at a certain height. The primary responsibility for providing this data lies with the chief of the field party. The data must be abstracted completely and correctly so that the computer does not have to search through the field records for missing data. The height of the telescope is entered in the column headed "Telescope above station=$t$" directly from the field book. The "Height of stand" should also be entered at the top of the form. This information is necessary when observations are being made from any type of observing stand or tower. The data in the column "Object above station=$o$" is obtained from the abstract for the station observed.

### 128. Explanation of Abstract of Zenith Distances

*a.* A detailed explanation of the abstract of zenith distances for station Anvik follows.

*b.* The station name, observer, location, instrument number, and height of stand are entered at the top of the form. The data concerning the signal shown to other stations is entered at the bottom of the form. On 19 June 1952, a target was shown to the observer at station Page. The target was 2.17 meters above station Anvik. On the same date a helio was shown to station Pole. The helio was 1.49 meters above station Anvik. Every time a signal is shown to a station, the date and height of the signal must be given. If a signal is changed in height on the same date, the time at which the signal was shown at each height must also be given. Next, the observations at Anvik are abstracted. On 19 June 1952, at 0821 hours (time entered in Hour column), the observer at Anvik sighted on station Page. The height of the instrument at the time these observations were made was 1.60 meters above station Anvik. This value, 1.60 meters, is entered in the column "Telescope above station=$t$". The observed zenith distances, 89°52′32″.0, 89°52′36″.0, 89°52′41″.5, and 89°52′37″.5, are entered in the column headed "Observed zenith distance." A mean value (89°52′36″.8) is found for the four observed values. The time of the last observation is entered in the Hour column. The same procedure is followed for the observations on station Pole. The height of the object observed at station Page is found at the bottom of the abstract for station Page. The target at Page shown to Anvik on 19 June 1952 was 2.05 meters above station Page. The same process is used to find the height of the object observed (2.23m) at station Pole. The "Difference of heights" ($t-o$) is now found and entered with the correct algebraic sign. For the observations to Page, this value is $1.60-2.05=-0.45$. For Pole, $(t-o)=-0.63$. The value ($t-o$) is necessary for "Reduction to line joining stations," which is used only for reciprocal observations. The reduction is essentially a vertical eccentric reduction and is computed from the formula:

$$\text{Reduction (in seconds)} = -\frac{t-o}{s \sin 1''}$$

in which ($t-o$) is the value previously explained, and $s$ is the distance between the two stations

| PROJECT | ABSTRACT OF ZENITH DISTANCES (TM 5-237) | |
|---|---|---|
| 3-8833 | | |
| LOCATION: Alaska | INSTR. (TYPE) (NO.): Wild T-2 #17554 | STATION: ANVIK, USC&GS, 1949 |
| ORGANIZATION: 30th Engrs | OBSERVER: Cpl. Wilkins | HEIGHT OF STAND: Tripod |

| Date | Hour | Object Observed | Object Above Station =o (Meters) | Telescope Above Station =t (Meters) | Diff. of Heights t−o (Meters) | Reduction to Line Joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| | | | | | | " | ° ′ ″ | ° ′ ″ |
| 19 Jun 52 | 0821 | Page (30th Eng. '52) | 2.05 | 1.60 | −0.45 | +04.4 | 89 52 32.0 | |
| | | | | | | | 36.0 | |
| | | | | | | | 41.5 | |
| | 0830 | | | | | | 37.5 | |
| | | | | | | Mean | 89 52 36.8 | 89 52 41.2 |
| | | | | | | | | |
| 19 Jun 52 | 0835 | Pole (30th Eng. '52) | 2.23 | 1.60 | −0.63 | +9.1 | 89 11 40.0 | |
| | | | | | | | 42.0 | |
| | | | | | | | 41.5 | |
| | 0846 | | | | | | 47.0 | |
| | | | | | | Mean | 89 11 42.6 | 89 11 51.7 |

| Date | ~~Target~~ ~~Light~~ Shown to Station | Height of ~~Target~~ ~~Light~~ Above Station (Meters) | Date | ~~Helio~~ ~~Light~~ Shown to Station | Height of ~~Helio~~ ~~Light~~* Above Station (Meters) |
|---|---|---|---|---|---|
| 19 Jun 52 | Page (30th Eng.) | 2.17 | 19 Jun 52 | Pole (30th Eng) | 1.49 |

*Height of Light (or object above station) should also be entered on Abstract of Zenith Distances of station to which light was shown.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.A. Smith | 17 Aug. 52 | W. C. Cumen | 17 Aug. 52 |

DA FORM 1 FEB 57 1943

*Figure 197. Abstract of zenith distances.*

| PROJECT | ABSTRACT OF ZENITH DISTANCES (TM 5-237) | |
|---|---|---|
| 3-8833 | | |
| LOCATION: Alaska | INSTR. (TYPE) (NO.): Wild T-2 #16950 | STATION: Page, 30th Engrs, 1952 |
| ORGANIZATION: 30th Engrs. | OBSERVER: Sgt. Austin | HEIGHT OF STAND: Tripod |

| Date | Hour | Object Observed | Object Above Station =o (Meters) | Telescope Above Station =t (Meters) | Diff. of Heights t−o (Meters) | Reduction to Line Joining Stations | Observed Zenith Distance | Corrected Zenith Distance |
|---|---|---|---|---|---|---|---|---|
| | | | | | | " | ° ′ ″ | ° ′ ″ |
| 19 Jun 52 | 0900 | ANVIK USC+GS | 2.17 | 1.35 | −0.82 | +8.0 | 90 17 26.0 | |
| | | | | | | | 25.0 | |
| | | | | | | | 25.5 | |
| | | | | | | | 26.0 | |
| | | | | | | | 25.0 | |
| | 0912 | | | | | | 25.0 | |
| | | | | | | Mean | 90 17 25.4 | 90 17 33.4 |

| Date | ~~Light~~ Target Shown to Station | Height of ~~Light~~ Target Above Station (Meters) | Date | Light Shown to Station | Height of Light* Above Station (Meters) |
|---|---|---|---|---|---|
| 19 Jun 52 | ANVIK USC+GS | 2.05 | | | |

*Height of Light (or object above station) should also be entered on Abstract of Zenith Distances of station to which light was shown.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.A. Smith | 17 Aug 52 | W. C. Aumen | 17 Aug. 52 |

DA FORM 1 FEB 57 1943

*Figure 197*—Continued.

| PROJECT | | ABSTRACT OF ZENITH DISTANCES (TM 5-237) | |
|---|---|---|---|
| 3-8833 | | | |
| LOCATION | INSTR. (TYPE) (NO.) | STATION | |
| Alaska | Wild T-2 #17750 | Pole, 30th Engrs, 1952 | |
| ORGANIZATION | OBSERVER | | HEIGHT OF STAND |
| 30th Engrs | Sgt Bradley | | Tripod |

| DATE | HOUR | OBJECT OBSERVED | OBJECT ABOVE STATION =o (Meters) | TELESCOPE ABOVE STATION =t (Meters) | DIFF. OF HEIGHTS t−o (Meters) | REDUCTION TO LINE JOINING STATIONS | OBSERVED ZENITH DISTANCE | CORRECTED ZENITH DISTANCE |
|---|---|---|---|---|---|---|---|---|
| | | | | | | " | ° ′ ″ | ° ′ ″ |
| 19 Jun 52 | 0915 | ANVIK USC+GS | 1.49 | 1.39 | -0.10 | +1.4 | 90 54 49.5 | |
| | | | | | | | 48.5 | |
| | | | | | | | 50.5 | |
| | | | | | | | 48.0 | |
| | | | | | | | 49.5 | |
| | 0926 | | | | | | 49.0 | |
| | | | | | | Mean | 90 54 49.2 | 90 54 50.6 |

| DATE | ~~Light~~ Target SHOWN TO STATION | HEIGHT OF ~~LIGHT~~ Target ABOVE STATION (Meters) | DATE | LIGHT SHOWN TO STATION | HEIGHT OF LIGHT* ABOVE STATION (Meters) |
|---|---|---|---|---|---|
| 19 Jun 52 | ANVIK USC+GS | 2.23 | | | |

*Height of Light (or object above station) should also be entered on Abstract of Zenith Distances of station to which light was shown.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R A Smith | 17 Aug. 52 | W. C. Annen | 17 Aug 52 |

DA FORM 1 FEB 57 1943

*Figure 197*—Continued.

(Anvik and Page or Pole). Notice the minus sign before the fraction. For the observations from Anvik to Page the reduction is +04″.4, found by using the values of $(t-o)=-0.45$, $s=21165.57$ meters ($s$ from preliminary triangle computations), and sin 1″. The sign of the reduction is always opposite the sign of $t-o$. The reduction is added algebraically to the mean observed zenith distance to obtain the "Corrected zenith distance." For Page, this is—

$$89°52'36''.8+04''.4=89°52'41''.2.$$

The same procedure is used to find a corrected zenith distance to all stations on all abstracts. The corrected zenith distances are used in the computation of the elevations.

## Section II. TRIGONOMETRIC ELEVATIONS FROM RECIPROCAL OBSERVATIONS

### 129. Forms Used

The observations used to illustrate the abstract of zenith distances described in paragraphs 127 and 128 will now be used to compute the elevations of stations Page and Pole on DA Form 1944, Computation of Elevations and Refractions from Reciprocal Observations (Logarithmic) (fig. 198), and on DA Form 1945, Computation of Elevations and Refractions from Reciprocal Observations (by Calculating Machine) (fig. 199). Station Anvik has a fixed elevation ($h_1$) of 254.8 meters.

### 130. Logarithmic Computation

*a.* The formula for the computation of elevations from reciprocal observations is:

$$h_2-h_1=s \tan \tfrac{1}{2}(\zeta_2-\zeta_1)\text{ABC}$$

in which $h_1$ is the known elevation of station 1 above mean sea level, $h_2$ is the unknown elevation of station 2, $s$ is the geodetic distance between the stations, $\zeta_1$ is the zenith distance of station 2 as observed from station 1, $\zeta_2$ is the zenith distance of station 1 as observed from station 2, and A, B, C are correction factors whose logarithms are given tables in TM 5–236, and in USC&GS Sp. Pubs. 138 and 247. When zenith distances are observed in both directions on a line, it is considered a reciprocal observation, and the elevation is computed by the above formula. The unit of meters is used throughout this computation (fig. 198).

*b.* Since observations were made in both directions on the lines Anvik-Page and Anvik-Pole, the elevations of Page and Pole will be computed as reciprocal observations. The station names are entered at the top of the form in the spaces indicated; the known station Anvik is number 1, and the unknown stations Page and Pole are number 2. The corrected zenith distances are taken from the abstracts. $\zeta_1$ for the line Anvik to Page is 89°52′41″.2; for the line Anvik to Pole it is 89°11′51″.7. These values are found on the abstract for station Anvik. For the line Anvik to Page, $\zeta_2$ is 90°17′33″.4 and is found on the abstract for station Page. A detailed description of the computation of Page will be given, but not for Pole as the procedure is the same for all. $\zeta_2-\zeta_1$ equals 90°17′33″.4 minus 89°52′41″.2 equals +0°24′52″.2. $\frac{1}{2}(\zeta_2-\zeta_1)=+0°12'26''.1$. For purposes of this computation, change $\frac{1}{2}(\zeta_2-\zeta_1)$ to seconds (equals +746″.1). From a table of logarithms of numbers find the log of 746″.1 (if number is negative, remember to indicate that log is negative). From the same page of the tables, obtain T for $\frac{1}{2}(\zeta_2-\zeta_1)$ in seconds. Log $s$ is obtained from triangle computations or geographic position sheets. If the elevations are to be used to reduce the horizontal angle computations to sea level, the distance $s$ from preliminary triangle computations will suffice. Add log $\frac{1}{2}(\zeta_2-\zeta_1)$ in seconds, T, and log $s$ to obtain log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$; in this example, 2.87280+4.68558+4.32563=1.88401 (the characteristic of T is 4._____−10). From the tables, take out the logs of A, B, C, observing the rules at the bottom of the tables regarding the algebraic signs of logs B and C. Log A carries the same sign as $h_1$. Add logs A, B, C, to log $[s \tan \frac{1}{2}(\zeta_2-\zeta_1)]$, to obtain log $(h_2-h_1)$ which is 1.88403, in this example. Find $(h_2-h_1)$ from the log and affix the correct algebraic sign. The sign of $(h_2-h_1)$ is the same as the sign of $(\zeta_2-\zeta_1)$. Add $(h_2-h_1)$ and $h_1$, algebraically, to obtain the unknown elevation. In the example, log $(h_2-h_1)$ equals 1.88403, from which $(h_2-h_1)$ equals 76.6. The sign is plus, as the sign of $(\zeta_2-\zeta_1)$ is plus. Adding +76.6 to the fixed elevation gives the elevation of $h_2$ (Page) as 331.4 meters.

*c.* The computation of the elevation is now complete, and the remainder of the form is used to solve for the weight to be assigned to the elevation

PROJECT: 3-8833

LOCATION: Alaska

ORGANIZATION: 30th Engrs.

DATE: 19 June 52

## COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS

(Logarithmic)
(TM 5-237)

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | ANVIK | ANVIK | | | | |
| Station 2, obs. | Page | Pole | | | | |
| | | | | | | |
| $\zeta_1$ | 89° 52' 41.2" | 89° 11' 51.7" | | | | |
| $\zeta_2$ | 90 17 33.4 | 90 54 50.6 | | | | |
| $\zeta_2-\zeta_1$ | +0 24 52.2 | 1 42 58.9 | | | | |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | +0 12 26.1 | 0 51 29.4 | | | | |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ in secs. | +746.1 | +3089.4 | | | | |
| log ditto | 2.87280 | 3.48987 | | | | |
| T | 4.68558 | 4.68561 | | | | |
| log $s$ | 4.32563 | 4.15367 | | | | |
| log [$s$ tan $\frac{1}{2}(\zeta_2-\zeta_1)$] | 1.88401 | 2.32915 | | | | |
| log $A$ | +2 | +2 | | | | |
| log $B$ | — | +1 | | | | |
| log $C$ | — | — | | | | |
| log $(h_2-h_1)$ | 1.88403 | 2.32918 | | | | |
| $h_2-h_1$ | +76.6 | +213.4 | | | | |
| $h_1$ | 254.8 | 254.8 | | | | |
| $h_2$ | 331.4 | 468.2 | | | | |
| 2 log $s$ | 8.65126 | 8.30734 | | | | |
| log $p=9-2$ log $s$ | 0.34874 | 0.69266 | | | | |
| $p$ of $(h_2-h_1)$ | 2.23 | 4.93 | | | | |
| $\alpha$ and mean $\phi$ | 70°9 62°7 | 120°5 62°8 | | | | |
| $\zeta_1+\zeta_2-180°$ | 10' 14.6" | 06' 42.3" | | | | |
| $\zeta_1+\zeta_2-180°$ in sec. | 614.6 | 402.3 | | | | |
| log ditto | 2.78859 | 2.60455 | | | | |
| log $\rho$ | 6.80580 | 6.80570 | | | | |
| colog $s$ | 5.67437 | 5.84633 | | | | |
| log $\frac{\sin 1''}{2}=4.38454$ | 4.38454 | 4.38454 | | | | |
| log $(0.5-m)$ | 9.65330 | 9.64112 | | | | |
| $(0.5-m)$ | 0.4501 | 0.4376 | | | | |
| $p$ of $(0.5-m)$* | 4.48 | 2.03 | | | | |

*Since $(0.5-m)$ varies as $s^2$, the weight $p=\frac{s^2}{N}$, where $N$ is constant for a set and is preferably a power of 10.

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R. L. Smith | 17 Aug 52 | W. C. Cramer | 17 Aug. 52 |

DA FORM 1 FEB 57 1944

*Figure 198. Elevations from reciprocal observations, logarithmic.*

if used in future adjustments and for the coefficient of refraction ($m$). The weight is inversely proportional to the square of the length of the line, and is most easily computed by the formula $\log p = 9 - 2 \log s$. For the line Anvik-Page, the computation is, $2 \log s = 2(4.32563) = 8.65126$. Log $p$ then equals $9 - 8.65126 = 0.34874$. From this logarithm, $p$ is found to be 2.23. One use of this weight, $p$, would be as follows. Suppose instead of one determination of the elevation of Page, two determinations were made, one from Anvik as shown in the example and another from Pole. The elevation computed from Anvik is 331.4 meters with a weight of 2.23. The elevation computed from Pole is 330.5 meters with a weight of 2.78. The weighted mean elevation of Page is then:

$$\frac{(2.23 \times 331.4) + (2.78 \times 330.5)}{2.23 + 2.78}$$

which equals 330.9 meters. This is an illustration of only one use of the weights.

*d.* The coefficient of refraction is used for nonreciprocal observation computation and is found from the formula:

$$0.5 - m = \frac{(\zeta_1 + \zeta_2 - 180°)\rho \sin 1''}{2s}$$

in which $\zeta_1$, $\zeta_2$, and $s$, are as previously defined, and $\rho$ is the radius of curvature in the azimuth and at the mean latitude of the line. Values of $\log \rho$ for the Clarke 1866 Spheroid are given in table IV (app. III) with the azimuth of the line ($\alpha$), and the mean latitude of the line as arguments. The azimuth and mean latitude can be obtained from geographic position sheets or from preliminary geographic position computations. Both the azimuth and mean latitude are needed only to the nearest tenth of a degree.

*e.* The computation of the refraction using the line Anvik-Page will be used to illustrate the procedure. $\alpha$ is 70°.9, and the mean $\phi$ is 62°.7. $\zeta_1 + \zeta_2 - 180°$ is 0°10′14″.6, which equals 614.6 seconds. The log of this number is 2.78859. Log $\rho$ from the tables is 6.80580. The colog of $s$ is 5.67437. The log of $\frac{\sin 1''}{2}$ is a constant printed on the form and equals 4.38454. The sum of the four logarithms (one is colog $s$) is the log of $(0.5 - m)$, and in this case is $2.78859 + 6.80580 + 5.67437 + 4.38454$ equals 9.65330 (actually 19.65330−20, as the −10 on log $\rho$ and log $\frac{\sin 1''}{2}$ is not shown). From this logarithm, $(0.5 - m)$ is found to equal 0.4501. Any value of $(0.5 - m)$ approaching or exceeding 0.5 must be rejected. Anything over 0.49 should probably be rejected. The weight of $(0.5 - m)$ varies as $s^2$, but since $s^2$ will ordinarily be quite large it will be more convenient to divide $s^2$ by a constant in each set of computations. Use a power of 10 for the constant. Logarithms need only be carried to five decimals, zenith distances to tenths of seconds, and elevations and differences to tenths of meters.

## 131. Natural Functions Computation

*a.* As in the logarithmic computation, Anvik is station 1, and Page and Pole are station 2. $\zeta_1$ for the line Anvik to Page is 89°52′41″.2, and $\zeta_2$ is 90°17′33″.4. $(\zeta_2 - \zeta_1)$ is +0°24′52″.2 and $\frac{1}{2}(\zeta_2 - \zeta_1)$ is +0°12′26″.1. The tangent of $\frac{1}{2}(\zeta_2 - \zeta_1)$ is taken from the tables (6 decimal places are sufficient), and the value for $s$, which is the distance between station 1 and station 2 in meters, is entered on the form. The tangent of $\frac{1}{2}(\zeta_2 - \zeta_1)$ is multiplied by $s$, the result being an approximate value of $(h_2 - h_1)$. Factors A, B, and C are taken from the tables. Factor A is a multiplier, the value for which is determined by the elevation of station 1 ($h_1$). Factor B is a multiplier whose value is determined by the approximate difference in elevation of station 1 and 2 $(h_2 - h_1)$. Factor C is a multiplier determined by $s$. The approximate value of $h_2 - h_1$ is then multiplied by the factors A, B, and C. In order to simplify this operation, $h_2 - h_1$ may be multiplied by A+B+C−2 when $h_2 - h_1$ is positive, or by A−B+C when $h_2 - h_1$ is negative. The resulting value is the difference in elevation between station 1 and station 2, or $h_2 - h_1$. A table for finding the correction to $h_2 - h_1$ directly is included in appendix III. This table can be used with differences of $h_2 - h_1$ up to 1,500 meters, and distances up to about 35,000 meters. The value of $h_2$ is the algebraic sum of $h_1$ and $(h_2 - h_1)$.

*b.* The weight of a computed difference in elevation is proportional to $\frac{1}{s^2}$. In order to get a convenient value, $s$ should first be divided by a multiple of 10. The same divisor must be used in determining the weights for all differences in elevation used in any one adjustment of elevation.

| PROJECT 3-8833 | | | | | | |
|---|---|---|---|---|---|---|
| LOCATION Alaska | | | | | | |
| ORGANIZATION 30th Engrs | | DATE 19 June 52 | | | | |

**COMPUTATION OF ELEVATIONS AND REFRACTIONS FROM RECIPROCAL OBSERVATIONS**
(By calculating machine)
(*TM 5-237*)

| Station 1, occ. | ANVIK | ANVIK | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| Station 2, obs. | Page | Pole | | | | |
| | | | | | | |
| $\zeta_1$ | 89° 52' 41.2" | 89° 11' 51.7" | | | | |
| $\zeta_2$ | 90 17 33.4 | 90 54 50.6 | | | | |
| $\zeta_2-\zeta_1$ | +0 24 52.2 | +1 42 58.9 | | | | |
| $\frac{1}{2}(\zeta_2-\zeta_1)$ | +0 12 26.1 | +0 51 29.4 | | | | |
| $\tan \frac{1}{2}(\zeta_2-\zeta_1)$ | 0.003618 | 0.014979 | | | | |
| $s$ | 21,165.57m. | 14,245.25 | | | | |
| $A$ | 1.000040 | 1.000040 | | | | |
| $B$ | 1.000006 | 1.000017 | | | | |
| $C$ | 1.000004 | 1.000002 | | | | |
| | | | | | | |
| $h_2-h_1$ | 76.6 m. | 213.4 m. | | | | |
| $h_1$ | 254.8 m. | 254.8 m. | | | | |
| $h_2$ | 331.4 m. | 468.2 m. | | | | |
| $\frac{1}{s^2}=p$ of $(h_2-h_1)$ | 2.23 | 4.93 | | | | |
| $\alpha$ and mean $\phi$ | 70°9 62°7 | 120°5 62°8 | | | | |
| $\zeta_1+\zeta_2-180°$ | 10' - 14.6" | 06' 42.3" | | | | |
| $\zeta_1+\zeta_2-180°$ in sec. | 614.6 | 402.3 | | | | |
| $\rho\frac{\sin 1''}{2}$ | 15.500 | 15.497 | | | | |
| $s$ | 21,165.57 | 14,245.25 | | | | |
| $(0.5-m)$ | 0.4501 | 0.4377 | | | | |
| $p$ of $(0.5-m)$* | 4.48 | 2.03 | | | | |

$$h_2-h_1=s \tan \tfrac{1}{2}(\zeta_2-\zeta_1)\, A\, B\, C$$

$$(0.5-m)=\frac{(\zeta_1+\zeta_2-180° \text{ in sec.})\,\rho\frac{\sin 1''}{2}}{s}$$

*Since $(0.5-m)$ varies as $s^2$, the weight $p=\frac{s^2}{N}$, where $N$ is constant for a set and is preferably a power of 10.

| COMPUTED BY W.C.A | DATE 17 Aug. 52 | CHECKED BY RAS | DATE 17 Aug. 52 |
|---|---|---|---|

DA FORM 1 FEB 57 1945

*Figure 199. Elevations from reciprocal observations, natural functions.*

A convenient value for $\frac{1}{s^2}$ for distances of 50,000 meters or less will be obtained by dividing $s$ (in meters) by 10,000 before finding the reciprocal. The result should then be multiplied by 10 and carried to 2 decimal places. This will give the same result as $\log p = 9 - 2 \log s$ on the logarithmic form. The application of weights to computed values is explained in the logarithmic computation (par. 130).

*c.* For natural functions computation (fig. 199), the formula for obtaining the refraction coefficient $(0.5-m)$ is as follows:

$$(0.5-m)=\frac{(\zeta_1+\zeta_2-180° \text{ in seconds})\ \rho\frac{\sin 1''}{2}}{s}$$

In the determination of $(0.5-m)$, $\alpha$ and mean $\phi$ are the azimuth from station 1 to station 2 and the mean latitude of the stations, respectively. The value of $\rho \sin 1''$ will be found in the table included in appendix III, and $\frac{\rho \sin 1''}{2}$ is one-half this value. The azimuths used in the table for $\rho \sin 1''$ are from 0° to 90°. For azimuths between 90° and 180°, subtract from 180°; between 180° and 270°, subtract 180° from azimuth; and between 270° and 360°, subtract azimuth from 360°. Apply formula for $(0.5-m)$. The weight $(p)$ of $(0.5-m)$ is proportional to $s^2$, and a convenient value may be obtained by dividing $s$ by 10,000 and squaring the result. This weight should be carried to 2 decimal places. The refraction coefficient $(0.5-m)$ is used only when nonreciprocal observations are made and is computed only if needed. All values of $(0.5-m)$ which are greater than 0.5, or which are very near 0.5, should be rejected.

## Section III. TRIGONOMETRIC ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS

### 132. Forms Used

For the purpose of illustration, the elevation of Pole will be set up from Anvik using the mean observed zenith distance from Anvik to Pole (89° 11′ 42″.6) as a nonreciprocal observation. This computation is made on DA Form 1946 (Computation of Elevations from Nonreciprocal Observations (Logarithmic)) (fig. 200); and on DA Form 1947 (Computation of Elevations from Nonreciprocal Observations (By Calculating machine)) (fig. 201).

### 133. Logarithmic Computation

*a.* The formula for the computation of elevations from nonreciprocal observations is as follows:

$$h_2-h_1=s \tan (90°-\zeta_1+k)\text{ABC}$$

in which $k=\frac{(0.5-m)s}{\rho \sin 1''}$; $\zeta_1$ is the mean *observed* zenith distance of the object sighted upon with no reduction for vertical eccentricity; the quantities $s$, A, B, and C are as previously defined in paragraph 130.

*b.* On the computation form (fig. 200), enter the station occupied (Anvik), the station observed (Pole), the object sighted (target), $\zeta_1$ (89° 11′ 42″.6 from the abstract for Anvik), the fixed elevation of Anvik ($h_1$=254.8 meters), and $t-o$ from the abstract for Anvik, equals −0.6 meters. The azimuth of the line ($\alpha$) and the mean latitude of the stations (mean $\phi$) are obtained as described in the reciprocal observation computation.

*c.* The first part of the computation is to solve for $k$. The value of $(0.5-m)$ is the weighted mean of all the values of $(0.5-m)$ computed from reciprocal observations involving station Anvik. In the previous example, two determinations of $(0.5-m)$ were made from observations involving Anvik. From the line Anvik-Page $(0.5-m)$ was computed as 0.4501 with a weight of 4.48. From the line Anvik-Pole $(0.5-m)$ was computed as 0.4376 with a weight of 2.03. The weighted mean of these values is $\frac{4.48\times0.4501+2.03\times0.4376}{4.48+2.03}$, which equals 0.4462. The log of this number is 9.64953. When $(0.5-m)$ is not observed use 9.63246 for this value. Log $s$ is found as in the reciprocal computation. Log $\rho$ is from the tables, but the colog is used in this computation. Colog sin 1″ is a constant printed on the form. The four logarithms (and cologs) for computing $k$ are then, log $(0.5-m)$= 9.64953, log $s$=4.15367, colog $\rho$=3.19430, and colog sin 1″=5.31443. The sum of the logs is 2.31193 [actually 22.31193, but log $(0.5-m)$ and colog sin 1″ both have a −10 understood in their characteristic] which is the log of $k$ (in seconds). From this log, $k$ is found to be 205.1 seconds. Subtract $\zeta_1$ from 90° and add $k$ to the result which would be, 90°−89° 11′ 42″.6=0° 48′ 17″.4=2897.4 seconds plus 205.1 seconds gives

| PROJECT | 3-8833 | | | | | | COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS (Logarithmic) (TM 5-237) |
|---|---|---|---|---|---|---|---|
| LOCATION | Alaska | | | | | | |
| ORGANIZATION | 30th Engrs. | | DATE | 19 June 52 | | | |
| Station 1, occ. | ANVIK | | | | | | |
| Station 2, obs. | Pole | | | | | | |
| | | | | | | | |
| Object sighted | Target | | | | | | |
| | | | | | | | |
| $\zeta_1$ | 89° 11' 42".6 | | | | | | |
| $\alpha$ and mean $\phi$ | 120°.5 62°.8 | | | | | | |
| | | | | | | | |
| log $(0.5-m)$ | 9.64953 | | | | | | |
| log $s$ | 4.15367 | | | | | | |
| colog $\rho$ | 3.19430 | | | | | | |
| colog sin 1'' | 5.31443 | 5.31443 | 5.31443 | 5.31443 | 5.31443 | 5.31443 | |
| log ($k$ in secs.) | 2.31193 | | | | | | |
| $k$ in secs. | 205.1 | | | | | | |
| $(90°-\zeta_1+k)$ in secs. | 3102.5 | | | | | | |
| | | | | | | | |
| log ditto | 3.49171 | | | | | | |
| T | 4.68561 | | | | | | |
| log $s$ | 4.15367 | | | | | | |
| log $[s \tan (90°-\zeta_1+k)]$ | 2.33099 | | | | | | |
| log $A$ | +2 | | | | | | |
| log $B$ | — | | | | | | |
| log $C$ | — | | | | | | |
| log $(h_2-h_1)$ | 2.33101 | | | | | | |
| | | | | | | | |
| $h_2-h_1$ | +214.3 | | | | | | |
| $h_1$ | 254.8 | | | | | | |
| $t-o$ | −0.6 | | | | | | |
| Corrected elevation | 468.5 | | | | | | |
| | | | | | | | |
| log $p=9-2$ log $s$ | 0.69266 | | | | | | |
| $p$ | 4.93 | | | | | | |
| Weighted mean elevation of sta. obs. | | | | | | | |
| COMPUTED BY | W.C.A. | DATE | 17 Aug. 52 | CHECKED BY | RAS | DATE | 17 Aug. 52 |

DA FORM 1, FEB 57 1946

*Figure 200. Elevations from nonreciprocal observations, logarithmic.*

| PROJECT | 3-8833 | COMPUTATION OF ELEVATIONS FROM NONRECIPROCAL OBSERVATIONS (By calculating machine) (TM 5-237) |
|---|---|---|
| LOCATION | Alaska | |
| ORGANIZATION | 30th Engrs. | DATE 19 June 52 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Station 1, occ. | ANVIK | | | | | |
| | | | | | | |
| Station 2, obs. | Pole | | | | | |
| | | | | | | |
| Object sighted | Target | | | | | |
| | | | | | | |
| $\zeta_1$ | 89° 11' 42."6 | | | | | |
| $\alpha$ and mean $\phi$ | 120°.5 62°.8 | | | | | |
| | | | | | | |
| $(0.5-m)$ | 0.4462 | | | | | |
| $s$ | 14,245.25m. | | | | | |
| $\rho \sin 1''$ | 30.994 | | | | | |
| $k$ in secs. | 205.1 | | | | | |
| $(90°-\zeta_1+k)$ | 0° 51' 42."5 | | | | | |
| tan $(90°-\zeta_1+k)$ | .015042 | | | | | |
| $A$ | 1.000040 | | | | | |
| $B$ | 1.000017 | | | | | |
| $C$ | 1.000002 | | | | | |
| $h_2-h_1$ | +214.3 | | | | | |
| $h_1$ | 254.8 | | | | | |
| $t-o$ | -0.6 | | | | | |
| Corrected elevation | 468.5 | | | | | |
| | | | | | | |
| $\frac{1}{s^2}=p$ of $(h_2-h_1)$ | 4.93 | | | | | |
| Weighted mean elevation of sta. obs. | | | | | | |

$$h_2-h_1=s \tan (90°-\zeta_1+k)\, A\, B\, C \qquad k \text{ in secs.}=\frac{(0.5-m)\, s}{\rho \sin 1''}$$

| COMPUTED BY | W. C. A. | DATE | 17 Aug 52 | CHECKED BY | RAS | DATE | 17 Aug. 52 |
|---|---|---|---|---|---|---|---|

DA FORM 1 FEB 57 1947

*Figure 201. Elevations from nonreciprocal observations, natural functions.*

an answer of 3102.5. Find the log of 3102.5 (=3.49171) from log tables, and on the same page of the tables find T (=4.68561). Enter these values on the form along with log $s$ previously used to find $k$. Add the three logs, 3.49171+4.68561+4.15367 to obtain log [$s$ tan $(90°-\zeta_1+k)$] equal to 2.33099. Add the correction factors A, B, C, from the tables to obtain the final log $(h_2-h_1)=2.33101$. From the logarithm, the value of $(h_2-h_1)$ is found to be 214.3 meters. The algebraic sign of $(h_2-h_1)$ is the same as $90°-\zeta_1$, which is plus in this case. The algebraic sum of $(h_2-h_1)+h_1+(t-o)$ equals the corrected elevation of station 2. In this example $(h_2-h_1)=+214.3$, $h_1=254.8$, and $(t-o)=-0.6$. The sum of these expressions equals the corrected elevation of Pole (station 2) (468.5 meters). The weight, $p$, is found as for the reciprocal observations. When used with reciprocal elevations in an adjustment, the nonreciprocal elevations should be given only one-third of the computed weight. The same rules for the number of decimal places carried apply to this computation as to the reciprocal observations.

### 134. Natural Functions Computation

*a.* As in the computation using logarithms, Anvik is station 1 and Pole is station 2. The object sighted on is the target. The fixed elevation of Anvik ($h_1$) is 254.8 meters, $\zeta_1$ is 89°11′42″.6, and $(t-o)$ from the abstract on Anvik is −0.6 meters. $\alpha$ and mean $\phi$ are the azimuth of station 1 to station 2 and the mean latitude of the stations, respectively. $(0.5-m)$ is the weighted mean refraction for station 1 obtained from all reciprocal observation computations for station 1. Two determinations of $(0.5-m)$ were made for Anvik in the previous computations. From the line Anvik-Page, $(0.5-m)$ was computed as 0.4501 with a weight of 4.48. From the line Anvik-Pole, $(0.5-m)$ was computed as 0.4377 with a weight of 2.03. The weighted mean of these values is

$$\frac{4.48\times0.4501+2.03\times0.4377}{4.48+2.03},$$ which equals 0.4462.

When $(0.5-m)$ is not observed, use 0.4290 for this value. $s$ is the geodetic distance (in meters) between stations 1 and 2. This is 14,245.25 meters. $\rho$ sin 1″ is taken from the tables in appendix III using $\alpha$ and mean $\phi$ as the argument. $\rho$ sin 1″ equals 30.994. Now, $k$ in seconds is found using the formula:

$$k \text{ in seconds}=\frac{(0.5-m)s}{\rho \sin 1''},$$

and

$k$ equals 205.1 seconds.

Now $(90°-\zeta_1+k)$ and tangent $(90°-\zeta_1+k)$ are found. In the example (fig. 141), $(90°-\zeta_1+k)$ equals 0°51′42″.5, and the tangent equals .015042. Tan $(90°-\zeta_1+k)$ is multiplied by $s$ to get an approximate $(h_2-h_1)$ which is used to determine the B factor from the tables. Factor A and factor C are also determined at this time. Now $(h_2-h_1)$ is computed using the following formula:

$$(h_2-h_1)=s \tan (90°-\zeta_1+k)\text{ABC}$$

*b.* In this example, $(h_2-h_1)$ equals +214.3. The corrected elevation of station 2 is found by adding algebraically $h_1$, $(h_2-h_1)$, and $(t-o)$. The corrected elevation in this example is 468.5 meters.

*c.* The weight, $p$, of $(h_2-h_1)$ is computed using the same procedure as in the reciprocal observation method. When used with reciprocal elevations in an adjustment, the nonreciprocal elevations should be given only one-third of the computed weight.

## Section IV. ADJUSTMENT OF TRIGONOMETRIC ELEVATIONS

### 135. Introduction

After all differences of elevations have been computed, both from the reciprocal and the nonreciprocal observations, and their weights determined, the next step (if the precision warrants) is the adjustment of the differences of elevations by the method of least squares.

### 136. Least Squares Adjustment

*a.* The adjustment of vertical observations is made by the means of observation equations. Elevations in even meters approximating the final values are first assumed for the different stations. To these assumed values are added $x$'s to be determined by the adjustment. Then

observation equations are formed by comparing the differences of the assumed elevations with the differences determined by computation. There are as many observation equations as there are computed differences of elevation.

*b.* Normal equations are formed from the observation equations using the computed weights from DA Form 1944 or 1945 [Computation of Elevations and Refractions from Reciprocal Observations] or if Computations from Nonreciprocal Observations (DA Form 1946, 1947) are included, use one-third of the computed weight for each nonreciprocal weight computation.

*c.* The system is then solved and the final elevations computed by applying the corrected differences of elevations.

*d.* A more thorough discussion and numerical example is given in USC&GS Sp. Pub. 138.

# CHAPTER 12
# ALTIMETER LEVELING

## Section I. SINGLE-BASE AND LEAP-FROG METHODS

### 137. Description

Figure 202 is an example of the single-base computation on DA Form 1948, Altimeter Leveling. In this single-base method, readings are made on one or more altimeters located on a base station of known elevation at fixed intervals of time, while simultaneous readings are being made on one or more roving altimeters located on points of unknown elevation. The differences in readings with certain corrections applied are the differences in elevation between the base station and the unknown stations. The correc-

**ALTIMETER LEVELING** (TM 5-237)

PROJECT: 231-05 | OBSERVER: Pfc. Olsen - Mr. Low | DATE: 29 Aug. 52 | ☒ Single Base Method ☐ Leapfrog Method

LOCATION: Yukon Delta, S.E. | RECORDER: Pfc. Vance

ORGANIZATION: 30th Engr. | WEATHER: Cool & Cloudy | INSTRUMENT NO. Set A {4077 / 3923 Set B {3921 / 3965

| (1) Field Station | (2) Time | (3) Mean Temp. Deg. F. | (4) Mean % Rel. Humidity | (5) Field Alt. Read. | (6) Index Diff. ± | (7) Field Indexed 5+6 | (8) Base Station Alt. Read. | (9) Indic. Diff. (7−8) ± | (10) Temp. and Humidity Corr. Factor | (11) Diff. Corr. (9×10) ± | (12) Unadj. Elev. | (13) Clos. Adj. ± | (14) Adj. Elev. | (15) Known Elev. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ Marshall | 1610 | | | | | | | | | | | | | 303.1 |
| PKDA-23-034A | 1610 | 52 | 84 | 898 | — | 898 | 1184 | -286 | 1.008 | -288.3 | 14.8 | +0.5 | 15.3 | |
| PKDA-23-034A | 1615 | | | | | | | | | | | | | |
| Δ Marshall | 1615 | 52 | 84 | 1183 | — | 1183 | 898 | +285 | 1.008 | +287.3 | 302.1 | +1.0 | 303.1 | 303.1 |
| Δ Marshall | 1625 | | | | | | | | | | | | | 303.1 |
| PKDA-23-034A | 1625 | 50 | 90 | 894 | — | 894 | 1181 | -287 | 1.004 | -288.1 | 15.0 | -1.1 | 13.9 | |
| PKDA-23-034A | 1630 | | | | | | | | | | | | | |
| Δ Marshall | 1630 | 50 | 97 | 1181 | — | 1181 | 892 | +289 | 1.005 | +290.4 | 305.4 | -2.3 | 303.1 | 303.1 |

COMPUTED BY: M.F.M. | DATE: 13 Oct. 52 | CHECKED BY: Erdman | DATE: 15 Oct. 52

DA FORM 1948

*Figure 202. Single-base method (DA Form 1948).*

tions applied in the single-base method are the calibration correction of the altimeter used (usually applied in the field book), and the air temperature and humidity correction factor which is multiplied by the difference in readings to get the difference in elevation.

## 138. Calibration

Calibration corrections must be made so that readings of the altimeters used in a set may be compared to a common mean. Altimeters are calibrated in sets of six or more, and a calibration correction diagram is made for each altimeter in the set. Figure 203 is an example of an altimeter calibration data sheet, and figure 204 is an example of an altimeter calibration graph. The correction from the calibration graph must be applied to the field reading of the altimeter before the reading is used in a computation.

## 139. Computation

The field information is entered at the top of the form (fig. 202). The computation is made starting at column 1 and continuing through column 15. Column 1, Field Station; column 2, Time; and column 3, Mean Temp., need no other explanation. Column 4, Mean % Rel. Humidity, contains the values for the mean humidity taken from the psychrometric chart (fig. 205), using wet-bulb temperature and dry-bulb temperature read in the field as the arguments. Column 5, Field Alt. Read., is the altimeter reading at the unknown station. Column 6 Index Diff., is the algebraic difference (base station instrument reading minus field instrument reading at base station). If the readings are the same, columns 5, Field Alt. Read., and 7, Field Indexed, have the same values. Column 8, Base Station Alt. Read., is the altimeter reading at the base station at the time of the field altimeter reading. Column 9, Indic. Diff. is the algebraic difference of column 7 minus column 8, and is the uncorrected difference in elevation between the base station and field station. Column 10, Temp. and Humidity Corr. Factor, contains the values taken from the cor-

| | | Altimeter Calibration Data | | | 29 Aug 52 | |
|---|---|---|---|---|---|---|
| 6-148 C | 6-151 C | 6-4280 C | 6-7762 C | 6-387 C | 6-482 C | Mean |
| 787 +10 | 799 -2 | 791 +6 | 803 -6 | 796 +1 | 806 -9 | 797 |
| 898 +11 | 911 -2 | 907 +2 | 913 -4 | 909 0 | 919 -10 | 909 |
| 991 +12 | 1008 -5 | 997 +6 | 1009 -6 | 1000 +3 | 1011 -8 | 1003 |

*Figure 203. Calibration data sheet.*

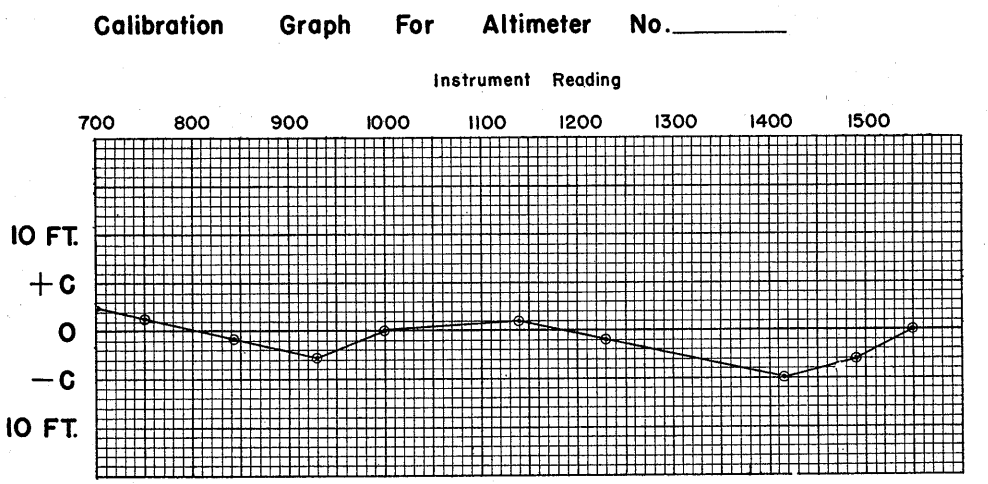



*Figure 204. Calibration graph.*

WET BULB TEMPERATURE F

| DRY BULB TEMPERATURE F | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 | 72 | 74 | 76 | 78 | 80 | 82 | 84 | 86 | 88 | 90 | 92 | 94 | 96 | 98 | 100 | 102 | 104 | 106 | 108 | 110 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 5 | 16 | 27 | 38 | 49 | 61 | 74 | 87 | 100 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 52 | | 9 | 19 | 29 | 40 | 51 | 63 | 75 | 87 | 100 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 54 | | 3 | 12 | 22 | 32 | 42 | 53 | 64 | 76 | 88 | 100 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 56 | | | 7 | 16 | 25 | 34 | 44 | 55 | 65 | 76 | 88 | 100 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 58 | | | 1 | 10 | 18 | 27 | 37 | 46 | 56 | 66 | 77 | 89 | 100 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 60 | | | | 5 | 13 | 21 | 30 | 39 | 48 | 58 | 68 | 78 | 89 | 100 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 62 | | | | | 8 | 16 | 24 | 32 | 41 | 50 | 59 | 69 | 79 | 89 | 100 | | | | | | | | | | | | | | | | | | | | | | | | |
| 64 | | | | | 4 | 11 | 18 | 26 | 34 | 43 | 51 | 60 | 70 | 79 | 90 | 100 | | | | | | | | | | | | | | | | | | | | | | |
| 66 | | | | | | 7 | 14 | 21 | 29 | 36 | 44 | 53 | 61 | 71 | 80 | 90 | 100 | | | | | | | | | | | | | | | | | | | | | |
| 68 | | | | | | 3 | 10 | 16 | 23 | 31 | 38 | 46 | 54 | 62 | 71 | 80 | 90 | 100 | | | | | | | | | | | | | | | | | | | | |
| 70 | | | | | | | 6 | 12 | 19 | 25 | 33 | 40 | 48 | 55 | 64 | 72 | 81 | 90 | 100 | | | | | | | | | | | | | | | | | | | | |
| 72 | | | | | | | 3 | 19 | 15 | 21 | 28 | 34 | 42 | 49 | 57 | 65 | 73 | 82 | 91 | 100 | | | | | | | | | | | | | | | | | | | |
| 74 | | | | | | | | 5 | 11 | 17 | 23 | 29 | 36 | 43 | 50 | 58 | 65 | 74 | 82 | 91 | 100 | | | | | | | | | | | | | | | | | | |
| 76 | | | | | | | | 3 | 8 | 13 | 19 | 25 | 31 | 38 | 44 | 51 | 59 | 66 | 74 | 82 | 91 | 100 | | | | | | | | | | | | | | | | | |
| 78 | | | | | | | | | 5 | 10 | 16 | 21 | 27 | 33 | 39 | 46 | 53 | 60 | 67 | 75 | 83 | 91 | 100 | | | | | | | | | | | | | | | | |
| 80 | | | | | | | | | 3 | 7 | 12 | 18 | 23 | 29 | 35 | 41 | 47 | 54 | 61 | 68 | 75 | 83 | 91 | 100 | | | | | | | | | | | | | | | |
| 82 | | | | | | | | | | 4 | 10 | 14 | 20 | 25 | 30 | 36 | 42 | 48 | 55 | 61 | 69 | 76 | 84 | 92 | 100 | | | | | | | | | | | | | | |
| 84 | | | | | | | | | | 1 | 7 | 12 | 16 | 21 | 26 | 32 | 37 | 43 | 49 | 56 | 62 | 69 | 76 | 84 | 92 | 100 | | | | | | | | | | | | | |
| 86 | | | | | | | | | | | 5 | 9 | 14 | 18 | 23 | 28 | 33 | 39 | 44 | 50 | 57 | 63 | 70 | 77 | 84 | 92 | 100 | | | | | | | | | | | | |
| 88 | | | | | | | | | | | 3 | 7 | 11 | 15 | 20 | 25 | 30 | 35 | 40 | 46 | 51 | 57 | 64 | 70 | 77 | 85 | 92 | 100 | | | | | | | | | | | |
| 90 | | | | | | | | | | | 1 | 5 | 9 | 13 | 17 | 22 | 26 | 31 | 36 | 41 | 47 | 52 | 58 | 65 | 71 | 78 | 85 | 92 | 100 | | | | | | | | | | |
| 92 | | | | | | | | | | | | 3 | 7 | 11 | 15 | 19 | 23 | 28 | 32 | 37 | 42 | 48 | 53 | 59 | 65 | 72 | 78 | 85 | 92 | 100 | | | | | | | | | |
| 94 | | | | | | | | | | | | | 5 | 9 | 12 | 16 | 20 | 24 | 29 | 33 | 38 | 43 | 49 | 54 | 60 | 66 | 72 | 79 | 85 | 93 | 100 | | | | | | | | |
| 96 | | | | | | | | | | | | | 3 | 7 | 10 | 14 | 18 | 22 | 26 | 30 | 35 | 39 | 44 | 50 | 55 | 61 | 66 | 73 | 79 | 86 | 93 | 100 | | | | | | | |
| 98 | | | | | | | | | | | | | 1 | 5 | 8 | 12 | 15 | 19 | 23 | 27 | 32 | 36 | 40 | 45 | 50 | 55 | 61 | 67 | 73 | 79 | 86 | 93 | 100 | | | | | | |
| 100 | | | | | | | | | | | | | | 3 | 7 | 10 | 13 | 17 | 21 | 24 | 28 | 33 | 37 | 41 | 46 | 51 | 56 | 62 | 68 | 73 | 80 | 86 | 93 | 100 | | | | | |
| 102 | | | | | | | | | | | | | | 1 | 5 | 8 | 11 | 15 | 18 | 22 | 26 | 30 | 34 | 38 | 42 | 47 | 52 | 57 | 62 | 68 | 74 | 80 | 86 | 93 | 100 | | | | |
| 104 | | | | | | | | | | | | | | | 3 | 7 | 10 | 13 | 16 | 20 | 23 | 27 | 31 | 35 | 39 | 43 | 48 | 53 | 58 | 63 | 69 | 74 | 80 | 87 | 93 | 100 | | | |
| 106 | | | | | | | | | | | | | | | | 5 | 8 | 11 | 14 | 17 | 21 | 24 | 28 | 32 | 36 | 40 | 44 | 49 | 53 | 58 | 64 | 69 | 75 | 81 | 87 | 93 | 100 | | |
| 108 | | | | | | | | | | | | | | | | 3 | 7 | 10 | 12 | 16 | 19 | 22 | 25 | 29 | 33 | 37 | 41 | 45 | 49 | 54 | 59 | 64 | 70 | 75 | 81 | 87 | 93 | 100 | |
| 110 | | | | | | | | | | | | | | | | | 5 | 8 | 11 | 14 | 17 | 20 | 23 | 26 | 30 | 34 | 38 | 42 | 46 | 50 | 55 | 60 | 65 | 70 | 75 | 81 | 87 | 93 | 100 |
| 112 | | | | | | | | | | | | | | | | | 4 | 7 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 31 | 35 | 38 | 42 | 47 | 51 | 55 | 60 | 65 | 70 | 76 | 81 | 87 | 94 |
| 114 | | | | | | | | | | | | | | | | | | 6 | 8 | 11 | 13 | 16 | 19 | 22 | 25 | 28 | 32 | 35 | 39 | 43 | 47 | 52 | 56 | 61 | 66 | 71 | 76 | 82 | 88 |
| 116 | | | | | | | | | | | | | | | | | | | 7 | 9 | 12 | 14 | 17 | 20 | 23 | 26 | 29 | 33 | 36 | 40 | 44 | 48 | 52 | 57 | 61 | 66 | 71 | 76 | 82 |
| 118 | | | | | | | | | | | | | | | | | | | 6 | 8 | 11 | 13 | 16 | 18 | 21 | 24 | 27 | 30 | 34 | 37 | 41 | 45 | 49 | 53 | 57 | 62 | 67 | 72 | 77 |
| 120 | | | | | | | | | | | | | | | | | | | | 6 | 9 | 12 | 14 | 17 | 19 | 22 | 25 | 28 | 31 | 34 | 38 | 41 | 45 | 49 | 53 | 58 | 62 | 67 | 72 |
| 122 | | | | | | | | | | | | | | | | | | | | | 8 | 10 | 13 | 15 | 18 | 20 | 23 | 26 | 29 | 32 | 35 | 39 | 42 | 46 | 50 | 54 | 58 | 63 | 67 |
| 124 | | | | | | | | | | | | | | | | | | | | | 7 | 9 | 11 | 14 | 16 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 43 | 47 | 50 | 54 | 59 | 63 |
| 126 | | | | | | | | | | | | | | | | | | | | | | 8 | 10 | 12 | 15 | 17 | 19 | 22 | 25 | 27 | 30 | 33 | 37 | 40 | 44 | 47 | 51 | 55 | 59 |
| | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 | 72 | 74 | 76 | 78 | 80 | 82 | 84 | 86 | 88 | 90 | 92 | 94 | 96 | 98 | 100 | 102 | 104 | 106 | 108 | 110 |

% RELATIVE-HUMIDITY TABLE (Barometric Pressure 29.92. Air Velocity 600 feet (or more) per minute)

*Figure 205. Psychrometric chart.*

rection factor chart (fig. 206), using the mean temperature and mean relative humidity as the arguments. Column 11, Diff. Corr., is the product of column 9 and column 10, and is the corrected difference in elevation between the base station and field station. Column 12, Unadj. Elev.; column 13, Clos. Adj.; column 14, Adj. Elev.; and column 15, Known Elev., need no further explanation.

## 140. Leap-Frog Method

Figure 207 is an example of the leap-frog method. This computation is made on the same form as the single-base method, and the same corrections must be applied to the field notes. The only difference between the leap-frog method and the single-base method is that in the leap-frog method the elevation can be computed for several unknown stations in one circuit.

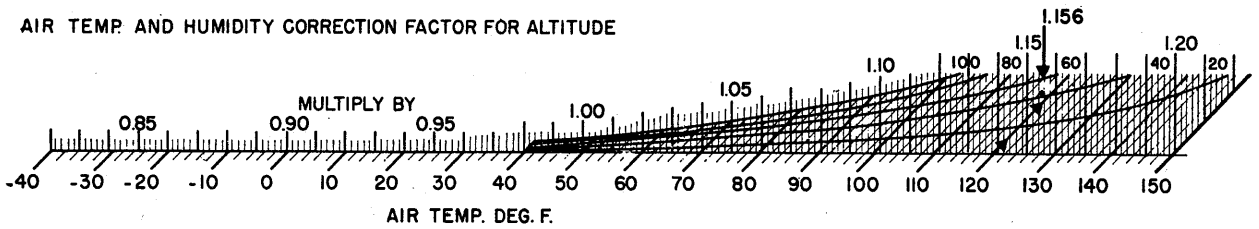



*Figure 206. Air temperature and relative humidity correction.*

PROJECT 231-05 | OBSERVER Sgt. Taylor - Sgt. Miller | **ALTIMETER LEVELING** (TM 5-237)

LOCATION Yukon Delta, S.E. | RECORDER Cpl. Fudd | DATE 18 July 52 | ☐ Single Base Method ☒ Leapfrog Method

ORGANIZATION 30th Engr. | WEATHER Clear & Windy | INSTRUMENT NO. Set A {4099 4092 Set B {4087 4086

| (1) Field Station | (2) Time | (3) Mean Temp. Deg. F. | (4) Mean % Rel. Humidity | (5) Field Alt. Read. | (6) Index Diff. ± | (7) Field Indexed 5+6 | (8) Base Station Alt. Read. | (9) Indic. Diff. (7−8) ± | (10) Temp. and Humidity Corr. Factor | (11) Diff. Corr. (9×10) ± | (12) Unadj. Elev. | (13) Clos. Adj. ± | (14) Adj. Elev. | (15) Known Elev. ft. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Δ Anvik | 1555 | | | | | | | | | | | | | 836.0 |
| PKDA-26-52-121A | 1555 | 68 | 78 | 888.6 | — | 888.6 | 1634.1 | -745.5 | 1.043 | -777.6 | 58.4 | -0.1 | 58.3 | |
| PKDA-26-52-121A | 1605 | | | | | | | | | | | | | |
| Turning Point | 1605 | 65 | 90 | 888.2 | — | 888.2 | 888.1 | +0.1 | 1.036 | +0.1 | 58.5 | -0.2 | 58.3 | |
| Turning Point | 1608 | | | | | | | | | | | | | |
| PKDA-26-52-121A | 1608 | 66 | 85 | 889.1 | — | 889.1 | 887.8 | +1.3 | 1.038 | +1.3 | 59.8 | -0.3 | 59.5 | |
| PKDA-26-52-121A | 1620 | | | | | | | | | | | | | |
| Δ Anvik | 1620 | 68 | 83 | 1636.6 | — | 1636.6 | 891.2 | +745.4 | 1.042 | +776.7 | 836.5 | -0.5 | 836.0 | 836.0 |

COMPUTED BY M. F. M. | DATE 13 Oct. 52 | CHECKED BY Erdman | DATE 15 Oct. 52

DA FORM 1 FEB 57 1948

*Figure 207. Leap-frog method (DA Form 1948).*

## Section II. TWO-BASE METHOD

### 141. Description

The third method is the two-base method. In this method, the density of the vertical column of air in the area to be surveyed is calibrated by placing altimeters at both extremes of altitude and taking readings at fixed intervals of time. This calibration replaces the correction factor for temperature and humidity necessary to the other two methods. The basic solution of the two-base method is a simple proportion (fig. 208). Readings are made simultaneously on the lower and upper bases, whose elevations are known, and on the station of unknown elevation. Using these readings, two determinations of elevation are computed. The elevation of the unknown point is then corrected by a proportional adjustment based on the altitude of the point with respect to the two bases.

### 142. Field Procedure

In the work selected as an example of the two-base method, six altimeters were used; two on the lower-base station, two on the upper-base station, and two roving altimeters to locate vertical control points. Simultaneous readings were made on all base altimeters at 5-minute intervals, and the roving altimeters were read on the even 5 minutes to coincide with the base altimeter readings. Calibration corrections were made in the field books by the recorders. All six altimeters were first read simultaneously on the lower base. Then the two upper-base altimeters and the two roving altimeters were taken to the upper base and simultaneous readings were made on all four altimeters. After the roving altimeters had completed the control points making two separate occupations of each point, they were returned to the upper base and another set of readings was made. These four altimeters were then returned to the lower base and all six altimeters were read again.

### 143. Computation

*a.* The procedure used to compute the elevations of the control points is as follows:

(1) Compute the lower-base comparison.
(2) Compute the upper-base comparison.
(3) Draw the correction graphs, one for each base altimeter and two for each roving altimeter.
(4) Correct the field readings using the graphs.
(5) Compute the elevations using the two-base method.
(6) Compute the final elevation for each point by meaning all acceptable computed elevations.

*b.* In the lower-base comparison (fig. 209), the time, altimeter numbers and the altimeter readings of the start and finish of the work are abstracted

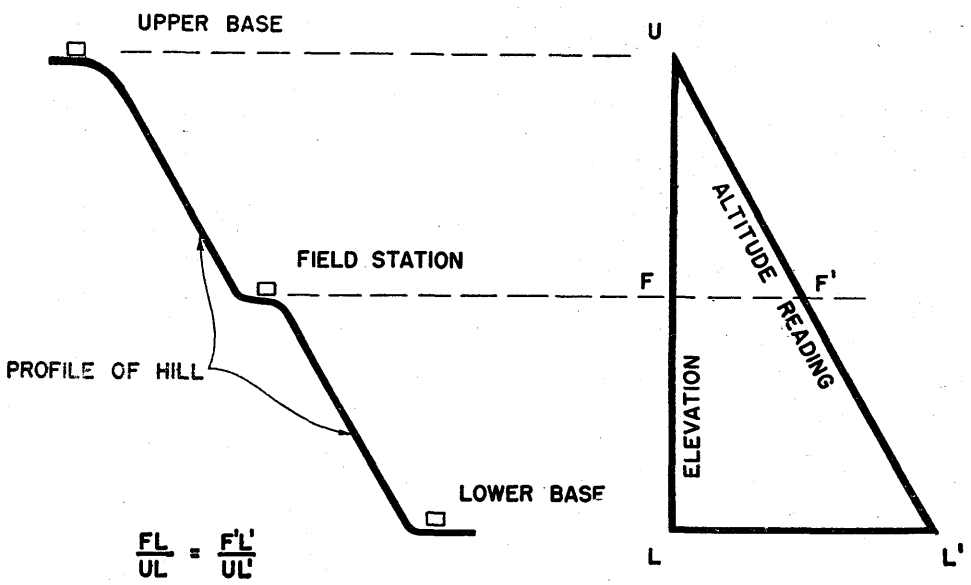



*Figure 208. Diagram, two-base method.*

① Lower Base Comparison

| Time | Alt. Number | Alt. Reading | Diff | Time | Alt. Number | Alt. Reading | Diff. |
|---|---|---|---|---|---|---|---|
| 1000 | 6-3921 | 1044.6 | -0.4 | 1505 | 6-3921 | 1079.7 | -0.2 |
| | 6-3965 | 1043.0 | +1.2 | | 6-3965 | 1078.8 | +0.7 |
| | 6-3393 | 1046.3 | -2.1 | | 6-3393 | 1080.1 | -0.6 |
| | 6-3923 | 1051.5 | -7.3 | | 6-3923 | 1083.7 | -4.2 |
| | 6-4077 | 1042.7 | +1.5 | | 6-4077 | 1079.2 | +0.3 |
| | 6-3426 | 1036.9 | +7.3 | | 6-3426 | 1075.6 | +3.9 |
| | Sum= | 6265.0 | | | Sum= | 6477.1 | |
| | Mean= | 1044.2 | | | Mean= | 1079.5 | |

② Upper Base Comparison

| Time | Alt. Number | Alt. Reading | Diff | Time | Alt. Number | Alt. Reading | Diff |
|---|---|---|---|---|---|---|---|
| 1105 | 6-3921 | 1652.3 | -2.7 | 1420 | 6-3921 | 1677.8 | -3.1 |
| | 6-3965 | 1648.5 | +1.1 | | 6-3965 | 1675.5 | -0.8 |
| | 6-4077 | 1650.1 | -0.5 | | 6-4077 | 1671.2 | +3.5 |
| | 6-3426 | 1647.6 | +2.0 | | 6-3426 | 1674.2 | +0.5 |
| | Sum= | 6598.5 | | | Sum= | 6698.7 | |
| | Mean= | 1649.6 | | | Mean= | 1674.7 | |

Lower-base comparison
Upper-base comparison

*Figure 209. Comparison of altimeters.*

from the field book. A mean reading is found and the difference between the mean reading and the reading on each altimeter is computed. In the upper-base comparison (fig. 209), the same process is repeated using the readings from the four altimeters compared at the upper-base.

*c.* Assuming the rate of change for density in each altimeter is constant, a graph (two graphs for the roving altimeters) is drawn showing the correction to be applied to the altimeter reading at the time of the reading to reduce the reading to the datum plane. The datum plane is defined by the straight line connecting the mean readings of the altimeters at the beginning and end of observations. See figures 210 through 217. Next, all field altimeter readings (previously corrected for calibration) are reduced to readings on the datum planes on DA Form 1949 (Corrected Altimeter Readings Prorated According to Time) (fig. 218). The corrections are taken from the graphs using the time of the station occupation as the argument. An example of the reduction is shown for altimeter 6–3393 (fig. 218).

*d.* Elevations for all points are now computed using DA Form 1950, Altimeter Leveling (Two-Base Method) (fig. 219). On this form, each step of the computation is numbered and a key to computation printed in each block. It often occurs that the elevation of the unknown station does not fall between the elevations of the bases. If the difference in elevation of the unknown station and either base falls outside the bases by more than 10 percent of the difference in elevation between bases, a special computation must be made. An example of this computation is performed on DA Form 1951, Altimeter Leveling (Two-Base Method) Special Case (fig. 220). I, II, and III for this computation are taken from figure 219.

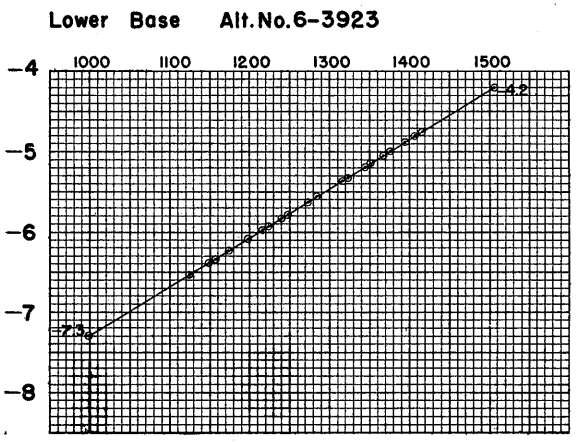



*Figure 210. Correction chart, altimeter 6-3923.*

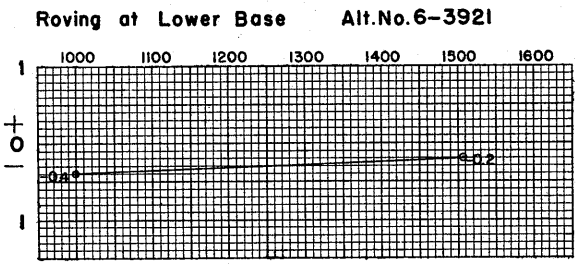



*Figure 211. Correction chart, altimeter 6-3921.*

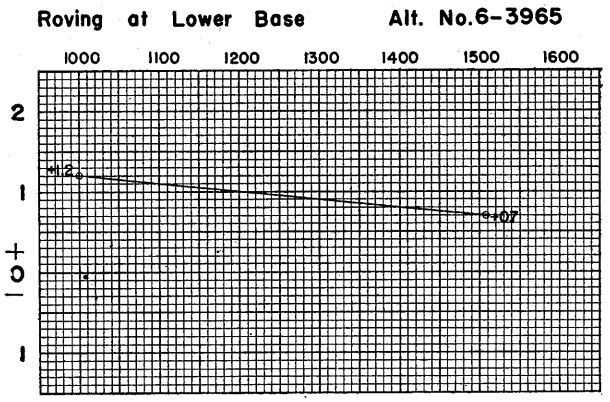



*Figure 212. Correction chart, altimeter 6-3965.*

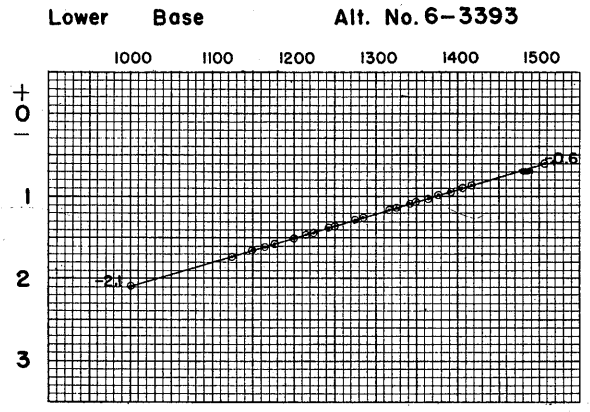



*Figure 213. Correction chart, altimeter 6-3393.*

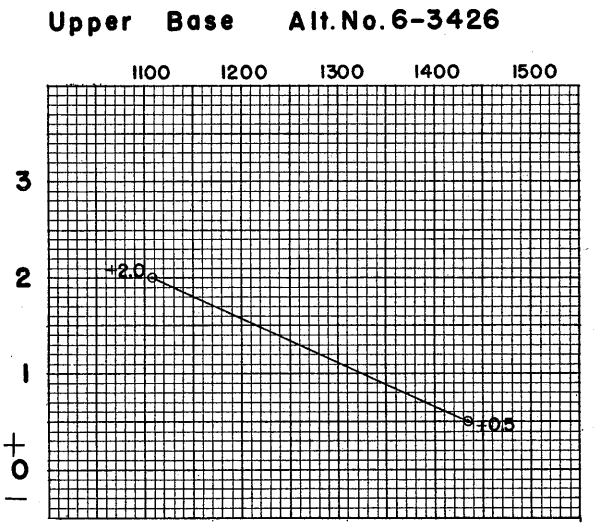



*Figure 214. Correction chart, altimeter 6-3426.*

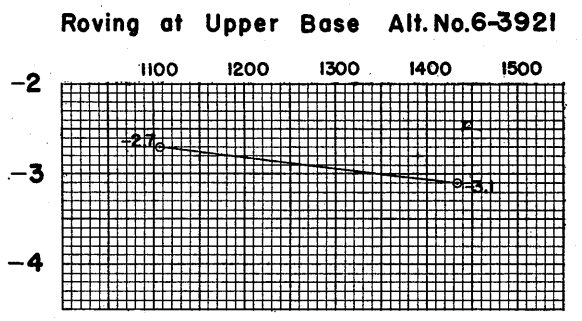



*Figure 215. Correction chart, altimeter 6-3921.*

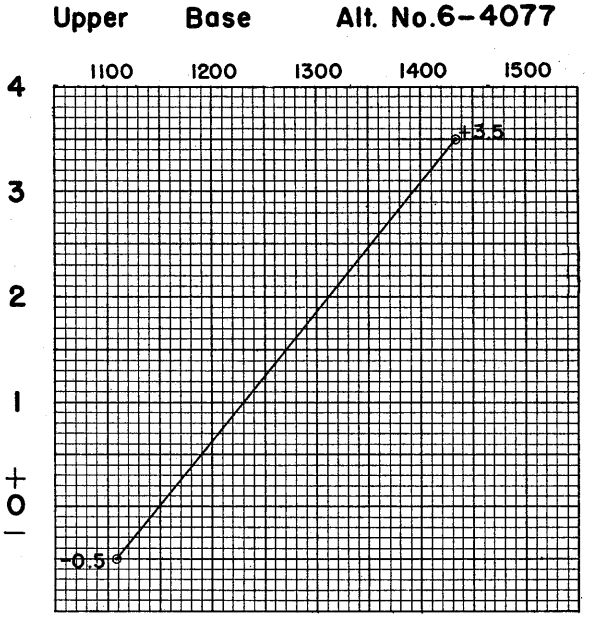



*Figure 216. Correction chart, altimeter 6-4077.*

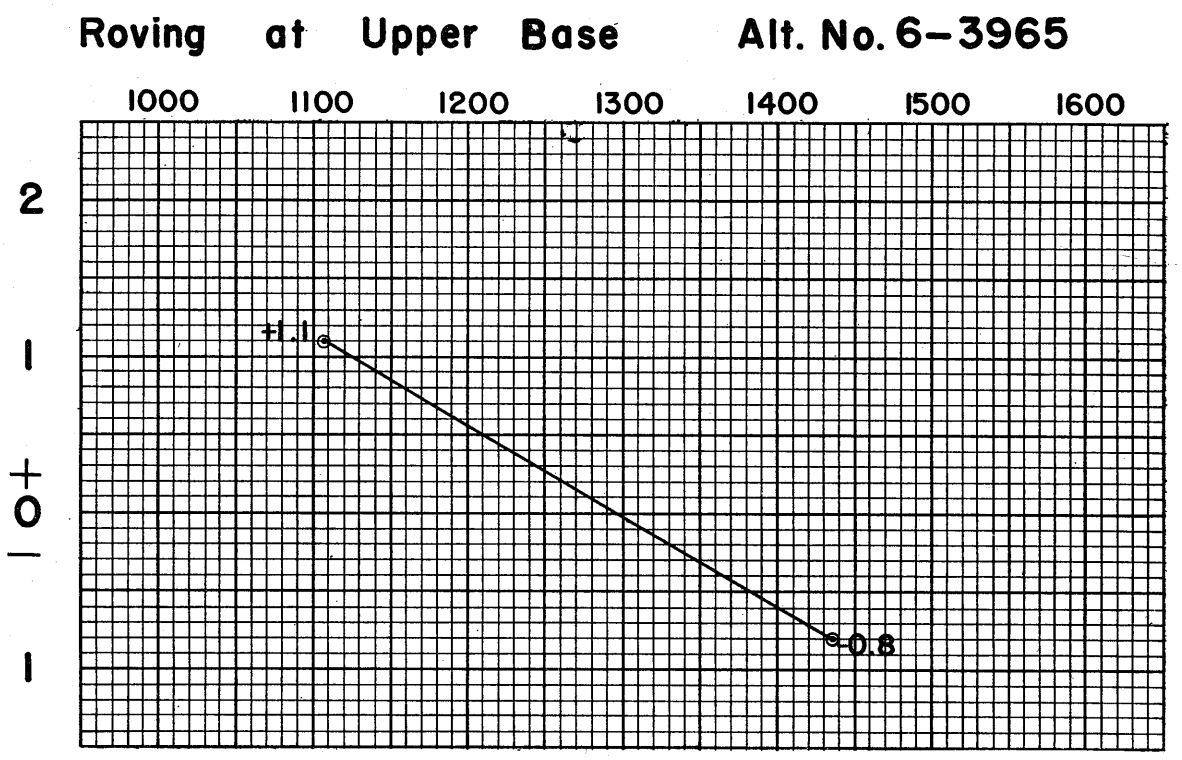



*Figure 217. Correction chart, altimeter 6–3965.*

*e.* Final elevations for the points are established by computing the mean of all acceptable elevations on DA Form 1952, Mean of Results, Altimeter Leveling (fig. 221). Acceptable elevations are those that do not vary from the mean by more than the specified accuracy of the work.

*f.* If only three altimeters are used, one to each base and one roving, three separate runs must be made. If any of these elevations is not acceptable, one or more additional runs must be made until three acceptable elevations for each point are computed.

PROJECT 1-249

## CORRECTED ALTIMETER READINGS PRORATED ACCORDING TO TIME

(TM 5-237)

LOCATION Yukon Delta, SE

ORGANIZATION 30th Engrs.

DATE OF OBSERVATIONS 28 August 52

| STATION | TIME | LOWER BASE Book: Page: | | | ROVING ALTIMETERS Book: Page: | | | | | UPPER BASE Book: Page: | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Instrument No. 6-3393 | Instrument No. 6-3923 | MEAN ① | Instrument No. 6-3921 Lower Comp. | Instrument No. 6-3921 Upper Comp. | Instrument No. 6-3965 Lower Comp. | Instrument No. 6-3965 Upper Comp. | MEAN ② | Instrument No. 6-4077 | Instrument No. 6-3426 | MEAN ③ |
| AP 43-38-0025 B | 1115 | 1040.6 | 1045.0 | 1042.8 | 1271.7 | 1269.3 | 1269.8 | 1269.7 | 1270.1 | 1648.8 | 1648.6 | 1648.7 |
| " -0025 A | 1130 | 1043.7 | 1046.1 | 1044.9 | 1028.7 | 1026.2 | 1031.2 | 1031.0 | 1029.3 | 1650.1 | 1648.5 | 1649.3 |
| AP 21 - 112 C | 1225 | 1050.9 | 1055.8 | 1053.4 | 922.6 | 920.0 | 920.8 | 920.1 | 920.9 | 1659.2 | 1655.9 | 1657.6 |
| AP 21 - 112 B | 1245 | 1053.0 | 1059.0 | 1056.0 | 1019.0 | 1016.4 | 1016.2 | 1015.4 | 1016.8 | 1660.7 | 1656.7 | 1658.7 |
| AP 21 - 112 B | 1250 | 1054.0 | 1059.0 | 1056.5 | 1017.0 | 1014.4 | 1015.2 | 1014.4 | 1015.2 | 1660.8 | 1654.7 | 1657.8 |
| AP 21 - 112 C | 1315 | 1055.2 | 1059.3 | 1057.2 | 924.7 | 922.0 | 924.7 | 923.6 | 923.8 | 1663.4 | 1661.3 | 1662.4 |
| AP 43-38-0025 A | 1405 | 1070.3 | 1069.8 | 1070.0 | 1051.3 | 1048.4 | 1049.8 | 1048.3 | 1049.4 | 1672.4 | 1667.9 | 1670.2 |
| " -0025 B | 1410 | 1071.3 | 1069.9 | 1070.6 | 1293.3 | 1290.4 | 1292.9 | 1291.4 | 1292.0 | 1673.5 | 1667.9 | 1670.7 |

Alt. No. 6-3393

| Time | Field Reading | Corr. | Corr. Reading |
|---|---|---|---|
| 1115 | 1042.3 | -1.7 | 1040.6 |
| 1130 | 1045.3 | -1.6 | 1043.7 |
| 1225 | 1052.3 | -1.4 | 1050.9 |
| 1245 | 1054.3 | -1.3 | 1053.0 |
| 1250 | 1055.3 | -1.3 | 1054.0 |
| 1315 | 1056.3 | -1.1 | 1055.2 |
| 1405 | 1071.2 | -0.9 | 1070.3 |
| 1410 | 1072.2 | -0.9 | 1071.3 |

Mean values ①, ②, and ③ are entered on two-base form as ①, ②, and ③.

COMPUTED BY RAS DATE 1 Sept 52 CHECKED BY W. C. A. DATE 3 Sept 52

DA FORM 1 FEB 57 1949

① Reduction of field altimeter readings ② Example of reduction

*Figure 218. Computation of corrected field readings (DA Form 1949).*

**PROJECT** 1249

## ALTIMETER LEVELING (Two-Base Method) (TM 5-237)

| | | |
|---|---|---|
| **LOCATION** Alaska | **ORGANIZATION** 30th Engrs. | **DATE** 28 Aug 1952 |
| **LOWER BASE** AP 21-112B (Aux.) | **WEATHER** Fair | **UPPER BASE** AP 43-38-0025C |
| **INST. NO.** 6-3393 & 6-3923 | **FIELD INST. NO.** 6-3921 & 6-3965 | **INST. NO.** 6-4077 & 6-3426 |

| I=Elev., Lower Base | III=II−I | II=Elev., Upper Base | I=Elev., Lower Base | III=II−I | II=Elev., Upper Base |
|---|---|---|---|---|---|
| 153.4 | 611.7 | 765.1 | 153.4 | 611.7 | 765.1 |

| Station AP 43-38-0025B | | Time 1115 | Station AP 43-38-0025B | | Time 1410 |
|---|---|---|---|---|---|
| ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading | ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading |
| 1042.8 | 1270.1 | 1648.7 | 1070.6 | 1292.0 | 1670.7 |
| ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ | ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ |
| +227.3 | 1.0096 | −378.6 | +221.4 | 1.0193 | −378.7 |
| Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II | Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II |
| 382.9 | 605.9 | 382.9 | 379.1 | 600.1 | 379.1 |
| Adj. Elev. | Known Elev. | | Adj. Elev. | Known Elev. | |

| Station AP 43-38-0025A | | Time 1130 | Station AP 43-38-0025A | | Time 1405 |
|---|---|---|---|---|---|
| ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading | ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading |
| 1044.9 | 1029.3 | 1649.3 | 1070.0 | 1049.4 | 1670.2 |
| ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ | ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ |
| −15.6 | 1.0121 | −620.0 | −20.6 | 1.0192 | −620.8 |
| Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II | Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II |
| 137.6 | 604.4 | 137.6 | 132.4 | 600.2 | 132.4 |
| Adj. Elev. | Known Elev. | | Adj. Elev. | Known Elev. | |

| Station AP 21-112B | | Time 1245 | Station AP 21-112B | | Time 1250 |
|---|---|---|---|---|---|
| ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading | ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading |
| 1056.0 | 1016.8 | 1658.7 | 1056.5 | 1015.2 | 1657.8 |
| ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ | ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ |
| −39.2 | 1.0149 | −641.9 | −41.3 | 1.0173 | −642.6 |
| Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II | Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II |
| 113.6 | 602.7 | 113.6 | 111.4 | 601.3 | 111.4 |
| Adj. Elev. | Known Elev. | | Adj. Elev. | Known Elev. | |

| Station | | Time | Station | | Time |
|---|---|---|---|---|---|
| ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading | ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading |
| ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ | ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ |
| Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II | Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II |
| Adj. Elev. | Known Elev. | | Adj. Elev. | Known Elev. | |

| Station | | Time | Station | | Time |
|---|---|---|---|---|---|
| ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading | ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading |
| ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ | ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ |
| Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II | Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II |
| Adj. Elev. | Known Elev. | | Adj. Elev. | Known Elev. | |

| Station | | Time | Station | | Time |
|---|---|---|---|---|---|
| ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading | ①=Lower Base Reading | ②=Field Reading | ③= Upper Base Reading |
| ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ | ④=−①+② | ⑦=III÷⑥ | ⑤=②−③ |
| Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II | Elev.=I+⑦·④ | ⑥=③−① | Elev.=⑦·⑤+II |
| Adj. Elev. | Known Elev. | | Adj. Elev. | Known Elev. | |

| | | | |
|---|---|---|---|
| **COMPUTED BY** R.B. Smith | **DATE** 1 Sept. 52 | **CHECKED BY** W. C. Cumen | **DATE** 3 Sept. 52 |

DA FORM 1 FEB 57 1950

*Figure 219. Two-base method (DA Form 1950).*

| PROJECT | ALTIMETER LEVELING (Two-Base Method) Special Case (TM 5-237) | |
|---|---|---|
| 1249 | | |
| LOCATION: Alaska | ORGANIZATION: 30th Engrs. | DATE: 28 Aug 52 |
| LOWER BASE: AP 21-112B (Aux) | WEATHER: Fair | UPPER BASE: AP 43-38-0025C |
| INST. NO.: 6-3393 & 6-3923 | FIELD INST. NO.: 6-3921 & 6-3965 | INST. NO.: 6-4077 & 6-3426 |

| | | | | | |
|---|---|---|---|---|---|
| I=Elev., Lower Base: 153.4 | | II=Elev., Upper Base: 765.1 | I=Elev., Lower Base: 153.4 | | II=Elev., Upper Base: 765.1 |
| Station: AP 21-112C | | Time: 1225 | Station: AP 21-112C | | Time: 1315 |
| ① Lower Base Reading: 1053.4 | ② Field Reading: 920.9 | ③ Upper Base Reading: 1657.6 | ① Lower Base Reading: 1057.2 | ② Field Reading: 923.8 | ③ Upper Base Reading: 1662.4 |
| ④=②−① : −132.5 | ⑤=②−③ : −736.7 | ⑥=④+⑤ : −869.2 | ④=②−① : −133.4 | ⑤=②−③ : −738.6 | ⑥=④+⑤ : −872.0 |
| ⑦=I+④ : 20.9 | ⑧=II+⑤ : 28.4 | ⑨=⑧−⑦ : +7.5 | ⑦=I+④ : 20.0 | ⑧=II+⑤ : 26.5 | ⑨=⑧−⑦ : +6.5 |
| ⑩=④÷⑥ : 0.1524 | ⑪=⑤÷⑥ : 0.8476 | | ⑩=④÷⑥ : 0.1530 | ⑪=⑤÷⑥ : 0.8470 | |
| Elev.=⑦+⑩×⑨ : 22.0 | Known Elev. | Elev.=⑧−⑪×⑨ : 22.0 | Elev.=⑦+⑩×⑨ : 21.0 | Known Elev. | Elev.=⑧−⑪×⑨ : 21.0 |
| Station | | Time | Station | | Time |
| ① Lower Base Reading | ② Field Reading | ③ Upper Base Reading | ① Lower Base Reading | ② Field Reading | ③ Upper Base Reading |
| ④=②−① | ⑤=②−③ | ⑥=④+⑤ | ④=②−① | ⑤=②−③ | ⑥=④+⑤ |
| ⑦=I+④ | ⑧=II+⑤ | ⑨=⑧−⑦ | ⑦=I+④ | ⑧=II+⑤ | ⑨=⑧−⑦ |
| ⑩=④÷⑥ | ⑪=⑤÷⑥ | | ⑩=④÷⑥ | ⑪=⑤÷⑥ | |
| Elev.=⑦+⑩×⑨ | Known Elev. | Elev.=⑧−⑪×⑨ | Elev.=⑦+⑩×⑨ | Known Elev. | Elev.=⑧−⑪×⑨ |
| Station | | Time | Station | | Time |
| ① Lower Base Reading | ② Field Reading | ③ Upper Base Reading | ① Lower Base Reading | ② Field Reading | ③ Upper Base Reading |
| ④=②−① | ⑤=②−③ | ⑥=④+⑤ | ④=②−① | ⑤=②−③ | ⑥=④+⑤ |
| ⑦=I+④ | ⑧=II+⑤ | ⑨=⑧−⑦ | ⑦=I+④ | ⑧=II+⑤ | ⑨=⑧−⑦ |
| ⑩=④÷⑥ | ⑪=⑤÷⑥ | | ⑩=④÷⑥ | ⑪=⑤÷⑥ | |
| Elev.=⑦+⑩×⑨ | Known Elev. | Elev.=⑧−⑪×⑨ | Elev.=⑦+⑩×⑨ | Known Elev. | Elev.=⑧−⑪×⑨ |
| Station | | Time | Station | | Time |
| ① Lower Base Reading | ② Field Reading | ③ Upper Base Reading | ① Lower Base Reading | ② Field Reading | ③ Upper Base Reading |
| ④=②−① | ⑤=②−③ | ⑥=④+⑤ | ④=②−① | ⑤=②−③ | ⑥=④+⑤ |
| ⑦=I+④ | ⑧=II+⑤ | ⑨=⑧−⑦ | ⑦=I+④ | ⑧=II+⑤ | ⑨=⑧−⑦ |
| ⑩=④÷⑥ | ⑪=⑤÷⑥ | | ⑩=④÷⑥ | ⑪=⑤÷⑥ | |
| Elev.=⑦+⑩×⑨ | Known Elev. | Elev.=⑧−⑪×⑨ | Elev.=⑦+⑩×⑨ | Known Elev. | Elev.=⑧−⑪×⑨ |

| | | | |
|---|---|---|---|
| COMPUTED BY: R A Smith | DATE: 1 Sept 52 | CHECKED BY: W. C. Aumen | DATE: 3 Sept. 52 |

DA FORM 1 FEB 57 1951

*Figure 220. Two-base method (special case) (DA Form 1951).*

PROJECT 1-249

**MEAN OF RESULTS, ALTIMETER LEVELING**
(TM 5-237)

LOCATION Yukon Delta, S.E. | ORGANIZATION 30th Engrs | 2 BASE METHOD | DATE 28 Aug 52

| STATION | FIRST RESULT | | | SECOND RESULT | | | THIRD RESULT | | | FOURTH RESULT | | | MEAN ELEV. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DATE | TIME | ELEV. | DATE | TIME | ELEV. | DATE | TIME | ELEV. | DATE | TIME | ELEV. | |
| IP 43-38-0025B | 28 Aug 52 | 1115 | 382.9 | 28 Aug 52 | 1410 | 379.1 | | | | | | | 381.0 |
| IP 43-38-0025A | " | 1130 | 137.6 | " | 1405 | 132.4 | | | | | | | 135.0 |
| IP 21-112C | " | 1225 | 22.0 | " | 1315 | 21.0 | | | | | | | 21.5 |
| IP 21-32-112B | " | 1245 | 113.6 | " | 1250 | 111.4 | | | | | | | 112.5 |

COMPUTED BY Robinett | DATE 1 Sept 52 | CHECKED BY W. C. Aumen | DATE 3 Sept 52

DA FORM 1 FEB 57 1952

*Figure 221. Computation of final elevations (DA Form 1952).*

# CHAPTER 13

# THE UNIVERSAL TRANSVERSE MECATOR GRID

## Section I. MATHEMATICS AND CONSTRUCTION OF THE UTM GRID

### 144. Discussion

The following is a brief discussion of the mathematical formulas and construction of the Universal Transverse Mercator (UTM) grid:

*a.* So long as calculations are made in E or N coordinates, or in terms of distance and bearing, there is no need to know the exact form of the earth. But if it is required, for example, to express a length measured along the equator as a difference of longitude, then it is necessary to know the length of a degree of longitude at the equator; this in turn means knowing $a$, the earth's equatorial radius, since $a \div \frac{180}{\pi}$ = length per degree of equatorial longitude. In the same way, other conversions require a knowledge of the polar radius $b$. From $a$ and $b$ can be derived the flattening, $f$, which is defined by the equation $\frac{a-b}{a}=f$. The quantities $a$ and $f$ are generally considered to define a spheroid.

*b.* Among the spheroids in present use are several for which Universal Transverse Mercator tables have been prepared (TM 5-241-series). These spheroids, with the accepted values of their dimensions, are listed below.

| *Name* | *a (International Meters)* | *1/f* | *TM 5-241-* |
|---|---|---|---|
| International | 6 378 388. 0000 | 297. 000000 | 3/1 & 3/2 |
| Clarke 1866 | 6 378 206. 4000 | 294. 978698 | 4/1 & 4/2 |
| Clarke 1880 | 6 378 249. 1450 | 293. 465000 | 6/1 & 6/2 |
| Everest | 6 377 276. 3452 | 300. 801700 | 7 |
| Bessell | 6 377 397. 1550 | 299. 152813 | 5/1 & 5/2 |

In the United States, the Clarke spheroid of 1866 is used.

*c.* The International meter is related to the American yard by the equation, 1 meter = 1.093 611 111 American yards, so that $a$ = 6,975,277.3879 American yards, regardless of the fact that Clarke himself considered that $a$ = 6,975,354 Imperial yards. The difference, which originated in the discrepancies that arose during the 19th century among various European standards of length, is not significant by comparison with other uncertainties in the size of the earth, hence the metric value and the corresponding figure in American yards are to be adhered to for the sake of consistency in computation. The value for $f$ is likewise not exactly Clarke's figure; it is derived from the value $b$ = 6,356,583.8000 International meters, which is used by the U.S. Coast and Geodetic Survey in the construction of the basic tables for the computation of triangulation.

### 145. Notation

*a.* The following elements of the Transverse Mercator projection are illustrated in figure 222:

P = the point under consideration
F = the foot of the perpendicular from P to the central meridian plane
O = the origin
OZ = the central meridian
LP = the parallel of the latitude of P
ZP = the meridian of P
OL = $k_0$S, the meridional arc from the equator
LF = the ordinate of curvature
OF = N, grid Northing
FP = E′, grid distance from the central meridian
GN = grid north
C = the convergence of the meridians, i.e., the angle at P between true north and grid north

*b.* In figure 222, P represents a point in the northern hemisphere east of the central meridian. By inverting the diagram, overturning it, or both, the figure can be made to represent the situation

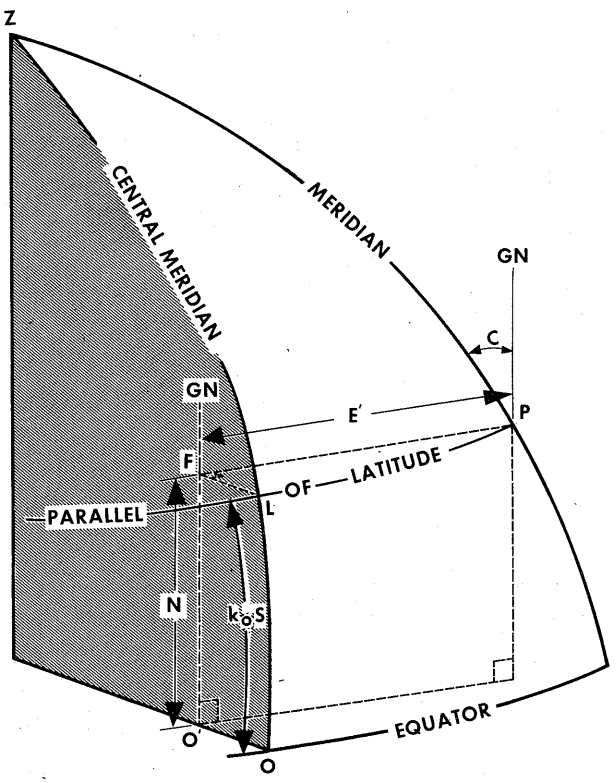



*Figure 222. Relationship of grid and geographic systems.*

in either hemisphere or on either side of the central meridian.

*c*. The adopted notation is as follows:

$\phi$=latitude

$\lambda$=longitude

$\phi'$=latitude of the foot of the perpendicular from the point to the central meridian

$\lambda_0$=longitude of the origin (the central meridian) of the projection

$\Delta\lambda$=difference of longitude from the central meridian

$=\lambda-\lambda_0$ in the Eastern Hemisphere

$=\lambda_0-\lambda$ in the Western Hemisphere

$a$=semimajor axis of the spheroid

$b$=semiminor axis of the spheroid

$f$=flattening or ellipticity$=\dfrac{a-b}{a}$

$e^2=(\text{eccentricity})^2=\dfrac{a^2-b^2}{a^2}$

$e'^2=\dfrac{a^2-b^2}{b^2}=\dfrac{e^2}{1-e^2}$

$n=\dfrac{a-b}{a+b}$

$\rho$=radius of curvature in the meridian$=\dfrac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}}$

$\nu$=radius of curvature in the prime vertical; also defined as the normal to the spheroid terminating at the minor axis

$\gamma=\dfrac{a}{(1-e^2\sin^2\phi)^{1/2}}=\rho[1+(e')^2\cos^2\phi]$

S=true meridional distance on the spheroid from the equator

$k_0$=central scale factor, an arbitrary reduction applied to all geodetic lengths to reduce the maximum scale distortion of the projection (for the UTM, $k_0$=0.9996)

$k$=scale factor at the working point on the projection

FN=False Northing

FE=False Easting

E′=grid distance from the central meridian (always positive)

E=grid Easting=E′+500,000 (when point is east of central meridian), 500,000−E′ (when point is west of central meridian)

N=grid Northing

$t$=plane azimuth (measured from north) } grid azimuth

T=projected geodetic azimuth (measured from grid north) } grid azimuth

$\alpha$=geodetic azimuth

C=convergence of the meridians; i.e., the angle between true north and grid north

$p=0.0001\Delta\lambda$

$q=0.000001E'$

$$(I)=Sk_0$$

$$(II)=\frac{\nu\sin\phi\cos\phi\sin^2 1''}{2}\cdot k_0\cdot 10^8$$

$$(III)=\frac{\sin^4 1''\nu\sin\phi\cos^3\phi}{24}(5-\tan^2\phi+9e'^2\cos^2\phi+4e'^4\cos^4\phi)\cdot k_0\cdot 10^{16}$$

$$(IV)=\nu\cos\phi\sin 1''\cdot k_0\cdot 10^4$$

$$(V)=\frac{\sin^3 1''\nu\cos^3\phi}{6}(1-\tan^2\phi+e'^2\cos^2\phi)\cdot k_0\cdot 10^{12}$$

$$(\text{VII})=\frac{\tan\phi'}{2\nu^2\sin 1''}(1+e'^2\cos^2\phi)\cdot\frac{1}{k_0^2}\cdot 10^{12}$$

$$(\text{VIII})=\frac{\tan\phi'}{24\nu^4\sin 1''}(5+3\tan^2\phi+6e'^2\cos^2\phi -6e'^2\sin^2\phi-3e'^4\cos^4\phi -9e'^4\cos^2\phi\sin^2\phi)\times\frac{1}{k_0^4}\times 10^{24}$$

$$(\text{IX})=\frac{\sec\phi'}{\nu\sin 1''}\cdot\frac{1}{k_0}\cdot 10^{6}$$

$$(\text{X})=\frac{\sec\phi'}{6\nu^3\sin 1''}(1+2\tan^2\phi+e'^2\cos^2\phi)\cdot\frac{1}{k_0^3}\cdot 10^{18}$$

$$(\text{XII})=\sin\phi\cdot 10^{4}$$

$$(\text{XIII})=\frac{\sin^2 1''\sin\phi\cos^2\phi}{3}(1+3e'^2\cos^2\phi+2e'^4\cos^4\phi)\cdot 10^{12}$$

$$(\text{XV})=\frac{\tan\phi'}{\nu\sin 1''}\cdot\frac{1}{k_0}\cdot 10^{6}$$

$$(\text{XVI})=\frac{\tan\phi'}{3\nu^3\sin 1''}(1+\tan^2\phi-e'^2\cos^2\phi -2e'^4\cos^4\phi)\cdot\frac{1}{k_0^3}\cdot 10^{18}$$

$$(\text{XVIII})=\frac{1+e'^2\cos^2\phi}{2\nu^2}\cdot\frac{1}{k_0^2}\cdot 10^{12}$$

$$(\text{XIX})=\frac{1+6e'^2\cos^2\phi+9e'^4\cos^4\phi+4e'^6\cos^6\phi}{24\nu^4}\cdot\frac{1}{k_0^4}\cdot 10^{24}$$

Terms Shown as Graphs

$$A_6=p^6\frac{\sin^6 1''\nu\sin\phi\cos^5\phi}{720}(61-58\tan^2\phi+\tan^4\phi +270e'^2\cos^2\phi-330e'^2\sin^2\phi)\times k_0\times 10^{24}$$

$$B_5=p^5\frac{\sin^5 1''\nu\cos^5\phi}{120}(5-18\tan^2\phi+\tan^4\phi +14e'^2\cos^2\phi-58e'^2\sin^2\phi)\cdot k_0\cdot 10^{20}$$

$$C_5=p^5\frac{\sin^4 1''\sin\phi\cos^4\phi}{15}(2-\tan^2\phi)\cdot 10^{20}$$

$$D_6=q^6\frac{\tan\phi'}{720\nu^6\sin 1''}(61+90\tan^2\phi+45\tan^4\phi +107e'^2\cos^2\phi-162e'^2\sin^2\phi -45e'^2\tan^2\phi\sin^2\phi)\cdot\frac{1}{k_0^6}\cdot 10^{36}$$

$$E_5=q^5\frac{\sec\phi'}{120\nu^5\sin 1''}(5+28\tan^2\phi+24\tan^4\phi +6e'^2\cos^2\phi+8e'^2\sin^2\phi)\cdot\frac{1}{k_0^5}\cdot 10^{30}$$

$$F_5=q^5\frac{\tan\phi'}{15\nu^5\sin 1''}(2+5\tan^2\phi+3\tan^4\phi)\cdot\frac{1}{k_0^5}\cdot 10^{30}$$

*d.* The Roman numerals in *c* above are the coefficients of $p$ and $q$ in the formulas given later in this chapter. The powers of 10 have been introduced in order to keep the decimal point within range of 10-bank calculating machines. The missing Roman numerals correspond to functions absorbed in the graphs.

## 146. Specifications

The Universal Transverse Mercator grid has the following specifications:

*a. Projection.* Transverse Mercator (Gauss-Kruger type), in zone 6° wide.

*b. Spheroid.* Clarke 1866 in North America; for other areas, see chart, TM 5-236.

*c. Longitude of Origin.* Central meridian of each zone.

*d. Latitude of Origin.* 0° (the equator).

*e. Unit.* Meter.

*f. False Northing.* 0 meters (10,000,000 for southern hemisphere).

*g. False Easting.* 500,000 meters.

*h. Scale Factor at the Central Meridian.* 0.9996.

*i. Zone Numbering.* Starting with 1 on the zone from 180° west to 174° west, and increasing eastward to 60 on the zone from 174° east to 180° east.

*j. Latitude Limits of System.* North, 84° N; south, 80° S.

*k. Limits of Zones and Overlap.* The zones are bounded by meridians whose longitudes are multiples of 6° west or east of Greenwich. On large-scale maps and in trig lists, an overlap of approxi-

mately 25 miles on either side of the junction is provided for engineer surveyors and for artillery survey and firing. This overlap is never used, however, in giving a grid reference.

## 147. Polar Areas

In the north and south polar areas, the Polar Stereographic grids will be used. These will extend from the poles to 83°30′ north or 79°30′ south, thus providing an overlap on the Universal Transverse Mercator.

## 148. Conformality

The Transverse Mercator is a conformal projection; that is, angles measured on the projection or computed from the grid coordinates closely approximate their true values; at any point, length corrections are the same in all directions. Conformality is an important advantage for all users of the numerical values of the grid, whether gunners, surveyors, or mathematicians.

## 149. Northern Hemisphere Computations

*a. Grid From Geographic Position.*

$N=(I)+(II)p^2+(III)p^4+A_6$ (north of equator)
$E=FE\pm E'$
$E'=(IV)p+(V)p^3+B_5$ (E′ is plus if point is east of the central meridian) (E′ is minus if point is west of the central meridian)
$p=0.0001(\Delta\lambda)$ $\Delta\lambda=CM-\lambda$
($p$ is always positive)

*b. Geographic Position From Grid.*

$\phi=\phi'-(VII)q^2+(VIII)q^4-D_6$
$\Delta\lambda=(IX)q-(X)q^3+E_5$
$\lambda=\lambda_0\pm\Delta\lambda$
$q=0.000001(E')$ $E'=500000-E$
($q$ is always positive)
C=convergence
$C=(XII)p+(XIII)p^3+C_5$
$C=(XV)q-(XVI)q^3+F_5$
Grid azimuth=geodetic azimuth$-C\pm180°$ (approximate)
(point east of CM: C is plus)
(point west of CM: C is minus)
$k$=scale factor
$k=k_0[1+(XVIII)q^2+(0.00003)q^4]$

*c. Tables.* A more complete explanation of the factors and accuracies is given in the front of each volume of tables. The tables for some spheroids come in two volumes. One volume is for converting geographic coordinates to UTM coordinates, and the second volume is for the reverse computation. The tables are set up with the latitude as the argument.

## 150. Southern Hemisphere Computations

*a.* In the southern hemisphere, the conversion of geographic coordinates to UTM coordinates requires that the Northing (N) obtained from the formula, $N=(I)+(II)p^2+(III)p^4+A_6$, be subtracted from a false Northing of 10,000,000. The Easting (E) is computed exactly as in the northern hemisphere. Grid convergence (C) is computed by the following formulas:

$$C=(XII)p+(XIII)p^3+C_5$$

or

$$C=(XV)q+(XVI)q^3+F_5$$

In the southern hemisphere, when the point is east of the central meridian, grid north is west of true north and C is negative; when the point is west of the central meridian, grid north is east of true north and C is positive.

*b.* The scale factor $k$ and the $t-T$ correction do not change from one hemisphere to another.

*c.* When converting from UTM coordinates to geographic coordinates, $\phi'$ in the formula, $\phi=\phi'-(VII)q^2+(VIII)q^4-D_6$, is obtained using 10,000,000−N as the argument for (I). The rest of the conversion is the same as that for the northern hemisphere.

## 151. Eastern Hemisphere Computations

No change in computations from those of the northern or southern hemisphere is involved. Remember that in the eastern hemisphere, longitude increases from west to east.

## Section II. CONVERSION OF GEOGRAPHIC COORDINATES TO UTM COORDINATES

### 152. Use of Special Tables

The conversion of geographic coordinates to UTM coordinates requires the solution of the basic formulas by the use of special tables. These formulas and tables are discussed in paragraphs 144 through 151.

### 153. Converting Coordinates

*a.* For the conversion to be illustrated, the basic formulas are—

(1) $N=(I)+(II)p^2+(III)p^4+A_6$

(2) $E=FE\pm E'$

Where $E'=(IV)p+(V)p^3+B_5$

$p=0.0001\ \Delta\lambda''$; $\Delta\lambda''=\Delta\lambda$ in seconds; $\Delta\lambda=\lambda_0-\lambda$

$\lambda_0$=central meridian; $\lambda$=longitude of point

$p$ is always considered positive

*b.* The conversion for station Hypothetical is shown in figure 223.

(1) The known date, i.e., station name, location, unit, latitude and longitude of the point, are tabulated. From the index map at the back of the special tables, the zone, spheroid of reference, and centra

*Universal Transverse Mercator Grid Coordinates*
*(Method 1)*

*Station: Hypothetical* *Location: Oklahoma* *Spheroid: Clarke 1866* *Zone: 14* *Unit: Meter*

*Latitude, ø* 34° 15′ 34.″742

*Tabular (IV)* (even min. of ø) 255 773.623

*Interpolation* for seconds of ø − 29.203

*Δ²(IV) from graph* + 0.003

*(IV)* = 255 744.423

*Tabular (V)* (even min. of ø) 37.041

*Interpolation* for seconds of ø − 0.036

*(V)* = 37.005

*(IV)p* = 272 031.302

*(V)p³* = 44.535

*from graph, B₅* = − 0.025

*E′* = 272 075.812

*FE* = 500 000.000

*E* = 772075.81

*p* = 1.0636842

*p²* = 1.1314241

*p³* = 1.203478

*p⁴* = 1.2801

*Longitude, λ* = 96° 02′ 43.″158

*Central Meridian, λ₀* = 99 00 00.000

*Δλ* = 02 57 16.842

*Tabular (I)* (even min of ø) 3 789 680.799

*Interpolation for seconds of ø* + 1 070.049

*(I)* = 3 790 750.848

*Tabular (II)* (even min. of ø) 3489.462

*Interpolation* for seconds of ø + 0.465

*(II)* = 3489.927

*(III)* = 2.137

*(II)p²* = 3 948.588

*(III)p⁴* = 2.736

*from graph, A₆* = 0.001

*N* = 3794 702.17

*Figure 223. Conversion of geographic coordinates to UTM grid coordinates.*

meridian of the zone are tabulated. The central meridian of the zone is halfway between the bounding meridians of the zone. $\Delta\lambda$ is found by subtracting the longitude of the point from the central meridian. Convert $\Delta\lambda$ to seconds and move the decimal 4 places to the left ($0.0001\ \Delta\lambda''$) to obtain $p$. List the first, second, third, and fourth powers of $p$. Using only the degrees and minutes of the latitude ($\phi$) of the point as the argument, take from the tables the tabulated values of functions (I), (II), (III), (IV), and (V) and list these values. Interpolate for these functions by multiplying the tabulated "Diff. 1′′′′" for each function by the seconds of $\phi$. List the $\Delta^2$ (IV) correction from the graph. Notice that the $\Delta^2$ (IV) graph rises from 0′′ to 30′′ back to 60′′=0′′. The argument for this graph is seconds of $\phi$. Apply the results of the interpolation to obtain the final value of functions (I), (II), (III), (IV), and (V). Observe the algebraic signs on the "Diff. 1′′′′", as some of the functions may decrease in numerical value as the latitude increases. Multiply (II) times $p^2$, and (III) times $p^4$. Add these results plus the graphic function $A_6$ to the final interpolated value of (I) to obtain the Northing (N) of the point. Multiply (IV) times $p$, and (V) times $p^3$. Add these results algebraically plus the graphic function $B_5$ to obtain E′. Add or subtract E′ from the false Easting (FE =500,000) following the rules to obtain the Easting (E) of the point. RULE: When point is east of the central meridian, *add* E′ to FE; when point is west of the central meridian, *subtract* E′ from FE.

(2) The $A_6$ graph has $\Delta\lambda$ as the argument. The $B_5$ nomogram has $\Delta\lambda$ as the argument at the left, and $\phi$ as the argument at the top. Apply the correct algebraic sign to all functions and interpolation differences.

*c.* DA Form 1932 (UTM Grid Coordinates from Geographic Coordinates) (fig. 224 ①) offers a much more compact arrangement for this conversion. On this form all interpolations for functions (I), (II), (III), (IV), and (V) are made directly on the calculating machine and only the final interpolated values are entered on the form. The $\Delta^2$(IV) correction is applied directly to the interpolated value of function (IV). The graphic functions $A_6$ and $B_5$ are entered on the form. The basic formulas are solved on the machine by accumulative or negative multiplication, and only the final Northing (N) and Easting (E) of the point are entered.

*d.* For the experienced computer desiring speed and efficiency, the procedure shown in figure 224 ② should be employed. This short computation follows a simple operation of factoring the basic formulas to the form:

$$N=(I)+p^2[(II)+(III)p^2]+A_6$$
$$E'=p[(IV)+(V)p^2]+B_5$$

Only the first and second powers of $p$ are required for this method. A little experimenting will readily develop the proper machine setup to solve these equations.

| PROJECT | U. T. M. GRID COORDINATES FROM GEOGRAPHIC COORDINATES (TM 5-237) | | | | |
|---|---|---|---|---|---|
| V-8059 | | | | | |
| LOCATION | ORGANIZATION | | | | |
| Oklahoma | A&S Inc. | | | | |
| SPHEROID | CENTRAL MERIDIAN | ZONE | UNIT | FE | |
| Clarke 1866 | 99°00' | 14 | Meter | 500,000 m. | |

| Station | Hypothetical (Method 2) | | | Station | Hypothetical (Method 3) | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | 34° 15' 34."742 | $\lambda$ | 96° 02' 43."158 | $\phi$ | 34° 15' 34."742 | $\lambda$ | 96° 02' 43."158 |
| (I) | 3 790 750.848 | $\Delta\lambda$ | 02° 57' 16."842 | (I) | | $\Delta\lambda$ | 02° 57' 16."842 |
| (II) | 3 489.927 | $p^2$ | 1.1314241 | (II) | | $p^2$ | 1.1314241 |
| (III) | 2.137 | $p^4$ | 1.2801 | (III) | | $p^4$ | |
| $A_6$ | 0.001 | | | $A_6$ | | | |
| (IV) | 255 744.423 | p | 1.0636842 | (IV) | | p | 1.0636842 |
| (V) | 37.005 | $p^3$ | 1.203478 | (V) | | $p^3$ | |
| $B_5$ | -0.025 | | | $B_5$ | | | |
| | | | | | | | |
| N | 3 794 702.17 | E | 772 075.81 | N | 3 794 702.17 | E | 772 075.81 |

| Station | ① | | | Station | ② | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | | $\lambda$ | | $\phi$ | | $\lambda$ | |
| (I) | | $\Delta\lambda$ | | (I) | | $\Delta\lambda$ | |
| (II) | | $p^2$ | | (II) | | $p^2$ | |
| (III) | | $p^4$ | | (III) | | $p^4$ | |
| $A_6$ | | | | $A_6$ | | | |
| (IV) | | p | | (IV) | | p | |
| (V) | | $p^3$ | | (V) | | $p^3$ | |
| $B_5$ | | | | $B_5$ | | | |
| | | | | | | | |
| N | | E | | N | | E | |

| Station | | | | Station | | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | | $\lambda$ | | $\phi$ | | $\lambda$ | |
| (I) | | $\Delta\lambda$ | | (I) | | $\Delta\lambda$ | |
| (II) | | $p^2$ | | (II) | | $p^2$ | |
| (III) | | $p^4$ | | (III) | | $p^4$ | |
| $A_6$ | | | | $A_6$ | | | |
| (IV) | | p | | (IV) | | p | |
| (V) | | $p^3$ | | (V) | | $p^3$ | |
| $B_5$ | | | | $B_5$ | | | |
| | | | | | | | |
| N | | E | | N | | E | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| W. C. A. | 8 Sept 53 | RAS | 8 Sept. 53 |

DA FORM 1 FEB 57 1932

① Using original equations
② Using factored equations

*Figure 224. Conversion of geographic coordinates to UTM grid coordinates.*

## Section III. CONVERSION OF UTM COORDINATES TO GEOGRAPHIC COORDINATES

### 154. Use of Special Tables

The conversion of UTM coordinates to geographic coordinates requires the solution of the basic formulas by the use of special tables. These formulas and tables are discussed in paragraphs 144 through 151.

### 155. Converting Coordinates

*a.* For the conversion to be illustrated, the basic formulas are—

(1) $\phi=\phi'-(\text{VII})q^2+(\text{VIII})q^4-D_6$
(2) $\lambda=\lambda_0+\Delta\lambda$

Where $\Delta\lambda=(\text{IX})q-(\text{X})q^3+E_5$
$q=0.000001E'$; $E'=E-500{,}000$
$q$ is always considered positive

*b.* For this conversion, the coordinate zone or the approximate geographic position must be known. The conversion for station Hypothetical is shown in figure 225. The known data, i.e., station name, location, UTM zone, spheroid of reference, unit of measure, and the Northing (N) and Easting (E) of the point are tabulated. The false Easting (FE=500,000) is subtracted from the Easting of the point to obtain E′. Move the decimal 6 places to the left in E′ to find $q$. List the first, second, third, and fourth power of $q$. Interpolate for $\phi'$ in the special tables, using the Northing of the point as the argument in the column headed "(I)." Using only the degrees and minutes of $\phi'$ as the argument, take out the tabulated values of functions (VII), (VIII), (IX), and (X), and list these values. Interpolate for

*Geographic Coordinates from Universal Transverse Mercator Grid Coordinates*
*(Method I)*

*Station: Hypothetical* *Location: Oklahoma* *Spheroid: Clarke 1866* *Zone: 14* *Unit: Meter*

$E$ = 772,075.81 $q$ = 0.27207581 $N$ = 3 794 702.17
FE = 500,000.00 $q^2$ = 0.07402525
$E'$ = 272,075.81 $q^3$ = 0.02014048
$q^4$ = 0.0054797

Tabular (IX) (even min. of $\phi'$) 39112.501
Interpolation for seconds of $\phi'$ 5.540
$\Delta^2$(IX) from graph −0.001
(IX) 39118.040 (IX)$q$ = 10643.0724
Tabular (X) (even min. of $\phi'$) 309.509
Interpolation for seconds of $\phi'$ 0.177
(X) 309.686 (X)$q^3$ = − 6.2372
$E_5$ = + 0.0068
$\Delta\lambda$ (seconds) = 10636.8420
East when E′ is +; West when −, $\Delta\lambda$ = 02° 57′ 16.″842
Central Meridian, $\lambda_0$ = 99° 00′ 00.000
LONGITUDE, $\lambda$ 96° 02′ 43.″158

Interpolating N in I, $\phi'$ = 34° 17′ 43.″032

Tabular (VII) (even min. of $\phi'$) 1733.958
Interpolation for seconds of $\phi'$ .773
(VII) 1734.731
(VII)$q^2$ (in sec.) 128.″4139 (VII)$q^2$ − 02′ 08.″4139
(VIII) 22.64 (VIII)$q^4$ + 0.1241
$D_6$ − 0.0001
LATITUDE, $\phi$ 34° 15′ 34.″742

*Figure 225. Conversion of UTM grid coordinates to geographic coordinates.*

| PROJECT | GEOGRAPHIC COORDINATES FROM U. T. M. GRID COORDINATES (TM 5-237) | | | | |
|---|---|---|---|---|---|
| V-8059 | | | | | |
| LOCATION | ORGANIZATION | | | | |
| Oklahoma | A & S, Inc. | | | | |
| SPHEROID | CENTRAL MERIDIAN | ZONE | UNIT | FE | |
| Clarke 1866 | 99°00′ | 14 | Meter | 500,000 m. | |

| Station | Hypothetical (Method 2) | | | Station | Hypothetical (Method 3) | | |
|---|---|---|---|---|---|---|---|
| N | 3 794 702.17 | E | 772 075.81 | N | 3 794 702.17 | E | 772 075.81 |
| $\phi'$ | 34° 17′ 43.″032 | | | $\phi'$ | 34° 17′ 43.″032 | | |
| (VII) | 1734.731 | $q^2$ | 0.07402525 | (VII) | | $q^2$ | 0.07402525 |
| (VIII) | 22.64 | $q^4$ | 0.0054797 | (VIII) | | $q^4$ | |
| $D_6$ | − 0.0001 | | | $D_6$ | | | |
| (IX) | 39118.040 | q | 0.27207581 | (IX) | | q | 0.27207581 |
| (X) | 309.686 | $q^3$ | 0.02014048 | (X) | | $q^3$ | |
| $E_5$ | + 0.0068 | | | $E_5$ | | | |
| $\Delta\phi$ | 02′ 08.″290 | $\Delta\lambda$ | 02° 57′ 16.″842 | $\Delta\phi$ | 02′ 08.″290 | $\Delta\lambda$ | 02° 57′ 16.″842 |
| $\phi$ | 34° 15′ 34.″742 | $\lambda$ | 96° 02′ 43.″158 | $\phi$ | 34° 15′ 34.″742 | $\lambda$ | 96° 02′ 43.″158 |

| Station | ① | | | Station | ② | | |
|---|---|---|---|---|---|---|---|
| N | | E | | N | | E | |
| $\phi'$ | | | | $\phi'$ | | | |
| (VII) | | $q^2$ | | (VII) | | $q^2$ | |
| (VIII) | | $q^4$ | | (VIII) | | $q^4$ | |
| $D_6$ | | | | $D_6$ | | | |
| (IX) | | q | | (IX) | | q | |
| (X) | | $q^3$ | | (X) | | $q^3$ | |
| $E_5$ | | | | $E_5$ | | | |
| $\Delta\phi$ | | $\Delta\lambda$ | | $\Delta\phi$ | | $\Delta\lambda$ | |
| $\phi$ | | $\lambda$ | | $\phi$ | | $\lambda$ | |

| Station | | | | Station | | | |
|---|---|---|---|---|---|---|---|
| N | | E | | N | | E | |
| $\phi'$ | | | | $\phi'$ | | | |
| (VII) | | $q^2$ | | (VII) | | $q^2$ | |
| (VIII) | | $q^4$ | | (VIII) | | $q^4$ | |
| $D_6$ | | | | $D_6$ | | | |
| (IX) | | q | | (IX) | | q | |
| (X) | | $q^3$ | | (X) | | $q^3$ | |
| $E_5$ | | | | $E_5$ | | | |
| $\Delta\phi$ | | $\Delta\lambda$ | | $\Delta\phi$ | | $\Delta\lambda$ | |
| $\phi$ | | $\lambda$ | | $\phi$ | | $\lambda$ | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| W. C. A. | 8 Sept. 53 | RAS | 8 Sept 53 |

DA FORM 1 FEB 57 1933

① Using original equations
② Using factored equations

*Figure 226. Conversion of UTM grid coordinates to geographic coordinates.*

these functions by multiplying the tabulated "Diff. 1″ " for each function by the seconds of $\phi'$. List the $\Delta^2$(IX) correction from the graph. Notice that the graph for $\Delta^2$(IX) rises from 0″ to 30″ back to 60″=0″. The argument for this graph is the seconds of $\phi'$. Apply the results of the interpolations to obtain the final values of functions (VII), (VIII), (IX), and (X). Multiply (VII) times $q^2$, the results being seconds of arc. Convert the seconds to degrees, minutes, and seconds and list this figure as a negative quantity. (VIII) times $q^4$ is a positive quantity in seconds of arc. From the graph, obtain $D_6$ and use as a negative quantity. The $D_6$ graph has $q$ as the argument. Add (VII)$q^2$, (VIII)$q^4$, and $D_6$ to $\phi'$, algebraically to obtain the latitude ($\phi$) of the point. Multiply (IX) times $q$ and list the results as a positive quantity in seconds of arc. (X) times $q^3$ is a negative quantity in seconds. $E_5$ is taken from the nomogram, using $q$ as the argument on the left, and $\phi'$ as the argument at the top of the graph. $E_5$ is a positive quantity in seconds. Add (IX)$q$, (X)$q^3$, and $E_5$, algebraically, to obtain $\Delta\lambda$ in seconds. Convert $\Delta\lambda$ to degrees, minutes, and seconds, and apply to the central meridian ($\lambda_0$) according to the rules. RULES: When point is east of the central meridian (E>500,000), *subtract* $\Delta\lambda$ from the central meridian; when point is west of the central meridian (E<500,000), *add* $\Delta\lambda$ to central meridian. (These rules are valid only in the western hemisphere. In the eastern hemisphere, the reverse is true.) The value of the central meridian is found in the index map at the back of the special tables. The central meridian of a zone is halfway between the bounding meridians of the zone.

*c.* A more compact computation can be made on DA Form 1933, Geographic Coordinates from UTM Grid Coordinates (fig. 226 ①). On this form, all interpolations are made on the calculating machine. The latitude computation is changed slightly by considering the basic equation as $\phi=\phi'-\Delta\phi$ in which $\Delta\phi=(\text{VII})q^2-(\text{VIII})q^4+D_6$. The $\Delta\lambda$ equation does not change form. The advantage of this form is that the solution of $\Delta\phi$ and $\Delta\lambda$ are performed on the calculating machine, and the separate multiplications are not written down on the form. $\Delta\phi$ in degrees, minutes, and seconds is subtracted from $\phi'$ to obtain the latitude ($\phi$) of the point. $\Delta\lambda$ is applied as before.

*d.* For an experienced computer, the fastest and most efficient method of making the conversion from UTM to geographic coordinates is to perform the conversion as shown in figure 226 ②. For this computation, the basic formulas are rewritten by factoring as:

$$\Delta\phi=q^2[(\text{VII})-(\text{VIII})q^2]+D_6$$

$$\Delta\lambda=q[(\text{IX})-(\text{X})q^2]+E_5$$

Notice that only the first and second powers of $q$ are required for this method. With a little experimenting, an experienced computer will readily find the easiest machine setup to solve these equations.

## Section IV. UTM GRID AZIMUTHS

### 156. Description

*a.* The construction of the UTM grid is such that true north and grid north do not coincide at any point in a zone except at the central meridian. This condition is caused by the fact that the meridians converge toward the poles while the north-south grid lines are parallel to the central meridian. Since geographic azimuths are referred to meridians, and grid azimuths are referred to north-south grid lines, it is evident that geographic azimuths and grid azimuths must differ by a certain amount which depends on the position of the point of origin of the azimuth in relation to the central meridian of the UTM zone.

*b.* The term geographic azimuth is used to denote azimuths on the geographic coordinate system of latitude and longitude. In this manual, geographic azimuths are symbolized as $\alpha$. Geographic azimuths are also subdivided into geodetic and astronomic azimuths. Geodetic azimuths are those obtained from triangulation, while astronomic azimuths are those obtained from celestial observations. Both geodetic and astronomic azimuths are symbolized as $\alpha$ with the subscripts G and A being used to differentiate between the two when necessary, such as $\alpha_G$ for geodetic and $\alpha_A$ for astronomic. The difference between UTM grid azimuths and geographic azimuths is known as the UTM grid convergence and is symbolized (C). For precise work on the grid, it is necessary to apply an additional correction which is known as ($t$—T).

*c.* There are two different UTM grid azimuths. One of these is the projected geodetic azimuth (T) which is directly related to the geographic azimuth by the grid convergence (C). The angle obtained from two projected geodetic azimuths is a true representation of an observed angle. The other UTM grid azimuth is the plane azimuth ($t$) which is a mathematical fiction and not directly connected to the geographic azimuth.

## 157. Conversion of Azimuth

*a.* To convert a geographic azimuth ($\alpha$) to a projected geodetic azimuth (T), the convergence (C) is applied to the geographic azimuth. In formula form, this relationship is—

$$T=\alpha\pm C\pm 180°$$

The 180° is applied when necessary to change the azimuth reference to north for the UTM grid. The grid convergence (C) is computed from geographic coordinates using the following formula:

$$C=(XII)p+(XIII)p^3+C_5$$

Functions (XII) and (XIII) and the graphic function $C_5$ are from the special tables mentioned in paragraph 144. $p$ is one ten-thousandth of $\Delta\lambda$ expressed in seconds. $\Delta\lambda$ is the difference in longitude between the point under consideration and the central meridian. In formula form—

$$p=0.0001\Delta\lambda''$$

*b.* The computation of (C) can be made on DA Form 1932 (fig. 227) by using the blank spaces for functions (XII) and (XIII) and entering $C_5$ in the place of $B_5$. $p$ and $p^3$ are entered on the form in the spaces so labelled. The products (XII)$p$ and (XIII)$p^3$ are accumulated on the calculating machine and then $C_5$ is added. The result is the convergence (C) in seconds. Change (C) to degrees, minutes, and seconds, and enter on the form in place of N or E.

*c.* It is possible to shorten this computation by factoring the basic equation which is—

$$C=(XII)p+(XIII)p^3+C_5$$

to read:

$$C=p[(XII)+(XIII)p^2]+C_5$$

Now only the first and second powers of $p$ are needed. By interpolating function (XII) on the calculating machine, accumulating the product of (XIII) times $p^2$ [interpolate (XIII) by inspection], multiplying this accumulation by $p$ and adding $C_5$, the value of C in seconds is found immediately without writing the functions (XII) and (XIII) or $C_5$ on the form at all.

d. The grid convergence (C) may also be computed from UTM coordinates using the following formula:

$$C=(XV)q-(XVI)q^3+F_5$$

Functions (XV) and (XVI) and the graphic function $F_5$ are from the special tables. $q$ equals 0.000001 E′ where E′ is the Easting of the point minus 500,000 (FE). The computation of C can be made on DA Form 1933 (fig. 228) by using the $\Delta\phi$ space for function (XV) and the $\Delta\lambda$ space for function (XVI). Function $F_5$ can be put in the $E_5$ space. $q$ and $q^3$ are entered in the spaces so labelled. The product (XV)$q$ is found on the calculating machine, and the product (XVI)$q^3$ is subtracted from it by negative multiplication. Function $F_5$ is added to give the answer for C in seconds. Change C to degrees, minutes, and seconds, and enter on the form in place of $\phi$ or $\lambda$.

*e.* When computing C from grid coordinates, remember that the argument for functions (XV) and (XVI) is $\phi'$ which is found by interpolating the Northing (N) of the point in the tables. The graphic function, $F_5$, has $q$ as the argument at the left of the graph. The basic equation for C from UTM coordinates can be factored to read—

$$C=q[(XV)-(XVI)q^2]+F_5$$

This is handled in a manner similar to the factored equation for C from geographic coordinates. Figure 227 illustrates the computation of C from geographic coordinates, and figure 228 illustrates the computation of C from grid coordinates.

*f.* The plane azimuth ($t$) is useful because of the ease with which it is computed. The computation of $t$ can be made on the left side of DA Form 1934, Grid Azimuths ($t$ and T) and ($t$—T) Correction from UTM Grid Coordinates (fig. 229), by use of an auxiliary angle ($\beta$) and the following formula:

$$\tan\beta=\frac{\Delta E}{\Delta N}$$

The relation of the auxiliary angle $\beta$ and $t$ is shown on the form. When $\Delta E$ is very large as compared to $\Delta N$, it is better to solve for—

$$\cot\beta=\frac{\Delta N}{\Delta E}$$

| PROJECT | ~~U. T. M. GRID COORDINATES~~ FROM GEOGRAPHIC COORDINATES (TM 5-237) | | | | |
|---|---|---|---|---|---|
| V-8059 Convergence | | | | | |
| LOCATION | ORGANIZATION | | | | |
| Oklahoma | A&S, Inc. | | | | |
| SPHEROID | CENTRAL MERIDIAN | ZONE | UNIT | FE | |
| Clarke 1866 | 99°00' | 14 | Meter | 500,000 m. | |

| Station | Hypothetical | | | Station | Hypothetical (Factored Equation) | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | 34° 15' 34."742 | $\lambda$ | 96° 02' 43."158 | $\phi$ | 34° 15' 34."742 | $\lambda$ | 96° 02' 43."158 |
| (I) | | $\Delta\lambda$ | 02° 57' 16."842 | (I) | | $\Delta\lambda$ | 02° 57' 16."842 |
| (II) | | $p^2$ | | (II) | | $p^2$ | 1.1314241 |
| (III) | | $p^4$ | | (III) | | $p^4$ | |
| $A_6$ | | | | $A_6$ | | | |
| (IV) | | p | 1.0636842 | (IV) | | p | 1.0636842 |
| (V) | | $p^3$ | 1.203478 | (V) | | $p^3$ | |
| ~~$B_5 C_5$~~ | +0.002 | (XIII) | 3.055 | $B_5$ | | | |
| (XII) | 5629.441 | C | +5991."626 | C | +5991."626 = | C | 1° 39' 51."63 |
| N | | ~~E~~ | C 1° 39' 51."63 | N | | E | |
| Station | ① | | | Station | ② | | |

① Using original equations
② Using factored equations

*Figure 227. Computation of grid convergence from geographic coordinates.*

| PROJECT | ~~GEOGRAPHIC COORDINATES~~ FROM U. T. M. GRID COORDINATES (TM 5-237) | | | | |
|---|---|---|---|---|---|
| V-8059 Convergence | | | | | |
| LOCATION | ORGANIZATION | | | | |
| Oklahoma | A&S, Inc. | | | | |
| SPHEROID | CENTRAL MERIDIAN | ZONE | UNIT | FE | |
| Clarke 1866 | 99°00' | 14 | Meter | 500,000 m. | |

| Station | Hypothetical | | | Station | Hypothetical (Factored Equation) | | |
|---|---|---|---|---|---|---|---|
| N | 3 794 702.17 | E | 772075.81 | N | 3 794 702.17 | E | 772075.81 |
| $\phi'$ | 34° 17' 43."032 | | | $\phi'$ | 34° 17' 43."032 | | |
| (VII) | | $q^2$ | | (VII) | | $q^2$ | 0.07402525 |
| (VIII) | | $q^4$ | | (VIII) | | $q^4$ | |
| $D_6$ | | | | $D_6$ | | | |
| (IX) | | q | 0.27207581 | (IX) | | q | 0.27207581 |
| (X) | | $q^3$ | 0.0201405 | (X) | | $q^3$ | |
| ~~$B_5 F_5$~~ | 0.007 | | | ~~$B_5 F_5$~~ | 0.007 | | |
| ~~$\Delta\phi$~~ XV | 22041.373 | ~~$\Delta\lambda$~~ | XVI 263.4 | $\Delta\phi$ | | $\Delta\lambda$ | |
| ~~$\phi$~~ C | +5991.626 | ~~$\lambda$~~ C | +1° 39' 51."63 | ~~$\phi$~~ C | +5991.626 | ~~$\lambda$~~ C | +1° 39' 51."63 |
| Station | ① | | | Station | ② | | |

① Using original equations
② Using factored equations

*Figure 228. Computation of grid convergence from grid coordinates.*

| PROJECT | | GRID AZIMUTHS (t and T) AND (t–T) CORRECTION FROM U. T. M. GRID COORDINATES (TM 5-237) | | | |
|---|---|---|---|---|---|
| 1-243 | | | | | |
| LOCATION | | | | | |
| California | | | | | |
| ORGANIZATION | | | CENTRAL MERIDIAN | | |
| 29th Engrs | | | 123°00' | | |
| SPHEROID | ZONE | UNIT | FE | | |
| Clarke 1866 | 10 | Meter | 500,000 m. | | |

| Grid Azimuth of Sta. (1) Point Reyes Hill | | | | to Sta. (2) BLACK | |
|---|---|---|---|---|---|
| $N_2$ | 4 214 597.169 | $E_2$ | 520662.728 | $2E'_1+E'_2$ | +44067.156 |
| $N_1$ | 4 214 473.597 | $E_1$ | 511 702.214 | (XVIII) | 0.012321 |
| $\Delta N$ | + 123.572 | $\Delta E$ | + 8 960.514 | (t–T) | -0.005 |
| cot. ~~tan~~ $\beta$ | 0.01379073 | | | T | 89° 12' 35".642 |
| $\beta$ | 89° 12' 35".637 | t | 89° 12' 35".637 | | |

| Grid Azimuth of Sta. (1) BLACK | | | | to Sta. (2) Point Reyes Hill | |
|---|---|---|---|---|---|
| $N_2$ | 4 214 473.597 | $E_2$ | 511702.214 | $2E'_1+E'_2$ | + 53027.670 |
| $N_1$ | 4 214 597.169 | $E_1$ | 520662.728 | (XVIII) | 0.012321 |
| $\Delta N$ | - 123.572 | $\Delta E$ | - 8 960.514 | (t–T) | +0.006 |
| cot. ~~tan~~ $\beta$ | 0.01379073 | | | T | 269° 12' 35".631 |
| $\beta$ | 89° 12' 35".637 | t | 269° 12' 35".637 | | |

| Grid Azimuth of Sta. (1) | | | | to Sta. (2) | |
|---|---|---|---|---|---|
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

| Grid Azimuth of Sta. (1) | | | | to Sta. (2) | |
|---|---|---|---|---|---|
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

$\tan\beta=\frac{\Delta E}{\Delta N}$ $\quad \cot\beta=\frac{\Delta N}{\Delta E}$

$t=\beta$ if $\Delta E+$, $\Delta N+$.

$t=180°-\beta$ if $\Delta E+$, $\Delta N-$.

$t=180°+\beta$ if $\Delta E-$, $\Delta N-$.

$t=360°-\beta$ if $\Delta E-$, $\Delta N+$.

$E'_1=E_1-500{,}000$ m.

$E'_2=E_2-500{,}000$ m.

$(t-T)=(-\Delta N)(2E'_1+E'_2)(\text{XVIII})\ 6.8755\times10^{-8}$

$T=t-(t-T)$

E′ is negative west of Central Meridian

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| W. C. A. | 25 Aug 55 | DAS | 25 Aug 55 |

DA FORM 1 FEB 57 1934

*Figure 229. Computation of grid traverse azimuth.*

The conversion of the plane azimuth ($t$) to the projected geodetic azimuth (T) is also made on DA Form 1934 (on the right side of the form). The conversion is made using the following formula:

$$T=t-(t-T).$$

The $(t-T)$ correction is computed from the formula:

$$(t-T)=(-\Delta N)(2E_1'+E_2')(XVIII)6.8755\times10^{-8}$$

in which—

$\Delta N=N_2-N_1$; $E_1'=E_1-500{,}000$; and
$E_2'=E_2-500{,}000$.

Function (XVIII) is from special tables. The algebraic signs of $\Delta N$, $E_1'$, and $E_2'$ are essential.

*g.* These two different azimuths should not be used indiscriminately, but according to definite rules which are:

(1) Use (T) for starting and ending traverses and short lines (less than 1 mile).
(2) Use ($t$) for triangulation.

These rules are stated because the plane azimuth ($t$) should be used only with grid angles, which means that observed directions must be corrected for $(t-T)$. For traverse computation, it is not feasible to make the extra effort to correct all the traverse angles for $(t-T)$; therefore, the observed angles are used with the projected geodetic azimuth.

## Section V. UTM SCALE FACTOR

### 158. Need for Scale Factor

Since the UTM grid is on a secant projection, geodetic and grid lengths are equal only along two approximately north-south lines in each zone. These two lines are 180,000 meters east and west of the central meridian of each zone. Grid lengths in the area between the two lines are shorter than geodetic lengths and, conversely, grid lengths in the areas outside the two lines are longer than geodetic lengths. Because of this situation, it is necessary to use a scale factor to relate geodetic lengths and grid lengths. The central meridian of each zone is given the scale factor of 0.9996, which then increases toward the edges of the zone to unity in the vicinity of the two north-south lines, and continues to increase beyond these lines to approximately 1.0010 at the zone boundaries (at the equator).

### 159. Computation of Scale Factor

*a.* According to the statements in paragraph 158 concerning geodetic and grid lengths, it is evident that geodetic lengths must be multiplied by a UTM scale factor to obtain a UTM grid length. From this, it follows that to convert a grid length to a geodetic length, it is necessary to divide the grid length by a UTM scale factor. Since the Transverse Mercator projection is conformal, the scale at any point is the same in all directions, but the scale varies according to the position of the point in regard to the central meridian. This means that the scale factors at the ends of any line (except a north-south line) are different. The difference is greatest for an east-west line.

*b.* The question now arises as to what value to use for the scale factor of a *line* when the scale factors at the ends of the line are different. There are several answers to this question, depending on the accuracy of the work being done, and the length of the line involved. Since all scale factors for lines require the solution of the scale factor for a point, that computation will be shown next. The formula for the scale factor at a point is—

$$k=k_0[1+(XVIII)q^2+0.00003q^4]$$

in which

$k$=scale factor at the point
$k_0$=scale factor at the central meridian
=0.9996
XVIII=function from special tables
$q$=0.000001(E′) where E′=Easting of point−FE

For surveys of a low order of accuracy (1:1,000), the scale factors can be ignored. For accuracy in the 1:1,000 to 1:5,000 range, scale factors can be taken from graphs or tables using the mid-Easting of the line or traverse. For third-order traverse or third-order triangulation lines, and for all other survey work requiring better than 1:5,000 accuracy, the scale factor should be computed from the formula. Acceptable scale factors for third-order traverses up to 15

miles in length or third-order triangulation lines up to 5 miles in length can be obtained by—

(1) A mean of the factors at the ends of the traverse or line, or

(2) The scale factor for the midpoint of the traverse or line.

*c.* A more accurate factor for use with longer lines is obtained by replacing $q^2$ in the formula by an average $q^2$ called $\overline{q}^2$ which comes from the formula:

$$\overline{q}^2=\frac{(q_1)^2+(q_1q_2)+(q_2)^2}{3}$$

in which $q_1$ and $q_2$ refer to the $q$'s at the ends of the line.

*d.* For very long triangulation lines, the scale factor should be computed from the following expression:

$$1/k=1/6(1/k_1+4/k_3+1/k_2)$$

in which $k_1$ equals the scale factor at the initial point of the line, $k_2$ at the end point, and $k_3$ at the midpoint.

*e.* As an example of the solution of the formula the scale factor at station Hypothetical is computed as follows:

$N=3,794,702.17 \qquad E=772,075.81$

$q=0.000001\ (E-FE)=(772,075.81 - 500,000)\ (0.000001)=0.27207581$

$q^2=0.07402525 \qquad q^4=0.0054797$

$XVIII=0.012331$ (from tables)

$k=k_0\ [1+(XVIII)q^2+0.00003\ q^4]$

$k=0.9996\ [1+0.000912805+0.000000164]$

$k=0.9996\ (1.000912969)=1.0005126$

*f.* As an example of the scale factor for a traverse or short triangulation line, the stations Hicks and Black are used as the fixed points at the ends of a traverse or short line. The first illustration will show the scale factor for the midpoint.

---

| *Station* | $\phi$ | $\lambda$ | $N$ | $E$ |
|---|---|---|---|---|
| Hicks | 38°07′46″.259 | 122°43′29″.366 | 4, 220, 015. 07 | 524, 117. 80 |
| Black | 38°04′50″.777 | 122°45′51″.846 | 4, 214, 597. 17 | 520, 662. 73 |
| Midpoint | ---------- | ---------- | 4, 217, 306. 12 | 522, 390. 26 |

Scale factor for midpoint:

$q=0.02239026 \qquad q^2=0.00050132 \qquad q^4=0.00000025$

$(XVIII)=0.012321 \qquad k=0.99960617$

For the same line a mean of the scale factors at the ends of the line is found.

Scale factor at Hicks:

$q=0.02411780 \qquad q^2=0.00058167 \qquad q^4=0.00000034$

$(XVIII)=0.012321 \qquad k_1=0.99960716$

Scale factor at Black:

$q=0.02066273 \qquad q^2=0.00042695 \qquad q^4=0.00000018$

$(XVIII)=0.012321 \qquad k_2=0.99960526$

Mean of $k_1$ and $k_2=0.99960621$

The same line used to illustrate the average $q^2$ method gives:

$q_1=0.02411780 \qquad q_2=0.02066273$

$\overline{q}^2=0.00050232 \qquad \overline{q}^4=0.00000025$

$(XVIII)=0.012321 \qquad k=0.99960619$

g. A better illustration of the average $q^2$ method is a long line such as the line between Point Reyes Head—2 and Ross Mountain, which is 57,236.34 meters or 35.6 miles long. The methods of average scale factor and midpoint scale factor are not satisfactory for this length of line. By the average $q^2$ method, the solution is:

| *Station* | $\phi$ | $\lambda$ | $N$ | $E$ |
|---|---|---|---|---|
| Pt. Reyes Head—2 | 37°59′47″.247 | 123°00′49″.529 | 4, 205, 216. 39 | 498, 791. 99 |
| Ross Mtn | 38°30′19″.432 | 123°07′07″.981 | 4, 261, 692. 21 | 489, 634. 11 |

$q_1=0.00120801$

$q_2=0.01036589$

$\overline{q}^2=\frac{(q_1)^2+(q_1q_2)+(q_2)^2}{3}=0.00004048$

$\overline{q}^2=0.00004048$

$\overline{q}^4=0.00000000$

$(XVIII)=0.012320 \qquad k=0.99960050$

*h.* For very long lines of 200 to 300 miles, the expression for $1/k$ should be used. For this illustration, a line 345,081,875 meters or 214.4 miles long between stations Macaya and Pico Turquino was selected.

| Station | $\phi$ | $\lambda$ | $N$ | $E$ |
|---|---|---|---|---|
| Macaya | 18°22′54″.230 | 74°01′28″.930 | 2,032,573.85 | 603,028.61 |
| Pico Turquino | 19°59′20″.684 | 76°50′10″.147 | 2,211,193.83 | 307,887.36 |
| Midpoint | ---------- | ---------- | 2,121,883.84 | 455,457.98 |

$q_1=0.10302861$ $q_2=0.19211264$ $q_3=0.04454202$

$(q_1)^2=0.01061489$ $(q_2)^2=0.03690727$ $(q_3)^2=0.00198399$

$(q_1)^4=0.00011268$ $(q_2)^4=0.00136215$ $(q_3)^4=0.00000394$

$k_1=0.9997312255$ $k_2=1.000056216$ $k_3=0.9996245242$

$1/k_1=1.000268846$ $1/k_2=0.9999437871$ $1/k_3=1.000375616$

$1/k=1/6(1/k_1+4/k_3+1/k_2)=1.000285850$

$k=0.999714232$

*i.* The three ways of computing scale factors discussed above cover the probable range of length and accuracy that will be encountered in triangulation or traverse by ordinary methods.

## Section VI. ZONE TO ZONE TRANSFORMATION ON THE UTM GRID

### 160. Methods

Two methods of transforming UTM coordinates from any zone into values in an adjacent zone are described below.

*a.* The first method is applicable when a large number of points in a small area are to be transferred to an adjacent zone. This method requires that the area involved be no greater than 60 miles square and that the center of the area be no more than 1 degree from the meridian where the two zones meet. The precision of this transformation will be such that the values of the transformed coordinates will be in error by no more than 0.5 meter. DA technical manuals for the transformation of UTM coordinates from geographic to grid coordinates and from grid to geographic coordinates are needed for these computations.

*b.* The second method to be given in this section will transform coordinates from any point in one zone to coordinates in either adjacent zone, but the method requires special tables published as TM 5-241-2, Zone to Zone Transformation Tables. A complete discussion of the method is presented in TM 5-241-2; therefore, in this manual, only an illustration of the procedure with a few pertinent remarks will be included.

### 161. Method 1

*a.* Assume the position of a point (call it $P_2$) with coordinates $N_2$ and $E_2$ near the center of the area in which the points to be transformed are located. Assume two other positions 100,000 meters equidistant north and south of $P_2$. Call these point $P_1$ and $P_3$ with coordinates $N_1$, $E_1$ and $N_3$, $E_3$, respectively. $E_1=E_2=E_3$. Always make $P_1$ the most southerly point. Obtain the geographic coordinates for these 3 points (fig. 230). Then obtain the UTM grid coordinates for these points in the zone to which all the points are to be transferred (fig. 231). Designate these converted coordinates as $(\overline{N}_1, \overline{E}_1)$, $(\overline{N}_2, \overline{E}_2)$ and $(\overline{N}_3\ \overline{E}_3)$. The formulas for converting all points in the area are as follows:

$$\overline{E}=\overline{E}_2+g_1\epsilon+h_1\eta-h_2(\epsilon^2-\eta^2)+2g_2\eta\epsilon$$

$$\overline{N}=\overline{N}_2+g_1\eta-h_1\epsilon-g_2(\epsilon^2-\eta^2)-2h_2\eta\epsilon$$

$\overline{N}$ and $\overline{E}$ are the transformed coordinates of the point.

Where: $g_1=\dfrac{\overline{N}_3-\overline{N}_1}{2}$ $\qquad h_1=\dfrac{\overline{E}_3-\overline{E}_1}{2}$

$g_2=\dfrac{\overline{N}_3-2\overline{N}_2+\overline{N}_1}{2}$ $\qquad h_2=\dfrac{\overline{E}_3-2\overline{E}_2+\overline{E}_1}{2}$

$\eta=0.00001(N-N_2)$ $\qquad \epsilon=0.00001(E-E_2)$

N and E are the original coordinates of the point to be transformed. Notice that the factors $g_1$, $g_2$, $h_1$, and $h_2$ remain constant for all the transformations in the group.

*b.* As an example, on the Clarke 1866 Spheroid,

| PROJECT | GEOGRAPHIC COORDINATES FROM U. T. M. GRID COORDINATES (TM 5-237) | | | | |
|---|---|---|---|---|---|
| 5-237 | | | | | |
| LOCATION | ORGANIZATION | | | | |
| Mississippi | A & S | | | | |
| SPHEROID | CENTRAL MERIDIAN | ZONE | UNIT | FE | |
| Clarke 1866 | 93° | 15 | Meter | 500,000 m. | |

| Station | $P_1$ | | | Station | $P_2$ | | |
|---|---|---|---|---|---|---|---|
| N | 3,590,000 | E | 780,000 | N | 3,690,000 | E | 780,000 |
| $\phi'$ | 32 26 55.864212 | | | $\phi'$ | 33 21 03.352782 | | |
| (VII) | | $q^2$ | 0.07840000 | (VII) | | $q^2$ | |
| (VIII) | | $q^4$ | | (VIII) | | $q^4$ | |
| $D_6$ | 0.00015 | | | $D_6$ | 0.00015 | | |
| (IX) | | q | 0.28000000 | (IX) | | q | |
| (X) | | $q^3$ | | (X) | | $q^3$ | |
| $E_5$ | 0.0068 | | | $E_5$ | 0.0071 | | |
| $\Delta\phi$ | 02' 06."711736 | $\Delta\lambda$ | 2° 58' 37."819 | $\Delta\phi$ | 02' 11."13544 | $\Delta\lambda$ | 3° 00' 26."818 |
| $\phi$ | 32 24 49.152 | $\lambda$ | 90 01 22.181 | $\phi$ | 33 18 52.217 | $\lambda$ | 89 59 33.182 |

| Station | $P_3$ | | | Station | | | |
|---|---|---|---|---|---|---|---|
| N | 3,790,000 | E | 780,000 | N | | E | |
| $\phi'$ | 34 15 10.363709 | | | $\phi'$ | | | |
| (VII) | | $q^2$ | | (VII) | | $q^2$ | |
| (VIII) | | $q^4$ | | (VIII) | | $q^4$ | |
| $D_6$ | 0.00015 | | | $D_6$ | | | |
| (IX) | | q | | (IX) | | q | |
| (X) | | $q^3$ | | (X) | | $q^3$ | |
| $E_5$ | 0.0078 | | | $E_5$ | | | |
| $\Delta\phi$ | 02' 15."649317 | $\Delta\lambda$ | 3° 02' 20."777 | $\Delta\phi$ | | $\Delta\lambda$ | |
| $\phi$ | 34 12 54.714 | $\lambda$ | 89 57 39.223 | $\phi$ | | $\lambda$ | |

*Figure 230. Conversion, UTM grid coordinates to geographic coordinates.*

the points A, B, and C will be transformed from Zone 15 to Zone 16. The coordinates of A, B, and C in Zone 15 are designated ($N_A$, $E_A$), ($N_B$, $E_B$), ($N_C$, $E_C$), respectively, with the values as follows:

$N_A$=3,688,232.69 $N_B$=3,696,422.78
$E_A$= 781,063.24 $E_B$= 780,692.16

$N_C$=3,707,421.27
$E_C$= 779,241.63

Assume the position of $P_2$ in Zone 15 as $N_2$= 3,690,000; $E_2$=780,000

$P_1$ is assumed; $N_1$=3,590,000
$E_1$= 780,000
$P_3$ is assumed; $N_3$=3,790,000
$E_3$= 780,000

Solve for the geographic coordinates of these three points, and then solve for the UTM coordinates in Zone 16. List the values.

| $P$ | $N$ | $E$ | $\phi$ | $\lambda$ | $\bar{N}$ | $\bar{E}$ |
|---|---|---|---|---|---|---|
| 1 | 3, 590, 000 | 780, 000 | 32°24′49.″152 | 90°01′22.″180 | 3, 590, 120. 658 | 215, 704. 383 |
| 2 | 3, 690, 000 | 780, 000 | 33°18′52.″217 | 89°59′33.″182 | 3, 689, 960. 046 | 221, 387. 623 |
| 3 | 3, 790, 000 | 780, 000 | 34°12′54.″714 | 89°57′39.″223 | 3, 789, 787. 571 | 227, 208. 020 |

The constant factors can now be computed.

$$g_1 = \frac{3{,}789{,}787.571 - 3{,}590{,}120.658}{2} = +99{,}833.456$$

$$g_2 = \frac{3{,}789{,}787.571 - 7{,}379{,}920.092 + 3{,}590{,}120.658}{2} = -5.932$$

$$h_1 = \frac{227{,}208.020 - 215{,}704.383}{2} = 5{,}751.818$$

$$h_2 = \frac{227{,}208.020 - 442{,}775.246 + 215{,}704.383}{2} = 68.578$$

**U. T. M. GRID COORDINATES FROM GEOGRAPHIC COORDINATES**
(TM 5-237)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| PROJECT: 5-237 | | | | ORGANIZATION: A&S, Inc. | | | |
| LOCATION: Mississippi | | | | | | | |
| SPHEROID: Clarke 1866 | | CENTRAL MERIDIAN: 87° | | ZONE: 16 | UNIT: Meter | | FE: 500,000 m. |

| Station | P1 | | | Station | P2 | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | 32 24 49.152 | $\lambda$ | 90 01 22.181 | $\phi$ | 33 18 52.217 | $\lambda$ | 89 59 33.182 |
| (I) | | $\Delta\lambda$ | 3 01 22.181 | (I) | | $\Delta\lambda$ | 2 59 33.182 |
| (II) | | $p^2$ | 1.1842186 | (II) | | $p^2$ | 1.1606145 |
| (III) | | $p^4$ | | (III) | | $p^4$ | |
| $A_6$ | +0.002 | | | $A_6$ | +0.0015 | | |
| (IV) | | p | 1.0882181 | (IV) | | p | 1.0773182 |
| (V) | | $p^3$ | | (V) | | $p^3$ | |
| $B_5$ | 0.019 | | | $B_5$ | -0.022 | | |
| | | | | | | | |
| N | 3,590,120.658 | E | 215,704.383 | N | 3,689,960.046 | E | 221,387.623 |

| Station | P3 | | | Station | | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | 34 12 54.714 | $\lambda$ | 89 57 39.223 | $\phi$ | | $\lambda$ | |
| (I) | | $\Delta\lambda$ | 2 57 39.223 | (I) | | $\Delta\lambda$ | |
| (II) | | $p^2$ | 1.1361903 | (II) | | $p^2$ | |
| (III) | | $p^4$ | | (III) | | $p^4$ | |
| $A_6$ | +0.001 | | | $A_6$ | | | |
| (IV) | | p | 1.0659223 | (IV) | | p | |
| (V) | | $p^3$ | | (V) | | $p^3$ | |
| $B_5$ | -0.023 | | | $B_5$ | | | |
| | | | | | | | |
| N | 3,789,787.571 | E | 227,208.020 | N | | E | |

*Figure 231. Conversion, geographic coordinates to UTM grid coordinates.*

Use the formulas for $\overline{N}$ and $\overline{E}$ for each point, A, B, and C. $\eta$ and $\epsilon$ must be solved separately for each point. The Zone 15 coordinates for point A are:

$N_A = 3{,}688{,}232.69$

$\eta = 0.00001(N_A - N_2) = -0.0176731$

$E_A = 781{,}063.24$

$\epsilon = 0.00001\ (E_A - E_2) = +0.0106324$

The factors $2\eta\epsilon$ and $(\epsilon^2 - \eta^2)$ can most easily be found at this point for later use.

$2\eta\epsilon = -0.00037581 \qquad (\epsilon^2 - \eta^2) = -0.0001993$

Solving the formulas, the coordinates for point A in Zone 16 are found to be:

$\overline{N}_A = 3{,}688{,}134.55 \qquad \overline{E}_A = 222{,}347.46$

For point B with Zone 15 coordinates:

$N_B = 3{,}696{,}422.78$
$E_B = 780{,}692.16$
$\eta = 0.00001(N_B - N_2) = +0.0642278$
$\epsilon = 0.00001(E_B - E_2) = +0.0069216$
$2\eta\epsilon = +0.0008891 \qquad (\epsilon^2 - \eta_2) = -0.0040773$

Solve the formulas for the Zone 16 coordinates of point B:

$\overline{N}_B = 3{,}696{,}332.23 \qquad \overline{E}_B = 222{,}448.33$

For point C with Zone 15 coordinates:

$\overline{N}_C = 3{,}707{,}421.27$
$\overline{E}_C = 779{,}241.63$
$\eta = 0.00001(N_C - N_2) = +0.1742127$
$\epsilon = 0.00001(E_C - E_2) = -0.0075837$
$2\eta\epsilon = -0.0026424 \qquad (\epsilon^2 - \eta^2) = -0.0302926$

Solve the formulas for the Zone 16 coordinates of point C:

$\overline{N}_C = 3{,}707{,}395.92 \qquad \overline{E}_C = 221{,}634.65$

This process is repeated for as many points as desired in the area.

## 162. Method 2

*a.* The points A, B, and C used to illustrate Method 1 will be used again to illustrate Method 2. Following the notations in TM 5-241-2, the given UTM coordinates for points A, B, and C in Zone 15 are designated by the lower-case letters $n$ and $e$. The transformed coordinates for these points in Zone 16 are designated by the capital letters N and E. For use as the argument in entering the tables $n$ and $e$ are rounded off to the nearest 100,000 meters and are then designated $n_t$ and $e_t$.

*Note.* Tables do not show subscript $t$ on $n$ and $e$.

The coefficients $n'$ and $e'$ are found from the expressions,

$$n' = (n - n_t)10^{-5} \text{ and } e' = (e - e_t)10^{-5}.$$

The formulas which are used are as follows:

$$A = b_1 + n'c_1 - e'c_2$$
$$B = b_2 + n'c_2 + e'c_1$$
$$C = a_1 + n'A - e'B$$
$$D = a_2 + n'B + e'A$$
$$N = N_0 + n'C - e'D$$
$$E = 500{,}000 + E_0 + n'D + e'C$$

In these formulas, $N_0$, $E_0$, $a_1$, $a_2$, $b_1$, $b_2$, $c_1$, and $c_2$ are obtained from the tables. The transformed coordinates will be correct to 0.01 meter.

*b.* The following transformation will illustrate Method 2:

| *Point* | *Zone 15 coord.* | $n_t, e_t$ | $n', e'$ |
|---|---|---|---|
| A---- | $n = 3{,}688{,}232.69$ | $n_t = 3{,}700{,}000$ | $n' = -0.1176731$ |
| | $e = 781{,}063.24$ | $e_t = 800{,}000$ | $e' = -0.1893676$ |

Since the points are to be transformed to a zone to the east, $e_t$ is used as the argument on the right hand side of the tables for the Clarke 1866 Spheroid, and also the upper sign of the function is used. The value of $n_t$ determines the table to use. For point A, the following values are obtained:

| $N_0$ | $E_0$ | $a_1$ | $a_2$ | $b_1$ | $b_2$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|---|---|
| +3, 698, 790. 428 | −258, 072. 765 | +99, 804, 893 | +5, 763. 425 | −05. 788 | +68. 483 | −0. 018 | −0. 245 |

Solving the formulas:

$A = -5.832 \qquad B = +68.515 \qquad C = +99{,}818.554 \qquad D = +5{,}756.467$
$N = 3{,}688{,}134.56 \qquad E = 222{,}347.45$

| *Point* | *Zone 15 coord.* | $n_t, e_t$ | $n', e'$ |
|---|---|---|---|
| B---- | $n = 3, 696, 422. 78$ | $n_t = 3, 700, 000$ | $n' = -0. 0357722$ |
| | $e = 780, 692. 16$ | $e_t = 800, 000$ | $e' = -0. 1930784$ |

| $N_0$ | $E_0$ | $a_1$ | $a_2$ | $b_1$ | $b_2$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|---|---|
| +3, 698, 790. 428 | −258, 072. 765 | +99, 804. 893 | +5, 763. 425 | −05. 788 | +68. 483 | −0. 018 | −0. 245 |

Solving the formulas:

A=−5.835 B=+68.495 C=+99,818.327 D=+5,762.101
N=3,696,332.24 E=222,448.35

| Point | Zone 15 coord. | $n_t$, $e_t$ | $n'$, $e'$ |
|---|---|---|---|
| C | $n$=3, 707, 421. 27<br>$e$= 779, 241. 63 | $n_t$=3, 700, 000<br>$e_t$= 800, 000 | $n'$ = +0. 0742127<br>$e'$ = −0. 2075837 |

| $N_0$ | $E_0$ | $a_1$ | $a_2$ | $b_2$ | $b_2$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|---|---|
| +3, 698, 790. 428 | −258, 072. 765 | +99, 804. 893 | +5, 763. 425 | −05. 788 | +68. 483 | −0. 018 | −0. 245 |

Solving the formulas:

A=−5.840 B=+68.469 C=+99,818.673 D=+5,769.719
N=3,707,395.94 E=221,634.69

*c.* Comparing these values for the transformed coordinates with the values found by Method 1, it will be found that the two methods have given results within 0.05 meter for Northings and Eastings. Notice that Method 2 is much simpler, more rapid, and has no restrictions on the area over which the points are distributed or the relation of the points to the bounding meridians of the zone. Method 2 is preferable for any transformations, but it does require special tables. Method 1 is restricted in application as previously stated, but it has the advantage of requiring only tables which are much more likely to be available to the computer.

*Note.* Tables are being prepared for the solution of this problem in high latitudes. The tables will transform points directly into a zone twice removed from a given UTM zone.

## 163. Zone to Zone Transformation of Projected Geodetic Azimuth (T)

*a.* There is no method for the transformation of the projected geodetic azimuth (T) from zone to zone which is comparable to Method 1 (para. 161) shown for the transformation of UTM coordinates. The only direct way to transform UTM azimuths is that shown in TM5–241–2. This method uses the special tables of TM5–241–2 as did Method 2 (para. 162) for the transformation of UTM coordinates.

*b.* The formulas used in TM5–241–2 for the transformation of azimuths are shown below.

(1) $\tau = T_2 - T_1$, where $T_1$=projected geodetic azimuth in the given zone; and $T_2$=transformed projected geodetic azimuth in the adjacent zone.

(2) $\tan \tau = \frac{A_2}{A_1}$ (in northern hemisphere)

$$\tan \tau = -\frac{A_2}{A_1} \text{ (in southern hemisphere)}$$

$A_1$ and $A_2$ are from the formulas:

$$F = 2b_1 + 3n'c_1 - 3e'c_2$$
$$G = 2b_2 + 3n'c_2 + 3e'c_1$$
$$A_1 = a_1 + n'F - e'G$$
$$A_2 = a_2 + n'G + e'F$$

*c.* The quantities $a_1$, $a_2$, $b_1$, $b_2$, $c_1$, and $c_2$ are tabulated in TM 5–241–2 and are found as in the transformation of UTM coordinates. The variables $n'$ and $e'$ are from the following formulas:

$$n' = (n - n_t)\ 10^{-5} \qquad e' = (e - e_t)\ 10^{-5}$$

*d.* The following sample problems are taken from TM 5–241–2:

(1) Take point P as the point with UTM Zone 32, International Spheroid, coordinates. $n$=4,181,865.42 and $e$=242,018.76. Given the projected geodetic azimuth, $T_1$, from P to some point Q as 117° 42′ 17″.13 referred to grid north of zone 32; find the projected geodetic azimuth, $T_2$, from P to Q referred to grid north of Zone 33.

$n_t$=4,200,000, $e_t$=200,000
$n' = (n - n_t)10^{-5} = -0.1813458$;
$e' = (e - e_t)10^{-5} = +0.4201876$

Since the azimuth is to be referred to a zone to the east, the International Spheroid section of the tables is entered using $e_t$ on the right side of the page, and the upper signs of the functions are employed. The values thus obtained are as follows:

| $a_1$ | $a_2$ | $b_1$ | $b_2$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|
| +100, 523. 020 | +6, 530. 277 | −10. 988 | −64. 981 | −0. 036 | −0. 288 |

Using the formulas, the following results are obtained:

$$F=2b_1+3n'c_1-3e'c_2=-21.593$$

$$G=2b_2+3n'c_2+3e'c_1=+130.073$$

$$A_1=a_1+n'F-e'G=+100,472.281$$

$$A_2=a_2+n'G+e'F=+6,497.616$$

$$\tan\tau=\frac{A_2}{A_1}=0.06467073$$

$$\tau=T_2-T_1=03°42'00''.75$$

$$T_2=117°42'17''.13+03°42'00''.75$$

$$=121°24'17''.88$$

(2) Find the projected geodetic azimuth, $T_2$, of the line of the previous example as referred to grid north of Zone 31.

$n_t=4,200,000$, $e_t=200,000$

$n'=(n-n_t)10^{-5}$

$=(4,181,865.42-4,200,000)10^{-5}$

$=-0.1813458$

$e'=(e-e_t)10^{-5}$

$=(242,018.76-200,000)10^{-5}$

$=+0.4201876$

Since the azimuth is to be referred to a zone to the west, the International Spheroid section of the tables is entered using $e_t$ on the left side of the page, and the lower signs of the functions are employed. The values thus obtained are as follows:

| $a_1$ | $a_2$ | $b_1$ | $b_2$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|
| +99, 746. 016 | −6, 428. 868 | −5. 964 | −64. 611 | −0.005 | +0.271 |

Using the formulas, the following results are obtained:

$$F=2b_1+3n'c_1-3e'c_2=-12.267$$

$$G=2b_2+3n'c_2+3e'c_1=-129.376$$

$$A_1=a_1+n'F-e'G=99,802.603$$

$$A_2=a_2+n'G+e'F=-6,410.561$$

$$\tan\tau=\frac{A_2}{A_1}=-0.06423240$$

$$\tau=T_2-T_1=-3°40'30''.71$$

$$T_2=117°42'17''.13-3°40'30''.71$$

$$=114°01'46''.42$$

*e.* To transform the azimuth without using the method just described, it is necessary to make the following computations:

(1) Obtain the grid convergence (C) in the given zone.

(2) Change the given projected geodetic azimuth to the geodetic azimuth.

(3) Transform the coordinates of the point from the given zone to the adjacent zone.

(4) Obtain the grid convergence (C) for the transformed point.

(5) Change the geodetic azimuth from (2) above, to the required transformed projected geodetic azimuth by applying the C found in (4) above.

*f.* It is evident that the process outlined in *e* above is very long and laborious if many azimuths must be transformed.

# CHAPTER 14

# HORIZONTAL CONTROL USING UTM GRID

## Section I. POSITION COMPUTATION ON THE UTM GRID

### 164. Introduction

The UTM grid position computation, which corresponds to the geographic position computation, is mainly encountered in grid triangulation and follows the computation of a grid triangle. The computation of grid azimuth and distance from grid coordinates is commonly called a grid inverse.

### 165. Position Computation (UTM Grid)

*a.* Figure 232 shows the known quantities in the problem and the basis of the solution.

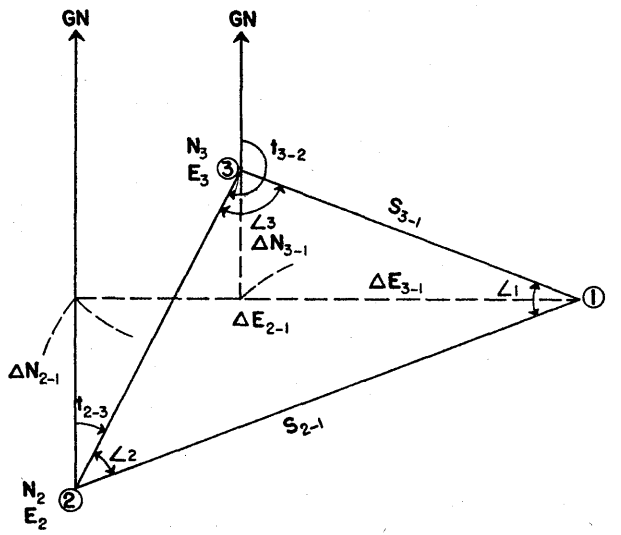



*Figure 232. Diagram of UTM grid position computation.*

*b.* DA Form 1938, Position Computation (UTM Grid) (fig. 233), is arranged and numbered to correspond with the Computation of Triangle (UTM Grid) form (DA Form 1937), in that the station to be set up is number 1 and the known stations are numbered 2 and 3. The known quantities are the grid coordinates of station 2 ($N_2$, $E_2$), and station 3 ($N_3$, $E_3$); the grid azimuths from 2 to 3 ($t_{2-3}$), and 3 to 2 ($t_{3-2}$); the quantities from the triangle computation, i.e. the angles at stations 1, 2, and 3 ($\angle 1$, $\angle 2$, $\angle 3$); and grid length from 2 to 1 ($S_{2-1}$) and 3 to 1 ($S_{3-1}$). These known quantities are all entered on the form in the space indicated. The grid azimuth from 2 to 1 is obtained by *adding* $\angle 2$ to $t_{2-3}$ (subtract 360° from $t_{2-1}$ if necessary). The azimuth from 3 to 1 is found by substracting $\angle 3$ from $t_{3-2}$ (add 360° to $t_{3-2}$ if necessary before substracting $\angle 3$). The sine and cosine of the grid azimuths $t_{2-1}$ and $t_{3-1}$ are taken from natural function tables. The algebraic signs for the trigonometric functions come from the following rules:

(1) Azimuths from 0° to 180°, sines are plus; for all other azimuths, sines are minus.
(2) Azimuths from 90° to 270°, cosines are minus; for all other azimuths, cosines are plus.

*c.* Multiply S times cosine $t$ to find $\Delta N$, and S times sine $t$ to find $\Delta E$ for each side of the form. Algebraically add the $\Delta N$'s and $\Delta E$'s to the Northings and Eastings of the fixed stations (2 and 3) to obtain the coordinates of station 1 on each side of the form. The Northings and Eastings from each side of the form should check within 1 or 2 in the last decimal, if an adjustment has been made of the angles in the triangle. To complete the computation, a check of the azimuths is made by adding $\angle 1$ to the azimuth $t_{2-1}$ which should then equal the azimuth $t_{3-1}$.

### 166. Inverse Position Computation (UTM Grid)

*a.* DA Form 1935, Grid Azimuth ($t$) and Grid Length (fig. 234), provides for six inverse com-

POSITION COMPUTATION (UTM GRID)
(TM 5-237)

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| PROJECT | I-243 | | | | | | | ORGANIZATION | 29th Engrs. | | | | | | | | |
| LOCATION | California | | | | | | | CENTRAL MERIDIAN | 123° 00' | | | | | | | | |
| SPHEROID | Clarke 1866 | | | ZONE | 10 | | | | | | | | | | | | |
| | | | t 2 to 3 | 31 | 50 | 39.4 | | | | | t 3 to 2 | 211 | 50 | 39.4 | | | |
| | | | ∠ 2 | + 49 | 00 | 56.2 | | | | | ∠ 3 | - 108 | 09 | 34.4 | | | |
| Sin t (2 to 1) | +0.98730284 | | t 2 to 1 | 80 | 51 | 35.6 | | Sin t (3 to 1) | +0.97161224 | | t 3 to 1 | 103 | 41 | 05.0 | | | |
| Cos t (2 to 1) | +0.15884929 | | ∠ 1 | 22 | 49 | 29.4 | | Cos t (3 to 1) | -0.23657908 | | | | | | | | |
| $N_2$ | 4220015.07 | 2 | HICKS | $E_2$ | 524117.80 | | | $N_3$ | 4224150.64 | 3 | LINCOLN | $E_3$ | 526086.39 | | | | |
| ΔN | + 1894.26 | S | 11924.892 | ΔE | + 11773.48 | | | ΔN | - 2241.31 | S | 9473.831 | ΔE | + 9204.89 | | | | |
| $N_1$ | 4221909.33 | 1 | Burdell | $E_1$ | 535891.28 | | | $N_1$ | 4221909.33 | 1 | Burdell | $E_1$ | 535891.28 | | | | |
| | | | t 2 to 3 | | | | | | | | t 3 to 2 | | | | | | |
| | | | ∠ 2 | | | | | | | | ∠ 3 | | | | | | |
| Sin t (2 to 1) | | | t 2 to 1 | | | | | Sin t (3 to 1) | | | t 3 to 1 | | | | | | |
| Cos t (2 to 1) | | | ∠ 1 | | | | | Cos t (3 to 1) | | | | | | | | | |
| $N_2$ | | 2 | | $E_2$ | | | | $N_3$ | | 3 | | $E_3$ | | | | | |
| ΔN | | S | | ΔE | | | | ΔN | | S | | ΔE | | | | | |
| $N_1$ | | 1 | | $E_1$ | | | | $N_1$ | | 1 | | $E_1$ | | | | | |
| | | | t 2 to 3 | | | | | | | | t 3 to 2 | | | | | | |
| | | | ∠ 2 | | | | | | | | ∠ 3 | | | | | | |
| Sin t (2 to 1) | | | t 2 to 1 | | | | | Sin t (3 to 1) | | | t 3 to 1 | | | | | | |
| Cos t (2 to 1) | | | ∠ 1 | | | | | Cos t (3 to 1) | | | | | | | | | |
| $N_2$ | | 2 | | $E_2$ | | | | $N_3$ | | 3 | | $E_3$ | | | | | |
| ΔN | | S | | ΔE | | | | ΔN | | S | | ΔE | | | | | |
| $N_1$ | | 1 | | $E_1$ | | | | $N_1$ | | 1 | | $E_1$ | | | | | |

ΔN=S cos t  ΔE=S sin t

| | | | |
|---|---|---|---|
| COMPUTED BY | W. C. A. | DATE | 29 Sept 53 |
| CHECKED BY | RAS | DATE | 29 Sept. 53 |

DA FORM 1 FEB 57 1938

*Figure 233. Computation of UTM grid position (DA Form 1938).*

putations, and also has the rules and formulas for the computations printed at the bottom of the form. The grid azimuth is computed by using an auxiliary angle ($\beta$) and the formula, $\tan \beta = \frac{\Delta E}{\Delta N}$. The relation between angle $\beta$ and the azimuth [plane azimuth ($t$) on UTM] is shown in the rules at the bottom of the form. In cases where ΔE is very large as compared to ΔN, it is more accurate to compute $\beta$ from $\cot \beta = \frac{\Delta N}{\Delta E}$.

*b.* ΔN and ΔE are obtained by subtracting the coordinates of station 1 from the coordinates of station 2. The azimuth then computed will be station 1 to station 2.

*c.* The grid length can be computed as the square root of the sum of the squares of ΔN and ΔE

$$S = \sqrt{\Delta N^2 + \Delta E^2}$$

or from the formulas printed at the bottom of the form which are $S = \frac{\Delta E}{\sin \beta} = \frac{\Delta N}{\cos \beta}$. The square root method is the most accurate, but may not always be the easiest.

*d.* When finding the grid distance from the formulas $S = \frac{\Delta E}{\sin \beta}$ and $S = \frac{\Delta N}{\cos \beta}$, a mean value of the two should be used as the final grid distance. This method is not good when either sin $\beta$ or cos $\beta$ becomes extremely small, as the S value found by using a small divisor becomes less and less accurate as the divisor becomes smaller and smaller. In this situation, the value of S found by using the larger divisor is preferred over a mean.

| PROJECT | GRID AZIMUTH (t) AND GRID LENGTH (TM 5-237) |
|---|---|
| I-243 | |
| LOCATION: California | ORGANIZATION: 29th Engrs. |
| GRID: UTM | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Sta. (1) | HICKS | To Sta. (2) | LINCOLN | Azimuth | | | 31° 50′ 39.4″ |
| $N_2$ | 4 224 150.64 | $E_2$ | 526 686.39 | tan β | 0.62109697 | β | 31° 50′ 39.45″ |
| $N_1$ | 4 220 015.07 | $E_1$ | 524 117.80 | ΔE | | ΔN | |
| ΔN | + 4 135.57 | ΔE | + 2568.59 | sin β | | cos β | |
| Grid length | 4868.326 (~~ft~~) (m) | | | S | | S | |
| Sta. (1) | | To Sta. (2) | | Azimuth | | | ° ′ ″ |
| $N_2$ | | $E_2$ | | tan β | | β | ° ′ ″ |
| $N_1$ | | $E_1$ | | ΔE | | ΔN | |
| ΔN | | ΔE | | sin β | | cos β | |
| Grid length | (ft) (m) | | | S | | S | |
| Sta. (1) | | To Sta. (2) | | Azimuth | | | ° ′ ″ |
| $N_2$ | | $E_2$ | | tan β | | β | ° ′ ″ |
| $N_1$ | | $E_1$ | | ΔE | | ΔN | |
| ΔN | | ΔE | | sin β | | cos β | |
| Grid length | (ft) (m) | | | S | | S | |
| Sta. (1) | | To Sta. (2) | | Azimuth | | | ° ′ ″ |
| $N_2$ | | $E_2$ | | tan β | | β | ° ′ ″ |
| $N_1$ | | $E_1$ | | ΔE | | ΔN | |
| ΔN | | ΔE | | sin β | | cos β | |
| Grid length | (ft) (m) | | | S | | S | |
| Sta. (1) | | To Sta. (2) | | Azimuth | | | ° ′ ″ |
| $N_2$ | | $E_2$ | | tan β | | β | ° ′ ″ |
| $N_1$ | | $E_1$ | | ΔE | | ΔN | |
| ΔN | | ΔE | | sin β | | cos β | |
| Grid length | (ft) (m) | | | S | | S | |
| Sta. (1) | | To Sta. (2) | | Azimuth | | | ° ′ ″ |
| $N_2$ | | $E_2$ | | tan β | | β | ° ′ ″ |
| $N_1$ | | $E_1$ | | ΔE | | ΔN | |
| ΔN | | ΔE | | sin β | | cos β | |
| Grid length | (ft) (m) | | | S | | S | |

AZ from North =;
β if ΔE+, ΔN+
180°−β if ΔE+, ΔN−
180°+β if ΔE−, ΔN−
360°−β if ΔE−, ΔN+

AZ from South =;
β if ΔE−, ΔN−
180°−β if ΔE−, ΔN+
180°+β if ΔE+, ΔN+
360°−β if ΔE+, ΔN−

$$\tan\beta = \frac{\Delta E}{\Delta N}$$

$$S = \frac{\Delta E}{\sin\beta} = \frac{\Delta N}{\cos\beta}$$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| W. C. A. | 19 Aug 55 | RHS | 21 Aug 55 |

DA, FORM 1 FEB 57 1935

*Figure 234. Computation of grid azimuth (t) and grid length (DA Form 1935).*

## Section II. TRIANGULATION ON THE UTM GRID

### 167. Conversions

For triangulation computation on the UTM grid, certain corrections must be applied to the observed data before it can be used. The corrections are made to change measured base lengths to grid lengths, and measured horizontal angles to grid angles. All other fixed data, such as positions, azimuths, and geodetic lengths, must also be converted to UTM values. Geodetic lengths and measured base lengths are converted to grid lengths by use of the scale factor. Geodetic azimuths are converted to grid azimuths by use of the grid convergence and the ($t$—T) correction. These conversions are more completely discussed in paragraphs 156 through 159.

### 168. Computation

*a.* The ($t$—T) correction must be applied to all observed horizontal directions before they can be used in UTM grid triangulation. The reason for this correction is more completely explained in TM 5-241-8. To obtain this correction, it is necessary to compute preliminary UTM coordinates for all the new stations established by the triangulation. These positions can be carried through a chain of simple triangles as the coordinates are only needed within 10 meters for the sake of computing ($t$—T) to 2 decimals.

*b.* For UTM grid triangulation, DA Form 1936 (List of Directions, UTM Grid) is used for the list of directions as it provides a column for the ($t$—T) correction. After reducing the observed directions for ($t$—T), the adjustment can be made by the standard methods used for geodetic triangulation. In grid triangulation, there is no spherical excess to be considered in the computation of the triangles. Triangle computations should be made on DA Form 1937 (Computation of Triangles (UTM Grid)). When the angles have been adjusted and the final lengths computed from the adjusted triangles, final position computations should be made on DA Form 1938. If possible, the fixed grid lengths and grid azimuths should be obtained by a grid inverse computed on DA Form 1935.

*c.* In the example shown for the UTM grid triangulation (fig. 235), the starting data was given as geographic positions for the three fixed stations. The positions were converted to UTM grid coordinates, and fixed grid lengths and grid azimuths obtained by inverse computation (fig. 236). The example is shown starting with the list of directions (fig. 237) as this is normally the point at which office computation would begin.

*d.* The preliminary computation of triangles (fig. 238) is made to obtain distances for computing preliminary positions (fig. 239) for the ($t$—T) reduction (fig. 240). These preliminary triangles can be computed using the observed angles to the nearest second. Only enough triangles need be computed to provide one distance from a previously established point to each new point.

*e.* The preliminary UTM coordinates can be computed from a single angle (azimuth) and distance, or by setting up the new point from two previously established points and using a mean value of the two positions thus obtained. These positions can be computed on DA Form 1938 or on DA Form 1940. Using these preliminary positions, the ($t$—T) reduction is computed as explained in paragraphs 156 and 157. If a line is observed in both directions, a ($t$—T) reduction must be found at each end of the line. When this is necessary, it is more expedient to perform computations for both ends of the line on the same page, since space is provided on DA Form 1934 for four ($t$—T) computations.

*f.* When all the ($t$—T) reductions are found, they are applied to the observed directions on the list of directions (fig. 237). This reduces the observed directions to grid directions, from which grid angles (plane angles) can be obtained. These angles are used in the adjustment (fig. 241). After the adjustment is made for angle, side, length, and azimuth conditions, the final computation of the grid triangles (fig. 242) is completed. From the final adjusted triangles, the necessary data is obtained to make the final position computations (figs. 243 and 244).

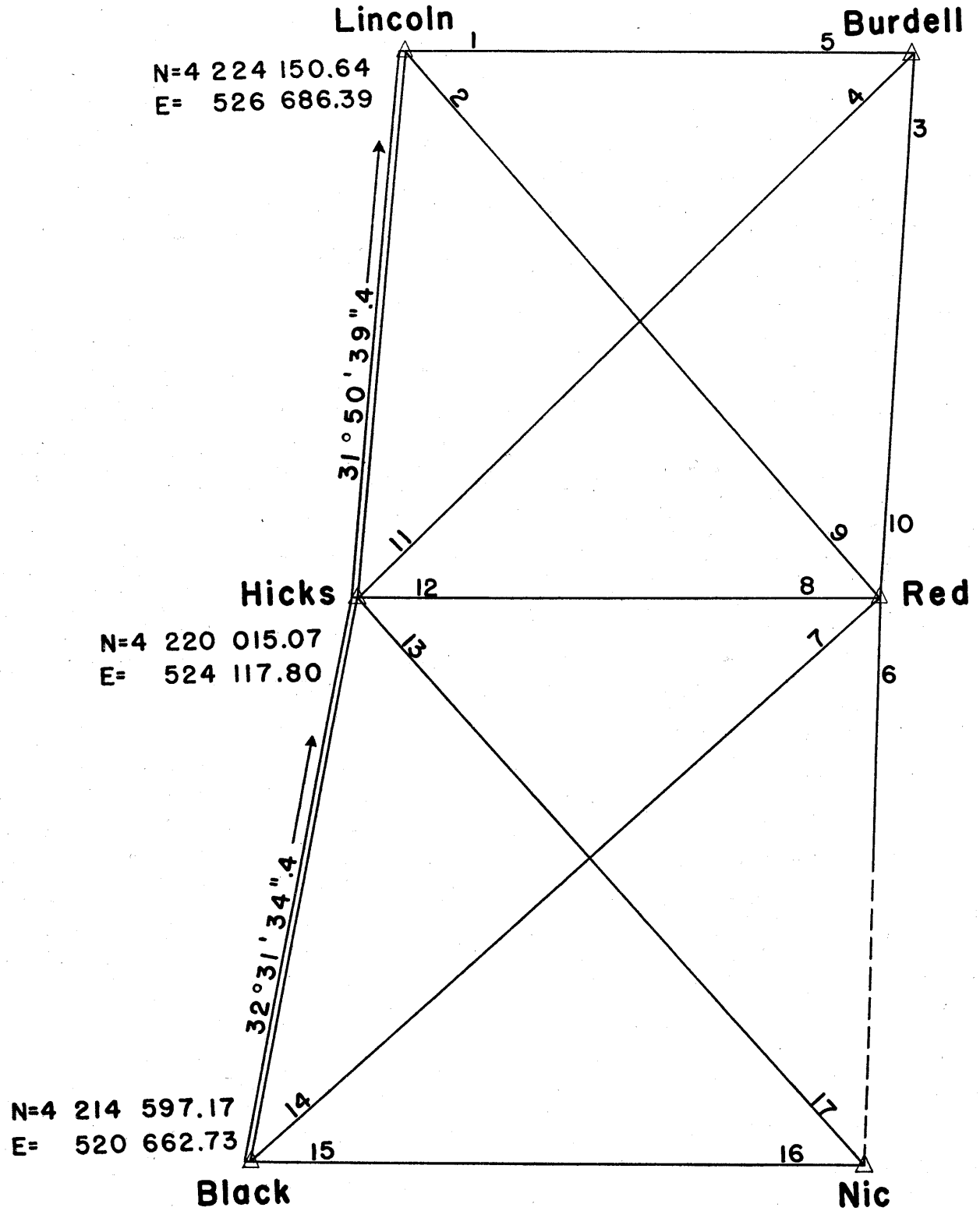



*Figure 235. Sketch, triangulation net.*

| PROJECT | | | | GRID AZIMUTH (t) AND GRID LENGTH (TM 5-237) | | | |
|---|---|---|---|---|---|---|---|
| I-243 | | | | | | | |
| LOCATION | | | | ORGANIZATION | | | |
| California | | | | 29th Engrs. | | | |
| GRID | | | | | | | |
| UTM | | | | | | | |
| Sta. (1) | HICKS | To Sta. (2) | LINCOLN | Azimuth | | | 31° 50′ 39.4″ |
| $N_2$ | 4 224 150.64 | $E_2$ | 526 686.39 | tan β | 0.62109697 | β | 31° 50′ 39.45″ |
| $N_1$ | 4 220 015.07 | $E_1$ | 524 117.80 | ΔE | | ΔN | |
| ΔN | + 4 135.57 | ΔE | + 2 568.59 | sin β | | cos β | |
| Grid length | 4868.326 | ~~(ft)~~ (m) | | S | | S | |
| Sta. (1) | BLACK | To Sta. (2) | HICKS | Azimuth | | | 32° 31′ 34.4″ |
| $N_2$ | 4 220 015.07 | $E_2$ | 524 117.80 | tan β | 0.63771387 | β | 32° 31′ 34.40″ |
| $N_1$ | 4 214 597.17 | $E_1$ | 520 662.73 | ΔE | | ΔN | |
| ΔN | + 5417.90 | ΔE | + 3455.07 | sin β | | cos β | |
| Grid length | 6425.819 | ~~(ft)~~ (m) | | S | | S | |

*Figure 236. Computation of azimuth and length (DA Form 1935).*

PROJECT: I-243

**LIST OF DIRECTIONS, U. T. M. GRID** (TM 5-237)

LOCATION: California | DATE: 22 Aug 55 | STATION: BLACK (USC+GS)
ORGANIZATION: 29th Engrs | ZONE: 10 | OBSERVER: Cpl. P.E. Smith
SPHEROID: Clarke 1866 | CENT. MER.: 123° | INSTRUMENT: K&E #57460

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | t-T REDUCTION | CORRECTED PLANE DIRECTION WITH ZERO INITIAL | ADJUSTED PLANE DIRECTION |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| HICKS (USC+GS) | 0 00 00.00 | — | -0.3 | 0 00 00.00 | 00.0 |
| (14) Red (29th Engrs) | 51 57 43.3 | — | -0.1 | 43.5 | 47.6 |
| (15) Nic (29th Engrs) | 90 39 09.9 | — | +0.2 | 10.4 | 09.5 |

LOCATION: California | DATE: 22 Aug 55 | STATION: Burdell (29th Engrs)
ORGANIZATION: 29th Engrs | ZONE: 10 | OBSERVER: Cpl. P.E. Smith
SPHEROID: Clarke 1866 | CENT. MER.: 123° | INSTRUMENT: K&E #57460

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | t-T REDUCTION | CORRECTED PLANE DIRECTION WITH ZERO INITIAL | ADJUSTED PLANE DIRECTION |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| (5) LINCOLN (USC+GS) | 0 00 00.00 | — | -0.2 | 0 00 00.00 | 02.5 |
| (3) Red (29th Engrs) | 294 48 01.2 | — | +0.6 | 02.0 | 47 59.7 |
| (4) HICKS (USC+GS) | 337 10 32.9 | — | +0.2 | 33.3 | 33.1 |

LOCATION: California | DATE: 22 Aug 55 | STATION: HICKS (USC+GS)
ORGANIZATION: 29th Engrs | ZONE: 10 | OBSERVER: Cpl. P.E. Smith
SPHEROID: Clarke 1866 | CENT. MER.: 123 | INSTRUMENT: K&E #57460

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | t-T REDUCTION | CORRECTED PLANE DIRECTION WITH ZERO INITIAL | ADJUSTED PLANE DIRECTION |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| LINCOLN (USC+GS) | 0 00 00.00 | — | -0.3 | 0 00 00.00 | 00.0 |
| (11) Burdell (29th Engr) | 49 00 55.2 | — | -0.1 | 55.4 | 56.2 |
| (12) Red (29th Engr) | 91 29 06.2 | — | +0.3 | 06.8 | 08.0 |
| (13) Nic (29th Engr) | 136 21 11.3 | — | +0.6 | 12.2 | 14.7 |
| BLACK (USC+GS) | 180 40 53.2 (54.5) | Fixed at 54.5 by previous adjustment | +0.2 | 55.0 | 55.0 |

LOCATION: California | DATE: 22 Aug 55 | STATION: LINCOLN (USC+GS)
ORGANIZATION: 29th Engrs | ZONE: 10 | OBSERVER: Cpl. P.E. Smith
SPHEROID: Clarke 1866 | CENT. MER.: 123° | INSTRUMENT: K&E #57460

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | t-T REDUCTION | CORRECTED PLANE DIRECTION WITH ZERO INITIAL | ADJUSTED PLANE DIRECTION |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| HICKS (USC+GS) | 0 00 00.00 | — | +0.3 | 0 00 00.00 | 00.0 |
| (1) Burdell (29th Engr) | 251 50 28.6 | — | +0.2 | 28.5 | 25.6 |
| (2) Red (29th Engr) | 302 11 14.7 | — | +0.6 | 15.0 | 11.4 |

LOCATION: California | DATE: 22 Aug 55 | STATION: Nic (29th Engrs)
ORGANIZATION: 29th Engrs | ZONE: 10 | OBSERVER: Cpl. P.E. Smith
SPHEROID: Clarke 1866 | CENT. MER.: 123 | INSTRUMENT: K&E #57460

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | t-T REDUCTION | CORRECTED PLANE DIRECTION WITH ZERO INITIAL | ADJUSTED PLANE DIRECTION |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| (16) BLACK (USC+GS) | 0 00 00.00 | — | -0.2 | 0 00 00.00 | 58.4 |
| (17) HICKS (USC+GS) | 45 01 07.4 | — | -0.6 | 07.0 | 08.6 |

*Figure 237. List of directions for UTM triangulation (DA Form 1936).*

| PROJECT | LIST OF DIRECTIONS, U. T. M. GRID (TM 5-237) | |
|---|---|---|
| 1-243 | | |
| LOCATION: California | DATE: 22 Aug 55 | STATION: Red (29th Engrs) |
| ORGANIZATION: 29th Engrs | ZONE: 10 | OBSERVER: Cpl. P.E. Smith |
| SPHEROID: Clarke 1866 | CENT. MER.: 123° | INSTRUMENT: K&E #57460 |

| OBSERVED STATION | OBSERVED DIRECTION | ECCENTRIC REDUCTION | t-T REDUCTION | CORRECTED PLANE DIRECTION WITH ZERO INITIAL | ADJUSTED PLANE DIRECTION |
|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ″ | ° ′ ″ | ′ ″ |
| (8) HICKS (USC+GS) | 0 00 00.00 | — | -0.3 | 0 00 00.00 | 58.2 |
| (9) LINCOLN (USC+GS) | 30 41 59.7 | — | -0.6 | 59.4 | 42 01.6 |
| (10) Burdell (29th Engrs) | 95 09 11.0 | — | -0.5 | 10.8 | 13.0 |
| (6) Nic (29th Engrs) | 284 17 14.0 | — | +0.3 | 14.6 | 12.4 |
| (7) BLACK (USC+GS) | 321 09 32.8 | — | +0.1 | 33.2 | 32.8 |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.D. Smith | 27 Aug. 55 | W. C. Aumen | 27 Aug. 55 |

DA, FORM 1 FEB 57 1936

*Figure 237*—Continued.

| PROJECT | COMPUTATION OF TRIANGLES (UTM GRID) Preliminary (TM 5-237) | |
|---|---|---|
| 1-243 | | |
| LOCATION | ORGANIZATION | |
| California | 29th Engrs. | |
| SPHEROID | ZONE | DATE |
| Clarke 1866 | 10 | 22 Aug 55 |

| | Station | U. T. M. Plane Angle | Adjustment Correction | Adjusted Plane Angle and Distance | Function: ~~Logarithm~~ or Natural. |
|---|---|---|---|---|---|
| | 2-3 HICKS - LINCOLN | | | | 4868.326 |
| | 1 Burdell | 22 49 27 | | | 0.38790438 |
| | 2 HICKS | 49 00 55 | | | 0.75488449 |
| | 3 LINCOLN | 108 09 31 | | | 0.95019743 |
| | 1-3 Burdell - LINCOLN | | | | 9474.046 |
| | 1-2 Burdell - HICKS | | | | 11925.286 |
| | | | | | |
| | 2-3 BLACK - HICKS | | | | 6425.819 |
| | 1 Red | 38 50 27 | | | 0.62715907 |
| | 2 BLACK | 51 57 43 | | | 0.78760166 |
| | 3 HICKS | 89 11 48 | | | 0.99990171 |
| | 1-3 Red - HICKS | | | | 8069.700 |
| | 1-2 Red - BLACK | | | | 10244.909 |
| | | | | | |
| | 2-3 BLACK - HICKS | | | | 6425.819 |
| | 1 Nic | 45 01 07 | | | 0.70733643 |
| | 2 BLACK | 90 39 10 | | | 0.99993510 |
| | 3 HICKS | 44 19 43 | | | 0.69877259 |
| | 1-3 Nic - HICKS | | | | 9083.940 |
| | 1-2 Nic - BLACK | | | | 6348.021 |
| | | | | | |
| | 2-3 | | | | |
| | 1 | | | | |
| | 2 | | | | |
| | 3 | | | | |
| | 1-3 | | | | |
| | 1-2 | | | | |
| | | | | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAS | 27 Aug 55 | W.C.A. | 27 Aug 55 |

DA FORM 1 FEB 57 1937

*Figure 238. Computation of triangles on UTM grid (DA Form 1937).*

PROJECT: I-243 | TOTAL ANGULAR CLOSURE | ~~TRAVERSE~~ COMPUTATION ON THE UNIVERSAL TRANSVERSE MERCATOR GRID (TM 5-237) — Preliminary Position

LOCATION: California | NR OF ANGLE STATIONS | FROM STATION | TO STATION | TRAVERSE NUMBER

ORGANIZATION: 29th Engrs. | ANGULAR CLOSURE PER STATION | BOOK NUMBER | LINEAR CLOSURE | LENGTH OF TRAVERSE | LINEAR CLOSURE RATIO | ZONE: 10 | UNIT: METERS | PAGE NR: 1 | NR OF PAGES: 1

| Station | Observed Angle | Adjusted Angle | Azimuths | Corrected Field Distances | Scale Factor × Sea Level Coefficient | Grid Distances | Cosine | Sine | Unadjusted ΔN | Unadjusted ΔE | Adjusted Grid Coordinates N | Adjusted Grid Coordinates E | Station |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| HICKS | | | | | | | | | | | | | |
| LINCOLN | 251 50 29 | | 31 50 39.4 | | | | | | | | 4 224 150.64 | 526 686.39 | LINCOLN |
| Burdell | | | 103 41 08 | | | 9474.046 | | | -2241.495 | +9205.066 | 4 221 909 | 535 891. | Burdell |
| LINCOLN | | | | | | | | | | | | | |
| HICKS | 49 00 55 | | 211 50 39.4 | | | | | | | | 4 220 015.07 | 524 117.80 | HICKS |
| Burdell | | | 80 51 34 | | | 11925.286 | | | +1894.415 | +11773.854 | 4 221 909 | 535 892 | Burdell |
| BLACK | | | | | | | | | | | | | |
| HICKS | 270 48 12 | | 32 31 34.4 | | | | | | | | 4 220 015.07 | 524 117.80 | HICKS |
| Red | | | 123 19 46 | | | 8069.700 | | | -4433.915 | +6742.437 | 4 215 581 | 530 860 | Red |
| HICKS | | | | | | | | | | | | | |
| BLACK | 51 57 43 | | 212 31 34.4 | | | | | | | | 4 214 597.17 | 520 662.73 | BLACK |
| Red | | | 84 29 17 | | | 10244.909 | | | + 984.057 | +10197.538 | 4 215 581 | 530 860 | Red |
| BLACK | | | | | | | | | | | | | |
| HICKS | 315 40 17 | | 32 31 34.4 | | | | | | | | 4 220 015.07 | 524 117.80 | HICKS |
| Nic | | | 168 11 51 | | | 9083.940 | | | -8891.892 | +1858.018 | 4 211 123 | 525 976 | Nic |
| HICKS | | | | | | | | | | | | | |
| BLACK | 90 39 10 | | 212 31 34.4 | | | | | | | | 4 214 597.17 | 520 662.73 | BLACK |
| Nic | | | 123 10 44 | | | 6348.021 | | | -3473.985 | +5313.078 | 4 211 123 | 525 976 | Nic |

COMPUTED BY: R.H.Smith | DATE: 27 Aug 55 | CHECKED BY: W. C. Aumen | DATE: 27 Aug. 55

DA FORM 1 FEB 57 1940

*Figure 239. Computation of preliminary grid positions (DA Form 1940).*

| PROJECT | | GRID AZIMUTHS (t and T) AND (t–T) CORRECTION FROM U. T. M. GRID COORDINATES (TM 5-237) | | | |
|---|---|---|---|---|---|
| 1-243 | | | | | |
| LOCATION | | | | | |
| California | | | | | |
| ORGANIZATION | | | CENTRAL MERIDIAN | | |
| 29th Engrs. | | | 123° | | |
| SPHEROID | ZONE | UNIT | FE | | |
| Clarke 1866 | 10 | Meter | 500,000 m. | | |

| Grid Azimuth of Sta. (1) Red | | | | to Sta. (2) Nic | |
|---|---|---|---|---|---|
| $N_2$ | 4 211 123 | $E_2$ | 525976 | $2E'_1+E'_2$ | + 87696 |
| $N_1$ | 4 215 581 | $E_1$ | 530860 | (XVIII) | 0.012321 |
| $\Delta N$ | - 4 458 | $\Delta E$ | | (t–T) | + 0.331 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

| Grid Azimuth of Sta. (1) BLACK | | | | to Sta. (2) Nic | |
|---|---|---|---|---|---|
| $N_2$ | 4211 123 | $E_2$ | 525976 | $2E'_1+E'_2$ | + 67302 |
| $N_1$ | 4 214 597 | $E_1$ | 520 663 | (XVIII) | 0.012321 |
| $\Delta N$ | - 3474 | $\Delta E$ | | (t–T) | + 0.198 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

| Grid Azimuth of Sta. (1) Nic | | | | to Sta. (2) BLACK | |
|---|---|---|---|---|---|
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 72615 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | - 0.214 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

| Grid Azimuth of Sta. (1) BLACK | | | | to Sta. (2) Red | |
|---|---|---|---|---|---|
| $N_2$ | 4 215 581 | $E_2$ | 530 860 | $2E'_1+E'_2$ | + 72186 |
| $N_1$ | 4 214 597 | $E_1$ | 520 663 | (XVIII) | 0. 012321 |
| $\Delta N$ | + 984 | $\Delta E$ | | (t–T) | - 0.060 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

| Grid Azimuth of Sta. (1) Red | | | | to Sta. (2) BLACK | |
|---|---|---|---|---|---|
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 82383 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | + 0. 069 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

| Grid Azimuth of Sta. (1) Burdell | | | | to Sta. (2) Red | |
|---|---|---|---|---|---|
| $N_2$ | 4 215 581 | $E_2$ | 530 860 | $2E'_1+E'_2$ | + 102644 |
| $N_1$ | 4 221 909 | $E_1$ | 535 892 | (XVIII) | 0. 012320 |
| $\Delta N$ | - 6 328 | $\Delta E$ | | (t–T) | + 0. 550 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

| Grid Azimuth of Sta. (1) Red | | | | to Sta. (2) Burdell | |
|---|---|---|---|---|---|
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 97612 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | - 0. 523 |

*Figure 240. Computation of (t – T) corrections (DA Form 1934).*

| PROJECT | GRID AZIMUTHS (t and T) AND (t–T) CORRECTION FROM U. T. M. GRID COORDINATES (TM 5-237) | |
|---|---|---|
| I-243 | | |
| LOCATION | | |
| California | | |
| ORGANIZATION | CENTRAL MERIDIAN | |
| 29th Engrs | 123° | |
| SPHEROID | ZONE | UNIT / FE |
| Clarke 1866 | 10 | Meter / 500,000 m. |

| | | | | | |
|---|---|---|---|---|---|
| Grid Azimuth of Sta. (1) HICKS | | | | to Sta. (2) BLACK | |
| $N_2$ | 4214597 | $E_2$ | 520663 | $2E'_1+E'_2$ | + 68899 |
| $N_1$ | 4220015 | $E_1$ | 524118 | (XVIII) | 0.012320 |
| $\Delta N$ | - 5418 | $\Delta E$ | | (t–T) | +0.316 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |
| Grid Azimuth of Sta. (1) BLACK | | | | to Sta. (2) HICKS | |
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 65444 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | -0.300 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |
| Grid Azimuth of Sta. (1) HICKS | | | | to Sta. (2) Nic | |
| $N_2$ | 4211123 | $E_2$ | 525976 | $2E'_1+E'_2$ | + 74212 |
| $N_1$ | 4220015 | $E_1$ | 524118 | (XVIII) | 0.012320 |
| $\Delta N$ | -8892 | $\Delta E$ | | (t–T) | +0.559 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |
| Grid Azimuth of Sta. (1) Nic | | | | to Sta. (2) HICKS | |
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 76070 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | -0.573 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |
| Grid Azimuth of Sta. (1) HICKS | | | | to Sta. (2) Red | |
| $N_2$ | 4 215 581 | $E_2$ | 530 860 | $2E'_1+E'_2$ | + 79096 |
| $N_1$ | 4 220 015 | $E_1$ | 524 118 | (XVIII) | 0.012320 |
| $\Delta N$ | - 4434 | $\Delta E$ | | (t–T) | +0.297 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |
| Grid Azimuth of Sta. (1) Red | | | | to Sta. (2) HICKS | |
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 85838 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | -0.322 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |
| Grid Azimuth of Sta. (1) HICKS | | | | to Sta. (2) Burdell | |
| $N_2$ | 4221 909 | $E_2$ | 535892 | $2E'_1+E'_2$ | + 84128 |
| $N_1$ | 4 220 015 | $E_1$ | 524 118 | (XVIII) | 0.012320 |
| $\Delta N$ | + 1 894 | $\Delta E$ | | (t–T) | -0.135 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |
| Grid Azimuth of Sta. (1) Burdell | | | | to Sta. (2) HICKS | |
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 95902 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t–T) | +0.154 |

*Figure 240*—Continued.

| PROJECT | GRID AZIMUTHS (t and T) AND (t–T) CORRECTION FROM U. T. M. GRID COORDINATES (TM 5-237) |
|---|---|
| 1-243 | |
| LOCATION | |
| California | |
| ORGANIZATION | CENTRAL MERIDIAN |
| 29th Engrs. | 123° |
| SPHEROID | ZONE / UNIT / FE |
| Clarke 1866 | 10 / Meter / 500,000 m. |

Grid Azimuth of Sta. (1) LINCOLN to Sta. (2) HICKS

| | | | | | |
|---|---|---|---|---|---|
| $N_2$ | 4 220 015 | $E_2$ | 524 118 | $2E'_1+E'_2$ | + 77490 |
| $N_1$ | 4 224 151 | $E_1$ | 526 686 | (XVIII) | 0.012320 |
| $\Delta N$ | − 4 136 | $\Delta E$ | | (t−T) | + 0.271 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

Grid Azimuth of Sta. (1) HICKS to Sta. (2) LINCOLN

| | | | | | |
|---|---|---|---|---|---|
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 74922 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t−T) | − 0.262 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

Grid Azimuth of Sta. (1) LINCOLN to Sta. (2) Red

| | | | | | |
|---|---|---|---|---|---|
| $N_2$ | 4 215 581 | $E_2$ | 530 860 | $2E'_1+E'_2$ | + 84232 |
| $N_1$ | 4 224 151 | $E_1$ | 526 686 | (XVIII) | 0.012320 |
| $\Delta N$ | − 8570 | $\Delta E$ | | (t−T) | + 0.611 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

Grid Azimuth of Sta. (1) Red to Sta. (2) LINCOLN

| | | | | | |
|---|---|---|---|---|---|
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 88406 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t−T) | − 0.642 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

Grid Azimuth of Sta. (1) LINCOLN to Sta. (2) Burdell

| | | | | | |
|---|---|---|---|---|---|
| $N_2$ | 4 221 909 | $E_2$ | 535 892 | $2E'_1+E'_2$ | + 89264 |
| $N_1$ | 4 224 151 | $E_1$ | 526 686 | (XVIII) | 0.012320 |
| $\Delta N$ | − 2 242 | $\Delta E$ | | (t−T) | + 0.170 |
| tan $\beta$ | | | | T | |
| $\beta$ | | t | | | |

Grid Azimuth of Sta. (1) Burdell to Sta. (2) LINCOLN

| | | | | | |
|---|---|---|---|---|---|
| $N_2$ | | $E_2$ | | $2E'_1+E'_2$ | + 98470 |
| $N_1$ | | $E_1$ | | (XVIII) | |
| $\Delta N$ | | $\Delta E$ | | (t−T) | − 0.187 |

*Figure 240*—Continued.

PROJECT 1-243

**SIDE/LENGTH EQUATIONS** (TM 5-237)

LOCATION California

ORGANIZATION 29th Engrs

DATE 22 Aug 55

| Symbol | Angle | Log. Sine | Tab. Diff. | Symbol | Angle | Log. Sine | Tab. Diff. |
|---|---|---|---|---|---|---|---|
| | Pole @ HICKS | | | | | | |
| -4 +5 | 22 49 26.7 | 9.58872319 | +5.01 | -1 | 108 09 31.5 | 9.97781350 | -0.69 |
| -8 +10 | 95 09 10.8 | 9.99824121 | -0.19 | -3 +4 | 42 22 31.3 | 9.82865008 | +2.31 |
| -2 | 57 48 45.0 | 9.92752917 | +1.32 | -8 +9 | 30 41 59.4 | 9.70803013 | +3.55 |
| | | 9.51449357 | | | | 9.51449371 | |
| | | 371 | | | | | |
| | | - 0.14 | | | | | |
| 0 = -0.14 -0.69(1) -1.32(2) +2.31(3) -7.32(4) +5.01(5) +3.74(8) -3.55(9) -0.19(10) | | | | | | | |
| | Pole @ BLACK | | | | | | |
| -7 +8 | 38 50 26.8 | 9.79737719 | +2.62 | -12 | 89 11 48.2 | 9.99995732 | +0.03 |
| +6 -7 +14 -15 | 104 26 14.5 | 9.98606410 | -0.55 | -6 +7 | 36 52 18.6 | 9.77817082 | +2.81 |
| -13 | 44 19 42.8 | 9.84433543 | +2.16 | -16 +17 | 45 01 07.0 | 9.84962603 | +2.10 |
| | | 9.62777672 | | | | 9.62775417 | |
| | | 75417 | | | | | |
| | | + 22.55 | | | | | |
| 0 = +22.55 +2.26(6) -4.88(7) +2.62(8) +0.03(12) -2.16(13) -0.55(14) +0.55(15) +2.10(16) -2.10(17) | | | | | | | |
| ÷10 | | | | | | | |
| 0 = +2.255 +0.226(6) -0.488(7) +0.262(8) +0.003(12) -0.216(13) -0.055(14) +0.055(15) +0.210(16) -0.210(17) | | | | | | | |
| | | LENGTH | | EQUATION | | | |
| | BLACK-HICKS | 3.80792848 | | | HICKS-LINCOLN | 3.68737965 | |
| -8 +9 | 30 41 59.4 | 9.70803013 | +3.55 | -2 | 57 48 45.0 | 9.92752917 | +1.32 |
| +14 | 51 57 43.5 | 9.89630744 | +1.65 | -7 +8 | 38 50 26.8 | 9.79737719 | +2.62 |
| | | 3.41226605 | | | | 3.41228601 | |
| | | 8601 | | | | | |
| | | - 19.96 | | | | | |
| 0 = -19.96 +1.32(2) +2.62(7) -6.17(8) +3.55(9) +1.65(14) | | | | | | | |
| ÷10 | | | | | | | |
| 0 = -1.996 +0.132(2) +0.262(7) -0.617(8) +0.355(9) +0.165(14) | | | | | | | |

COMPUTED BY R A Smith — DATE 27 Aug 55

CHECKED BY W. C. Aumen — DATE 27 Aug. 55

DA FORM 1 FEB 57 1926

*Figure 241. Condition equations and solution of condition equations (DA Form 1926).*

## Condition Equations

① $0 = -4.1 - (1) + (2) - (3) + (5) - (9) + (10)$

② $0 = -8.8 - (2) - (8) + (9) + (12)$

③ $0 = -6.5 - (3) + (4) - (8) + (10) - (11) + (12)$

④ $0 = -1.5 - (7) + (8) - (12) + (14)$

⑤ $0 = +0.2 - (13) + (15) - (16) + (17)$

⑥ $0 = -0.14 - 0.69(1) - 1.32(2) + 2.31(3) - 7.32(4) + 5.01(5) + 3.74(8) - 3.55(9) - 0.19(10)$

⑦ $0 = -1.996 + 0.132(2) + 0.262(7) - 0.617(8) + 0.355(9) + 0.165(14)$

⑧ $0 = +2.255 + 0.226(6) - 0.488(7) + 0.262(8) + 0.003(12) - 0.216(13) - 0.055(14) + 0.055(15) + 0.210(16) - 0.210(17)$

Correlate Equations

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | $\Sigma_c$ | $v$ | Adopt. $v$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | | | | | -0.69 | | | -1.69 | -2.89154 | -2.9 | 1 |
| 2 | +1 | -1 | | | | -1.32 | +0.132 | | -1.188 | -3.58722 | -3.6 | 2 |
| 3 | -1 | | -1 | | | +2.31 | | | +0.31 | -2.34404 | -2.3 | 3 |
| 4 | | | +1 | | | -7.32 | | | -6.32 | -0.17383 | -0.2 | 4 |
| 5 | +1 | | | | | +5.01 | | | +6.01 | +2.51788 | +2.5 | 5 |
| 6 | | | | | | | | +0.226 | +0.226 | -2.17848 | -2.2 | 6 |
| 7 | | | | -1 | | | +0.262 | -0.488 | -1.226 | -0.40676 | -0.4 | 7 |
| 8 | | -1 | -1 | +1 | | +3.74 | -0.617 | +0.262 | +2.385 | -1.80262 | -1.8 | 8 |
| 9 | -1 | +1 | | | | -3.55 | +0.355 | | -3.195 | +2.22896 | +2.2 | 9 |
| 10 | +1 | | +1 | | | -0.19 | | | +1.81 | +2.15890 | +2.2 | 10 |
| 11 | | | -1 | | | | | | -1 | +0.81309 | +0.8 | 11 |
| 12 | | +1 | +1 | -1 | | | | +0.003 | +1.003 | +1.18127 | +1.2 | 12 |
| 13 | | | | | -1 | | | -0.216 | -1.216 | +2.49114 | +2.5 | 13 |
| 14 | | | | +1 | | | +0.165 | -0.055 | +1.110 | +4.07706 | +4.1 | 14 |
| 15 | | | | | +1 | | | +0.055 | +1.055 | -0.93922 | -0.9 | 15 |
| 16 | | | | | -1 | | | +0.210 | -0.790 | -1.61519 | -1.6 | 16 |
| 17 | | | | | +1 | | | -0.210 | +0.790 | +1.61519 | +1.6 | 17 |

Normal Equations

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | n | $\Sigma_n$ | C | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | +6 | -2 | +2 | — | — | +5.43 | -0.223 | — | -4.1 | +7.107 | +2.95540 | 1 |
| 2 | | +4 | +2 | -2 | — | -5.97 | +0.840 | -0.259 | -8.8 | -12.189 | +6.17447 | 2 |
| 3 | | | +6 | -2 | — | -13.560 | +0.617 | -0.259 | -6.5 | -11.702 | -0.81309 | 3 |
| 4 | | | | +4 | — | +3.74 | -0.714 | +0.692 | -1.5 | +2.218 | +4.15119 | 4 |
| 5 | | | | | +4 | — | — | -0.149 | +0.2 | +4.051 | -0.40906 | 5 |
| 6 | | | | | | +112.8633 | -3.74207 | +0.97988 | -0.14 | +99.60111 | -0.08233 | 6 |
| 7 | | | | | | | +0.62001 | -0.29858 | -1.996 | -4.89664 | -3.66235 | 7 |
| 8 | | | | | | | | +0.49878 | +2.255 | +3.46008 | -9.63928 | 8 |

*Figure 241*— Continued.

| | | | | Forward | Solution | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | n | $\Sigma n$ |
| +6 | −2 | +2 | — | — | +5.43 | −0.223 | — | −4.1 | +7.107 |
| | | | | | | | | | |
| $C_1$ | +0.33333 | −0.33333 | — | — | −0.90500 | +0.03717 | — | +0.68333 | −1.18450 |
| | +4 | +2 | −2 | — | −5.97 | +0.840 | −0.259 | −8.8 | −12.189 |
| | +3.33333 | +2.66667 | −2 | — | −4.16000 | +0.76566 | −0.259 | −10.16667 | −9.82000 |
| | $C_2$ | −0.80000 | +0.60000 | — | +1.24800 | −0.22970 | +0.07770 | +3.05000 | +2.94600 |
| | | +6 | −2 | — | −13.560 | +0.617 | −0.259 | −6.5 | −11.702 |
| | | +3.20000 | −0.40000 | — | −12.04200 | +0.07881 | −0.05180 | +3.00000 | −6.21499 |
| | | $C_3$ | +0.12500 | — | +3.76312 | −0.02463 | +0.01619 | −0.93750 | +1.94218 |
| | | | +4 | — | +3.74 | −0.714 | +0.692 | −1.5 | +2.218 |
| | | | +2.75000 | — | −0.26125 | −0.24475 | +0.53012 | −7.22500 | −4.45088 |
| | | | $C_4$ | — | +0.09500 | +0.08900 | −0.19277 | +2.62727 | +1.61850 |
| | | | | +4 | — | — | −0.149 | +0.2 | +4.051 |
| | | | | +4 | — | — | −0.149 | +0.2 | +4.051 |
| | | | | $C_5$ | — | — | +0.03725 | −0.05000 | −1.01275 |
| | | | | | +112.8633 | −3.74207 | +0.97988 | −0.14 | +99.60111 |
| | | | | | +57.41716 | −2.31134 | +0.51205 | +1.48548 | +57.10335 |
| | | | | | $C_6$ | +0.04026 | −0.00892 | −0.02587 | −0.99453 |
| | | | | | | +0.62001 | −0.29858 | −1.996 | −4.89664 |
| | | | | | | +0.31907 | −0.17001 | −0.47023 | −0.32117 |
| | | | | | | $C_7$ | +0.53283 | +1.47375 | +1.00658 |
| | | | | | | | +0.49878 | +2.255 | +3.46008 |
| | | | | | | | +0.27492 | +2.65003 | +2.92495 |
| | | | | | | | $C_8$ | −9.63928 | −10.63928 |

*Figure 241*— Continued.

| PROJECT | COMPUTATION OF TRIANGLES (UTM GRID) (TM 5-237) | | |
|---|---|---|---|
| 1-243 | | | |
| LOCATION: California | ORGANIZATION: 29th Engrs. | | |
| SPHEROID: Clarke 1866 | ZONE: 10 | DATE: 22 Aug 55 | |

| | Station | U. T. M. Plane Angle | Adjustment Correction | Adjusted Plane Angle and Distance | Function: ~~Logarithm~~ or Natural |
|---|---|---|---|---|---|
| | 2-3 HICKS - LINCOLN | | | | 4868.326 |
| -4+5 | 1 Burdell | 22 49 26.7 | +2.7 | 29.4 | 0.38791511 |
| +11 | 2 HICKS | 49 00 55.4 | +0.8 | 56.2 | 0.75488831 |
| -1 | 3 LINCOLN | 108 09 31.5 | +2.9 | 34.4 | 0.95019229 |
| | 1-3 Burdell - LINCOLN | | | | 9473.831 |
| | 1-2 Burdell - HICKS | | | | 11924.892 |
| | | 53.6 | +6.4 | | |
| | 2-3 HICKS - LINCOLN | | | | 4868.326 |
| -8+9 | 1 Red | 30 41 59.4 | +4.0 | 42 03.4 | 0.51055709 |
| +12 | 2 HICKS | 91 29 06.8 | +1.2 | 08.0 | 0.99966389 |
| -2 | 3 LINCOLN | 57 48 45.0 | +3.6 | 48.6 | 0.84631870 |
| | 1-3 Red - LINCOLN | | | | 9532.117 |
| | 1-2 Red - HICKS | | | | 8069.921 |
| | | 51.2 | +8.8 | | |
| | 2-3 HICKS - Burdell | | | | 11924.892 |
| -8+10 | 1 Red | 95 09 10.8 | +4.0 | 14.8 | 0.99595667 |
| -11+12 | 2 HICKS | 42 28 11.4 | +0.4 | 11.8 | 0.67520336 |
| -3+4 | 3 Burdell | 42 22 31.3 | +2.1 | 33.4 | 0.67399229 |
| | 1-3 Red - Burdell | | | | 8084.415 |
| | 1-2 Red - HICKS | | | | 8069.915 |
| | | 53.5 | +6.5 | | |
| | 2-3 LINCOLN - Burdell | | | | 9473.831 |
| -9+10 | 1 Red | 64 27 11.4 | — | 11.4 | 0.90223309 |
| -1+2 | 2 LINCOLN | 50 20 46.5 | -0.7 | 45.8 | 0.76991276 |
| -3+5 | 3 Burdell | 65 11 58.0 | +4.8 | 12 02.8 | 0.90778317 |
| | 1-3 Red - Burdell | | | | 8084.411 |
| | 1-2 Red - LINCOLN | | | | 9532.109 |
| | | 55.9 | +4.1 | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R45 | 28 Aug 55 | W. C. A. | 28 Aug 55 |

DA FORM 1 FEB 57 1937

*Figure 242. Computation of triangles on UTM grid (final) (DA Form 1937).*

| PROJECT | COMPUTATION OF TRIANGLES (UTM GRID) (TM 5-237) | | |
|---|---|---|---|
| 1-243 | | | |
| LOCATION | ORGANIZATION | | |
| California | 29th Engrs. | | |
| SPHEROID | ZONE | DATE | |
| Clarke 1866 | 10 | 22 Aug 55 | |

| | Station | U. T. M. Plane Angle | Adjustment Correction | Adjusted Plane Angle and Distance | Function: ~~Logarithm~~ or Natural |
|---|---|---|---|---|---|
| | 2-3 BLACK - HICKS | | | | 6425.819 |
| -7+8 | 1 Red | 38 50 26.8 | -1.4 | 25.4 | 0.62715303 |
| +14 | 2 BLACK | 51 57 43.5 | +4.1 | 47.6 | 0.78761540 |
| -12 | 3 HICKS | 89 11 48.2 | -1.2 | 47.0 | 0.99990164 |
| | 1-3 Red - HICKS | | | | 8069.919 |
| | 1-2 Red - BLACK | | | | 10245.007 |
| | | 58.5 | +1.5 | | |
| | 2-3 BLACK - HICKS | | | | 6425.819 |
| -16+17 | 1 Nic | 45 01 07.0 | +3.2 | 10.2 | 0.70734740 |
| +15 | 2 BLACK | 90 39 10.4 | -0.9 | 09.5 | 0.99993512 |
| -13 | 3 HICKS | 44 19 42.8 | -2.5 | 40.3 | 0.69876322 |
| | 1-3 Nic - HICKS | | | | 9083.800 |
| | 1-2 Nic - BLACK | | | | 6347.837 |
| | | 00.2 | -0.2 | | |
| | 2-3 BLACK - Red | | | | 10245.007 |
| +6-7 +14-15 | 1 Nic | (104 26 14.5) | +3.2 | 17.7 | 0.96841692 |
| -14+15 | 2 BLACK | 38 41 26.9 | -5.0 | 21.9 | 0.62509849 |
| -6+7 | 3 Red | 36 52 18.6 | +1.8 | 20.4 | 0.60003401 |
| | 1-3 Nic - Red | | | | 6612.997 |
| | 1-2 Nic - BLACK | | | | 6347.837 |
| | | | 0.0 | | |
| | 2-3 HICKS - Red | | | | 8069.919 |
| +6-8 +12-13 | 1 Nic | (59 25 09.2) | -1.7 | 07.5 | 0.86090856 |
| -12+13 | 2 HICKS | 44 52 05.4 | +1.3 | 06.7 | 0.70548238 |
| -6+8 | 3 Red | 75 42 45.4 | +0.4 | 45.8 | 0.96907055 |
| | 1-3 Nic - Red | | | | 6612.997 |
| | 1-2 Nic - HICKS | | | | 9083.800 |
| | | | 0.0 | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.D. Smith | 28 Aug. 55 | W. C. Aumen | 28 Aug. 55 |

DA FORM 1 FEB 57 1937

*Figure 242*—Continued.

**POSITION COMPUTATION (UTM GRID)** (TM 5-237)

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| PROJECT | I-243 | | | | | | ORGANIZATION | 29th Engrs. | | | | | | | |
| LOCATION | California | | | | | | CENTRAL MERIDIAN | 123° | | | | | | | |
| SPHEROID | Clarke 1866 | | | ZONE | 10 | | | | | | | | | | |
| | | | t 2 to 3 | 31 | 50 | 39.4 | | | | t 3 to 2 | 211 | 50 | 39.4 | | |
| | | | ∠ 2 | 49 | 00 | 56.2 | | | | ∠ 3 | −108 | 09 | 34.4 | | |
| Sin t (2 to 1) | +0.98730284 | | t 2 to 1 | 80 | 51 | 35.6 | Sin t (3 to 1) | +0.97161224 | | t 3 to 1 | 103 | 41 | 05.0 | | |
| Cos t (2 to 1) | +0.15884929 | | ∠ 1 | 22 | 49 | 29.4 | Cos t (3 to 1) | −0.23657908 | | | | | | | |
| $N_2$ | 4220 015.07 | 2 | HICKS | $E_2$ | 524 117.80 | | $N_3$ | 4 224 150.64 | 3 | LINCOLN | $E_3$ | 526 686.39 | | | |
| ΔN | + 1 894.26 | S | 11.924.892 | ΔE | +11 773.48 | | ΔN | − 2 241.31 | S | 9473.831 | ΔE | + 9 204.89 | | | |
| $N_1$ | 4 221 909.33 | 1 | Burdell | $E_1$ | 535 891.28 | | $N_1$ | 4 221 909.33 | 1 | Burdell | $E_1$ | 535 891.28 | | | |
| | | | t 2 to 3 | 31 | 50 | 39.4 | | | | t 3 to 2 | 211 | 50 | 39.4 | | |
| | | | ∠ 2 | 91 | 29 | 08.0 | | | | ∠ 3 | 57 | 48 | 48.6 | | |
| Sin t (2 to 1) | +0.83552138 | | t 2 to 1 | 123 | 19 | 47.4 | Sin t (3 to 1) | +0.43788827 | | t 3 to 1 | 154 | 01 | 50.8 | | |
| Cos t (2 to 1) | −0.54945794 | | ∠ 1 | 30 | 42 | 03.4 | Cos t (3 to 1) | −0.89902940 | | | | | | | |
| $N_2$ | 4 220 015.07 | 2 | HICKS | $E_2$ | 524 117.80 | | $N_3$ | 4 224 150.64 | 3 | LINCOLN | $E_3$ | 526 686.39 | | | |
| ΔN | − 4 434.08 | S | 8069.918 | ΔE | + 6 742.59 | | ΔN | − 8 569.65 | S | 9532.113 | ΔE | + 4 174.00 | | | |
| $N_1$ | 4 215 580.99 | 1 | Red | $E_1$ | 530 860.39 | | $N_1$ | 4 215 580.99 | 1 | Red | $E_1$ | 530 860.39 | | | |
| | | | t 2 to 3 | 80 | 51 | 35.6 | | | | t 3 to 2 | 260 | 51 | 35.6 | | |
| | | | ∠ 2 | 42 | 28 | 11.8 | | | | ∠ 3 | − 42 | 22 | 33.4 | | |
| Sin t (2 to 1) | +0.83552138 | | t 2 to 1 | 123 | 19 | 47.4 | Sin t (3 to 1) | −0.62229531 | | t 3 to 1 | 218 | 29 | 02.2 | | |
| Cos t (2 to 1) | −0.54945794 | | ∠ 1 | 95 | 09 | 14.8 | Cos t (3 to 1) | −0.78278257 | | | | | | | |
| $N_2$ | 4 220 015.07 | 2 | HICKS | $E_2$ | 524 117.80 | | $N_3$ | 4 221 909.33 | 3 | Burdell | $E_3$ | 535 891.28 | | | |
| ΔN | − 4 434.08 | S | 8069.918 | ΔE | + 6 742.59 | | ΔN | − 6 328.34 | S | 8084.413 | ΔE | − 5030.89 | | | |
| $N_1$ | 4 215 580.99 | 1 | Red | $E_1$ | 530 860.39 | | $N_1$ | 4 215 580.99 | 1 | Red | $E_1$ | 530 860.39 | | | |

N=S cos t  E=S sin t

| COMPUTED BY | RAS | DATE | 25 Aug 55 | CHECKED BY | W.C.A. | DATE | 27 Aug 55 |
|---|---|---|---|---|---|---|---|

DA FORM 1938 FEB 57

*Figure 243. UTM grid position computation (final) (DA Form 1938).*

**POSITION COMPUTATION (UTM GRID)** (TM 5-237)

PROJECT: I-243
LOCATION: California
SPHEROID: Clarke 1866
ZONE: 10
ORGANIZATION: 29th Engrs.
CENTRAL MERIDIAN: 123°

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | t 2 to 3 | 123 | 19 | 47.4 | | | t 3 to 2 | 303 | 19 | 47.4 | | | | |
| | | ∠ 2 | + 44 | 52 | 06.7 | | | ∠ 3 | - 75 | 42 | 45.8 | | | | |
| Sin t (2 to 1) | + 0.20452406 | t 2 to 1 | 168 | 11 | 54.1 | Sin t (3 to 1) | - 0.73865668 | t 3 to 1 | 227 | 37 | 01.6 | | | | |
| Cos t (2 to 1) | - 0.97886154 | ∠ 1 | 59 | 25 | 07.5 | Cos t (3 to 1) | - 0.67408182 | | | | | | | | |
| $N_2$ | 4 220 015.07 | 2 | HICKS | $E_2$ | 524 117.80 | $N_3$ | 4 215 580.99 | 3 | Red | $E_3$ | 530 860.39 | | | | |
| $\Delta N$ | - 8 891.78 | S | 9083.800 | $\Delta E$ | 1857.86 | $\Delta N$ | - 4 457.70 | S | 6612.997 | $\Delta E$ | - 4 884.73 | | | | |
| $N_1$ | 4 211 123.29 | 1 | Nic | $E_1$ | 525 975.66 | $N_1$ | 4 211 123.29 | 1 | Nic | $E_1$ | 525 975.66 | | | | |
| | | t 2 to 3 | 32 | 31 | 34.4 | | | t 3 to 2 | 212 | 31 | 34.4 | | | | |
| | | ∠ 2 | + 90 | 39 | 09.5 | | | ∠ 3 | - 44 | 19 | 40.3 | | | | |
| Sin t (2 to 1) | + 0.83696628 | t 2 to 1 | 123 | 10 | 43.9 | Sin t (3 to 1) | + 0.20452406 | t 3 to 1 | 168 | 11 | 54.1 | | | | |
| Cos t (2 to 1) | - 0.54725446 | ∠ 1 | 45 | 01 | 10.2 | Cos t (3 to 1) | - 0.97886154 | | | | | | | | |
| $N_2$ | 4 214 597.17 | 2 | BLACK | $E_2$ | 520 662.73 | $N_3$ | 4 220 015.07 | 3 | HICKS | $E_3$ | 524 117.80 | | | | |
| $\Delta N$ | - 3 473.88 | S | 6347.837 | $\Delta E$ | + 5 312.93 | $\Delta N$ | - 8 891.78 | S | 9083.800 | $\Delta E$ | 1857.86 | | | | |
| $N_1$ | 4 211 123.29 | 1 | Nic | $E_1$ | 525 975.66 | $N_1$ | 4 211 123.29 | 1 | Nic | $E_1$ | 525 975.66 | | | | |
| | | t 2 to 3 | 32 | 31 | 34.4 | | | t 3 to 2 | 212 | 31 | 34.4 | | | | |
| | | ∠ 2 | + 51 | 57 | 47.6 | | | ∠ 3 | - 89 | 11 | 47.0 | | | | |
| Sin t (2 to 1) | + 0.99537852 | t 2 to 1 | 84 | 29 | 22.0 | Sin t (3 to 1) | + 0.83552138 | t 3 to 1 | 123 | 19 | 47.4 | | | | |
| Cos t (2 to 1) | + 0.09602913 | ∠ 1 | 38 | 50 | 25.4 | Cos t (3 to 1) | - 0.54945794 | | | | | | | | |
| $N_2$ | 4 214 597.17 | 2 | BLACK | $E_2$ | 520 662.73 | $N_3$ | 4 220 015.07 | 3 | HICKS | $E_3$ | 524 117.80 | | | | |
| $\Delta N$ | + 983.82 | S | 10 245.007 | $\Delta E$ | +10 197.66 | $\Delta N$ | - 4 434.08 | S | 8069.919 | $\Delta E$ | + 6 742.59 | | | | |
| $N_1$ | 4 215 580.99 | 1 | Red | $E_1$ | 530 860.39 | $N_1$ | 4 215 580.99 | 1 | Red | $E_1$ | 530 860.39 | | | | |

N=S cos t  E=S sin t

COMPUTED BY: R A Smith  DATE: 25 Aug 55
CHECKED BY: W. C. Asmen  DATE: 27 Aug. 55

DA FORM 1 FEB 57 1938

*Figure 243*—Continued.

| PROJECT | GEOGRAPHIC COORDINATES FROM U. T. M. GRID COORDINATES (TM 5-237) | | | | |
|---|---|---|---|---|---|
| 1-243 | | | | | |
| LOCATION | California | ORGANIZATION | 29th Engrs. | | |
| SPHEROID | Clarke 1866 | CENTRAL MERIDIAN | 123° | ZONE 10 | UNIT Meter / FE 500,000 m. |

| Station | Burdell | | | Station | Red | | |
|---|---|---|---|---|---|---|---|
| N | 4,221,909.33 | E | 535,891.28 | N | 4,215,580.99 | E | 530 860.39 |
| $\phi'$ | 38 08 48.8812 | | | $\phi'$ | 38 05 23.5489 | | |
| (VII) | | $q^2$ | 0.00128818 | (VII) | | $q^2$ | 0.000 952 36 |
| (VIII) | | $q^4$ | | (VIII) | | $q^4$ | |
| $D_6$ | — | | | $D_6$ | — | | |
| (IX) | | q | 0.03589128 | (IX) | | q | 0. 03086039 |
| (X) | | $q^3$ | | (X) | | $q^3$ | |
| $E_5$ | — | | | $E_5$ | — | | |
| $\Delta\phi$ | 02."571 | $\Delta\lambda$ | 0° 24' 34."561 | $\Delta\phi$ | 01."897 | $\Delta\lambda$ | 0° 21' 06."890 |
| $\phi$ | 38 08 46.310 | $\lambda$ | 122 35 25.439 | $\phi$ | 38 05 21.652 | $\lambda$ | 122 38 53.110 |

| Station | Nic | | | Station | | | |
|---|---|---|---|---|---|---|---|
| N | 4,211,123.29 | E | 525,975.66 | N | | E | |
| $\phi'$ | 38 02 58.9110 | | | $\phi'$ | | | |
| (VII) | | $q^2$ | 0.00067473 | (VII) | | $q^2$ | |
| (VIII) | | $q^4$ | | (VIII) | | $q^4$ | |
| $D_6$ | — | | | $D_6$ | | | |
| (IX) | | q | 0.02597566 | (IX) | | q | |
| (X) | | $q^3$ | | (X) | | $q^3$ | |
| $E_5$ | — | | | $E_5$ | | | |
| $\Delta\phi$ | 01."342 | $\Delta\lambda$ | 0° 17' 45."780 | $\Delta\phi$ | | $\Delta\lambda$ | |
| $\phi$ | 38 02 57.569 | $\lambda$ | 122 42 14.220 | $\phi$ | | $\lambda$ | |

*Figure 244. Conversion, UTM grid coordinates to geographic coordinates (DA Form 1933).*

## Section III. TRAVERSE ADJUSTMENTS ON THE UTM GRID

### 169. Conversion to Grid Data

The first step in computing a traverse (fig. 245) on the UTM grid is to convert all geodetic data (fig. 246) to grid data (fig. 247). This means that geographic coordinates are converted to grid coordinates (paras. 152 and 153) and geodetic azimuths are converted to projected geodetic azimuths (paras. 156 and 157). The projected geodetic azimuth (T) is used for traverse computations. Do not use the plane azimuth ($t$) for traverse computations unless all the observed angles are corrected for the ($t-T$) reductions (paras. 156 and 157).

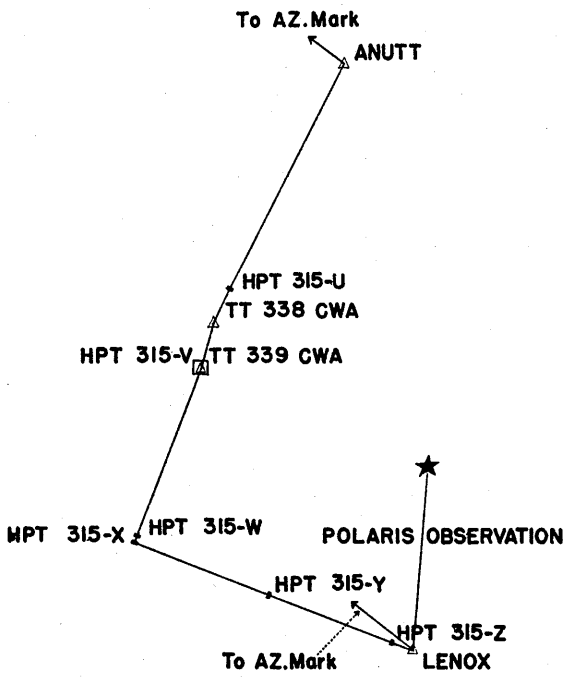



*Figure 245. Traverse diagram.*

### 170. Computation

*a.* Field distances are corrected for temperature, tape, and inclination on DA Form 1939 (Reduction of Taped Distances) (fig. 248) and then multiplied by the combined factor for scale and sea-level reduction. Distances must be in meters for UTM computations.

*b.* Traverse computations on the UTM grid are made on DA Form 1940 (Transverse Computation on the Universal Transverse Mercator Grid) (fig. 249). The observed traverse angles are entered on the form beside the station at which they were turned. Starting with the fixed azimuth and always adding the clockwise traverse angle to the back azimuth, an azimuth closure is found on the fixed closing azimuth. From this closure, a correction is found for each angle and applied to the observed angles. These adjusted angles are entered in the third column on the form. Using the adjusted angles, the azimuths are computed as before and entered on the form. The stations labelled "thru" in the example are points on line between two angle stations. These "thru" stations are picture points or check points for which a position is desired but at which no angle is turned.

*c.* The sine and cosine of each azimuth are taken from tables (6 decimals sufficient for third-order traverse) and entered on the form. Multiplying the grid distance by the cosine and sine gives the $\Delta N$ and $\Delta E$ to 3 or 4 places for the purpose of later adjustment. The algebraic sign of $\Delta N$ or $\Delta E$ is determined by the following rules:

(1) For azimuths between 0° and 180°, $\Delta E$ is plus; for all other azimuths, $\Delta E$ is minus.

(2) For azimuths between 90° and 270°, $\Delta N$ is minus; for all other azimuths, $\Delta N$ is plus.

(3) Azimuths are referred to grid north.

*d.* After the $\Delta N$'s and $\Delta E$'s are computed for the individual courses, an algebraic summation is made of the $\Delta N$ column and the $\Delta E$ column. These summations are compared with the differences of Northing and Easting of the fixed positions at the ends of the line. Subtracting the fixed difference from the summation gives the error of closure in Northing and Easting respectively. The square root of the sum of the squares of the error in Northing and the error in Easting gives the linear error of closure. The linear closure ratio is an indication of the accuracy of the traverse, and is the ratio of the linear closure to the total length of traverse.

*e.* The adjustment is completed by the compass rule which proportions the errors in $\Delta N$ and $\Delta E$ according to the length of the course. The corrections to the $\Delta N$'s and $\Delta E$'s are written in units of the last decimal carried in the original computation. These corrections are written above the original numbers. The final coordinates are obtained by starting with the fixed coordinates of the beginning station and successively adding the

| COUNTRY | TYPE OF MARK | STATION | |
|---|---|---|---|
| UNITED STATES | DISK | LENOX (USC&GS 1947) | |
| LOCALITY | STAMPING ON MARK | AGENCY (CAST IN MARK) | ELEVATION (FT) (M) |
| LENOX, MISSOURI | LENOX 1947 | USC&GS | |
| LATITUDE | LONGITUDE | DATUM | DATUM |
| N 37° 38' 47".807 | W 91° 44' 17".976 | 1927 NAD | |
| (NORTHING)(EASTING) (FT) (M) | (EASTING)(NORTHING) (FT) (M) | GRID AND ZONE | ESTABLISHED BY (AGENCY) |
| 4 167 150.96 | 611 306.05 | UTM 15 | |
| (NORTHING)(EASTING) (FT) (M) | (EASTING)(NORTHING) (FT) (M) | GRID AND ZONE | DATE / ORDER |

TO OBTAIN U. T. M. GRID AZIMUTH, ADD 179 ° 13' 45".49 TO THE GEODETIC AZIMUTH

TO OBTAIN GRID AZ. (ADD)(SUB.) ° ' " TO THE GEODETIC AZIMUTH

| OBJECT | AZIMUTH OR DIRECTION (GEODETIC) (GRID) (MAGNETIC) | BACK AZIMUTH | GEOD. DISTANCE (METERS) (FEET) | GRID DISTANCE (METERS) (FEET) |
|---|---|---|---|---|
| MAPLES 1939 | 28° 17 11".76 | 208° 15' 09".05 | 10 414.78 | |
| AZIMUTH MARK | 101 30 46.1 | | | |
| EDGAR | 120 38 57.09 | 300 34 14.11 | 13 182.48 | |
| ANUTT | 183 53 09.97 | 3 53 20.17 | 6 030.55 | |
| SPRING | 244 52 33.91 | 64 57 21.08 | 12 715.17 | |
| HOLT | 291 57 19.95 | 112 01 12.52 | 10 072.82 | |
| ZION 1939 | 349 39 22.63 | 169 40 27.78 | 14 610.44 | |

DA FORM 1 OCT 64 1961 REPLACES DA FORM 1961, 1 FEB 57, WHICH IS OBSOLETE. GEODETIC DATA (TM 5-237)

| COUNTRY | TYPE OF MARK | STATION | |
|---|---|---|---|
| UNITED STATES | DISK | ANUTT (USC&GS 1947) | |
| LOCALITY | STAMPING ON MARK | AGENCY (CAST IN MARK) | ELEVATION (FT) (M) |
| ROLLA, MISSOURI | ANUTT 1947 | USC&GS | |
| LATITUDE | LONGITUDE | DATUM | DATUM |
| N 37° 42' 02".964 | W 91° 44' 01".292 | 1927 NAD | |
| (NORTHING)(EASTING) (FT) (M) | (EASTING)(NORTHING) (FT) (M) | GRID AND ZONE | ESTABLISHED BY (AGENCY) |
| 4 173 171.12 | 611 633.67 | UTM 15 | |
| (NORTHING)(EASTING) (FT) (M) | (EASTING)(NORTHING) (FT) (M) | GRID AND ZONE | DATE / ORDER |

TO OBTAIN U. T. M. GRID AZIMUTH, ADD 179 ° 13' 31".89 TO THE GEODETIC AZIMUTH

TO OBTAIN GRID AZ. (ADD)(SUB.) ° ' " TO THE GEODETIC AZIMUTH

| OBJECT | AZIMUTH OR DIRECTION (GEODETIC) (GRID) (MAGNETIC) | BACK AZIMUTH | GEOD. DISTANCE (METERS) (FEET) | GRID DISTANCE (METERS) (FEET) |
|---|---|---|---|---|
| EDGAR | 93° 25 45".19 | 273 20 51".83 | 11 770.70 | |
| AZIMUTH MARK | 121 31 51.9 | | | |
| PILOT | 156 35 58.95 | 336 33 38.74 | 14 102.15 | |
| LECOMA | 199 08 17.18 | 19 09 58.91 | 12 397.93 | |

DA FORM 1 OCT 64 1961 REPLACES DA FORM 1961, 1 FEB 57, WHICH IS OBSOLETE. GEODETIC DATA (TM 5-237)

*Figure 246. Geographic position tabulation (DA Form 1961).*

**U. T. M. GRID COORDINATES FROM GEOGRAPHIC COORDINATES**
(TM 5-237)

| PROJECT | | ORGANIZATION | | | | | |
|---|---|---|---|---|---|---|---|
| Test | | A&S | | | | | |
| LOCATION | | SPHEROID | CENTRAL MERIDIAN | ZONE | UNIT | FE | |
| Missouri | | Clarke 1866 | 93° | 15 | Meter | 500,000 m. | |

| Station | ANUTT 1947 | | | Station | LENOX 1947 | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | 37° 42' 02."964 | $\lambda$ | 91° 44' 01."292 | $\phi$ | 37° 38' 47.807 | $\lambda$ | 91° 44' 17."976 |
| (I) | | $\Delta\lambda$ | 1° 15' 58.708 | (I) | | $\Delta\lambda$ | 1° 15' 42."024 |
| (II) | | $p^2$ | 0.2078182 | (II) | | $p^2$ | 0.2062998 |
| (III) | | $p^4$ | | (III) | | $p^4$ | |
| $A_6$ | | | | $A_6$ | | | |
| (IV) | | p | 0.4558708 | (IV) | | p | 0.4542024 |
| (V) | | $p^5$ | | (V) | | $p^5$ | |
| $B_6$ | | | | $B_6$ | | | |
| C | + 2788."113 | C | +0° 46' 28.113 | C | + 2774.505' | C | +0° 46' 14."505' |
| N | 4173171.12 | E | 611633.67 | N | 4167150.96 | E | 611306.05 |

| Station | ANUTT | | | Station | LENOX | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | | $\lambda$ | | $\phi$ | | $\lambda$ | |
| (I) | | $\Delta\lambda$ | | (I) | | $\Delta\lambda$ | |
| (II) | | ~~$p^2$~~ | $q^2$ = 0.01246208 | (II) | | ~~$p^2$~~ | $q^2$ = 0.01238904 |
| (III) | | $p^4$ | | (III) | | $p^4$ | |
| $A_6$ | | | | $A_6$ | | | |
| (IV) | | ~~p~~ | q = 0.11163367 | (IV) | | ~~p~~ | q = 0.11130605 |
| (V) | | $p^5$ | | (V) | | $p^5$ | |
| $B_6$ | | | | $B_6$ | | | |
| k | 0.99975348 | | | k | 0.99975258 | | |
| N | | E | mean k = 0.99975303 | N | | E | |

*Figure 247. Computation of UTM grid coordinates, convergence, and scale factor (DA Form 1932).*

$\Delta$N's + corrections and $\Delta$E's + corrections of each course.

*f.* In some types of work, angles and distances may be measured to points not on the traverse itself. Points set up in this manner are known as side shots. The azimuth to start the computation should theoretically be obtained from a grid inverse between adjusted coordinates, but for practical purposes this is not necessary and the adjusted azimuth from the traverse computation is used instead. The measured distance to the side shot must still be reduced to the grid distance by using the combined factor for scale and sea level. The actual computation of the position is made on the same form as used for the traverse. No closing error can be found or adjustment made for points set up in this manner.

*g.* At this point, a word of caution is needed regarding loop traverses. Loop traverses begin and end on the same point and the same azimuth with no azimuth check or position tie except at the end point. Under these circumstances it is possible to have good azimuth and position closures and still have a large error in the positions of the traverse points. This can occur because the starting data is also the closing data, and an error in the starting data cannot be discovered by a bad closure. A good closure of a loop traverse merely indicates that the field work is good and does not guarantee that the fixed data is correct; therefore, the fixed data for a loop traverse should be very carefully checked.

| PROJECT | BOOK NR. | REDUCTION OF TAPED DISTANCES (TM 5-237) | | | |
|---|---|---|---|---|---|
| Test Traverse | Vol. 30 | | | | |
| LOCATION | TRAVERSE NR. | FROM STA. | | TO STA. | |
| Missouri | 3TT - 1 | LENOX 1947 | | ANUTT 1947 | |
| ORGANIZATION | TAPE | CAL. LENGTH | TEMP. | TENSION | SUPPORT T |
| A & S | 6602 | | | | |
| DATE | | CAL. LENGTH | TEMP. | TENSION | SUPPORT 2 |
| 13 June 54 | | 199.956 | 68° | 20# | |

| SECTION | SUPPORT | MEASURED DISTANCE | TEMP. | TEMP. CORRECTION | CALIBRATION CORRECTION | DIFFERENCE OF ELEVATION % | SLOPE CORRECTION | CORRECTED FIELD DISTANCE |
|---|---|---|---|---|---|---|---|---|
| LENOX | 2' | 200.000' | 57° | | | 4.0' | -0.160' | LENOX |
| to | | 200.000' | | | | 4.0' | -0.160' | to #1 |
| #1 | | 200.000' | | | | – | | |
| | | 200.000' | | total | total | – | total | |
| | | 78.000' | | -0.062 | -0.176 | – | -0.320' | 877.442 |
| | | | | | | | | |
| #1 | 2' | 200.000' | 60.5 | | | – | | #1 |
| to | | 200.000' | | | | 2.0' | -0.040' | to |
| #2 | | 200.000 | | total | total | – | total | #2 |
| | | 150.865 | | -0.036' | -0.148' | – | -0.040' | 750.641 |
| | | | | | | | | |
| #2 | 2' | 200.000' | 60.5 | | | 2.0' | -0.040' | #2 |
| to | | 200.000' | | | | 2.4' | -0.058' | to |
| #3 | | 200.000' | | total | total | 2.2' | -0.048' | #3 |
| | | 89.100' | | -0.033 | -0.132' | | -0.146' | 688.789 |
| | | | | | | | | |
| #3 | 2' | 200.000' | 59.5 | | | – | | |
| to | | 200.000' | | | | – | | #3 |
| #4 | | 200.000' | | | | – | | to |
| | | 200.000' | | | | – | | #4 |
| | | 200.000' | | total | total | – | total | |
| | | 190.000' | | -0.065' | -0.257' | – | – | 1189.678 |
| | | | | | | | | |
| #4 | 2' | 200.000' | 57.0 | | | – | | #4 |
| to | | 200.000' | | | | 2.2 | -0.048 | to |
| #5 | | 200.000' | | total | total | – | total | #5 |
| | | 42.810' | | -0.046' | -0.130 | – | -0.048' | 642.586 |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| W. C. A. | 17 June 54 | RAS | 17 June 54 |

DA FORM 1 FEB 57 1939

U. S. GOVERNMENT PRINTING OFFICE : 1957 O—420803

*Figure 248. Computation of traverse distances (DA Form 1939).*

**TRAVERSE COMPUTATION ON THE UNIVERSAL TRANSVERSE MERCATOR GRID** (TM 5-237)

PROJECT: Test Traverse
LOCATION: Ft Leonard Wood, Missouri
ORGANIZATION: AMS
TOTAL ANGULAR CLOSURE: -42.1
NR OF ANGLE STATIONS: 24
ANGULAR CLOSURE PER STATION: 1.75 = 11@1.7, 13@1.8
FROM STATION: LENOX 1947 USCGS
TO STATION: ANUTT 1947 USCGS
BOOK NUMBER: Vols 30 & 50
LINEAR CLOSURE: 0.269154 m.
LENGTH OF TRAVERSE: 8192.827 m.
LINEAR CLOSURE RATIO: 1/32,000
ZONE: 15
UNIT: METERS
TRAVERSE NUMBER: 3π-1
PAGE NR: 1
NR OF PAGES: 2

| Station | Observed Angle ° ′ ″ | Adjusted Angle ′ ″ | Azimuths ° ′ ″ | Corrected Field Distances (meters) | Scale Factor × Sea Level Coefficient | Grid Distances | Cosine | Sine | Unadjusted ΔN | Unadjusted ΔE | Adjusted Grid Coordinates N | Adjusted Grid Coordinates E | Station |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LENOX Az. Mk | | | (Back Az.) | | | | | Corr. per meter = | -0.000020140 | +0.000035022 | | | |
| LENOX | 359 51 59.5 (1.8) | | 280 44 31.6 | | | | | | | | 4 167 150.96 | 611 306.05 | LENOX |
| HPT 315-Z Sta. #1 | (thru) | 52 01.3 | 280 36 32.9 | 267.445 | | 267.363 | 0.184109 | 0.982906 | +49.224 -5 | -262.793 +7 | | | HPT 315-Z |
| Sta. #2 | 188 34 34.8 (1.7) | 36.5 | " | 228.796 | | 228.726 | " | " | +42.111 -5 | -224.816 +6 | 4 167 200.18 | 611 043.26 | Sta. #1 |
| Sta. #3 | 168 15 25.5 (1.8) | 27.3 | 289 11 09.4 | 209.943 | | 209.878 | 0.328633 | 0.944458 | +68.973 -4 | -198.221 +5 | 4 167 242.28 | 610 818.45 | Sta. #2 |
| HPT 315-Y Sta. #4 | (thru) | (thru) | 277 26 36.7 | 362.615 | | 362.503 | 0.129550 | 0.991573 | +46.962 -7 | -359.408 +9 | 4 167 311.25 | 610 620.24 | Sta. #3 |
| Sta. #5 | 189 43 02.8 (1.7) | 04.5 | " | 183.213 | S.F. x S.L. = 0.999692 | 183.157 | " | " | +23.728 -4 | -181.614 +5 | 4 167 358.21 | 610 260.80 | HPT 315-Y Sta. #4 |
| Sta. #6 | 181 10 48.8 (1.8) | 50.6 | 287 09 41.2 | 195.861 | | 195.801 | 0.295064 | 0.955477 | +57.774 -4 | -187.083 +5 | 4 167 381.93 | 610 079.19 | Sta. #5 |
| Sta. #7 | 196 36 34.0 (1.7) | 35.7 | 288 20 31.8 | 373.047 | | 372.932 | 0.314692 | 0.949194 | +117.359 -8 | -353.985 +9 | 4 167 439.70 | 609 892.11 | Sta. #6 |
| Sta. #8 | 216 56 06.5 (1.8) | 08.3 | 304 57 07.5 | 245.021 | | 244.946 | 0.572893 | 0.819630 | +140.328 -5 | -200.765 +6 | 4 167 557.05 | 609 538.14 | Sta. #7 |
| HPT 315-W Sta. #9 | 201 49 55.5 (1.7) | 57.2 | 341 53 15.8 | 182.234 | | 182.178 | 0.950449 | 0.310879 | +173.151 -6 | -56.635 +5 | 4 167 697.38 | 609 337.38 | Sta. #8 |
| Sta. #10 | 192 04 36.8 (1.8) | 38.6 | 03 43 13.0 | 261.521 | Mean Scale Factor = 0.999753 | 261.440 | 0.997893 | 0.064885 | +260.889 -5 | +16.964 +7 | 4 167 870.52 | 609 280.75 | HPT 315-W Sta. #9 |
| Sta. #11 | 185 55 52.0 (1.8) | 54.6 | 15 47 51.6 | 366.013 | | 365.900 | 0.962229 | 0.272243 | +352.080 -7 | +99.614 +9 | 4 168 131.41 | 609 297.72 | Sta. #10 |
| Sta. #12 | 185 32 48.0 (1.8) | 49.8 | 21 43 46.2 | 261.563 | | 261.482 | 0.928942 | 0.370224 | +242.902 -5 | +96.807 +7 | 4 168 483.48 | 609 397.34 | Sta. #11 |
| CWA 339 Sta. #13 | 157 38 42.5 (1.7) | 44.2 | 27 16 36.0 | 481.453 | | 481.305 | 0.888804 | 0.458288 | +427.786 -9 | +220.576 +12 | 4 168 726.38 | 609 494.16 | Sta. #12 |
| CWA 338 Sta. #14 | 200 41 43.5 (1.8) | 45.3 | 04 55 20.2 | 574.654 | Sea Level Coefficient = 0.999939 | 574.477 | 0.996312 | 0.085803 | +572.358 -12 | +49.292 +14 | 4 169 154.16 | 609 714.74 | CWA 339 Sta. #13 |
| HPT 315-U Sta. #15 | (thru) | (thru) | 25 37 05.5 | 354.675 | | 354.566 | 0.901694 | 0.432374 | +319.710 -7 | +153.305 +9 | 4 169 726.50 | 609 764.05 | CWA 338 Sta. #14 |
| Sta. #16 | 187 42 20.0 (1.7) | 21.7 | " | 185.892 | | 185.835 | " | " | +167.566 -4 | +80.350 +5 | 4 170 046.20 | 609 917.36 | HPT 315-U Sta. #15 |
| Sta. #17 | 173 05 04.8 (1.8) | 06.6 | 33 19 27.2 | 227.678 | | 227.608 | 0.835576 | 0.549375 | +190.184 -5 | +125.042 +6 | 4 170 213.77 | 609 997.72 | Sta. #16 |
| Sta. #18 | 176 57 53.5 (1.7) | 55.2 | 26 24 33.8 | 538.015 | | 537.849 | 0.895638 | 0.444783 | +481.718 -11 | +239.226 +13 | 4 170 403.94 | 610 122.77 | Sta. #17 |
| Sta. #19 | 173 43 45.5 (1.8) | 47.3 | 23 22 29.0 | 523.361 | | 523.200 | 0.917930 | 0.396743 | +480.261 -10 | +207.576 +13 | 4 170 885.65 | 610 362.00 | Sta. #18 |
| Sta. #20 | 178 21 38.2 (1.7) | 39.9 | 17 06 16.3 | 289.152 | | 289.063 | 0.955770 | 0.294114 | +276.278 -6 | +85.017 +7 | 4 171 365.90 | 610 569.59 | Sta. #19 |
| Sta. #21 | 186 38 19.2 (1.8) | 21.0 | 15 27 56.2 | 427.053 | | 426.921 | 0.963791 | 0.266659 | +411.463 -9 | +113.842 +11 | 4 171 642.18 | 610 654.62 | Sta. #20 |
| Sta. #22 | 209 55 33.0 (1.7) | 34.7 | 22 06 17.2 | 336.675 | | 336.571 | 0.926498 | 0.376301 | +311.832 -7 | +126.652 +12 | 4 172 053.63 | 610 768.47 | Sta. #21 |
| Sta. #23 | 172 25 55.8 (1.8) | 57.6 | 52 01 51.9 | 555.202 | | 555.031 | 0.615234 | 0.788345 | +341.474 -11 | +437.556 +16 | 4 172 365.45 | 610 895.13 | Sta. #22 |
| Sta. #24 | 166 08 29.8 (1.7) | 31.5 | 44 27 49.5 | 242.959 | | 242.884 | 0.713692 | 0.700460 | +173.344 -5 | +170.131 +8 | 4 172 706.92 | 611 332.70 | Sta. #23 |
| Sta. #25 | 166 19 20.8 (1.8) | 22.6 | 30 36 21.0 | 171.164 | | 171.111 | 0.860690 | 0.509129 | +147.274 -3 | +87.118 +4 | 4 172 880.26 | 611 502.84 | Sta. #24 |
| ANUTT | 103 49 38.5 (1.7) | 40.2 | 16 55 43.6 | 150.146 | | 150.100 | 0.956667 | 0.291185 | +143.596 -3 | +43.707 +4 | 4 173 027.53 | 611 589.96 | Sta. #25 |
| ANUTT Az. Mk | | | 300 45 23.8 Fixed | 8195.351 | Total | 8192.827 | | Comp. | Σ=+6020.325 | Σ=+327.415 | 4 173 171.12 | 611 633.67 | ANUTT |
| | | | Comp. 44 41.7 | | | | | Fixed | +6020.16 | +327.62 | | | |
| | | | Error -42.1 | | | | | Error | +0.165 | -0.205 | | | |
| | | | Corr. = +1.75 sec/omk | | | | | | | | | | |

COMPUTED BY: R.A. Smith — DATE: 6 June 54
CHECKED BY: W. C. Aumen — DATE: 6 June 54

DA FORM 1 FEB 57 1940

*Figure 249. Computation of traverse on UTM grid (DA Form 1940).*

## TRAVERSE COMPUTATION ON THE UNIVERSAL TRANSVERSE MERCATOR GRID

(TM 5-237)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| PROJECT: Test Traverse | TOTAL ANGULAR CLOSURE | | | | | | | TRAVERSE NUMBER: Side Shots 3TT-1 | |
| LOCATION: Ft. Leonard Wood, Missouri | NR OF ANGLE STATIONS | FROM STATION: LENOX 1947 USC&GS | | | TO STATION: ANUTT 1947 USC&GS | | | | |
| ORGANIZATION: AMS | ANGULAR CLOSURE PER STATION | BOOK NUMBER: Vols. 30 & 50 | LINEAR CLOSURE | LENGTH OF TRAVERSE | LINEAR CLOSURE RATIO | ZONE: 15 | UNIT: METERS | PAGE NR: 2 | NR OF PAGES: 2 |

| Station | Observed Angle | Adjusted Angle | Azimuths | Corrected Field Distances | Scale Factor × Sea Level Coefficient | Grid Distances | Cosine | Sine | Unadjusted ΔN | Unadjusted ΔE | Adjusted Grid Coordinates N | Adjusted Grid Coordinates E | Station |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sta. #7 | | | | | | | | | | | | | |
| Sta. #8 | 185 13 31 | ——— | 304 57 08 | Subtense dist | | | | | | | | | |
| NPT 345 K Sta. #8-A | | | 310 10 39 | 76.240 | | 76.217 | 0.645158 | 0.764049 | + 49.172 | - 58.234 | 4 167 697.38 | 609 337.38 | Sta. #8 |
| | | | | | 0.999692 | | | | | | 4 167 746.55 | 609 279.15 | NPT 315 K Sta. #8-A |
| Sta. #12 | | | | | | | | | | | | | |
| CWA 339 Sta. #13 | 11 27 02 | ——— | 27 16 36 | | | | | | | | | | CWA 339 Sta. #13 |
| NPT 345 V Sta. #13-A | | | 218 43 38 | 43.215 | | 43.202 | 0.780133 | 0.625613 | -33.703 | - 27.028 | 4 169 154.16 | 609 714.74 | NPT 315 V Sta. #13-A |
| | | | | | | | | | | | 4 169 120.46 | 609 687.71 | |

COMPUTED BY: Aldmith — DATE: 6 June 54 — CHECKED BY: W. C. Aumen — DATE: 6 June 54

DA, FORM 1, FEB 57 1940

*Figure 249*—Continued.

# CHAPTER 15

# COMPUTATIONS ON THE UNIVERSAL POLAR STEREOGRAPHIC GRID

## Section I. UNIVERSAL POLAR STEREOGRAPHIC TRANSFORMATIONS

### 171. Tables Needed

The transformation of geographic coordinates to Universal Polar Stereographic (UPS) grid coordinates and the reverse computation (UPS grid coordinates to geographic coordinates) can best be performed on DA Form 1953, Universal Polar Stereographic Transformations (fig. 250). The procedure is very similar to UTM transformation procedure, in that, part of the computation involves interpolation for values in previously prepared tables. A table of the natural trigonometric functions, sine, cosine, tangent, and cotangent, preferably to eight decimals, is required for the conversions. TM 5–241–9 tabulates the value of R and scale factor for each minute of latitude from 79°30′ to 90° with the difference for 1 second of latitude for each function tabulated in a separate column.

### 172. Geographic Coordinates to UPS Grid Coordinates

*a.* To transform geographic coordinates to UPS grid coordinates, enter the location and zone (north or south) at the top of the form for the station or stations involved. Enter the station name, latitude ($\phi$) and longitude ($\lambda$) for each station in the spaces provided. Be sure to denote the longitude as east or west. Obtain the sine and cosine of $\lambda$ from the 8-place natural functions table. Obtain the value of R from TM 5–241–9 interpolating for seconds of latitude. Enter the final value of R on the form. N′, which is to be added or subtracted from the false Northing, is the product of R times cos $\lambda$. In the North zone, N′ is subtracted from the FN if $\lambda$ is less than 90°. N′ is added to the FN if $\lambda$ is greater than 90°. E′, which is added or subtracted from the false Easting, is the product of R times sin $\lambda$. In both north and south zones, E′ is added to the FE if $\lambda$ is east, and subtracted from the FE if $\lambda$ is west. In the south zone, N′ is added to the FN if $\lambda$ is less than 90°. N′ is subtracted from the FN if $\lambda$ is more than 90°. In both the north and south zones, the false Northing is 2,000,000 meters and the false Easting is 2,000,000 meters.

*b.* TM 5–241–9 states the precision attainable in this computation.

### 173. UPS Grid Coordinates to Geographic Coordinates

*a.* To transform UPS grid coordinates to geographic coordinates, again use DA Form 1953 (fig. 250). Enter the location, station name, zone, Northing, and Easting of the station. N′ and E′ are the differences between N and the FN, and E and the FE, respectively. The longitude of the station is found from the expressions:

$$\tan \lambda = \frac{E'}{N'} \text{ or } \cot \lambda = \frac{N'}{E'}.$$

If N′ is greater than E′, use the expression $\tan \lambda = \frac{E'}{N'}$; but if E′ is greater than N′, use $\cot \lambda = \frac{N'}{E'}$. The angle corresponding to these tangents or cotangents is now taken from the table of natural functions. In the north zone, if N is less than the FN, the angle obtained from the table is the longitude. If N is greater than the FN, subtract the angle from the table from 180° to obtain the longitude. If E is greater than the FE, the longitude is east. If E is less than the FE, the longitude is west. Find the value of R by dividing E′ by the sine of the longitude. Using this value of R, find the latitude of the station by inverse interpolation in the tables. For the determination of the longitude in the south zone, use the same formulas for

| PROJECT | | UNIVERSAL POLAR STEREOGRAPHIC TRANSFORMATIONS (TM 5-237) | | | |
|---|---|---|---|---|---|
| 62234 | | | | | |
| LOCATION | | ORGANIZATION | | | |
| Alaska | | A&S | | | |
| ZONE | UNIT | FALSE NORTHING (FN) | | FALSE EASTING (FE) | |
| North | Meter | 2,000,000 Meters | | 2,000,000 Meters | |

Geographic to Grid

| STATION | Hypo | | | STATION | | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | 84° 17′ 14.″042 N | | | $\phi$ | | | |
| $\lambda$ | 132° 14′ 52.″761 W | | | $\lambda$ | | | |
| sin $\lambda$ | 0.74024173 | cos $\lambda$ | 0.67234083 | sin $\lambda$ | | cos $\lambda$ | |
| tan $\lambda$ | | cot $\lambda$ | | tan $\lambda$ | | cot $\lambda$ | |
| R | 634,791.746 | | | R | | | |
| N′ | 426,796.41 | E′ | -469,899.34 | N′ | | E′ | |
| N | 2,426,796.41 | E | 1,530,100.66 | N | | E | |

Grid to Geographic

| STATION | Hypo | | | STATION | | | |
|---|---|---|---|---|---|---|---|
| $\phi$ | 84° 17′ 14.″042 N | | | $\phi$ | | | |
| $\lambda$ | 132° 14′ 52.″760 W | | | $\lambda$ | | | |
| sin $\lambda$ | 0.74024173 | cos $\lambda$ | | sin $\lambda$ | | cos $\lambda$ | |
| tan $\lambda$ | | cot $\lambda$ | 0.90827199 | tan $\lambda$ | | cot $\lambda$ | |
| R | 634,791.746 | | | R | | | |
| N′ | 426,796.41 | E′ | -469,899.34 | N′ | | E′ | |
| N | 2,426,796.41 | E | 1,530,100.66 | N | | E | |

GEOGRAPHIC COORDINATES TO UPS GRID COORDINATES

| NORTH ZONE | SOUTH ZONE |
|---|---|
| $\lambda$ less than 90° subtract N′ from FN | $\lambda$ less than 90° add N′ to FN |
| $\lambda$ greater than 90° add N′ to FN | $\lambda$ greater than 90° subtract N′ from FN |
| $\lambda$ east, E′ is plus $\lambda$ west, E′ minus | $\lambda$ east, E′ plus $\lambda$ west, E′ minus |

$N = FN + N'$ $\quad N' = R \cos \lambda$ $\quad E = FE + E'$ $\quad E' = R \sin \lambda$

UPS GRID COORDINATES TO GEOGRAPHIC COORDINATES

BOTH ZONES

$N' = N - 2{,}000{,}000$ $\quad$ If N′ greater than E′ use $\tan \lambda = \frac{E'}{N'}$

$E' = E - 2{,}000{,}000$ $\quad$ If E′ greater than N′ use $\cot \lambda = \frac{N'}{E'}$

| NORTH ZONE | SOUTH ZONE |
|---|---|
| If N less than FN use $\lambda$ as solved | If N less than FN subtract $\lambda$ from 180° |
| If N greater than FN subtract $\lambda$ from 180° | If N greater than FN use $\lambda$ as solved |
| If E less than FE $\lambda$ is west | If E less than FE $\lambda$ is west |

$R = \frac{E'}{\sin \lambda}$ $\quad$ $\phi$ by inverse interpolation of R

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R A Smith | 13 June 54 | W. C. Aumen | 13 June 54 |

DA FORM 1 FEB 57 1953

*Figure 250. Conversion of geographic coordinates to UPS coordinates and conversion of UPS coordinates to geographic coordinates (DA Form 1933).*

tan $\lambda$ and cot $\lambda$. If N is less than the FN, subtract the angle from the table from 180°. If N is greater than the FN, the longitude is the angle taken from the table. The same rule for determining east and west longitude applies for both north and south zones. The latitude is found as in the north zone.

*b.* Always use the decimal part of the seconds of longitude when looking up the sine and cosine values.

## Section II. UPS SCALE FACTOR AND CONVERGENCE

### 174. Scale Factor

The scale factor ($k$) on the UPS grid is extracted and interpolated from the tables in TM 5–241–9 with the latitude of the station as the argument. As in the UTM grid, the scale factor is a multiplier used to convert geodetic distances to grid distances.

### 175. Convergence

*a.* Convergence of the meridians on the Universal Polar Stereographic grid differs from convergence on other grids because of the position of the pole at the center of the projection. In a stereographic projection of this type, the meridians converge with angular values numerically equal to the difference in longitude of two given positions. Grid declinations, therefore, increase from 0° at the zero meridian to 180° at the 180° meridian in both an east and west direction.

*b.* The grid declinations (convergences) carry a directional postscript such as E or W. In the north zone, the postscripts read the same as the longitude of any given position. For example, the grid convergence (C) at $\phi$=85° N, $\lambda$=35° E, reads C=35° E. In the south zone, the postscripts read the opposite of the given longitude. For example, the grid convergence at $\phi$=85° S, $\lambda$=35° E, reads C=35° W.

# CHAPTER 16

# OTHER GRID SYSTEMS

## Section I. TRANSVERSE MERCATOR PROJECTION

### 176. Tables Needed

*a.* The basic formulas given for the Universal Transverse Mercator grid in chapter 13 apply to any Transverse Mercator Projection. The UTM is treated separately because it is world-wide in application and the tables for use on the UTM grid are designed expressly for UTM grid and no other.

*b.* The Transverse Mercator Projection is particularly adaptable for use in areas with greatest extent in a north-south direction. It is, therefore, one of the three projections used for the state plane grid systems in the United States. Since each state system is a separate projection, tables have been prepared by the USC&GS for the conversion of geographic to grid coordinates and from grid to geographic coordinates for each state, including Alaska and Hawaii. Tables such as these are practically a necessity for computations on any Transverse Mercator grid.

### 177. Conversion of Geographic to State Plane Grid Coordinates (Transverse Mercator)

*a.* The example is taken from USC&GS Sp. Pub. 256, the projection tables for the state of Maine. Using similar tables, conversion of coordinates can be made on any Transverse Mercator projection using the forms illustrated, which are DA Form 1954, Plane Coordinates from Geographic Coordinates on the Transverse Mercator Projection (Calculating Machine Computation) (fig. 251), and DA Form 1955, Geographic Coordinates from Transverse Mercator Grid Coordinates (Calculating Machine Method) (fig. 252).

*b.* The basic formulas for use with this type of table, when converting geographic to plane grid coordinates, are:

$$X = x' + FE$$

$$X = H\Delta\lambda'' \pm ab$$

$$Y = y_0 + V\left(\frac{\Delta\lambda''}{100}\right)^2 + c$$

Grid azimuth = geodetic azimuth $-\Delta\alpha$ — second term

$$\Delta\alpha'' = \Delta\lambda'' \sin\phi + g$$

The functions H, V, and $a$ are based on the latitude of the geographic position as the argument. The functions $b$, $c$, and $g$ are based on the numerical value of $\Delta\lambda''$ ($\Delta\lambda$ in seconds) as the argument. $\Delta\lambda$ = central meridian $-\lambda$.

*c.* The algebraic sign of $\Delta\lambda$ is required for the computation of X. The functions $a$ and $b$ each carry an algebraic sign which affects the computation of X. When the product of $ab$ is minus ($-$), H times $\Delta\lambda$ is reduced numerically; and when the product of $ab$ is plus ($+$), H times $\Delta\lambda$ is increased numerically.

*d.* On the form, $y_0$ is designated as "Tabular $y$".

*e.* All state plane grid coordinates are in feet, and all state plane grid azimuths are referred to south.

### 178. Conversion of State Plane Grid Coordinates to Geographic Coordinates (Transverse Mercator)

*a.* To convert state plane grid coordinates to geographic coordinates, the following equations are used:

$$P(x'/10{,}000)^2 + d = V(\Delta\lambda''/100)^2 + C$$

$$y_0 = Y - P(x'/10{,}000)^2 - d$$

PROJECT 5-4291

ORGANIZATION A&S

LOCATION Maine

**PLANE COORDINATES FROM GEOGRAPHIC COORDINATES ON THE TRANSVERSE MERCATOR PROJECTION. (CALCULATING MACHINE COMPUTATION)**
(TM 5-237)

ZONE East

CENTRAL MERIDIAN 68° 30′ 00.″000

| STATION | Libby, 1941 | Michaud, (USGS) 1942 | | | |
|---|---|---|---|---|---|
| $\phi$ | 46° 32′ 46.″920 | 47° 02′ 12.″659 | | | |
| $\lambda$ | 68 24 25.489 | 68° 37 29.366 | | | |
| $\Delta\lambda$=Central mer.−$\lambda$ | + 0 05 34.511 | − 0 07 29.366 | | | |
| $\Delta\lambda''$ | + 334.511 | − 449.366 | | | |
| $\left(\frac{\Delta\lambda''}{100}\right)^2$ | 11.190 | 20.193 | | | |
| $H$ | 69.892342 | 69.260175 | | | |
| $V$ | 1.230114 | 1.228831 | | | |
| $a$ \| $b$ | +0.635 \| +0.173 | +0.837 \| +0.231 | | | |
| $x'=H\cdot\Delta\lambda \pm ab$ | + 23379.87 | − 31123.36 | | | |
| $V\left(\frac{\Delta\lambda''}{100}\right)^2 \pm c$ | 13.763 | 24.810 | | | |
| Tabular $y$ | 989111.64 | 1167981.76 | | | |
| $X$ | 523379.87 | 468876.64 | | | |
| $Y$ | 989125.40 | 1168006.57 | | | |
| $\Delta\alpha''$ | + 242.83 | − 328.84 | | | |
| $\Delta\alpha$ | + 04 02.8 | − 05 28.8 | | | |
| Geod. Az. to Az. Mk. | 281 27 50.4 | 187 11 59.5 | | | |
| Grid Az. to Az. Mk. | 281 23 48 | 187 17 28 | | | |

$X=x'+500{,}000$

$Y=$Tab. $y+V\left(\frac{\Delta\lambda''}{100}\right)^2 \pm c$

When $ab$ is $\begin{matrix}-\\+\end{matrix}$, decrease / increase $H\cdot\Delta\lambda$ numerically

$\Delta\alpha''=\Delta\lambda''\sin\phi+g$ — $g$ increases $\Delta\lambda''\sin\phi$ numerically

Grid Az.=Geod. Az.−$\Delta\alpha$

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.Smith | 12 Nov 55 | W. C. Aumen | 12 Nov. 55 |

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*Figure 251. Conversion of geographic coordinates to Transverse Mercator coordinates (DA Form 1954).*

*b*. The latitude of the point is then found from the tables of $y_0$. Obtain H and $a$ from the tables using the latitude as the argument.

$$x'=\text{X}-500{,}000$$

$$\text{Approximate } \Delta\lambda''=\frac{x'}{\text{H}}$$

Obtain $b$ from the tables using the approximate $\Delta\lambda''$ as the argument. Then:

$$\Delta\lambda''=\frac{x'\pm ab}{\text{H}}$$

$$\lambda=\text{central meridian}-\Delta\lambda$$

When the product of $ab$ is plus (+), it numerically increases $x'$; and when the product of $ab$ is minus (−), it numerically decreases $x'$.

*c*. In the computation, the factor P is taken from the table using Y as the argument, and $d$ is taken from the table using $x'$ as the argument. Notice the algebraic sign of $d$.

| PROJECT | GEOGRAPHIC COORDINATES FROM TRANSVERSE MERCATOR GRID COORDINATES. (CALCULATING MACHINE METHOD) (TM 5-237) | |
|---|---|---|
| 5-4291 | | |
| ORGANIZATION: A&S | | |
| LOCATION: Maine | ZONE: East | CENTRAL MERIDIAN: 68° 30' 00".000 |

| STATION | Dun, 1944 | | |
|---|---|---|---|
| $X$ | 592 192.30 | $Y$ | 204 303.46 |
| $C$ | − 500 000.00 | $P\left(\frac{x'}{10,000}\right)^2+d$ | − 198.52 |
| $x'$ | + 92 192.30 | $y_o$ | 204 104.94 |
| $P$ | 2.33545 | Approx.$\Delta\lambda = x' \div H$ | +1270" |
| $d$ | +0.02 | $\Delta\lambda = (x' \mp ab) \div H$ | +1269.768 |
| $H$ | 72.605763 | $\Delta\lambda$ | + 0° 21' 09".768 |
| $a$ \| $b$ | −0.292 \| +0.611 | Central Meridian | 68° 30' 00".000 |
| $\phi$ | 44° 23' 35".807 | $\lambda$=C.M.−$\Delta\lambda$ | 68° 08' 50".232 |

| STATION | Pendleton, 1934 | | |
|---|---|---|---|
| $X$ | 397 824.29 | $Y$ | 170 788.98 |
| $C$ | − 500 000.00 | $P\left(\frac{x'}{10,000}\right)^2+d$ | − 243.06 |
| $x'$ | − 102 175.71 | $y_o$ | 170 545.92 |
| $P$ | 2.32800 | Approx.$\Delta\lambda = x' \div H$ | −1405" |
| $d$ | +0.02 | $\Delta\lambda = (x' \mp ab) \div H$ | −1405.069 |
| $H$ | 72.719511 | $\Delta\lambda$ | − 0° 23' 25".069 |
| $a$ \| $b$ | −0.333 \| +0.665 | Central Meridian | 68° 30' 00".000 |
| $\phi$ | 44° 18' 04".381 | $\lambda$=C.M.−$\Delta\lambda$ | 68° 53' 25".069 |

| STATION | | | |
|---|---|---|---|
| $X$ | | $Y$ | |
| $C$ | − | $P\left(\frac{x'}{10,000}\right)^2+d$ | − |
| $x'$ | | $y_o$ | |
| $P$ | | Approx.$\Delta\lambda = x' \div H$ | " |
| $d$ | | $\Delta\lambda = (x' \mp ab) \div H$ | " |
| $H$ | | $\Delta\lambda$ | ° ' " |
| $a$ \| $b$ | | Central Meridian | |
| $\phi$ | ° ' " | $\lambda$=C.M.−$\Delta\lambda$ | ° ' " |

When $ab$ is $\frac{+, \text{decrease}}{-, \text{increase}}$ $x'$ numerically

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| R.A. Smith | 12 Nov. 55 | W. C. Aumen | 12 Nov. 55 |

DA FORM 1 FEB 57 1955

*Figure 252. Conversion of Transverse Mercator coordinates to geographic coordinates (DA Form 1955).*

## Section II. LAMBERT CONICAL CONFORMAL PROJECTION

### 179. Data Needed

*a.* The Lambert Conical Conformal type of projection is used with one or two standard parallels in areas extending mainly in an east-west direction. The projection provides very simple formulas for the conversion of geographic to grid coordinates and vice versa, once certain basic tables are computed. This projection is one of the two types used by the USC&GS for the state plane grid systems in the United States.

*b.* One of the basic tables contains the R's, the radii of the developed parallels. For the state grids, certain constants are also computed. They are $R_b$, the radius of the developed parallel of the lower limit of the projection and $l$. In the simplest projection where there is only one standard parallel, $l$ is the sine of the latitude of the parallel of tangency of the cone and the spheroid. When there are two standard parallels, the expression for $l$ is a little more complex.

### 180. Plane Grid Coordinates From Geographic Coordinates (Lambert)

*a.* Having the basic table of R's and the constants $R_b$ and $l$, the equations for computing X and Y coordinates from geographic coordinates are:

$X = R \sin \theta + C$

$Y = R_b - R \cos \theta$

Grid azimuth = geodetic azimuth $- \theta +$ second term

$\theta$ is the mapping angle for the longitude of the station and is computed from $\theta$ (in seconds) $= \Delta\lambda$ (in seconds) times $l$

$\Delta\lambda$ = central meridian $- \lambda$

C = false Easting for the system

State plane grid coordinates are in feet, and state plane grid azimuths are referred to south.

*b.* USC&GS tables for computations on the Lambert projection contain a table of $\theta$ angles with the longitude as the argument.

*c.* The sample computation shown on DA Form 1956, Plane Coordinates from Geographic Coordinates on the Lambert Projection (Calculating Machine Computation) (fig. 253), was taken from USC&GS Sp. Pub. 274, the projection tables for Massachusetts. Notice that the algebraic sign for X is used as previously stated. For this computation, the sine and cosine of $\theta$ should be carried to 10 decimal places.

### 181. Geographic Coordinates From Plane Grid Coordinates (Lambert)

*a.* For computing geographic coordinates from plane grid coordinates, the formulas are:

$$x' = X - C$$

$$\tan \theta = x' \div (R_b - y)$$

$$R = (R_b - y) \div \cos \theta$$

$$\Delta\lambda = \theta \div l$$

$$\lambda = \text{central meridian} - \Delta\lambda$$

*b.* DA Form 1957, Geographic Coordinates from Lambert Grid Coordinates (Calculating Machine Computation) (fig. 254), is used for this computation. The example was again taken from USC&GS Sp. Pub. 274.

*c.* The latitude of the point is interpolated from the table of R's. By affixing the correct algebraic sign to $x'$ and carrying the same sign through tan $\theta$, $\theta$, and $\Delta\lambda$, the longitude of the point will automatically come out of the equation, $\lambda = CM - \Delta\lambda$. Tan $\theta$ and cos $\theta$ should be carried to 10 decimal places. A table of sines, cosines, and tangents to 10 decimal places for angles from 0° to 6° is available as USC&GS Sp. Pub. 246, and the same tables are reproduced in TM 5–236.

*d.* As long as the appropriate tables are available, computations for any Lambert Conical projections can be made by these formulas.

| PROJECT | PLANE COORDINATES FROM GEOGRAPHIC COORDINATES ON THE LAMBERT PROJECTION (CALCULATING MACHINE COMPUTATION) (TM 5-237) |
|---|---|
| 2-6154 | |
| ORGANIZATION: A&S Inc. | |
| LOCATION: Massachusetts | ZONE: Mainland |

| $C$ | 600 000.00 | $R_b$ | 23 549 477.32 |
|---|---|---|---|

| STATION | Latitude / Longitude | $R$ / $\theta$ | sin $\theta$ / cos $\theta$ | $X$ / $Y$ |
|---|---|---|---|---|
| Pond, 1934 | 41 40 15.808 | 23 304 946.92 | +0.0123074278 | 886 823.95 |
| | 70 27 00.716 | +0 42 18.6533 | 0.9999242607 | 246 295.50 |
| | Grid azimuth to azimuth mark | | | 339° 23' 29" |
| Bald Peak 1836, r. 1938 | 42 06 06.860 | 23 147 948.59 | -0.0226615283 | 75 432.11 |
| | 73 25 59.173 | -1 17 54.6759 | 0.9997431946 | 407 473.25 |
| | Grid azimuth to azimuth mark | | | 9° 33' 11" |
| | | | | |
| | | | | |
| | Grid azimuth to azimuth mark | | | |
| | | | | |
| | | | | |
| | Grid azimuth to azimuth mark | | | |

| $X=R \sin \theta + C$ | $Y=R_b - R \cos \theta$ | Grid Az.=Geod. Az.$-\theta$ |
|---|---|---|

| COMPUTED BY | DATE | CHECKED BY | DATE |
|---|---|---|---|
| RAdmith | 18 Oct. 55 | W. C. Auman | 18 Oct. 55 |

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*Figure 253. Conversion of geographic coordinates to Lambert coordinates.*

| PROJECT | 2-6154 | GEOGRAPHIC COORDINATES FROM LAMBERT GRID COORDINATES. (CALCULATING MACHINE COMPUTATION) (TM 5-237) | |
|---|---|---|---|
| LOCATION | Massachusetts | | |
| ORGANIZATION | A&S | ZONE Mainland | $l$ 0.6717286561 |

| STATION | Pond, 1934 | | |
|---|---|---|---|
| $C$ | − 600 000.00 | $R_b$ | 23 549 477.32 |
| $X$ | 886 823.95 | $Y$ | − 246 295.50 |
| $x'=X-C$ | + 286 823.95 | $R_b-Y$ | 23 303 181.82 |
| $\tan\theta=x'\div(R_b-Y)$ | +0.0123083600 | $\theta$ | + 2538.6533 |
| $\theta$ | + 0° 42′ 18.6533″ | $\Delta\lambda=\theta\div l$ | + 3779.284 |
| $\cos\theta$ | 0.9999242607 | $\Delta\lambda$ | + 1° 02′ 59.284″ |
| $R=(R_b-Y)\div\cos\theta$ | 23 304 946.92 | Central Meridian | 71° 30′ 00.000″ |
| $\phi$ | 41° 40′ 15.808″ | $\lambda$=C. M.−$\Delta\lambda$ | 70° 27′ 00.716″ |

| STATION | Bald Peak, 1836, r. 1938 | | |
|---|---|---|---|
| $C$ | − 600 000.00 | $R_b$ | 23 549 477.32 |
| $X$ | 75 432.11 | $Y$ | − 407 473.25 |
| $x'=X-C$ | −524 567.89 | $R_b-Y$ | 23 142 004.07 |
| $\tan\theta=x'\div(R_b-Y)$ | −0.0226673493 | $\theta$ | −4674.6759″ |
| $\theta$ | − 1° 17′ 54.6759″ | $\Delta\lambda=\theta\div l$ | −6959.173″ |
| $\cos\theta$ | 0.9997431946 | $\Delta\lambda$ | − 1° 55′ 59.173″ |
| $R=(R_b-Y)\div\cos\theta$ | 23 147 948.59 | Central Meridian | 71° 30′ 00.000″ |
| $\phi$ | 42° 06′ 06.860″ | $\lambda$=C. M.−$\Delta\lambda$ | 73° 25′ 59.173″ |

| STATION | | | |
|---|---|---|---|
| | | | |

| $\cos\theta$ | | $\Delta\lambda$ | |
|---|---|---|---|
| $R=(R_b-Y)\div\cos\theta$ | | Central Meridian | ° ′ ″ |
| $\phi$ | ° ′ ″ | $\lambda$=C. M.−$\Delta\lambda$ | ° ′ ″ |

| COMPUTED BY | Rofsmith | DATE 18 Oct. 55 | CHECKED BY | W. C. Aumen | DATE 18 Oct. 55 |
|---|---|---|---|---|---|

DA FORM 1 FEB 57 1957

*Figure 254. Conversion of Lambert coordinates to geographic coordinates.*

## Section III. STATE PLANE COORDINATE SYSTEMS IN ALASKA

### 182. Description

*a.* For the purpose of establishing a state plane coordinate system, Alaska is divided into 10 zones. The coordinate system in each zone is based on a map projection which is determined by the orientation of the zone relative to the earth's meridian. It is impossible to reproduce exactly in a plane any portion of the earth's surface, but any curve on this surface can be mapped in its true length. A narrow strip on either side of this curve will be sufficiently undistorted to serve as a map on which distances can be measured to satisfy the demands of surveying accuracy without resorting to scale corrections.

*b.* In zone 1, the curve in question is the geodetic line passing through an arbitrary central point of the area and roughly bisecting the strip. The

resulting projection is an oblique Mercator projection of the spheroid. Zones 2–9 all have the same set of plane coordinates based on the Transverse Mercator projection in which the strips extend to both sides of a central meridian. The Transverse Mercator is a "Universal" projection in that the same set of coordinates can be used for any narrow strip following any arbitrary meridian, designated the central meridian.

*c.* In zone 10 the center line is a parallel of the spheroid near the center of the zone and the resulting projection is the well-known Lambert conformal.

### 183. Computation

The U.S. Department of Commerce, Coast and Geodetic Survey has printed Publications 65–1, Parts 49, 50, and 51 containing (2½-minute) Plane Coordinate Intersection Tables with interpolation formulas for all zones. No direct method of computation is available.

## Section IV. WORLD POLYCONIC PROJECTION

### 184. Description

Although the Universal Transverse Mercator grid system has been adopted as standard by the United States Army, many maps still carry as a secondary grid the former military grid system which is based on the polyconic projection and the Clarke 1866 Spheroid.

*a.* The World Polyconic system is divided into five bands of 73° longitude each, including a 1° overlap on each side. The limits of the bands are shown below.

| | | |
|---|---|---|
| Band I | 68°30′ W | 141°30′ W |
| Band II | 140°30′ W | 146°30′ E |
| Band III | 147°30′ E | 74°30′ E |
| Band IV | 75°30′ E | 2°30′ E |
| Band V | 3°30′ E | 69°30′ W |

*b.* Each band is divided into nine zones of 9° each, including 1° overlap on each side. The zones are lettered A through J omitting I, from right to left. For example, in Band 1 the limits of the zones are as follows:

| | | |
|---|---|---|
| Zone A | 68°30′ W | 77°30′ W |
| Zone B | 76°30′ W | 85°30′ W |
| Zone C | 84°30′ W | 93°30′ W |
| Zone D | 92°30′ W | 101°30′ W |
| Zone E | 100°30′ W | 109°30′ W |
| Zone F | 108°30′ W | 117°30′ W |
| Zone G | 116°30′ W | 125°30′ W |
| Zone H | 124°30′ W | 133°30′ W |
| Zone J | 132°30′ W | 141°30′ W |

*c.* Each zone has a central meridian. The central meridians are multiples of 8° beginning at 1° west of Greenwich and proceeding east or west. The Roman numeral of each band is followed by the letter N or S, to indicate northern or southern hemisphere.

*d.* The book necessary for the computations (AMS Key No. 201160) is Tables for Transformation of Geographic Coordinates to the Military Grid, which also contains the tables necessary for grid to geographic coordinates conversion. The tables are an assembly of three sections: 0° to 28°, 24° to 49°30′, and 48°30′ to 72°. In the northern hemisphere, the tables from 0° to 28° have values based on the origin at the equator, while the tables from 24° to 72° are based on an origin of 2,000,000 yards south of latitude 40°30′. All X origins are 1,000,000 yards west of the central meridian of the zone. The tables overlap 4° between 24° and 28°, and 1° along the 49th parallel. These overlaps provide for maps and surveys which may extend into the overlapping areas. *Caution should be exercised when entering overlapping parts of the tables to select values which are continuous within one of the sections of the tables.*

### 185. Geographic Coordinates to Grid Coordinates (Polyconic)

*a.* The formulas for the conversion of geographic coordinates to grid coordinates are as follows:

$$X = 1{,}000{,}000 + [H(\Delta\lambda') + ab]$$

$$Y = y_0 + V(\Delta\lambda')^2$$

Where:

$\Delta\lambda'$ = difference of longitude in minutes between the central meridian of the zone and the point

H = the approximate number of yards per minute of longitude at the latitude of the point

$ab$ = correction term for $H(\Delta\lambda')$, $a$ being the latitude function and $b$ a factor based on the value of $(\Delta\lambda')$ ($b$ takes its algebraic sign from $\Delta\lambda$; $a$ is always positive)

V = coefficient applied to $(\Delta\lambda')^2$ to obtain the correction for curvature of the parallel

$y_0$ = distance in yards from the origin to the point measured on the central meridian

*b.* The computation procedure is described below.

(1) Subtract algebraically the longitude of the point from that of the central meridian to obtain $\Delta\lambda$.
(2) Reduce $\Delta\lambda$ to minutes and decimals of a minute ($\Delta\lambda'$).
(3) Square $\Delta\lambda'$ and retain 2 decimal places.
(4) Enter the table for the value of H using the latitude as the argument, interpolating by multiplying $\Delta$H by the number of seconds.
(5) Obtain the value for $ab$. The value of $a$ may be read directly from the table using the latitude as the argument. $b$ is found in the table on the last page of each section using $\Delta\lambda'$ as the argument. The value of $b$ is obtained for the whole number of minutes, interpolating by multiplying $\Delta b$ by the decimals of a minute.

| Station: POINT | Location: Ontario | Bond: IN | Zone: A |
|---|---|---|---|
| Latitude | 47° 52' 12".742 | Longitude | 77° 05' 56".311 |
| H (minutes) | 1362.958 | Central Meridian | 73 00 00.000 |
| $\Delta$H/sec. x secs. (Lat) | .093 | $\Delta\lambda$ | -4 05 56.311 |
| H | 1362.865 | $\Delta\lambda'$ (min. & dec.) | -245.93852 |
| b (minutes of $\Delta\lambda$) | -33.8 | $(\Delta\lambda')^2$ | 60 485.76 |
| $\Delta b$ | 1.1 | V (minutes) | .147047 |
| b | -32.7 | $\Delta V10^6$/sec. x secs. (Lat) | - 2 |
| a | .985 | V | .147045 |
| ab | -32.21 | $Y_0$ (minutes) | 2 895 171.9 |
| H($\Delta X'$) | -335 181.00 | $\Delta Y_0$/sec. x secs. (Lat) | 430.39 |
| x' | -335 213.21 | $Y_0$ | 2 895 602.29 |
| | +1 000 000.00 | V($\Delta X'$)$^2$ | 8894.13 |
| X (yards) | 664 786.8 | Y (yards) | 2 904 496.4 |

*Figure 255. Conversion of geographic coordinates to Polyconic coordinates.*

(6) Determine the value of X by inserting in the basic formula the value determined above.
(7) Enter the table for the value of V using the latitude as the argument, interpolating by multiplying $\Delta V10^6$ by the number of seconds.
(8) Enter the table for $y_0$ using the latitude as the argument, interpolating by multiplying $\Delta y_0$ by the number of seconds.
(9) Determine the value of Y by inserting in the basic formula the values determined previously.

*c.* Figure 255 is an example of the conversion of geographic to grid coordinates.

## 186. Grid Coordinates to Geographic Coordinates (Polyconic)

*a.* It is often necessary to transform grid coordinates to geographic coordinates. On the World Polyconic grid system, the formulas for this transformation are:

$$P(x'/10{,}000)^2+ef=V(\Delta\lambda')^2$$

$$y_0=Y-V(\Delta\lambda')^2$$

$$\phi=y_0$$

$$\Delta\lambda=\frac{x'-ab}{H}$$

$\lambda$=longitude of central meridian $\pm\Delta\lambda$

*b.* The computation procedure is described below.

(1) Subtract 1,000,000 yards from X to obtain $x'$.
(2) Interpolate for P in the tables, using Y as the argument.
(3) Interpolate for $e$ in the tables, using Y as the argument.
(4) Interpolate for $f$ in the tables, using $x'$ as the argument; $f$ is found on the last page of each section.
(5) Divide $x'$ by 10,000 and square the result.
(6) Multiply P by $(x'/10{,}000)^2$ and add algebraically the product of $ef$.
(7) Subtract this result from Y to obtain $y_0$.
(8) Interpolate for $\phi$ (latitude) in the tables, using $y_0$ as the argument. This is an approximate $\phi$ and must be corrected later in the computation.
(9) Interpolate for H, V, and $a$, using $\phi$ as the argument.
(10) Divide $x'$ by H to obtain an approximate $\Delta\lambda'$.
(11) Interpolate for $b$, using the approximate $\Delta\lambda'$ as the argument.
(12) Subtract algebraically the product of $ab$ from $x'$ (this always reduces $x'$ numerically) and divide the result by H to obtain $\Delta\lambda'$ which takes the same sign as $x'$.
(13) Check the value of $b$ for this new $\Delta\lambda'$, and further correct $\Delta\lambda'$ if necessary.
(14) Square $\Delta\lambda'$ and multiply by V. Use this value to find a new $y_0$ and a final $\phi$.
(15) Convert $\Delta\lambda'$ to degrees, minutes, and seconds. Subtract algebraically the result from the longitude of the central meridian to obtain $\lambda$ (longitude).

| Station: POINT | Location: Ontario | Band: IN | Zone: A |
|---|---|---|---|
| x (yards) | 664 768.8 | y (yards) | 2 904 496.4 |
| | - 1 000 000 | $P(x'/10{,}000)^2+ef$ | - 8 894.13 |
| $x'$ | - 335 213.2 | $y^\circ$ | 2 895 602.27 |
| $a$ +0.985 | | $P$ (even 10,000 yds.) | 7.9038 |
| $b$ -32.7 $ab$ | - 32.21 | $\Delta P$ x fraction of $y$ | +103 |
| $x'-ab=H(\Delta\lambda')$ | - 335 180.99 | $P$ | 7.9141 |
| $H$ (minutes) | 1362.958 | $(x'/10{,}000)^2$ | 1123.679 |
| $\Delta H$ x seconds of $\phi$ | - .093 | $P(x'/10{,}000)^2$ | 8892.91 |
| $H$ | 1362.865 | $e$ =+1.67 $f$=+0.8 $ef$ | +1.34 |
| $V$ (minutes) | .147047 | $x'/H=(\Delta\lambda')$ approx. | -245.96 |
| $\Delta V$ x seconds of $\phi$ | - 2 | $(x'-ab)/H=(\Delta\lambda')$ | -245.93851 |
| $V$ | .147045 | Central Meridian | 73° 00′ 00″ |
| $(\Delta\lambda')^2$ | 60 485.75 | $\Delta\lambda$ | +4 05 56.311 |
| $V\times(\Delta\lambda')^2$ | 8894.13 | $\lambda$ (Longitude) | 77° 05′ 56.″311 |
| $\phi$ (Latitude) | 47° 52′ 12.″742 | | |

*Figure 256. Conversion of Polyconic coordinates to geographic coordinates.*

*c.* Figure 256 is an example of the conversion of grid to geographic coordinates.

*d.* For conversions from geographic coordinates to grid coordinates in the southern hemisphere, the equator has been given a Y value of 10,000,000 yards, in order that all Y values may increase toward the north. The values computed from the tables may be used to obtain coordinates in the southern hemisphere by subtracting these values from constants as follows:

Y*sh*=10,000,000—values obtained from section 1 (par. 184*d*) (0° to 28°)

Y*sh*=7,095,332.8—values obtained from section 2 (24° to 49°30′)

Y*sh*=7,095,331.7—values obtained from section 3 (48°30′ to 72°).

*e.* Scale corrections and corrections for reduction of azimuths are tabulated in the tables, and a complete explanation is included in the front of the tables.

*f.* For computations in the World Polyconic system using logarithms, the book, Military Grid Tables (AMS Key No. 201271) has been published. Complete explanation of all computations is included in the front of the tables.

# CHAPTER 17

# GRIDS AND DECLINATIONS FOR MAPS

## Section I. DIMENSIONS OF A GRID

### 187. Discussion

A grid is a network composed of two sets of uniformly spaced straight lines intersecting in right angles. To construct a map on a grid, the location of the four corners of the map sheet on the grid must be known. The information needed by the computer is the scale of the map, the type of grid, and the coordinates (geographic or grid) of the corners of the map sheet. If the coordinates are geographic, they must be transformed to grid coordinates, using the proper spheroid.

### 188. Computation

*a. Development of Formulas.* All formulas used in the computation of neatline lengths and grid plotting distances are developed using the Pythagorean theorem and the principle of similar triangles. The basic formula is—

$$L=CF\sqrt{(\Delta E)^2+(\Delta N)^2}$$

L=Neatline in terms of the map distance.
ΔE=Difference in Easting between two sheet corners.
ΔN=Difference in Northing between two sheet corners.
CF=Conversion factor necessary to change grid distance to map distance at a given scale.

$$CF=\frac{\text{unit}}{\text{scale}}$$

Using ratios from similar triangles and substituting in the basic formula, the following refined formulas are obtained:

$$n=n'\sqrt{1+\frac{(\Delta E)^2}{(\Delta N)^2}}\times(CF)\}\text{For East and West sides}$$

$$s=s'\sqrt{1+\frac{(\Delta E)^2}{(\Delta N)^2}}\times(CF)\}\text{For East and West sides}$$

$$x=x'\sqrt{1+\frac{(\Delta N)^2}{(\Delta E)^2}}\times(CF)\}\text{For North and South sides}$$

$$w=w'\sqrt{1+\frac{(\Delta N)^2}{(\Delta E)^2}}\times(CF)\}\text{For North and South sides}$$

$n$=Map distance along neatline from corner north to first full northing grid line.
$s$=Map distance along neatline from corner south to first full northing grid line.
$x$=Map distance along neatline from corner east to first full easting grid line.
$w$=Map distance along neatline from corner west to first full easting grid line.
$n'$=Grid distance from corner north to first full northing grid line.
$s'$=Grid distance from corner south to first full northing grid line.
$x'$=Grid distance from corner east to first full easting grid line.
$w'$=Grid distance from corner west to first full easting grid line.

Normally, the term under the radical in the above formulas is sufficiently close to 1 and may be omitted. The formulas for general use are:

$L=\sqrt{(\Delta E)^2+(\Delta N)^2}(CF)$.
$n=n'(CF)\}$For East and West sides.
$s=s'(CF)\}$For East and West sides.
$x=x'(CF)\}$For North and South sides.
$w=w'(CF)\}$For North and South sides.

*b. Procedure.* The procedure used generally starts with entering the grid coordinates on the form. From these coordinates, determine the

value of the first full grid line (grid line that cuts both sides of the sheet) nearest each corner of the sheet. Next, compute the conversion factor for changing grid distance to map distance of the given scale. This is found by dividing the number of inches in the grid unit (meter, yard, or foot) by the scale of the map. Eight decimal places are sufficient for this value. Now multiply the difference between the coordinates of the grid line and the corner of the sheet by the conversion factor to find the distance in inches. All these dimensions plus the total length (in inches) of each side of the sheet are entered on the form for use by the draftsman.

*c. Example.* Figure 257 is an example of the computation on DA Form 1941 (Grid and Declination Computations). It is at a scale of 1/25,000 on the UTM (Clarke 1866) grid in Zone 18, with a 1,000-meter grid interval. The coordinates of the sheet corners are taken from TM 5-241-11. The full grid lines are designated and the conversion factor computed. Then all dimensions are computed and entered. The boxed value is the total length of each edge (neatline) of the sheet. Be very careful to use the correct unit (meter, yard, or foot) when computing the conversion factor.

*d. Special Case.* When a map sheet is tilted to the grid, the refined formulas must be used, where:

$$s=n\sqrt{1+\left(\frac{\Delta E}{\Delta N}\right)^2} \text{ and}$$

$$s=e\sqrt{1+\left(\frac{\Delta N}{\Delta E}\right)^2}$$

$s$=Neatline distance from the map corner to the first EW or NS grid line. These grid lines do not fall wholly within the sheet.

$n$=Grid distance in northing from the corner to the first EW grid line.

$e$=Grid distance in easting from the corner to the first NS grid line.

Figure 258 shows a map tilted to the grid, with the solid lines representing the map and the dash lines representing the grid. The first NORTH-SOUTH or EAST-WEST grid line from the corner of the map sheet does not fall wholly within the sheet, but instead cuts the corner. The sheet sizes are found by using the Pythagorean theorem.

$$L=\sqrt{(\Delta E)^2+(\Delta N)^2}$$

Referring to figure 258 and to the formula for neatline grid distance, the following formulas are written:

$$S_1=e_1\sqrt{1+\left(\frac{N_1-N_4}{E_1-E_4}\right)^2}$$

$$S_2=e_1\sqrt{1+\left(\frac{N_2-N_1}{E_2-E_1}\right)^2}$$

$$S_3=n_2\sqrt{1+\left(\frac{E_2-E_1}{N_2-N_1}\right)^2}$$

$$S_4=n_2\sqrt{1+\left(\frac{E_3-E_2}{N_3-N_2}\right)^2}$$

$$S_5=e_3\sqrt{1+\left(\frac{N_2-N_3}{E_2-E_3}\right)^2}$$

$$S_6=e_3\sqrt{1+\left(\frac{N_3-N_4}{E_3-E_4}\right)^2}$$

$$S_7=n_4\sqrt{1+\left(\frac{E_3-E_4}{N_3-N_4}\right)^2}$$

$$S_8=n_4\sqrt{1+\left(\frac{E_1-E_4}{N_1-N_4}\right)^2}$$

All grid distances must be converted to the map distance of a given scale.

*e. Example of Tilted Sheet.* Figure 259 is an example of the grid computation of a tilted map sheet. Except for the neatline grid distance (the difference between the coordinates of the grid line and the corner of the sheet), the computations are identical to that of figure 257 discussed in *b* and *c* above.

Following are the computations for the neatline distances using the formulas in *d* above.

$$CF=0.0007874$$

$$S_1=559.75\sqrt{1+\left(\frac{37186.81}{26971.22}\right)^2}$$

$$=953.38M\times CF=0.751 \text{ inch}$$

$$S_2=559.75\sqrt{1+\left(\frac{37245.02}{54384.22}\right)^2}$$

$$=678.43M\times CF=0.534 \text{ inch}$$

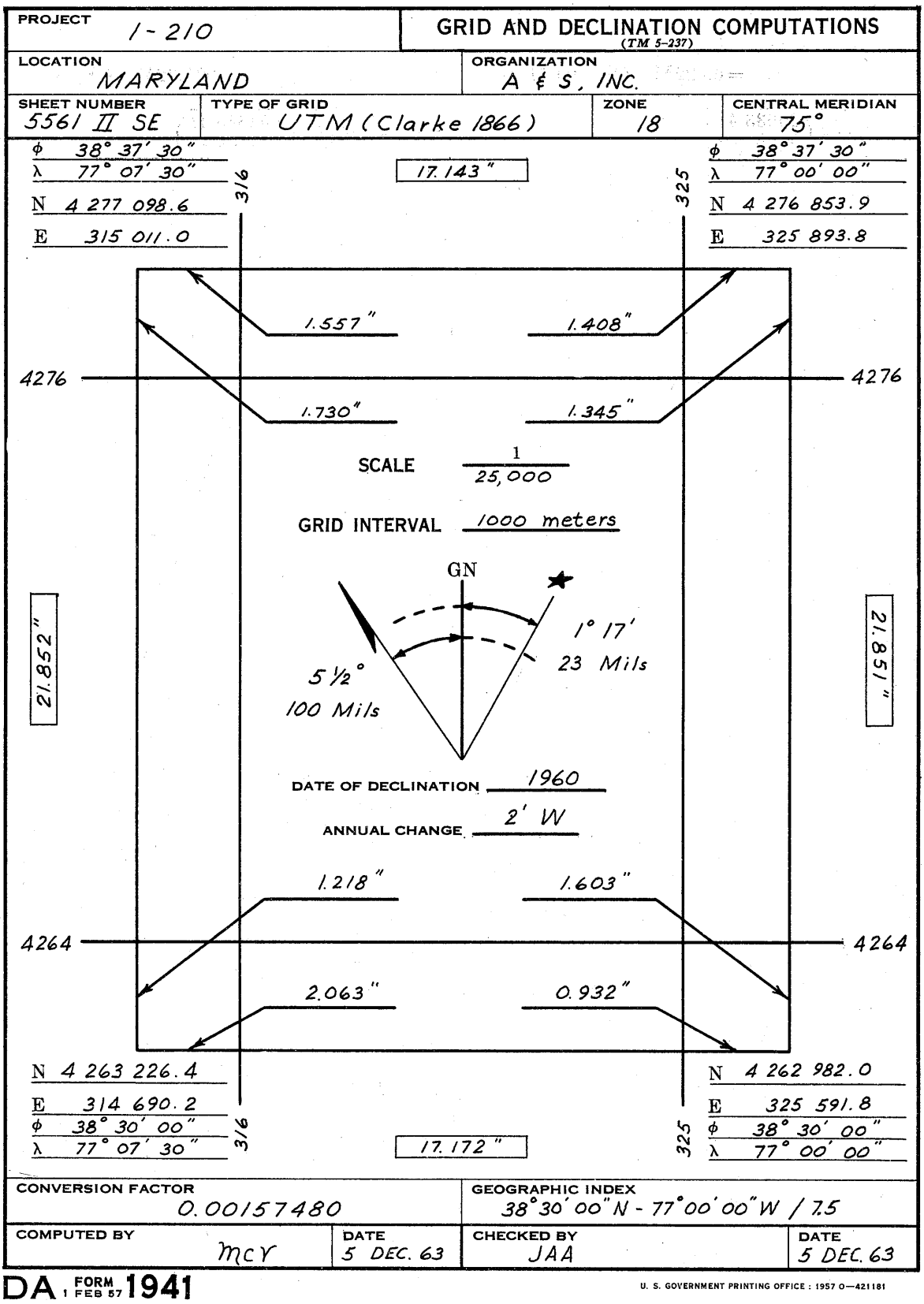



*Figure 257. Grid and declination computations.*

$$S_3 = 36.85\sqrt{1+\left(\frac{54383.22}{37245.02}\right)^2}$$

$$= 65.22M \times CF = 0.051 \text{ inch}$$

$$S_4 = 36.85\sqrt{1+\left(\frac{26933.41}{36363.24}\right)^2}$$

$$= 45.86M \times CF = 0.036 \text{ inch}$$

$$S_5 = 757.88\sqrt{1+\left(\frac{36363.24}{26933.41}\right)^2}$$

$$= 1273.33M \times CF = 1.003 \text{ inches}$$

$$S_6 = 757.88\sqrt{1+\left(\frac{38068.59}{54346.41}\right)^2}$$

$$= 925.32M \times CF = 0.729 \text{ inch}$$

$$S_7 = 394.98\sqrt{1+\left(\frac{54346.41}{38068.59}\right)^2}$$

$$= 688.45M \times CF = 0.542 \text{ inch}$$

$$S_8 = 394.98\sqrt{1+\left(\frac{26971.22}{37186.81}\right)^2}$$

$$= 487.93M \times CF = 0.384 \text{ inch}$$

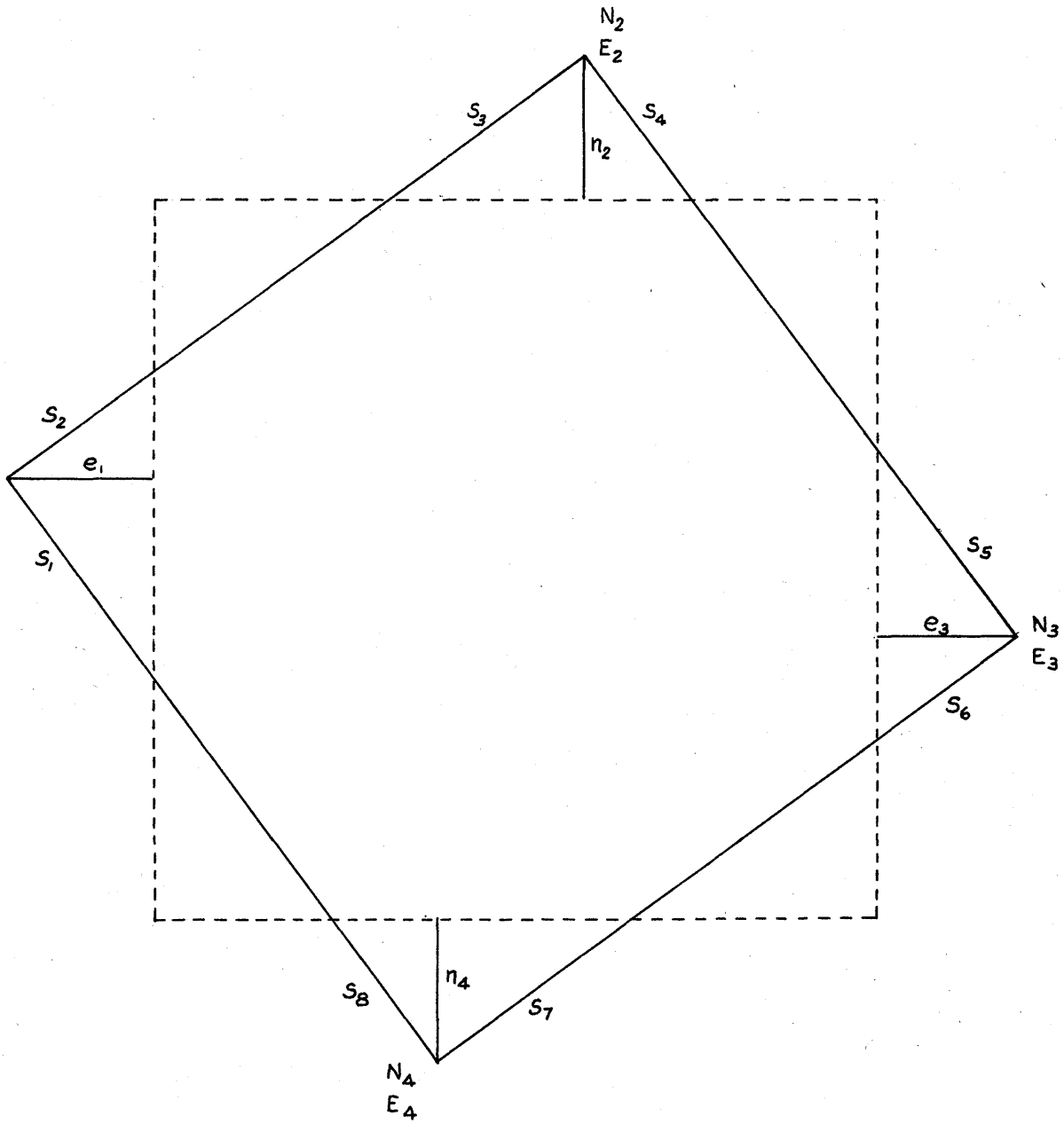



*Figure 258. Map sheet tilted to the grid.*

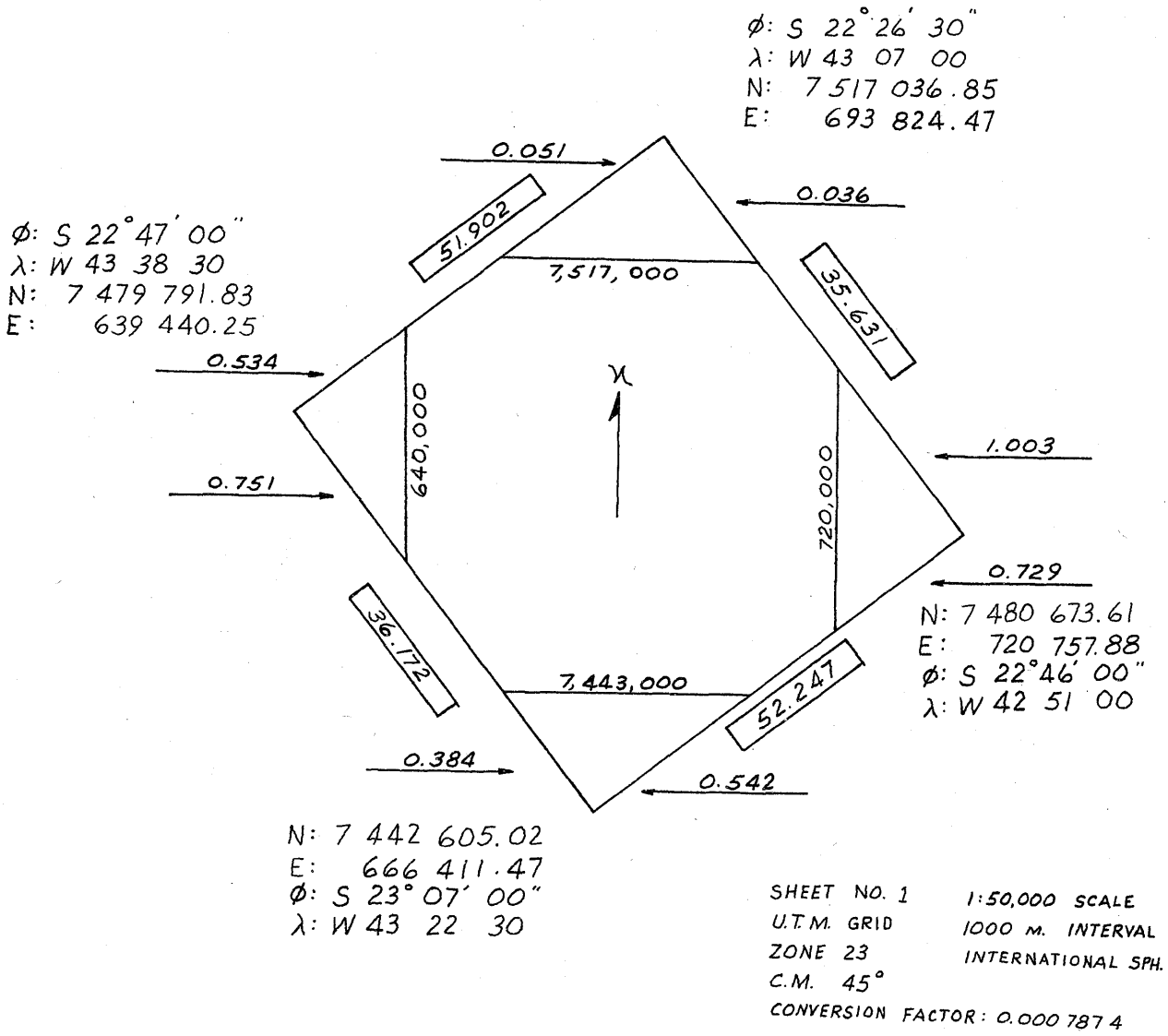



*Figure 259. Grid computation of tilted map sheet.*

## Section II. GRID AND MAGNETIC DECLINATION

### 189. Description

To be of most value to the user, one of the parts a map must have is a declination diagram, giving the relationship of magnetic north and true north to grid north, for the center of the sheet. The date of declination reflects the latest epoch for which up-to-date isogonic data have been prepared. Isogonic charts are published every 5 years—1960, 1965, 1970, etc.

### 190. Computation

*a.* Figure 257 gives the values and directions of the grid declination and the grid-magnetic angle to be used on maps larger than 1:100,000 scale. The grid declination is the angle between grid north and true north at the center of the sheet. It is found by the formula $C=\Delta\lambda_m \sin \phi_m$ where C is the grid declination in minutes, $\Delta\lambda_m$ is the difference in longitude in minutes between the longitude of the center of the sheet and the central meridian of the zone, and $\phi_m$ is the mean latitude of the sheet. A five place table of sines may be used. The declination is east or west depending on whether the center of the sheet is to the east or west of the central meridian. Grid declination is taken to the nearest minute and the nearest mil equivalent.

*b.* Before the grid magnetic angle can be computed, the magnetic declination must be found. Magnetic declination is the angle between magnetic north and true north. This value to the nearest minute is scaled from the isogonic chart using the latitude and longitude of the center of the map sheet as the arguments and it will be either easterly or westerly. At the same time, list the annual magnetic change from the chart. The grid magnetic angle can be computed from the magnetic and grid declinations. This value is taken to the nearest one half degree (10 mils).

*c.* A three pronged declination diagram is drawn (fig. 257) listing the values for the grid declination and the grid magnetic angle in both degrees and mils. On the diagram, the prongs are designated by the symbols, GN for grid north, a five point star for true north, and a half-arrowhead for magnetic north. The date of the isogonic chart used and the annual magnetic change are listed below the diagram.

*d.* On maps at 1:100,000 and smaller scales, the magnetic declination to the nearest one half degree (10 mils) from true north for the centers of the west and east sheet edges are scaled from the latest isogonic chart. The annual magnetic change is also recorded. On sheets where there is no variation between the west and east edges, the magnetic declination, to the nearest one half degree (10 mils), from true north is scaled for the center of the sheet.

# CHAPTER 18

# DATUM OR SPHEROID SHIFT BY TRANSFORMATION OF GRID COORDINATES

## 191. Basic Method

In many areas of the world, it is often necessary to shift coordinates from one spheroid or datum to a different spheroid or datum. One good approximate method of solving the problem is by finding the most probable values for rotation and translation of points on one datum to make them fit another datum. The most probable values are found by a least-squares solution using all points common to the two systems. To obtain acceptable results, the points to be transformed should be consistent within themselves, such as the stations of an adjusted triangulation net.

## 192. Transformation

*a.* The basic formulas which are used in the solution are—

$$x=BY+AX+C$$

$$y=AY-BX+D$$

*b.* The least-squares solution solves the coefficients A, B, C, and D. $x$ and $y$ are the coordinates desired; X and Y are the coordinates to be transformed (fig. 260).

*c.* The problem to be solved involves stations on the island of Jamaica, for which geographic positions are available on a local datum referred to the Clarke 1880 Spheroid. Two of the stations in the net were also used in the triangulation connection of Jamaica, Cuba, and Hispaniola. Positions for these two stations, therefore, are available on the North American 1927 Datum (NAD), referred to the Clarke 1866 Spheroid. Because the two stations are common to two systems, they can be used to transform all the positions on Jamaica to the North American 1927 Datum, Clarke 1866 Spheroid. This method requires the use of grid coordinates which suggests the use of the UTM grid system because of its world-wide application. The only problem arising from the use of UTM's develops when the area involved is of very large extent in an east-west direction, which could cause points to fall in several zones. Since it is assumed that any large-scale jobs would not be computed by topographic units, the UTM grid is still the logical choice.

*d.* For the problem in question, it was necessary to convert the given geographic coordinates on the Jamaica datum to UTM coordinates. Both the geographic and grid coordinates are referred to the Clarke 1880 Spheroid. The 1927 NAD geographic positions for the two common stations were converted to UTM coordinates referred to the Clarke 1866 Spheroid.

*e.* The coefficients A, B, C, and D in the basic formulas are solved so that the 1880 UTM's of the two common points can be transformed to 1866 UTM's. The basic formulas can then be used to transform all the 1880 UTM's to 1866 UTM's, thereby effecting both a datum shift and a spheroid shift in one computation.

*f.* Since the example problem is shown in great detail, no written explanation of the computation is given.

*g.* Note that in this example the sum of the residuals of the least-squares solution came out to exactly zero, which is in accordance with the theory of least-squares. Because the systems were originally adjusted by using observation data, it cannot be assumed that the residuals of the least-squares solution will all be zero, even though the sum of the residuals should be zero. The residuals for the common points should be less than the internal errors of the systems involved, and any large residuals indicate either an error in the solution or a poor selection of points. Remember that this method does not replace the standard arc and net adjustments for the extension of control, but it is satisfactory for mapping, fire control, and so forth.

| | Station | $X = E_1$ (1880 UTM's) | $Y = N_1$ | $x = E_2$ | $y = N_2$ | $\Delta x = x - X$ | $\Delta y = y - Y$ |
|---|---|---|---|---|---|---|---|
| ① | Blue Mt. M.P. | 332 754.148 | 1 995646.454 | 332 843.62 | 1 995 909.34 | +89.472 | +262.886 |
| ② | Mt. Denham | 231 820.743 | 2016550.129 | 231 910.91 | 2 016 816.64 | +90.167 | +266.511 |
| | $\Sigma$ | 564574.891 | 4 012196.583 | 564 754.53 | 4 012 725.98 | +179.639 | +529.397 |

| | $X^2$ / $Y^2$ $\times 10^{-12}$ | $X\Delta x$ / $Y\Delta y$ $\times 10^{-8}$ | $-X\Delta y$ / $Y\Delta x$ $\times 10^{-8}$ | Computed $x = E_2$ | Computed $y = N_2$ | Residual E | Residual N |
|---|---|---|---|---|---|---|---|
| ① | 0.110 725323<br>3.982 604 769 | +0.297 721 791<br>+5.246 275 137 | −0.874 764 070<br>+1.785 544 795 | 332 843.620 | 1 995 909.34 | 0.00 | 0.00 |
| ② | 0.053 740 857<br>4.066 474 423 | +0.209 025 809<br>+5.374 327 914 | −0.617 827 780<br>+1.818 262 755 | 231 910.910 | 2 016 816.64 | 0.00 | 0.00 |
| $\Sigma$ | 8.213 545 372 | +11.127 350 651 | +2.111 215 700 | | | | |

$$A = 1 + \frac{Q}{P}$$

$$B = \frac{R}{P}$$

$$C = \frac{\Sigma\Delta x - (A-1)(\Sigma X) - B\Sigma Y}{n}$$

$$D = \frac{(\Sigma y - \Sigma Y) - (A-1)\Sigma Y + B\Sigma X}{n}$$

$$P = \Sigma(X^2 + Y^2) - \left[\frac{(\Sigma X)^2 + (\Sigma Y)^2}{n}\right]$$

$$Q = \Sigma(X\Delta x + Y\Delta y) - \left[\frac{(\Sigma X)(\Sigma\Delta x) + (\Sigma Y)(\Sigma\Delta y)}{n}\right]$$

$$R = \Sigma(Y\Delta x - X\Delta y) - \left[\frac{(\Sigma Y)(\Sigma\Delta x) - (\Sigma X)(\Sigma\Delta y)}{n}\right]$$

$$S = \Sigma[(\Delta x)^2 + (\Delta y)^2] - \left[\frac{(\Sigma\Delta x)^2 + (\Sigma\Delta y)^2}{n}\right]$$

Check

$$\Sigma(r_x^2 + r_y^2) = S - \left(\frac{Q^2 + R^2}{P}\right)$$

$r_x$ = residual of $x$

$r_y$ = residual of $y$

$n$ = number of stations common to both systems

*Figure 260. Computation and application of coefficients for transformation of coordinates.*

$$A = 1 + \frac{Q}{P} = 1 + \frac{0.000028135 \times 10^{8}}{0.005312258 \times 10^{12}} = +1.000000529624 1$$

$$B = \frac{R}{P} = \frac{+0.001902058 \times 10^{8}}{+0.005312258 \times 10^{12}} = +0.0000358050757$$

$$C = \frac{\Sigma \Delta x - (A-1)(\Sigma X) - B\Sigma Y}{n}$$

$\Sigma \Delta x = +179.639$

$(A-1)(\Sigma X) = (+0.000000529624 1)(564574.891) = +0.29901$

$B\Sigma Y = (+0.0000358050757)(4012196.583) = +143.65700$

$n = 2$

$$C = \frac{+179.639 - 0.29901 - 143.65700}{2} = +17.84150$$

$$D = \frac{(\Sigma y - \Sigma Y) - (A-1)(\Sigma Y) + B\Sigma X}{n}$$

$\Sigma y - \Sigma Y = 4012725.98 - 4012196.583 = +529.397$

$(A-1)(\Sigma Y) = (+0.000000529624 1)(4012196.583) = +2.12496$

$B\Sigma X = (+0.0000358050757)(564574.891) = +20.21465$

$n = 2$

$$D = \frac{529.397 - 2.12496 + 20.21465}{2} = +273.74334$$

$$P = \Sigma(X^2 + Y^2) - \left[\frac{(\Sigma X)^2 + (\Sigma Y)^2}{n}\right]$$

$\Sigma(X^2 + Y^2) = 8.213545372 \times 10^{12}$

$\Sigma X^2 + \Sigma Y^2 = (564574.891)^2 + (4012196.583)^2$

$= 0.318744808 \times 10^{12} + 16.097721421 \times 10^{12} = 16.416466229 \times 10^{12}$

$n = 2$

$$\frac{\Sigma X^2 + \Sigma Y^2}{n} = \frac{16.416466229 \times 10^{12}}{2} = 8.208233114 \times 10^{12}$$

$P = 8.213545372 \times 10^{12} - 8.208233114 \times 10^{12} = +0.005312258 \times 10^{12}$

*Figure 260*—Continued.

$$Q = \Sigma(X\Delta x + Y\Delta y) - \left[\frac{(\Sigma X)(\Sigma\Delta x)+(\Sigma Y)(\Sigma\Delta y)}{n}\right]$$

$\Sigma(X\Delta x + Y\Delta y) = +11.127350651 \times 10^8$

$(\Sigma X)(\Sigma\Delta x) = (564574.891)(+179.639) = 1.014196688 \times 10^8$

$(\Sigma Y)(\Sigma\Delta y) = (4012196.583)(+529.397) = 21.240448345 \times 10^8$

$(\Sigma X)(\Sigma\Delta x)+(\Sigma Y)(\Sigma\Delta y) = 22.254645033 \times 10^8$

$\frac{(\Sigma X)(\Sigma\Delta x)+(\Sigma Y)(\Sigma\Delta y)}{n} \quad (n=2) = 11.127322516 \times 10^8$

$Q = (+11.127350651 - 11.127322516) \times 10^8 = +0.000028135 \times 10^8$

$$R = \Sigma(Y\Delta x - X\Delta y) - \left[\frac{(\Sigma Y)(\Sigma\Delta x)-(\Sigma X)(\Sigma\Delta y)}{n}\right]$$

$\Sigma(Y\Delta x - X\Delta y) = +2.111215700 \times 10^8$

$(\Sigma Y)(\Sigma\Delta x) = (4012196.583)(+179.639) = 7.207469820 \times 10^8$

$(\Sigma X)(\Sigma\Delta y) = (564574.891)(+529.397) = 2.988842536 \times 10^8$

$(\Sigma Y)(\Sigma\Delta x)-(\Sigma X)(\Sigma\Delta y) = 4.218627284 \times 10^8$

$\frac{(\Sigma Y)(\Sigma\Delta x)-(\Sigma X)(\Sigma\Delta y)}{n} \quad (n=2) = 2.109313642 \times 10^8$

$R = (+2.111215700 - 2.109313642) \times 10^8 = +0.001902058 \times 10^8$

$$S = \Sigma[(\Delta x)^2+(\Delta y)^2] - \left[\frac{(\Sigma\Delta x)^2+(\Sigma\Delta y)^2}{n}\right]$$

$\Sigma[(\Delta x)^2+(\Delta y)^2] = (89.472)^2+(90.167)^2+(262.886)^2+(266.511)^2 = 156272.48879$

$(\Sigma\Delta x)^2 = (179.639)^2 = 32270.17032$

$(\Sigma\Delta y)^2 = (529.397)^2 = 280261.18361$

$(\Sigma\Delta x)^2+(\Sigma\Delta y)^2 = 312531.35393$

$\frac{(\Sigma\Delta x)^2+(\Sigma\Delta y)^2}{n} \quad (n=2) = 156265.67696$

$S = 156272.48879 - 156265.67696 = +6.81183$

*Figure 260*—Continued.

The coefficients A, B, C, and D. are now used in the basic formulas to transform the coördinates of the common points.

Blue Mt. M.P. 1880 UTM's $N_1 = Y = 1\,995\,646.454$

$E_1 = X = 332\,754.148$

Using the basic formulas

$y = N_2 = AY - BX + D$

$= (+1.000000529624)(1995646.454) - (+0.000035805075)(332754.148) + 273.74334$

$y = N_2 = 1995909.34$

$x = E_2 = BY + AX + C$

$= (+0.000035805075)(1995646.454) + (+1.000000529624)(332754.148) + 17.84150$

$x = E_2 = 332\,843.620$

(Follow the same procedure for Mt. Denham)

Residuals are found by subtracting the computed coördinates (from the formulas) from the fixed coördinates.

To check the solution:

$$\Sigma(r_x^2 + r_y^2) = S - \frac{Q^2 + R^2}{P}$$

$$0 = +6.81183 - \frac{79157\,82.25 + 36178246353.64}{5312258000}$$

$$0 = 0$$

*Figure 260*—Continued.

# CHAPTER 19

# REDUCTION OF GRAVITY OBSERVATIONS

## 193. Types of Measurements

*a.* Measurements of gravity at the earth's surface are accomplished by the following two techniques:

(1) Absolute measurements—these determine the total intensity of gravity at a station.

(2) Relative measurements—these determine the gravity differential between two stations and can be easily accomplished with compact portable instruments called gravimeters. Specific instructions in this chapter refer to long range gravimeters.

*b.* Gravity surveys can be classified by two procedures—the loop survey where the survey begins and ends on the same base reference station, or a line survey where the survey begins on one base reference station and ends on another base reference station. In a line survey where a number of gravity stations are being observed, intermediate base reference stations should be interspersed among the gravity stations.

## 194. Abstracting Gravity Data

The field notes are abstracted on DA Form 2859 (Gravity Survey Computation) (fig. 261).

PROJECT: AMS GRAVITY BASE — LOCATION: WASHINGTON, D.C. — GRAVITY SURVEY COMPUTATION (TM 5-237)

| STATION | UNIVERSAL 1962 DATE | UNIVERSAL TIME | Δ TIME MIN. | COUNTER READING | COUNTER MGALS DIAL RDG | COUNTER MGALS DIAL FACTOR MGALS/D.U. | OBSERVED MGALS | CORRECTIONS 1 | 2 | 3 | 4 | CORRECT MGALS | ΔG MGALS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AMS GS-3 | 26 NOV. | 1813 | ΔT ΣΔT | 35 | 76.18 | 3673.23 / 1.05037 | 3753.25 | -.02 | | | | 3753.23 | 00.00 |
| USC&GS (G.M.) | | 1840 | 27 | 35 | 88.30 | | 3765.98 | -.03 | | +.01 | | 3765.96 | + 12.73 |
| USC&GS (ELEV. PIER) | | 1904 | 24 51 | 35 | 88.89 | | 3766.60 | -.04 | | +.01 | | 3766.57 | + 13.34 |
| USC&GS (FLOOR) | | 1921 | 17 68 | 35 | 89.04 | | 3766.75 | -.05 | | +.02 | | 3766.72 | + 13.49 |
| USC&GS (G.M.) | | 2000 | 39 107 | 35 | 88.31 | | 3765.99 | -.06 | | +.03 | | 3765.96 | + 12.73 |
| AMS GS-3 | | 2036 | 36 143 | 35 | 76.20 | | 3753.27 | -.08 | | +.04 | | 3753.23 | 00.00 |

1. EARTH TIDE
2. DRIFT BETWEEN TRIPS
3. DRIFT DURING TRIPS
4. TARES

OBSERVER AND ORGANIZATION: D. Kahler - AMS
INSTRUMENT (Type and No.): L & R G-13
COMPUTED BY: G. Tennis - AMS
CHECKED BY: D. Haddox - AMS
DATE: JAN. '63 (JAN. '63)
PAGE NO.: 1
NO. OF PAGES: 1

DA FORM 2859, 1 OCT 64

*Figure 261. Gravity survey computation (AMS line)*

PROJECT: DITTO — LOCATION: U.S.A. — GRAVITY SURVEY COMPUTATION (TM 5-237)

| STATION | UNIVERSAL 1962 DATE | UNIVERSAL TIME | Δ TIME MIN. | COUNTER READING DIAL RDG | COUNTER MGALS DIAL FACTOR MGALS/D.U. | OBSERVED MGALS | CORRECTIONS 1 | 2 | 3 | 4 | CORRECT MGALS | Δg MGALS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 28 NOV. | 1740 | | 06 87.30 | 603.89 / 1.0063 | 691.74 | +.01 | | | | 691.75 | 00.00 |
| | | | 286 | | | | | | | | | |
| 2 | | 2226 | | 04 63.97 | 402.60 / 1.0064 | 466.98 | -.07 | | -.16 | | 466.75 | -225.00 |
| 2 | 2 DEC. | 1618 | | 04 64.07 | | 467.08 | -.01 | -.16 | -.16 | | 466.75 | |
| | | | 313 | | | | | | | | | |
| 3 | | 2131 | | 07 64.13 | 704.52 / 1.0063 | 769.05 | -.06 | -.16 | -.33 | | 768.50 | + 76.75 |
| 3 | 4 DEC. | 1902 | | 07 64.37 | | 769.30 | +.01 | -.48 | -.33 | | 768.50 | |
| | | | 158 | | | | | | | | | |
| 4 | | 2140 | | 08 11.17 | 805.15 / 1.0063 | 816.39 | -.03 | -.48 | -.42 | | 815.46 | +123.71 |
| 4 | 8 DEC. | 1836 | | 08 11.60 | | 816.82 | -.05 | -.89 | -.42 | | 815.46 | |
| | | | 138 | | | | | | | | | |
| 5 | | 2054 | | 07 65.30 | 704.52 / 1.0063 | 770.23 | -.01 | -.89 | -.49 | | 768.84 | + 77.09 |
| | | | 191 | | | | | | | | | |
| 1 | 9 DEC. | 0005 | | 06 88.80 | 603.89 / 1.0063 | 693.25 | -.01 | -.89 | -.60 | | 691.75 | 00.00 |

1. EARTH TIDE
2. DRIFT BETWEEN TRIPS
3. DRIFT DURING TRIPS
4. TARES

OBSERVER AND ORGANIZATION: J. WILLIAMS - AMS
INSTRUMENT (Type and No.): L & R G-1
COMPUTED BY: D. Haddox - AMS
CHECKED BY: J. Love - AMS
DATE: JAN '63 / DATE: JAN '63
PAGE NO.: 1
NO. OF PAGES: 1

DA FORM 2859, 1 OCT 64

*Figure 262. Gravity survey computation, DA Form 2859 (sample)*

*a.* At the top of the form, in the spaces designated, enter the LOCATION of the survey and the name of the PROJECT.

*b.* Column 1—enter the name of the observed STATION.

*c.* Columns 2 and 3—enter the DATE and TIME. If more than one reading is taken, a mean of the observed times is used. All local time should be converted to UNIVERSAL time before entering on the form.

*d.* Columns 5 and 6—enter the COUNTER READING and DIAL RDG. Normally, a valid reading is considered to be the mean of a minimum of three consecutive readings in which each reading is the result of a reset of the instrument reading dial.

## 195. Gravity Computations

*a.* The computations of ΔT and instrument corrections are as follows:

(1) Column 4— the Δ TIME is obtained by tabulating the elapsed time between successive stations. Note that the elapsed time between a repeated station is not included since it has no effect on the instrument drift during trip (fig 262).

(2) Column 7—COUNTER MGALS, in upper space, and DIAL FACTOR MGALS/D.U., in lower space are taken directly from the calibration sheet for the instrument (fig. 263). This calibration is for a LaCoste and Romberg gravimeter. Calibrations for other types of gravimeters may have a different format.

*Example:* If counter reading is 35, look under column headed Counter Reading for 3500. Opposite 3500 and in column headed Value in Milligals,

Approximate Milligal Values for LaCoste and Romberg
Gravity Meter No. G 13 (small)
Corrected October, 1960
With Correction Factor of K-1.0009 from Original Table

| Counter Reading | Value in Milligals | Interval Factor | Counter Reading | Value in Milligals | Interval Factor |
|---|---|---|---|---|---|
| 000 | 000.00 | 1.05006 | 3600 | 3778.26 | 1.05043 |
| 100 | 105.01 | 1.04998 | 3700 | 3883.31 | 1.05045 |
| 200 | 210.00 | 1.04990 | 3800 | 3988.35 | 1.05046 |
| 300 | 314.99 | 1.04979 | 3900 | 4093.40 | 1.05043 |
| 400 | 419.97 | 1.04964 | 4000 | 4198.44 | 1.05033 |
| 500 | 534.94 | 1.04947 | 4100 | 4303.47 | 1.05019 |
| 600 | 629.88 | 1.04932 | 4200 | 4408.49 | 1.05007 |
| 700 | 734.82 | 1.04921 | 4300 | 4513.50 | 1.04999 |
| 800 | 839.74 | 1.04913 | 4400 | 4618.50 | 1.04997 |
| 900 | 944.65 | 1.04907 | 4500 | 4723.49 | 1.05001 |
| 1000 | 1049.56 | 1.04904 | 4600 | 4828.50 | 1.05005 |
| 1100 | 1154.46 | 1.04903 | 4700 | 4933.50 | 1.05009 |
| 1200 | 1259.36 | 1.04905 | 4800 | 5038.51 | 1.05011 |
| 1300 | 1364.27 | 1.04907 | 4900 | 5143.52 | 1.05012 |
| 1400 | 1469.18 | 1.04912 | 5000 | 5248.53 | 1.05009 |
| 1500 | 1574.09 | 1.04922 | 5100 | 5353.54 | 1.05003 |
| 1600 | 1679.01 | 1.04930 | 5200 | 5458.54 | 1.04992 |
| 1700 | 1783.94 | 1.04938 | 5300 | 5563.54 | 1.04979 |
| 1800 | 1888.88 | 1.04942 | 5400 | 5668.52 | 1.04962 |
| 1900 | 1993.82 | 1.04944 | 5500 | 5773.48 | 1.04946 |
| 2000 | 2098.76 | 1.04943 | 5600 | 5778.42 | 1.04928 |
| 2100 | 2203.71 | 1.04942 | 5700 | 5983.35 | 1.04909 |
| 2200 | 2308.65 | 1.04940 | 5800 | 6088.26 | 1.04891 |
| 2300 | 2413.59 | 1.04938 | 5900 | 6193.15 | 1.04874 |
| 2400 | 2518.53 | 1.04937 | 6000 | 6298.03 | 1.04859 |
| 2500 | 2623.46 | 1.04938 | 6100 | 6402.88 | 1.04844 |
| 2600 | 2728.40 | 1.04940 | 6200 | 6507.73 | 1.04829 |
| 2700 | 2833.34 | 1.04943 | 6300 | 6612.56 | 1.04814 |
| 2800 | 2938.29 | 1.04952 | 6400 | 6717.37 | 1.04796 |
| 2900 | 3043.24 | 1.04962 | 6500 | 6822.17 | 1.04777 |
| 3000 | 3148.20 | 1.04974 | 6600 | 6926.94 | 1.04754 |
| 3100 | 3253.19 | 1.04988 | 6700 | 7031.70 | 1.04730 |
| 3200 | 3358.18 | 1.05003 | 6800 | 7136.43 | 1.04703 |
| 3300 | 3463.18 | 1.05017 | 6900 | 7241.13 | 1.04677 |
| 3400 | 3568.20 | 1.05028 | 7000 | 7345.81 | |
| 3500 → | 3673.23 → | 1.05037 | | | |

*Figure 263. Gravity meter calibration sheet.*

read 3673.23 (Counter Mgals). Again opposite 3500 and in column headed Interval Factor, read 1.05037 (Dial Factor Mgals/D.U.).

(3) Column 8—OBSERVED MGALS is the product of the DIAL RDG. and DIAL FACTOR MGALS/D.U. plus the COUNTER MGALS.

*Example:* If the meter reading is 3588.30 and the COUNTER MGALS is 3673.23 and the DIAL FACTOR MGALS/D.U. is 1.05037, then the OBSERVED MGALS is 88.30 (1.05037) +3673.23=3765.98 (fig. 261).

*b.* The Earth Tide correction is applied to the Observed Mgals.

(1) The Earth Tide correction is the correction for the effect of sun and moon attraction on observed gravity. Because of this attraction of the sun and the moon, the gravity of the earth, as well as the plumb line direction, fluctuates from its normal.

(*a*) The tidal corrections are the vertical component of attraction between the elastic earth and the celestial bodies and is defined by the following equations.

(*b*) Tidal correction.

$$\Delta g_i = 3\,(g)\,\frac{M_i}{M_E}\,\frac{r_\phi^3}{D_i^3}\,\frac{R_E^2}{r_\phi^2}\left[(\sin\phi\,\sin\delta_i+\cos\phi\,\cos\delta_i\cos t_{Li})^2-\left(\frac{1}{3}\right)\right]\left[1-\left(\frac{3}{2}\right)k+h\right]P$$

(*c*) Total tidal correction $\Delta g_t = \Delta g_s + \Delta g_m$.

(*d*) Where: $g$=The gravity in Mgals at given latitude
$\Delta g_i$=Total tidal correction for celestial body $i$
$\Delta g_t$=Total tidal correction
$\Delta g_s$=Tidal correction due to attraction of sun
$\Delta g_m$=Tidal correction due to attraction of moon
$k=0.32$, $h=0.66$ } Love numbers: depend on the elastic deformation of the earth under the influence of tidal force
$M_i$=Mass of celestial body
$M_s$=Mass of sun
$M_m$=Mass of moon
$M_E$=Mass of earth
$R_E$=Mean radius of earth
$r_\phi$=Earth radius in given latitude
$D_i$=Distance from center of earth to celestial body
$D_s$=Distance to sun
$D_m$=Distance to moon
$\phi$=Latitude of station point
$\delta_i$=Declination of celestial body
$\delta_s$=Declination of sun
$\delta_m$=Declination of moon
$T_{Li}$=Local hour angle of celestial body
$T_{Ls}$=Local hour angle of sun
$T_{Lm}$=Local hour angle of moon
$f$=Ratio of flattening of the reference ellipsoid
$a$=Equatorial radius of ellipsoid
$e$=Eccentricity of ellipsoid
P=Parallax factor (moon only)

(*e*) Ratios used.

$$\frac{M_s}{M_E}=3.334\times10^5$$

$$\frac{r_\phi^3}{D_s^3}=\left[\frac{1}{(2.35\times10^4)}\right]^3$$

Spheroid factor equation—

$$\left(\frac{R_E}{r_\phi}\right)^2=\frac{\left[\frac{2(1/f)\,a-a}{2(1/f)}\right]^2}{\left[\frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{1.5}}\right]^2}$$

$$\frac{M_m}{M_E}=\frac{1}{81}$$

$$\left(\frac{r_\phi}{D_m}\right)^3=\left(\frac{1}{60}\right)^3$$

(*f*) Substituting in the equation (*b*) above, we have for the sun—

$$\Delta g_s = 9.094 \times 10^{-8}(g) \frac{\left(\frac{2(1/f)a - a}{2(1/f)}\right)^2}{\left[\frac{a(1-e^2)}{(1-e^2 \sin^2 \phi)^{1.5}}\right]^2} \left[(\sin \phi \sin \delta_s + \cos \phi \cos \delta_s \cos t_{Ls})^2 - \frac{1}{3}\right] \text{mgal}$$

and for the moon—

$$\Delta g_m = 2.192 \times 10^{-7}(g) \frac{\left(\frac{2(1/f)a - a}{2(1/f)}\right)^2}{\left[\frac{a(1-e^2)}{(1-e^2 \sin^2 \phi)^{1.5}}\right]^2} \left[(\sin \phi \sin \delta_m + \cos \phi \cos \delta_m \cos t_{Lm})^2 - \frac{1}{3}\right] \text{P mgal}$$

---

(*g*) The above equations are generally rearranged in some tabular form such as "Tidal Gravity Effect Tables" by J. B. Damrel, published by Houston Technical Laboratories; or "Tidal Gravity Corrections for *Year*" by Service Hydrographique De La Marine and Compagnie General De Geophysique with introduction by Jean Goguel, published each year in the Journal of European Association of Exploration Geophysist (December issue).

(2) For field computation of tide correction the tables, "Tidal Gravity Corrections for *Year*" are generally used for field reduction. Table No. 1 gives the tidal correction at the rotational poles in the column for the month of the year and in the row for the day of the month in hundredths of a milligal to the nearest half unit. This value may be used if only the difference in correction value is desired for intervals of several hours, as is often the case in field gravimetric observations. This correction can be improved by further corrections for the effect of longitude by use of tables No. 2 and No. 3. Table No. 2 can be used for longitudes 45° N to 75° E (covering Europe, Africa, and the Near East), and for longitudes ranging from 135° E to 105° W, table No. 3 can be used for longitudes ranging from 15° W to 135° W (covering the American continent), and for longitudes ranging from 45° E to 165° E (covering the Far East). The tables can be used to cover wide areas on both sides of the meridians by applying the local time at the observation point. Often it will be quite sufficient to take the time of the central meridian of the time zone (take care to allow for daylight saving time). If greater accuracy is desired, add to the civil time 4 minutes for each degree to the west. The form of table No. 2 and No. 3 indicates which of the two must be used. From either table No. 2 or table No. 3 correction values N and S are obtained for latitude 45° N and 45° S respectively for the month, day, and hour. At any other latitude ($\phi$), they must be multiplied by the following coefficient:

Coefficient of N: $\cos \phi \ (\cos \phi + \sin \phi)$
Coefficient of S: $\cos \phi \ (\cos \phi - \sin \phi)$
The total tidal correction then is given by—

$$C_T = P + N[\cos \phi \ (\cos \phi + \sin \phi)] + S \ [\cos \phi \ (\cos \phi - \sin \phi)]$$

or approximately—$C_{T1} = P$ or $C_{T2} = P + N + S$

(3) The office computation of tide corrections may be computed on DA Form 2860 (Earth Tide Gravity Correction) (fig. 264). In the first two columns, enter the latitude (to nearest even degree) and the longitude (to the nearest degree). The longitude should always be West; therefore, East longitude should be subtracted from 360°.

(*a*) *Example:* To find the tide correction on 26 November 1962 at 1813 universal time for station AMS GS–3. Enter

latitude and longitude as stated above. In column 3, record 26 November and in column 4, record 1800 and 1900. From the *Air Almanac*, obtain and record the G.H.A. of the sun, the declination of the sun, the G.H.A. of the moon, and the declination of the moon. The parallax factor is found in the *American Ephemeris and Nautical Almanac* for the appropriate year. The parallax factor is recorded as the semidiameter of the moon.

(*b*) For 26 November 1962 at 1800, the sun is found to have a G.H.A. of 93° (take out to nearest degree) and a declination (to the nearest even degree) of −20°. For the same time, the moon has a G.H.A. of 98° and a declination of −16°.

(*c*) The L.H.A. is obtained by taking the difference between G.H.A. and longitude without respect to sign. That is, subtract the smaller from the larger. If the L.H.A. happens to be greater than 180°, subtract this value from 360° and record resultant value as the L.H.A. For the example above, the L.H.A. for the sun is 16° and the L.H.A. for the moon is 21°. Repeat for 1900 hours.

(*d*) This information is all that is needed to obtain the corrections from Damrel's *Tidal Gravity Effect Tables*. Turning to the proper latitude (38°) and looking under the proper declination (−20°) and the proper L.H.A. (16°), the correction for the sun is found to be −.007 mgals. This value is recorded under SUN, CORR. MGALS. Using the proper values for the moon, declination −16°, L.H.A. 21°, the correction for the moon is found to be −.008 mgals. This value is recorded under MOON, CORR. MGALS. Looking in the *American Ephermeris and Nautical Almanac for 1962*, it is found by interpolation, on 26 November at 1800, the semidiameter of the moon is 15′.0. This value is recorded under MOON, PARALLAX FACTOR. Repeat for 1900 hours. If, however, the declination of the sun exceeds 22° then the value given in the Damrel's tables is that of the moon and must be multiplied by 0.456 to obtain the correction for the sun.

(*e*) Using the tables in the back of Damrel's *Tidal Gravity Effect Tables* book, looking in the left hand vertical columns to 8, trace this across to the proper parallax factor, 15′.0, and record the value 8 (−.008 mgals). At 1900, the MOON, CORR. MGALS was −.026 and parallax factor 15′.0. Using the tables and looking in the left hand column to 26, trace this across to the proper parallax factor, 15′.0, and record the value 24 (−.024) under MOON, CORR. FOR PARALLAX MGALS.

(*f*) Adding the corrections for the sun and moon, the total correction at 1800 is found to be −.015 mgals and at 1900 to be −.038 mgals. These values are recorded under TOTAL CORR. MGALS.

(*g*) The tidal correction for 1813 can now be computed by interpolating between 1800 and 1900. For example, 13/60 $\times(-.023)+(-.015)=-.020$. This value is recorded on DA Form 2859, Gravity Survey Computation, in column 1 under corrections (fig. 261).

*c.* The drift and tare corrections are determined as follows:

(1) *Static drift.* If all observations are taken without a repeat within a loop, there is no static drift correction. When two successive sets of readings have been taken at the same station and the instrument has not been in transit, a static drift correction must be found. First apply the earth tide correction to the OBSERVED MGALS, then subtract the second reading from the first. This correction is carried on to the succeeding readings. If another station is repeated, it is combined algebraically to the first reading and carried on to the following readings. (See fig. 262.) Station 2 is repeated; and after adding in the tide correction, the difference is −.16. This value is carried on to the following readings and combined algebraically with the next repeat station and so to the end.

PROJECT: AMS GRAVITY BASE

**EARTH TIDE GRAVITY CORRECTION**
(TM 5-237)

DATE: NOV. 1962 | LOCATION: WASHINGTON, D.C. | ORGANIZATION: USAMS

| LATITUDE | LONGITUDE | DATE | UNIVERSAL TIME | SUN: GREENWICH HOUR ANGLE | SUN: DECLINATION | SUN: LOCAL HOUR ANGLE | SUN: CORRECTION (MGals) | MOON: GREENWICH HOUR ANGLE | MOON: DECLINATION | MOON: LOCAL HOUR ANGLE | MOON: CORRECTION (MGals) | MOON: PARALLAX FACTOR | MOON: CORRECTION FOR PARALLAX (MGals) | TOTAL CORRECTION (MGals) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| N | W | | AMS GS-3 | — | | | USC & GS (G.M.) | | | | | | | |
| 38 | 77 | 26 NOV. | 1800 | 93 | -20 | 16 | -.007 | 98 | -16 | 21 | -.008 | 15.0 | -.008 | -.015 |
| | | | 1900 | 108 | -20 | 31 | -.014 | 113 | -16 | 36 | -.026 | 15.0 | -.024 | -.038 |
| | | | | | | | | | | | | | | |
| | | | USC & GS (ELEV. PIER) | — | | | USC & GS (FLOOR) | | | | | | | |
| 38 | 77 | 26 NOV. | 1900 | 108 | -20 | 31 | -.014 | 113 | -16 | 36 | -.026 | 15.0 | -.024 | -.038 |
| | | | 2000 | 123 | -20 | 46 | -.022 | 127 | -16 | 50 | -.046 | 15.0 | -.042 | -.064 |
| | | | | | | | | | | | | | | |
| | | | USC & GS (G.M.) | — | | | AMS GS-3 | | | | | | | |
| | | | | | | | | | | | | | | |
| 38 | 77 | 26 NOV. | 2000 | 123 | -20 | 46 | -.022 | 127 | -16 | 50 | -.046 | 15.0 | -.042 | -.064 |
| | | | 2100 | 138 | -20 | 61 | -.028 | 142 | -16 | 65 | -.061 | 15.0 | -.055 | -.083 |

COMPUTED BY: D. Haddox - AMS | DATE: JAN. '63 | CHECKED BY: J. Love - AMS | DATE: JAN. '63

DA FORM 2860, 1 OCT 64

*Figure 264. Earth Tide Gravity Correction (DA Form 2860).*

(2) *Tare correction.* If there is a change in the instrument drift rate, this may be caused by a tare (jump in instrument reading). Plot the station closures as a function of running time between closures. If there are no tares or changes in the drift rate, the plot will define a straight line whose slope is the dynamic drift rate. If the plot defines two of more parallel lines, a tare is indicated whose magnitude is the offset between the lines and whose location is indicated by the stations between which the offset occurs. Although it is not possible to tell from the closure plot whether the tare occurred on the going or back leg of the loop, it is known that trouble exists. The magnitude of the tares and the observations which are critical to the correction are defined; however, earlier measurements or new measurements will help isolate the legs on which the tares occurred.

(3) *Dynamic drift (drift during trips).* This is obtained from the closure plot by measuring the slope of the tare reduced Linear Regression Line. Gravity specifications should be followed for a specific survey for maximum closure, but the general objective is a station gravity value of $\pm 1$ mgal accuracy. This is done by finding the ratio of the change in gravity closure, ($\Delta C_g$), with respect to the corresponding change in cumulative time ($\Delta T_c$), i.e., $\Delta C_g/\Delta T_c$. If, however, there are no tares or the tare magnitude is less than three tenths of a milligal (0.3), the total dynamic drift is determined by finding the difference between the first and the closing readings which are corrected by tidal effect and static drift. The total drift is then distributed to each observation in the same ratio as a particular observation time bears to the total elapsed times. (See fig. 262.) *Example:* The first reading, plus tide and static drift corrections, is 691.75 (691.74+0.01) and the last closing observation, plus tide and static drift corrections, is 692.35 (693.25 −0.01−0.89). The difference is determined (691.75−692.35=−0.60). The second dynamic drift correction is then given by the rule above. The elapsed time for the second reading, 286 minutes, divided by the total accumulated time, 1086 minutes, and multiplied by the total drift, −0.60, is $\frac{286}{1086}\times -0.60 = -0.16$. The dynamic drift is recorded in column 3 under CORRECTIONS.

*d.* The concluding computations are the corrected observation, the gravity change from the base station, $\Delta g$, and the adjustment to the base station.

(1) The result of all corrections added algebraically to the OBSERVED MGALS is recorded under the column heading CORRECT MGALS of DA Form 2859 (fig. 262). This value is the corrected observation in terms of the instrument gravity reading.

(2) The corrected observation for the base station is subtracted from the absolute value of the base station to obtain the results in absolute terms. This difference is applied to every observation.

(3) The gravity change is determined by subtracting the base station value from every CORRECTED OBSERVATION. This gravity change, $\Delta g$, is recorded in the last column of DA Form 2859 (fig. 262).

## 196. Instrument Calibration

*a.* The calibration of absolute and relative gravimetric instruments is a complex subject for which a complete treatment is beyond the scope of this manual. However, there will be frequent occasions when the computer will be required to obtain calibration corrections to the manufacturers tables (fig. 263).

*b.* The data for the computation of the calibration correction is abstracted (in the same manner as described in para. 194) from the field books of an instrument survey over a calibration line such as THE NORTH AMERICAN CALIBRATION LINE, or THE EAST COAST CALIBRATION LINE in The Western Hemisphere, or THE EUROPEAN CALIBRATION LINE in The Eastern Hemisphere.

*c.* The reduction computation proceeds in the same manner as is described in paragraph 195.

*d.* The calibration scale correction is obtained

LOCATION: NACL | PROJECT: | DATE: SEPT. '63

**LEAST SQUARES ADJUSTMENT OF GRAVIMETER COMPARISONS** (TM 5-237)

| n | STATION | PUBLISHED GRAVITY (MGALS) (1) | INST. 46 OBSERVED GRAVITY (MGALS) (2) | (1) MINUS (2) y (MGALS) (3) | x (GALS) (4) | $x^2$ (5) | xy (6) | Y CORR. TO (2) (MGALS) (7) | (2) PLUS (7) (MGALS) (8) | (1) MINUS (8) (MGALS) (9) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | CARNEGIE | 980 100.60 | 980 100.60 | .00 | .000000 | | | - .01 | 980 100.59 | + .01 |
| 2 | FRIENDSHIP A.P. | 980 103.19 | 980 103.26 | - .07 | +.002660 | | | - .01 | 980 103.25 | - .06 |
| 3 | HOUSTON A.P. | 979 293.35 | 979 294.03 | - .68 | $\bar{.}806570$ | | | - .65 | 979 293.38 | - .03 |
| 4 | RICE UNIVERSITY | 979 298.35 | 979 299.08 | - .73 | $\bar{.}801515$ | | | - .64 | 979 298.44 | - .09 |
| 5 | SAN ANTONIO (GATE 10) | 979 197.60 | 979 198.27 | - .67 | $\bar{.}902325$ | | | - .72 | 979 197.55 | + .05 |
| 6 | SAN ANTONIO (STREET) | 979 197.30 | 979 197.97 | - .67 | $\bar{.}902625$ | | | - .72 | 979 197.25 | + .05 |
| 7 | MONTERREY A.P. | 978 861.75 | 978 862.63 | - .88 | $\bar{1}.237970$ | | | - .99 | 978 861.64 | + .11 |
| 8 | MEXICO A.P. | 977 970.11 | 977 971.74 | - 1.63 | $\bar{2}.128860$ | 1 | 7 | - 1.69 | 977 970.05 | + .06 |
| 9 | UNIV. OF MEXICO | 977 941.41 | 977 943.10 | - 1.69 | $\bar{2}.157495$ | 2.028911 | 15.93017 | - 1.71 | 977 941.39 | + .02 |
| 10 | PASO de CORTEZ | 977 571.11 | 977 573.23 | - 2.12 | $\bar{2}.527370$ | 20. | 15. | - 2.00 | 977 571.23 | - .12 |
| | | | | | | | | | | |
| | | | Σ | - 9.16 | $\bar{1}1.462070$ | + | + | | | .00 |

EQUATIONS

$nb + \Sigma xm = \Sigma y$

$\Sigma xb + \Sigma x^2 m = \Sigma xy$

$Y = mx + b$

$b = \dfrac{(\Sigma y)(\Sigma x^2) - (\Sigma xy)(\Sigma x)}{(n)(\Sigma x^2) - (\Sigma x)(\Sigma x)} = -.01266$ MGALS

$m = \dfrac{(n)(\Sigma xy) - (\Sigma x)(\Sigma y)}{(n)(\Sigma x^2) - (\Sigma x)(\Sigma x)} = +.78812$ $\frac{\text{MGALS}}{1000\ \text{MGALS}}$

$1 + m =$ 1.000788

COMPUTED BY: J. Love - AMS | DATE: Oct. '63

REMARKS

CHECKED BY: D. Haddox - AMS | DATE: Oct. '63

DA FORM 2861, 1 OCT 64

*Figure 265. Least Squares Adjustment of Gravimeter Comparisons (DA Form No. 2861).*

by a regression line fit of the observed values relative to the fixed gravity values.

(1) The station names or numbers are entered on DA Form 2861, Least Squares Adjustment of Gravimeter Comparisons (fig. 265).

(2) The published absolute gravity values are entered in column (1).

(3) The corrected changes in gravity are determined and added to the base station absolute value; the values are then listed in column (2) as observed gravity values.

(4) The difference between the fixed and observed gravity is recorded in column (3).

(5) The observed gravity intervals, in gals, from the base published absolute value of the calibration line station are recorded in column (4).

(6) Column (3) is designated $y$ and column (4), $x$. Compute $\Sigma y$, $\Sigma x$, $\Sigma xy$ and $\Sigma x^2$. Compute the intercept, $b$, and the instrument calibration factor, $m$, using the equations on DA Form 2861, Least Squares Adjustment of Gravimeter Comparisons.

(7) With the intercept ($b$) and the slope ($m$), the correction, Y, to the observed value is obtained, and recorded in column (7).

(8) The Y correction (column 7) is added to the observed value (column 2) to obtain the adjusted gravity value (column 8).

(9) The residual error is obtained by subtracting column (8) from column (1) and recording in column (9).

(10) All subsequent observations are to be corrected by the new scale factor by multiplying the values on the calibration chart by the factor $(1+m)$.

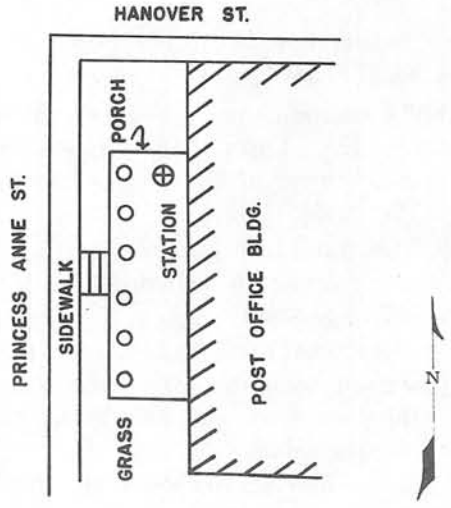
| COUNTRY | TYPE OF STATION | STATION | |
|---|---|---|---|
| USA | Calibration | Fredericksburg | |
| LOCATION | TYPE OF MARK | ESTABLISHED BY | GRAVITY VALUE (g) |
| Fredericksburg, Va. | Disc | USAMS | 980 029.62 |
| LATITUDE | LONGITUDE | DATUM/REFERENCE | ELEV. (FT) ~~(M)~~ |
| 38° 18.1 N | 77° 27.6 W | AMS 1/250,000 | 59.764 |
| REMARKS | | | ORDER: 1st; EST. BY: USC & GS |
| | | | DATUM: MSL |

| DATE | OBSERVER | INSTRUMENT | BASE | BASE VALUE (g) | Δ g |
|---|---|---|---|---|---|
| Jun '63 | J.D. Love | L & RG 41 | Commerce O.M. | 980 118.98 | -89.32 |
| Jun '63 | F. Wilson | L & RG 45 | Commerce O.M. | 980 118.98 | -89.33 |
| Jun '63 | F. Wilson | L & RG 46 | Commerce O.M. | 980 118.98 | -89.43 |
| | | | | | |
| | | | | | |
| | | | | | |

The gravity station is located at the U.S. Post Office Building, Fredericksburg, Virginia. The Post Office is at the S.E. corner of the intersection of Princess Anne Street and Hanover Street. Observations were made at the brass disc which is 0.5 meter from the North edge of the front porch of the Post Office Building.

The brass disc is a standard USC & GS BM marker stamped "C 1 1926."






DATE OF PHOTO: 19 Jun 1963

SKETCH

| (Described) ~~(Recovered)~~ BY | AGENCY | DATE |
|---|---|---|
| J. Sureddin | AMS | Jun 63 |

DA FORM 2862, 1 OCT 64

DESCRIPTION OF GRAVITY STATION (TM 5-237)

*Figure 266. Description of Gravity Station (DA Form 2862).*

## 197. Description of Gravity Station

*a. Importance.* The description, sketch, and numerical information for gravity stations are the final product of the expenditure of man-hours and money. The Chief of Party is responsible for the accuracy and completeness of this information. If the record does not provide sufficient information to make the station readily recoverable, most of the effort of the survey has been wasted. Every reasonable effort should be made to insure that—

(1) The type of mark and the exact location at which a gravity value was observed are described.
(2) Sufficient measurements are made from identifiable surrounding physical or cultural features which would enable the station to be reoccupied in the future.
(3) The station descriptions are written at the site.

*b. Control Card.* DA Form 2862, Description of Gravity Station (fig. 266), is to be used for all gravity stations described.

(1) After verifying the information recorded by the surveyor, tabulate the remaining data which have been computed, such as the gravity value.
(2) The position and elevations should be the best available; however, if no horizontal control is available, scale the position from a map and give the publisher and scale of the map in the reference block.
(3) Designate the type of station (excenter, national base, calibration, etc.).
(4) Note additional information which is applicable in the remarks block.
(5) The section of the form composed of columns headed Date, Observer, Instrument, Base, Base value, and $\Delta g$ receives an entry each time a station is occupied or reoccupied. If more space is needed, extend the columns into a portion of the unlined section of the card.
(6) Complete the description, photograph, and sketch of the station area. Give the name and organization of persons making the description, and the date of the description or recovery.

# APPENDIX I

# REFERENCES

## 1. Army Regulations

| | |
|---|---|
| AR 117–5 | Military Mapping and Surveying |
| AR 320–5 | Dictionary of United States Army Terms |
| AR 320–50 | Authorized Abbreviations and Brevity Codes |

## 2. DA Pamphlets

| | |
|---|---|
| DA Pam 310-series | Military Publications and Index |

## 3. Field Manuals

| | |
|---|---|
| FM 5–1 | Engineer Troop Organizations and Operations |
| FM 5–30 | Engineer Intelligence |
| FM 5–146 | Engineer Topographic Units |

## 4. Technical Manuals

| | |
|---|---|
| TM 5-230 | General Drafting |
| TM 5–231 | Mapping Functions of the Corps of Engineers |
| TM 5–232 | Elements of Surveying |
| TM 5–233 | Construction Surveying |
| TM 5–235 | Special Surveys |
| TM 5–236 | Surveying Tables and Graphs |
| TM 5–236–(Year) | American Ephemeris and Nautical Almanac |
| TM 5–241–1 | Grids and Grid References |
| TM 5–241–2 | UTM Grid Zone to Zone Transformation Tables |
| TM 5–241–3/1 | UTM Grid Tables for Latitudes 0°–80°, International Spheroid. Transformation of Coordinates from Geographic to Grid |
| TM 5–241–3/2 | UTM Grid Tables for Latitudes 0°–80°, International Spheroid. Transformation of Coordinates from Grid to Geographic |
| TM 5–241–4/1 | UTM Grid Tables for Latitudes 0°–80°, Clarke 1866 Spheroid. Transformation of Coordinates from Geographic to Grid |
| TM 5–241–4/2 | UTM Grid Tables for latitudes 0°–80°, Clarke 1866 Spheroid. Transformation of Coordinates from Grid to Geographic |
| TM 5–241–5/1 | UTM Grid Tables for latitudes 0°–80°, Bessel Spheroid. Transformation of Coordinates from Geographic to Grid |
| TM 5–241–5/2 | UTM Grid Tables for latitudes 0°–80°, Bessel Spheroid. Transformation of Coordinates from Grid to Geographic |
| TM 5–241–6/1 | UTM Grid Tables for latitudes 0°–80°, Clarke 1880 Spheroid. Transformation of Coordinates from Geographic to Grid |
| TM 5–241–6/2 | UTM Grid Tables for latitudes 0°–80°, Clarke 1880 Spheroid. Transformation of Coordinates from Grid to Geographic |
| TM 5–241–7 | UTM Grid Tables for latitudes 0°–45°, Everest Spheroid. Transformation of Coordinates from Geographic to Grid and Grid to Geographic |

| | |
|---|---|
| TM 5–241–8 | UTM Grid |
| TM 5–241–9 | Universal Polar Stereographic Grid |
| TM 5–241–11 | Clarke 1866 Spheroid Coordinates for 7½ Minute Intersections |
| TM 5–241–12 | Clarke 1866 Spheroid Coordinates for 5 Minute Intersections |
| TM 5–241–13 | International Spheroid Coordinates for 5 Minute Intersections |
| TM 5–241–14 | Bessel Spheroid Coordinates for 5 Minute Intersections |
| TM 5–241–15 | Clarke 1880 Spheroid Coordinates for 5 Minute Intersections |
| TM 5–241–16 | Everest Spheroid Coordinates for 5 Minute Intersections |
| TM 5–241–17 | Latitude functions, Hayford Spheroid (International Spheroid) |
| TM 5–241–18 | Latitude functions, Clarke 1866 Spheroid |
| TM 5–241–19 | Latitude functions, Bessel Spheroid |
| TM 5–241–20 | Latitude functions, Clarke 1880 Spheroid |
| TM 5–241–21 | Latitude functions, Everest Spheroid |
| TM 5–241–22 | Latitude transformation, Clarke 1866 Spheroid (geodetic-rectifying latitudes) |
| TM 5–241–23 | Latitude transformation, International Spheroid (geodetic-rectifying latitudes) |
| TM 5–241–24 | Latitude transformation, Clarke 1880 Spheroid (geodetic-isometric latitudes) |
| TM 5–241–25 | Latitude transformation, Bessel Spheroid (geodetic-isometric latitudes) |
| TM 5–241–26 | Latitude transformation, Clarke 1866 Spheroid (geodetic-authalic latitudes) |
| TM 5–241–27 | Latitude transformation, Clarke 1866 Spheroid (geodetic-isometric latitudes) |
| TM 5–241–28 | Latitude transformation, Everest Spheroid (geodetic-isometric latitudes) |
| TM 5–241–29 | Latitude transformation, International Spheroid (geodetic-isometric latitudes) |
| TM 5–241–30 | UTM Grid Tables for latitudes 72°–84°, International Spheroid (meters) |
| TM 5–241–31 | UTM Grid Tables for latitudes 72°–84°, Clarke 1866 Spheroid |
| TM 5–6675–202–15 | Tellurometer, Master and Remote |
| TM 5–6675–204–15 | Geodimeter, Mapping and Surveying—30 km Range |
| TM 5–6675–206–15 | Geodimeter, Mapping and Surveying—50 km Range |
| TM 5–6675–210–15 | Theodolite, directional—$\frac{1}{10}$ second |
| TM 5–9421 | Altimeter, surveying, Wallace and Tierman |
| TM 5–441 | Topographic Surveying |

## 5. Technical Bulletins

| | |
|---|---|
| TB Eng 23 | Use of the Tellurometer in Military Surveying |

## 6. Department of Commerce Publications (Coast and Geodetic Survey)

| | |
|---|---|
| Sp. Pub. No. 5 | Tables for a polyconic projection of maps and lengths of terrestrial arcs of meridians and parallels (Clarke 1866) |
| Sp. Pub. No. 8 | Formulas and tables for the computation of geodetic positions |
| 28 | Application of the theory of least-squares to the adjustment of triangulation |
| 62–1 | Tellurometer manual |
| 62–2 | Geodimeter manual |
| 65–1 | Plane coordinate intersection tables |
| 129 | Geodetic level and rod |
| 135 | Tidal datum planes |
| 137 | Manual of first order traverse |
| 138 | Manual of triangulation computation and adjustment |
| 193 | Manual of plane coordinate computation |
| 196 | Manual of tide observations |
| 200 | Formulas and tables for the computation of geodetic positions on the International Ellipsoid |
| 237 | Manual of geodetic astronomy |
| 240 | Manual of leveling computation and adjustment |

| | |
|---|---|
| 241 | Natural tables for the computation of geodetic positions, Clarke spheroid of 1866 |
| 242 | Definitions of terms used in geodetic and other surveys |
| 246 | Tables of sines cosines and tangents to 10 decimal places (0°–6°) |
| 247 | Manual of geodetic triangulation |
| 251 | Conformal projections in Geodesy and Cartography |
| 256 | Projection tables (State of Maine) |
| 274 | Projection tables (State of Massachusetts) |
| G–56 | Elevations from zenith distances (Machine computation) |
| G–58 | Natural tables for computing geographic positions on the International Ellipsoid |

## 7. Army Map Service Publications

| | |
|---|---|
| 201160 | Tables for Transformation of Geographic Coordinates to the Military Grid |

## 8. Department of the Navy (Navy Observatory)

The Air Almanac (published annually in three parts)

## 9. Other Publications

(Heidelberg: Astronomisches Rechen-Institut)
Apparent Places of Fundamental Stars
(Carnegie Institution of Washington, Washington, D.C.)
General Catalogue of 33342 Stars for the Epoch 1950
(Houston Technical Laboratories)
Tidal Gravity Effect Tables
(Journal of European Association of Exploration Geophysicist)
December issue
Tidal Gravity Corrections for (year)

## 10. DA Forms

All forms prescribed in this manual may be requested through normal publications supply channels.

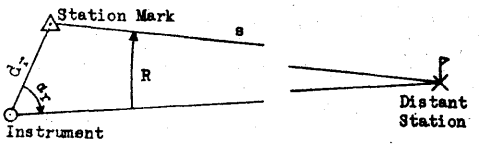
# APPENDIX II

# CHARTS AND GRAPHS

## EXPLANATION -- ECCENTRICITY NOMOGRAM

### Reduction to Center -- Eccentric Instrument

$d_r$ = distance in feet, instrument to station mark,
$\alpha_r$ = clockwise angle from local station mark to distant station.
s = approximate geodetic distance between stations.
All distances horizontal.




The total reduction to center = $\frac{d_r \sin \alpha_r}{s \sin 1''}$ = R, and is applied to the locally observed direction irrespective of the position of the local signal.

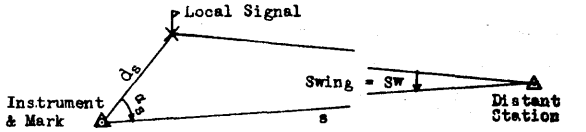
### Swing -- Eccentric Local Signal

If the signal is eccentric, three cases are possible: (1) instrument on center, signal eccentric; (2) instrument and signal both eccentric in the same position; (3) instrument and signal both eccentric in different positions. In each case, the correction is computed for the local eccentric station, but is applied to the direction observed at the distant station:

$d_s$ = distance in feet, instrument to local signal, horizontal,
$\alpha_s$ = clockwise angle from local signal to distant station,
s = approximate geodetic distance between stations.

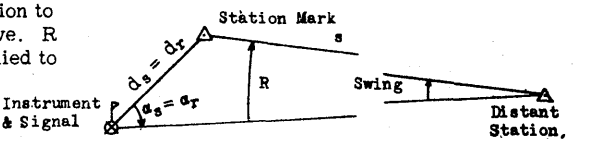
Case 1. - Instrument on center, local signal eccentric.

The total swing = $-\frac{d_s \sin \alpha_s}{s \sin 1''}$ = Sw



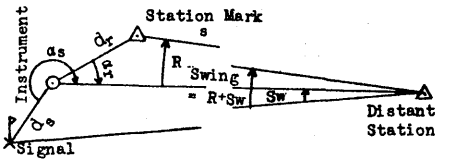

Case 2. - Instrument and signal both eccentric in same position.

Total swing is identical with reduction to center, and is equal to R as determined above. R is applied to local observation; swing is applied to distant observation.




Case 3. - Instrument and local signal eccentric in different positions.

Total swing consists of two partial corrections: Sw, to swing the line from the eccentric signal to the eccentric instrument; and R, to complete the swing from the eccentric instrument to the station mark. These two partial corrections, R and Sw, must be added algebraically.




### TO USE THE NOMOGRAM

1. Place a straight edge across the three inner scales, connecting the value of d on the inner left scale with the value of $\alpha$ on the inner right scale; mark the crossing point on the center (y) scale; it is not necessary to read the value of y.

2. Holding as a pivot the point on the center (y) scale so determined, swing the straight edge to the value of s (or log s ) on the outer left scale; the intersected value on the outer right scale is the correction for reduction to center or the partial correction for swing, with the sign indicated by the legend on the $\alpha$ scale.

If the procedure of step 2 brings the straight edge beyond the limits of the outer right scale, multiply s by 10 or add 1.00 to log s; then multiply the resulting R or Sw by 10.

*CHART 1*

# NOMOGRAM FOR ECCENTRICITY CORRECTIONS
## (REDUCTION TO CENTER; SWING)
## GEODETIC TRIANGULATION

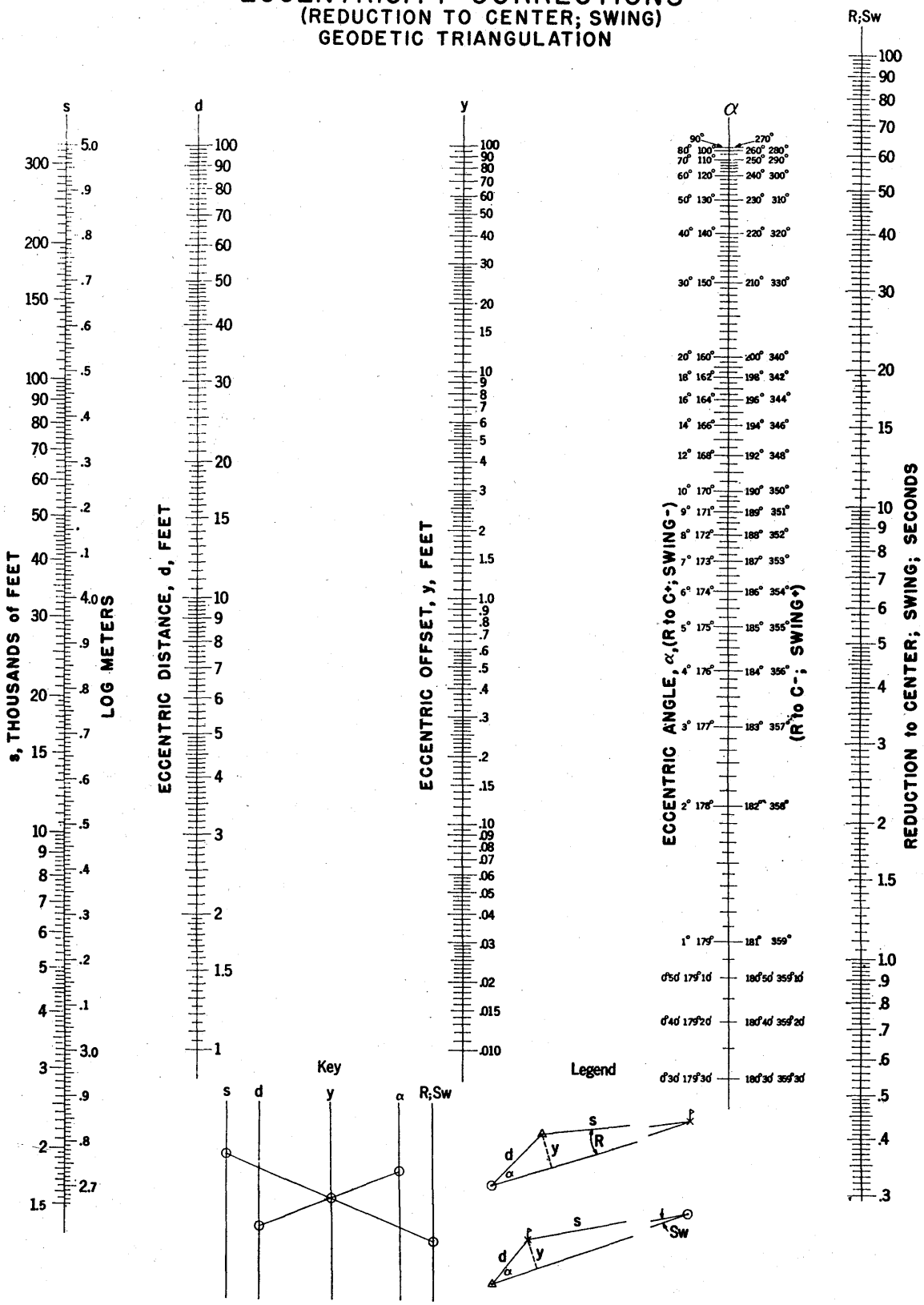



*CHART 2*

# SEA LEVEL REDUCTION CHART

## DIAGRAM NO. 2

CORRECTION IN SECONDS

.335
.325
.315
.305
.295
.285
.275
.265
.255
.245
.235
.225
.215
.205
.195
.185
.175
.165
.155
.145
.135
.125
.115
.105
.095
.085
.075
.065
.055
.045
.035
.025
.015
.000

**USE OF CHART**

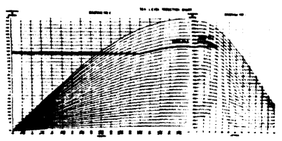
**EXAMPLE** To obtain the sea level reduction for a station whose elevation is 2440 meters, azimuth is 32°50' and the latitude is 15°.

**STEP 1:** Enter Diagram No. 1 at the correct elevation (2440 meters). Follow along between the curves of elevation to the correct line of latitude (15°). From this point, proceed horizontally to the left margin of Diagram No. 1. (See sketch below.)

**STEP 2:** Extend the horizontal line of Diagram No. 1 to the right margin of Diagram No. 2. Then follow along and proportionately between the curves to the given azimuth (32°50'). From this point, proceed horizontally to the left margin of Diagram No. 2, reading the sea level correction in thousandths of a second: in this example, 0.230". (See sketch below.)

The sign of the correction (+ or -) is determined by the azimuth. Where the azimuth is less than 90°, or between 180° and 270°, the sign is plus. For all other azimuths, it is minus.

For elevations greater than 3000 meters, divide the elevation by 2 and multiply the resulting correction by 2 for correct reduction.



0° 90° | 2°30' 87°30' | 5° 85° | 7°30' 82°30' | 10° 80° | 12°30' 77°30' | 15° 75° | 17°30' 72°30' | 20° 70° | 22°30' 67°30' | 25° 65° | 27°30' 62°30' | 30° 60° | 32°30' 57°30' | 35° 55° | 37°30' 52°30' | 40° 50° | 42°30' 47°30' | 45°

AZIMUTH

## DIAGRAM NO. 1

ELEVATION IN METERS

3000
2900
2800
2700
2600
2500
2400
2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100

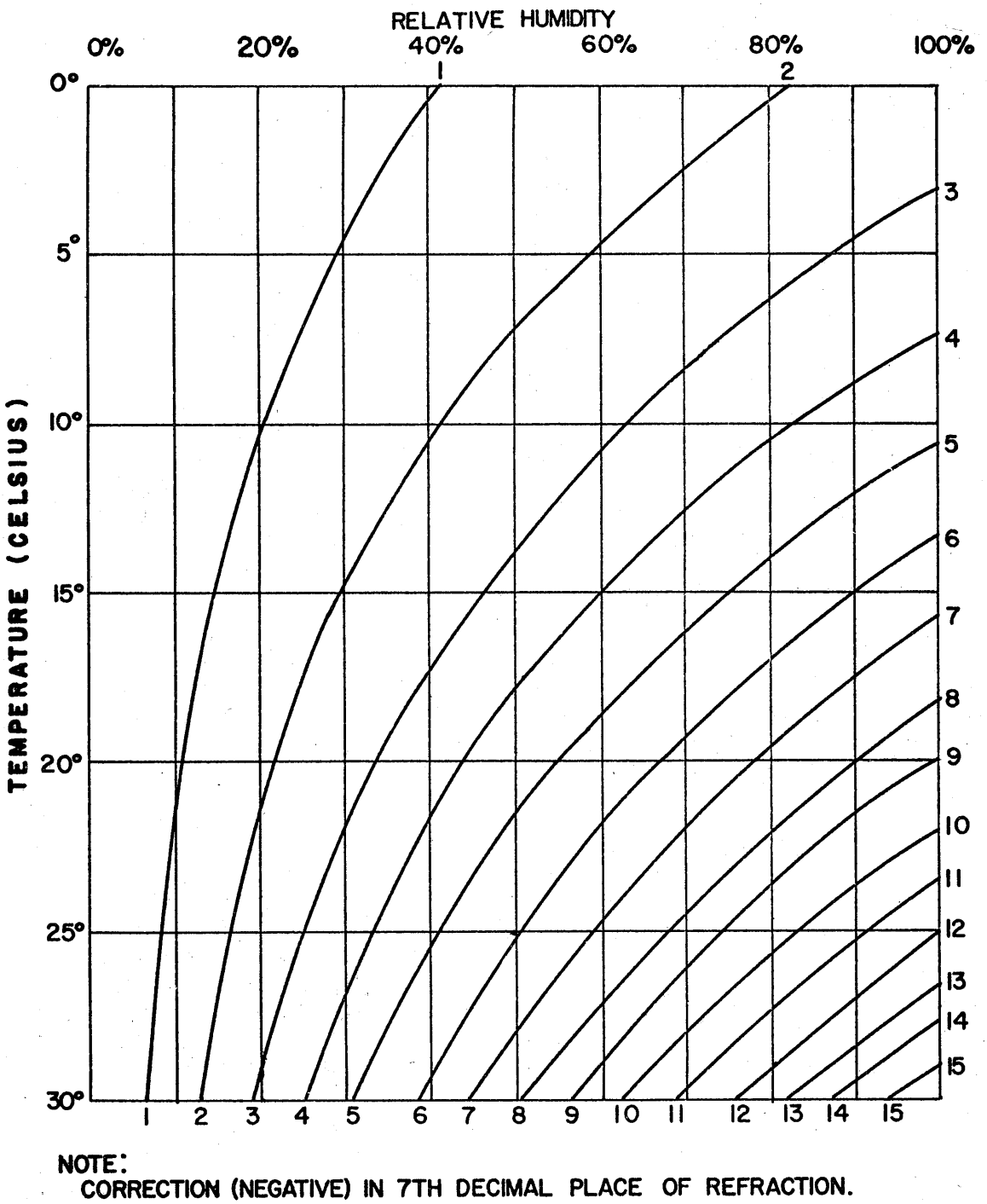
0 5 10 15 20 25 30 35 40 45 50 55 60

LATITUDE

*CHART 3*

## CORRECTION FOR HUMIDITY (LIGHT WAVES)

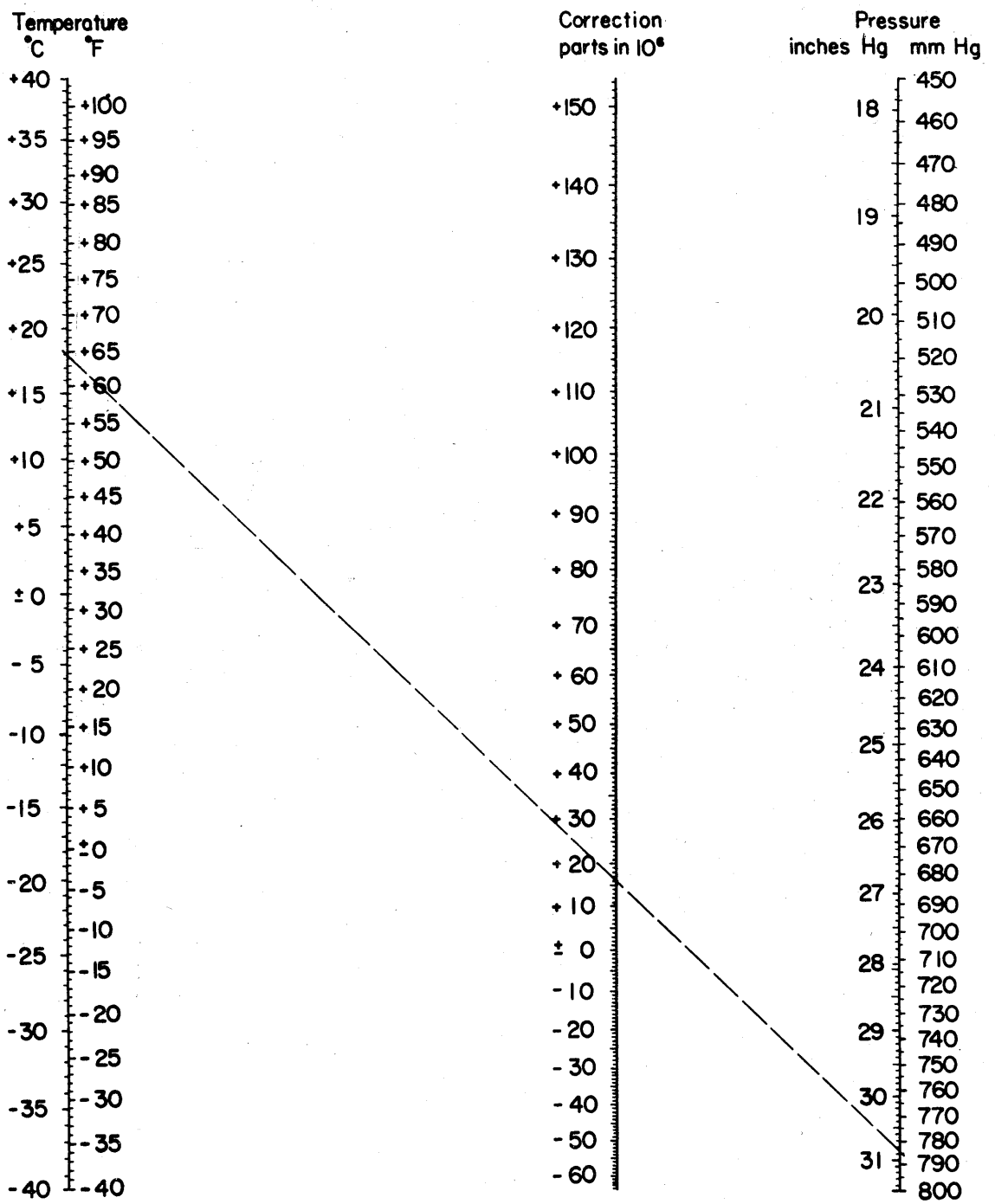
## GRAPHIC REPRESENTATION OF $-\frac{1.5026e10^{-5}}{273.2+t}$




NOTE:
CORRECTION (NEGATIVE) IN 7TH DECIMAL PLACE OF REFRACTION.

*CHART 4. CORRECTION FOR HUMIDITY (LIGHT WAVES)*

TEMPERATURE - PRESSURE CORRECTION - MODEL 4 GEODIMETER



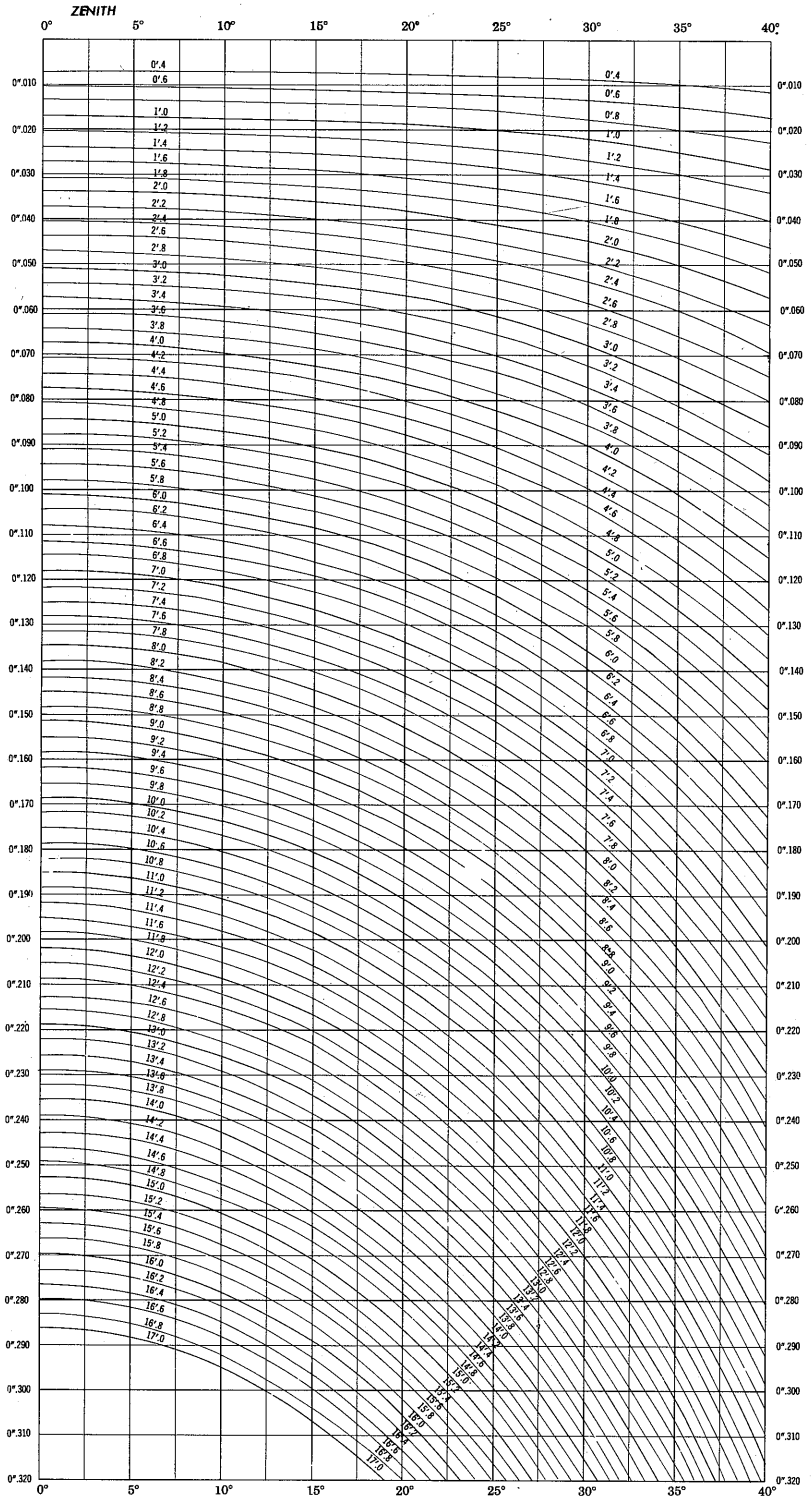

**Nomogram for the atmospheric correction ($t_o$ = - 6°C, $p_o$ = 760 mm Hg).**

**Ex: t = + 18°C, p = 785 mm Hg, corr. = + 16 x $10^{-6}$ x distance.**

**If the barometer has a millibar graduation use the relation:**

**1000 m bar = 750 mm Hg.**

*CHART 5. TEMPERATURE-PRESSURE CORRECTION—MODEL 4 GEODIMETER*




CORRECTION TO LATITUDE FOR DIFFERENTIAL REFRACTION

The curved lines represent one-half difference of zenith distance. The vertical lines represent zenith distance of a star, and on outside margin lines are corrections which can be read quickly to nearest two thousandth of a second. This correction is to be multiplied by (Ct. Bt.) which are dependent upon temperature and barometric pressure. Example: ½ difference of zenith is 9.7 minutes; zenith is 36°-15′, what is differential refraction. Interpolate between 9.6 and 9.8 on curve line and follow to 36°-15′. This intersection shows correction on horizontal line to be .252 seconds.

*CHART 6. CORRECTION TO LATITUDE FOR DIFFERENTIAL REFRACTION*

# APPENDIX III

# TABLES

*Table I. Factors for Use in Computing Elevations by Vertical Angles (Machine Method)*

| Elevation of $h_1$ | A | $s \tan \frac{1}{2}(\zeta_2 - \zeta_1)$ | B | $s$ | C |
|---|---|---|---|---|---|
| *meters* | | *meters* | | *meters* | |
| 0 | 1.000000 | 0 | 1.000000 | 0 | 1.000000 |
| 100 | 16 | 100 | 8 | 75,000 | 12 |
| 200 | 31 | 200 | 16 | 130,000 | 35 |
| 300 | 47 | 300 | 24 | 168,000 | 58 |
| 400 | 63 | 400 | 31 | 198,000 | 81 |
| 500 | 78 | 500 | 39 | 225,000 | 104 |
| 600 | 94 | 600 | 47 | 250,000 | 127 |
| 700 | 110 | 700 | 55 | 270,000 | 150 |
| 800 | 125 | 800 | 63 | 290,000 | 173 |
| 900 | 141 | 900 | 71 | | |
| 1000 | 1.000157 | 1000 | 1.000079 | | |
| 1100 | 172 | 1100 | 86 | | |
| 1200 | 188 | 1200 | 94 | | |
| 1300 | 204 | 1300 | 102 | | |
| 1400 | 219 | 1400 | 110 | | |
| 1500 | 235 | 1500 | 118 | | |
| 1600 | 251 | 1600 | 126 | | |
| 1700 | 267 | 1700 | 133 | | |
| 1800 | 282 | 1800 | 141 | | |
| 1900 | 298 | 1900 | 149 | | |
| 2000 | 1.000314 | 2000 | 1.000157 | | |
| 2100 | 329 | 2100 | 165 | | |
| 2200 | 345 | 2200 | 173 | | |
| 2300 | 361 | 2300 | 181 | | |
| 2400 | 376 | 2400 | 188 | | |
| 2500 | 392 | 2500 | 196 | | |
| 2600 | 408 | 2600 | 204 | | |
| 2700 | 423 | 2700 | 212 | | |
| 2800 | 439 | 2800 | 220 | | |
| 2900 | 455 | 2900 | 228 | | |
| 3000 | 1.000470 | 3000 | 1.000236 | | |
| 3100 | 486 | 3100 | 243 | | |
| 3200 | 502 | 3200 | 251 | | |
| 3300 | 517 | 3300 | 259 | | |
| 3400 | 533 | 3400 | 267 | | |
| 3500 | 549 | 3500 | 275 | | |
| 3600 | 564 | 3600 | 283 | | |
| 3700 | 580 | 3700 | 290 | | |
| 3800 | 596 | 3800 | 298 | | |
| 3900 | 611 | 3900 | 306 | | |
| 4000 | 1.000627 | 4000 | 1.000314 | | |

*Table II. Corrections to $h_2-h_1$ in Centimeters*

(Correction always added numerically to $h_2-h_1$)

$h_1$ in meters

| $h_2-h_1$ in meters | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | $h_2-h_1$ in meters |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +1500 | 18 | 20 | 22 | 25 | 27 | 29 | 32 | 34 | 36 | 39 | 41 | 44 | 46 | 48 | 51 | 53 | +1500 |
| +1400 | 15 | 18 | 20 | 22 | 24 | 26 | 29 | 31 | 33 | 35 | 37 | 40 | 42 | 44 | 46 | 48 | +1400 |
| +1300 | 13 | 15 | 17 | 19 | 21 | 23 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | +1300 |
| +1200 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | +1200 |
| +1100 | 9 | 11 | 13 | 15 | 16 | 18 | 20 | 22 | 23 | 25 | 27 | 28 | 30 | 32 | 34 | 35 | +1100 |
| +1000 | 8 | 9 | 11 | 13 | 14 | 16 | 17 | 19 | 20 | 22 | 24 | 25 | 27 | 28 | 30 | 31 | +1000 |
| +900 | 6 | 8 | 9 | 11 | 12 | 13 | 15 | 16 | 18 | 19 | 20 | 22 | 23 | 25 | 26 | 28 | +900 |
| +800 | 5 | 6 | 8 | 9 | 10 | 11 | 13 | 14 | 15 | 16 | 18 | 19 | 20 | 21 | 23 | 24 | +800 |
| +700 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | +700 |
| +600 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | +600 |
| +500 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 13 | 14 | +500 |
| +400 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 | 6 | 7 | 8 | 8 | 9 | 9 | 10 | 11 | +400 |
| +300 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 | +300 |
| +200 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | +200 |
| +100 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | +100 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| −100 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | −100 |
| −200 | | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | −200 |
| −300 | | | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 6 | 6 | −300 |
| −400 | | | | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 | 6 | 7 | 8 | 8 | −400 |
| −500 | | | | | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | −500 |
| −600 | | | | | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 9 | 10 | 11 | −600 |
| −700 | | | | | | | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 13 | −700 |
| −800 | | | | | | | | 4 | 5 | 6 | 8 | 9 | 10 | 11 | 13 | 14 | −800 |
| −900 | | | | | | | | | 5 | 6 | 8 | 9 | 11 | 12 | 13 | 15 | −900 |
| −1000 | | | | | | | | | | 6 | 8 | 9 | 11 | 13 | 14 | 16 | −1000 |
| −1100 | | | | | | | | | | | 8 | 9 | 11 | 13 | 15 | 16 | −1100 |
| −1200 | | | | | | | | | | | | 9 | 11 | 13 | 15 | 17 | −1200 |
| −1300 | | | | | | | | | | | | | 11 | 13 | 15 | 17 | −1300 |
| −1400 | | | | | | | | | | | | | | 13 | 15 | 18 | −1400 |
| −1500 | | | | | | | | | | | | | | | 15 | 18 | −1500 |
| | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | |

$h_1$ in meters

| $h_2-h_1$ in meters | 1500 | 1600 | 1700 | 1800 | 1900 | 2000 | 2100 | 2200 | 2300 | 2400 | 2500 | 2600 | 2700 | 2800 | 2900 | 3000 | $h_2-h_1$ in meters |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +1500 | 53 | 55 | 58 | 60 | 62 | 65 | 67 | 69 | 72 | 74 | 77 | 79 | 81 | 84 | 86 | 88 | +1500 |
| +1400 | 48 | 51 | 53 | 55 | 57 | 59 | 62 | 64 | 66 | 68 | 70 | 73 | 75 | 77 | 79 | 81 | +1400 |
| +1300 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 | 72 | 74 | +1300 |
| +1200 | 40 | 41 | 43 | 45 | 47 | 49 | 51 | 53 | 55 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | +1200 |
| +1100 | 35 | 37 | 39 | 41 | 42 | 44 | 46 | 47 | 49 | 51 | 53 | 54 | 56 | 58 | 60 | 61 | +1100 |
| +1000 | 31 | 33 | 35 | 36 | 38 | 39 | 41 | 42 | 44 | 46 | 47 | 49 | 50 | 52 | 53 | 55 | +1000 |
| +900 | 28 | 29 | 30 | 32 | 33 | 35 | 36 | 37 | 39 | 40 | 42 | 43 | 44 | 46 | 47 | 49 | +900 |
| +800 | 24 | 25 | 26 | 28 | 29 | 30 | 31 | 33 | 34 | 35 | 36 | 38 | 39 | 40 | 41 | 43 | +800 |
| +700 | 20 | 21 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 34 | 35 | 36 | 37 | +700 |
| +600 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | +600 |
| +500 | 14 | 15 | 15 | 16 | 17 | 18 | 18 | 19 | 20 | 21 | 22 | 22 | 23 | 24 | 25 | 26 | +500 |
| +400 | 11 | 11 | 12 | 13 | 13 | 14 | 14 | 15 | 16 | 16 | 17 | 18 | 18 | 19 | 19 | 20 | +400 |
| +300 | 8 | 8 | 9 | 9 | 10 | 10 | 11 | 11 | 12 | 12 | 12 | 13 | 13 | 14 | 14 | 15 | +300 |
| +200 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 8 | 9 | 9 | 9 | 10 | +200 |
| +100 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | +100 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| −100 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | −100 |
| −200 | 4 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 8 | 9 | 9 | −200 |
| −300 | 6 | 7 | 7 | 8 | 8 | 9 | 9 | 10 | 10 | 11 | 11 | 12 | 12 | 12 | 13 | 13 | −300 |
| −400 | 8 | 9 | 9 | 10 | 11 | 11 | 12 | 13 | 13 | 14 | 14 | 15 | 16 | 16 | 17 | 18 | −400 |
| −500 | 10 | 11 | 11 | 12 | 13 | 14 | 15 | 15 | 16 | 17 | 18 | 18 | 19 | 20 | 21 | 22 | −500 |
| −600 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 24 | 25 | −600 |
| −700 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | −700 |
| −800 | 14 | 15 | 16 | 18 | 19 | 20 | 21 | 23 | 24 | 25 | 26 | 28 | 29 | 30 | 31 | 33 | −800 |
| −900 | 15 | 16 | 18 | 19 | 20 | 22 | 23 | 25 | 26 | 28 | 29 | 30 | 32 | 33 | 35 | 36 | −900 |
| −1000 | 16 | 17 | 19 | 20 | 22 | 24 | 25 | 27 | 28 | 30 | 31 | 33 | 35 | 36 | 38 | 39 | −1000 |
| −1100 | 16 | 18 | 20 | 22 | 23 | 25 | 27 | 28 | 30 | 32 | 34 | 35 | 37 | 39 | 41 | 42 | −1100 |
| −1200 | 17 | 19 | 21 | 23 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 41 | 43 | 45 | −1200 |
| −1300 | 17 | 19 | 21 | 23 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | −1300 |
| −1400 | 18 | 20 | 22 | 24 | 26 | 29 | 31 | 33 | 35 | 37 | 40 | 42 | 44 | 46 | 48 | 51 | −1400 |
| −1500 | 18 | 20 | 22 | 25 | 27 | 29 | 32 | 34 | 36 | 39 | 41 | 44 | 46 | 48 | 51 | 53 | −1500 |
| | 1500 | 1600 | 1700 | 1800 | 1900 | 2000 | 2100 | 2200 | 2300 | 2400 | 2500 | 2600 | 2700 | 2800 | 2900 | 3000 | |

$h_1$ in meters

## *Table III. Tables for Refraction Computations*

$\rho \sin 1''$

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 32° | 33° | 34° | 35° | 36° | 37° | 38° | 39° | 40° |
| 0 | 30.801 | 30.806 | 30.811 | 30.816 | 30.821 | 30.826 | 30.831 | 30.837 | 30.842 |
| 5 | .801 | .806 | .811 | .817 | .822 | .827 | .833 | .838 | .843 |
| 10 | .805 | .810 | .815 | .820 | .825 | .830 | .835 | .840 | .846 |
| 15 | .811 | .816 | .820 | .825 | .830 | .835 | .840 | .845 | .850 |
| 20 | .818 | .823 | .828 | .832 | .837 | .842 | .847 | .852 | .857 |
| 25 | .828 | .832 | .836 | .840 | .845 | .850 | .855 | .860 | .864 |
| 30 | .838 | .843 | .847 | .851 | .855 | .860 | .864 | .868 | .873 |
| 35 | .850 | .854 | .858 | .862 | .866 | .870 | .874 | .878 | .882 |
| 40 | .862 | .867 | .870 | .874 | .877 | .881 | .885 | .889 | .893 |
| 45 | .876 | .879 | .882 | .886 | .889 | .893 | .897 | .900 | .904 |
| 50 | .889 | .892 | .895 | .898 | .902 | .904 | .908 | .911 | .914 |
| 55 | .902 | .904 | .907 | .910 | .913 | .916 | .919 | .921 | .925 |
| 60 | .914 | .916 | .919 | .921 | .924 | .926 | .929 | .932 | .934 |
| 65 | .924 | .926 | .930 | .931 | .934 | .936 | .939 | .941 | .944 |
| 70 | .934 | .936 | .938 | .940 | .942 | .944 | .946 | .949 | .951 |
| 75 | .941 | .944 | .946 | .947 | .949 | .951 | .953 | .955 | .957 |
| 80 | .947 | .949 | .951 | .952 | .954 | .956 | .958 | .960 | .961 |
| 85 | .951 | .952 | .954 | .956 | .957 | .959 | .961 | .963 | .964 |
| 90 | .951 | .954 | .955 | .956 | .959 | .960 | .962 | .963 | .966 |

| Azimuth (degrees) | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° | 49° |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 30.848 | 30.853 | 30.858 | 30.864 | 30.870 | 30.875 | 30.880 | 30.886 | 30.891 |
| 5 | .848 | .854 | .860 | .865 | .870 | .775 | .881 | .887 | .892 |
| 10 | .851 | .857 | .862 | .867 | .872 | .878 | .883 | .889 | .894 |
| 15 | .855 | .861 | .866 | .871 | .877 | .882 | .887 | .892 | .897 |
| 20 | .862 | .867 | .872 | .877 | .882 | .887 | .892 | .897 | .902 |
| 25 | .869 | .874 | .878 | .883 | .888 | .893 | .898 | .902 | .907 |
| 30 | .877 | .882 | .887 | .891 | .896 | .900 | .905 | .909 | .914 |
| 35 | .887 | .891 | .895 | .899 | .904 | .908 | .912 | .916 | .921 |
| 40 | .897 | .901 | .905 | .909 | .913 | .916 | .921 | .925 | .929 |
| 45 | .907 | .911 | .914 | .918 | .922 | .926 | .929 | .933 | .936 |
| 50 | .918 | .921 | .924 | .928 | .931 | .934 | .938 | .941 | .944 |
| 55 | .928 | .931 | .934 | .936 | .940 | .943 | .946 | .949 | .952 |
| 60 | .937 | .940 | .943 | .945 | .948 | .951 | .954 | .956 | .959 |
| 65 | .946 | .949 | .952 | .954 | .956 | .958 | .961 | .963 | .967 |
| 70 | .954 | .956 | .958 | .960 | .962 | .964 | .966 | .969 | .971 |
| 75 | .959 | .961 | .963 | .966 | .968 | .969 | .971 | .973 | .976 |
| 80 | .963 | .966 | .968 | .969 | .971 | .973 | .975 | .977 | .979 |
| 85 | .966 | .968 | .970 | .972 | .973 | .976 | .978 | .979 | .981 |
| 90 | .967 | .969 | .971 | .973 | .975 | .976 | .978 | .980 | .982 |

*Table III. Tables for Refraction Computations*—Continued

$\rho \sin 1''$

| Azimuth (degrees) | Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 50° | 51° | 52° | 53° | 54° | 55° | 56° | 57° | 58° |
| 0 | 30.897 | 30.902 | 30.907 | 30.913 | 30.918 | 30.923 | 30.929 | 30.934 | 30.939 |
| 5 | .897 | .903 | .908 | .914 | .919 | .924 | .929 | .934 | .939 |
| 10 | .899 | .904 | .910 | .915 | .920 | .925 | .930 | .935 | .940 |
| 15 | .902 | .908 | .913 | .918 | .923 | .928 | .933 | .938 | .942 |
| 20 | .907 | .911 | .916 | .921 | .926 | .931 | .936 | .941 | .945 |
| 25 | .912 | .917 | .921 | .926 | .931 | .936 | .940 | .944 | .949 |
| 30 | .919 | .923 | .927 | .932 | .936 | .941 | .945 | .949 | .954 |
| 35 | .925 | .929 | .934 | .938 | .942 | .946 | .950 | .954 | .958 |
| 40 | .933 | .936 | .941 | .944 | .948 | .952 | .956 | .959 | .963 |
| 45 | .940 | .944 | .947 | .951 | .954 | .958 | .961 | .965 | .968 |
| 50 | .948 | .951 | .954 | .958 | .961 | .963 | .967 | .970 | .973 |
| 55 | .955 | .958 | .961 | .963 | .967 | .970 | .973 | .976 | .978 |
| 60 | .962 | .965 | .967 | .970 | .973 | .975 | .978 | .981 | .983 |
| 65 | .968 | .971 | .973 | .976 | .978 | .980 | .983 | .985 | .987 |
| 70 | .973 | .976 | .978 | .980 | .982 | .984 | .986 | .988 | .991 |
| 75 | .978 | .980 | .982 | .984 | .986 | .988 | .990 | .992 | .993 |
| 80 | .981 | .983 | .985 | .987 | .988 | .991 | .992 | .994 | .996 |
| 85 | .983 | .985 | .986 | .988 | .991 | .992 | .993 | .996 | .997 |
| 90 | .983 | .986 | .987 | .989 | .991 | .993 | .994 | .996 | .998 |

| Azimuth (degrees) | 59° | 60° | 61° | 62° | 63° | 64° | 65° | 66° | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 30.944 | 30.948 | 30.953 | 30.958 | 30.962 | 30.966 | 30.971 | 30.975 | |
| 5 | .944 | .949 | .954 | .958 | .962 | .966 | .971 | .975 | |
| 10 | .945 | .950 | .954 | .959 | .964 | .968 | .972 | .976 | |
| 15 | .947 | .951 | .956 | .961 | .965 | .969 | .974 | .977 | |
| 20 | .950 | .954 | .959 | .963 | .967 | .971 | .975 | .979 | |
| 25 | .955 | .958 | .962 | .966 | .970 | .974 | .978 | .981 | |
| 30 | .957 | .961 | .965 | .969 | .973 | .976 | .980 | .984 | |
| 35 | .961 | .966 | .969 | .973 | .976 | .980 | .984 | .986 | |
| 40 | .966 | .970 | .973 | .976 | .980 | .984 | .986 | .989 | |
| 45 | 30.971 | 30.974 | 30.978 | 30.981 | 30.984 | 30.986 | 30.990 | 30.992 | |
| 50 | .976 | .979 | .982 | .985 | .988 | .990 | .993 | .996 | |
| 55 | .981 | .983 | .986 | .988 | .991 | .993 | .996 | 30.998 | |
| 60 | .985 | .988 | .990 | .992 | .995 | 30.997 | 30.999 | 31.001 | |
| 65 | .989 | .991 | .993 | .996 | 30.998 | 31.000 | 31.002 | .003 | |
| 70 | .993 | .995 | .996 | 30.998 | 31.001 | .002 | .004 | .006 | |
| 75 | .996 | .997 | 30.999 | 31.001 | .003 | .004 | .006 | .008 | |
| 80 | .998 | .999 | 31.001 | .003 | .004 | .006 | .007 | .008 | |
| 85 | .999 | 31.001 | .002 | .003 | .005 | .006 | .008 | .009 | |
| 90 | 30.999 | 31.001 | 31.003 | 31.004 | 31.006 | 31.007 | 31.008 | 31.010 | |

*Table IV. Logarithms of Radii of Curvature of the Earth's Surface (in Meters)*

[Based upon Clarke's spheroid of 1866 as expressed in meters]

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0° | 1° | 2° | 3° | 4° | 5° | 6° | 7° | 8° | |
| 0 | 6.80175 | 6.80175 | 6.80175 | 6.80176 | 6.80177 | 6.80178 | 6.80180 | 6.80181 | 6.80183 | 180 |
| 5 | 177 | 177 | 178 | 178 | 179 | 180 | 182 | 184 | 186 | 175 |
| 10 | 184 | 184 | 184 | 185 | 186 | 187 | 188 | 190 | 192 | 170 |
| 15 | 195 | 195 | 195 | 196 | 197 | 198 | 199 | 201 | 203 | 165 |
| 20 | 209 | 209 | 210 | 210 | 211 | 212 | 214 | 215 | 217 | 160 |
| 25 | 227 | 228 | 228 | 228 | 229 | 230 | 232 | 233 | 235 | 155 |
| 30 | 248 | 249 | 249 | 250 | 250 | 251 | 252 | 254 | 256 | 150 |
| 35 | 272 | 272 | 272 | 273 | 273 | 274 | 276 | 277 | 278 | 145 |
| 40 | 296 | 297 | 297 | 297 | 298 | 299 | 300 | 301 | 303 | 140 |
| 45 | 322 | 322 | 322 | 323 | 324 | 324 | 325 | 326 | 328 | 135 |
| 50 | 348 | 348 | 348 | 348 | 349 | 350 | 351 | 352 | 353 | 130 |
| 55 | 373 | 373 | 373 | 373 | 374 | 374 | 375 | 376 | 377 | 125 |
| 60 | 396 | 396 | 396 | 396 | 397 | 398 | 398 | 399 | 400 | 120 |
| 65 | 417 | 417 | 417 | 418 | 418 | 418 | 419 | 420 | 421 | 115 |
| 70 | 435 | 435 | 436 | 436 | 436 | 437 | 437 | 438 | 439 | 110 |
| 75 | 450 | 450 | 450 | 450 | 451 | 451 | 452 | 452 | 453 | 105 |
| 80 | 461 | 461 | 461 | 461 | 462 | 462 | 463 | 463 | 464 | 100 |
| 85 | 468 | 468 | 468 | 468 | 468 | 469 | 469 | 470 | 470 | 95 |
| 90 | 470 | 470 | 470 | 470 | 471 | 471 | 472 | 472 | 473 | 90 |

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 8° | 9° | 10° | 11° | 12° | 13° | 14° | 15° | 16° | |
| 0 | 6.80183 | 6.80186 | 6.80188 | 6.80191 | 6.80194 | 6.80197 | 6.80201 | 6.80204 | 6.80208 | 180 |
| 5 | 186 | 188 | 190 | 193 | 196 | 199 | 203 | 206 | 210 | 175 |
| 10 | 192 | 194 | 197 | 200 | 202 | 206 | 209 | 213 | 217 | 170 |
| 15 | 203 | 205 | 207 | 210 | 213 | 216 | 219 | 223 | 227 | 165 |
| 20 | 217 | 219 | 222 | 224 | 227 | 230 | 233 | 236 | 240 | 160 |
| 25 | 235 | 237 | 239 | 242 | 244 | 247 | 250 | 254 | 257 | 155 |
| 30 | 256 | 257 | 260 | 262 | 264 | 267 | 270 | 273 | 276 | 150 |
| 35 | 278 | 280 | 282 | 284 | 287 | 289 | 292 | 295 | 298 | 145 |
| 40 | 303 | 304 | 306 | 308 | 310 | 313 | 315 | 318 | 321 | 140 |
| 45 | 328 | 329 | 331 | 333 | 335 | 337 | 339 | 342 | 344 | 135 |
| 50 | 353 | 354 | 356 | 358 | 359 | 361 | 364 | 366 | 368 | 130 |
| 55 | 377 | 379 | 380 | 382 | 383 | 385 | 387 | 389 | 391 | 125 |
| 60 | 400 | 401 | 403 | 404 | 406 | 407 | 409 | 411 | 413 | 120 |
| 65 | 421 | 422 | 423 | 424 | 426 | 427 | 429 | 430 | 432 | 115 |
| 70 | 439 | 440 | 441 | 442 | 443 | 444 | 446 | 447 | 449 | 110 |
| 75 | 453 | 454 | 455 | 456 | 457 | 458 | 460 | 461 | 463 | 105 |
| 80 | 464 | 465 | 466 | 467 | 468 | 469 | 470 | 471 | 473 | 100 |
| 85 | 470 | 471 | 472 | 473 | 474 | 475 | 476 | 478 | 479 | 95 |
| 90 | 473 | 474 | 474 | 475 | 476 | 477 | 478 | 480 | 481 | 90 |

*Table IV. Logarithms of Radii of Curvature of the Earth's Surface (in Meters)*—Continued

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 16° | 17° | 18° | 19° | 20° | 21° | 22° | 23° | 24° | |
| 0 | 6.80208 | 6.80213 | 6.80217 | 6.80222 | 6.80226 | 6.80232 | 6.80237 | 6.80242 | 6.80248 | 180 |
| 5 | 210 | 215 | 219 | 224 | 228 | 234 | 239 | 244 | 250 | 175 |
| 10 | 217 | 221 | 225 | 230 | 234 | 239 | 244 | 250 | 255 | 170 |
| 15 | 227 | 231 | 235 | 239 | 244 | 249 | 254 | 259 | 264 | 165 |
| 20 | 240 | 244 | 248 | 252 | 257 | 262 | 266 | 271 | 277 | 160 |
| 25 | 257 | 261 | 265 | 269 | 273 | 277 | 282 | 287 | 292 | 155 |
| 30 | 276 | 280 | 284 | 287 | 292 | 296 | 300 | 305 | 309 | 150 |
| 35 | 298 | 301 | 305 | 308 | 312 | 316 | 320 | 324 | 329 | 145 |
| 40 | 321 | 324 | 327 | 330 | 334 | 338 | 341 | 345 | 350 | 140 |
| 45 | 344 | 347 | 350 | 353 | 357 | 360 | 364 | 367 | 371 | 135 |
| 50 | 368 | 371 | 373 | 376 | 379 | 382 | 386 | 389 | 392 | 130 |
| 55 | 391 | 394 | 396 | 398 | 401 | 404 | 407 | 410 | 413 | 125 |
| 60 | 413 | 415 | 417 | 419 | 422 | 424 | 427 | 430 | 432 | 120 |
| 65 | 432 | 434 | 436 | 438 | 440 | 443 | 445 | 448 | 450 | 115 |
| 70 | 449 | 451 | 453 | 454 | 456 | 459 | 461 | 463 | 465 | 110 |
| 75 | 463 | 464 | 466 | 468 | 470 | 472 | 473 | 476 | 478 | 105 |
| 80 | 473 | 474 | 476 | 478 | 479 | 481 | 483 | 485 | 487 | 100 |
| 85 | 479 | 480 | 482 | 483 | 485 | 487 | 489 | 490 | 492 | 95 |
| 90 | 481 | 482 | 484 | 485 | 487 | 489 | 490 | 492 | 494 | 90 |

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 24° | 25° | 26° | 27° | 28° | 29° | 30° | 31° | 32° | |
| 0 | 6.80248 | 6.80254 | 6.80260 | 6.80266 | 6.80272 | 6.80279 | 6.80285 | 6.80292 | 6.80299 | 180 |
| 5 | 250 | 256 | 262 | 268 | 274 | 280 | 287 | 294 | 300 | 175 |
| 10 | 255 | 261 | 267 | 273 | 279 | 285 | 292 | 298 | 305 | 170 |
| 15 | 264 | 270 | 276 | 282 | 288 | 294 | 300 | 306 | 313 | 165 |
| 20 | 277 | 282 | 288 | 293 | 299 | 305 | 311 | 317 | 324 | 160 |
| 25 | 292 | 297 | 302 | 308 | 313 | 319 | 325 | 331 | 337 | 155 |
| 30 | 309 | 314 | 319 | 324 | 330 | 335 | 340 | 346 | 352 | 150 |
| 35 | 329 | 333 | 338 | 343 | 348 | 353 | 358 | 363 | 369 | 145 |
| 40 | 350 | 354 | 358 | 362 | 367 | 372 | 377 | 382 | 386 | 140 |
| 45 | 371 | 375 | 379 | 383 | 387 | 391 | 396 | 400 | 405 | 135 |
| 50 | 392 | 396 | 399 | 403 | 407 | 411 | 415 | 419 | 423 | 130 |
| 55 | 413 | 416 | 420 | 423 | 426 | 430 | 434 | 437 | 441 | 125 |
| 60 | 432 | 435 | 438 | 442 | 445 | 448 | 451 | 455 | 458 | 120 |
| 65 | 450 | 453 | 455 | 458 | 461 | 464 | 467 | 470 | 473 | 115 |
| 70 | 465 | 468 | 470 | 473 | 475 | 478 | 481 | 484 | 486 | 110 |
| 75 | 478 | 480 | 482 | 484 | 487 | 489 | 492 | 494 | 497 | 105 |
| 80 | 487 | 489 | 491 | 493 | 495 | 498 | 500 | 502 | 505 | 100 |
| 85 | 492 | 494 | 496 | 498 | 501 | 503 | 505 | 507 | 510 | 95 |
| 90 | 494 | 496 | 498 | 500 | 502 | 504 | 507 | 509 | 511 | 90 |

*Table IV. Logarithms of Radii of Curvature of the Earth's Surface (in Meters)*—Continued

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 32° | 33° | 34° | 35° | 36° | 37° | 38° | 39° | 40° | |
| 0 | 6.80299 | 6.80306 | 6.80313 | 6.80320 | 6.80327 | 6.80335 | 6.80342 | 6.80350 | 6.80357 | 180 |
| 5 | 300 | 307 | 314 | 322 | 329 | 336 | 344 | 351 | 359 | 175 |
| 10 | 305 | 312 | 319 | 326 | 333 | 340 | 348 | 355 | 363 | 170 |
| 15 | 313 | 320 | 326 | 333 | 340 | 348 | 355 | 362 | 369 | 165 |
| 20 | 324 | 330 | 337 | 343 | 350 | 357 | 364 | 371 | 378 | 160 |
| 25 | 337 | 343 | 349 | 355 | 362 | 368 | 375 | 382 | 388 | 155 |
| 30 | 352 | 358 | 364 | 370 | 376 | 382 | 388 | 394 | 401 | 150 |
| 35 | 369 | 374 | 380 | 385 | 391 | 397 | 402 | 408 | 414 | 145 |
| 40 | 386 | 392 | 397 | 402 | 407 | 412 | 418 | 423 | 429 | 140 |
| 45 | 405 | 410 | 414 | 419 | 424 | 429 | 434 | 439 | 444 | 135 |
| 50 | 423 | 428 | 432 | 436 | 441 | 445 | 450 | 454 | 459 | 130 |
| 55 | 441 | 445 | 449 | 453 | 457 | 461 | 465 | 469 | 474 | 125 |
| 60 | 458 | 462 | 465 | 469 | 472 | 476 | 480 | 484 | 487 | 120 |
| 65 | 473 | 476 | 480 | 483 | 486 | 489 | 493 | 496 | 500 | 115 |
| 70 | 486 | 489 | 492 | 495 | 498 | 501 | 504 | 507 | 510 | 110 |
| 75 | 497 | 500 | 502 | 505 | 508 | 510 | 513 | 516 | 519 | 105 |
| 80 | 505 | 507 | 510 | 512 | 515 | 517 | 520 | 523 | 525 | 100 |
| 85 | 510 | 512 | 514 | 517 | 519 | 522 | 524 | 527 | 529 | 95 |
| 90 | 511 | 514 | 516 | 518 | 521 | 523 | 526 | 528 | 531 | 90 |

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 40° | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° | |
| 0 | 6.80357 | 6.80365 | 6.80373 | 6.80380 | 6.80388 | 6.80396 | 6.80404 | 6.80411 | 6.80419 | 180 |
| 5 | 359 | 366 | 374 | 382 | 389 | 397 | 404 | 412 | 420 | 175 |
| 10 | 363 | 370 | 378 | 385 | 393 | 400 | 408 | 415 | 423 | 170 |
| 15 | 369 | 376 | 384 | 391 | 398 | 406 | 413 | 420 | 428 | 165 |
| 20 | 378 | 385 | 392 | 399 | 406 | 413 | 420 | 427 | 434 | 160 |
| 25 | 388 | 395 | 402 | 408 | 415 | 422 | 429 | 436 | 442 | 155 |
| 30 | 401 | 407 | 413 | 420 | 426 | 433 | 439 | 446 | 452 | 150 |
| 35 | 414 | 420 | 426 | 432 | 438 | 444 | 450 | 456 | 462 | 145 |
| 40 | 429 | 434 | 440 | 446 | 451 | 457 | 462 | 468 | 474 | 140 |
| 45 | 444 | 449 | 454 | 459 | 464 | 470 | 475 | 480 | 485 | 135 |
| 50 | 459 | 464 | 468 | 473 | 478 | 482 | 487 | 492 | 496 | 130 |
| 55 | 474 | 478 | 482 | 486 | 490 | 495 | 499 | 503 | 508 | 125 |
| 60 | 487 | 491 | 495 | 499 | 502 | 506 | 510 | 514 | 518 | 120 |
| 65 | 500 | 503 | 507 | 510 | 514 | 517 | 520 | 524 | 528 | 115 |
| 70 | 510 | 514 | 517 | 520 | 523 | 526 | 529 | 532 | 536 | 110 |
| 75 | 519 | 522 | 525 | 528 | 531 | 534 | 536 | 539 | 542 | 105 |
| 80 | 525 | 528 | 531 | 534 | 536 | 539 | 542 | 544 | 547 | 100 |
| 85 | 529 | 532 | 534 | 537 | 540 | 542 | 545 | 548 | 550 | 95 |
| 90 | 531 | 533 | 536 | 538 | 541 | 544 | 546 | 549 | 551 | 90 |

*Table IV. Logarithms of Radii of Curvature of the Earth's Surface (in Meters)*—Continued

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 48° | 49° | 50° | 51° | 52° | 53° | 54° | 55° | 56° | |
| 0 | 6.80419 | 6.80426 | 6.80434 | 6.80442 | 6.80449 | 6.80457 | 6.80464 | 6.80471 | 6.80479 | 180 |
| 5 | 420 | 428 | 435 | 443 | 450 | 458 | 465 | 472 | 479 | 175 |
| 10 | 423 | 430 | 438 | 445 | 453 | 460 | 467 | 474 | 481 | 170 |
| 15 | 428 | 435 | 442 | 450 | 457 | 464 | 471 | 478 | 485 | 165 |
| 20 | 434 | 441 | 448 | 455 | 462 | 469 | 476 | 483 | 489 | 160 |
| 25 | 442 | 449 | 456 | 463 | 469 | 476 | 482 | 489 | 495 | 155 |
| 30 | 452 | 458 | 465 | 471 | 477 | 484 | 490 | 496 | 502 | 150 |
| 35 | 462 | 468 | 474 | 480 | 486 | 492 | 498 | 503 | 509 | 145 |
| 40 | 474 | 479 | 485 | 490 | 496 | 501 | 506 | 512 | 517 | 140 |
| 45 | 485 | 490 | 495 | 500 | 505 | 510 | 515 | 520 | 525 | 135 |
| 50 | 496 | 501 | 506 | 510 | 515 | 520 | 524 | 528 | 533 | 130 |
| 55 | 508 | 512 | 516 | 520 | 524 | 528 | 533 | 537 | 541 | 125 |
| 60 | 518 | 522 | 526 | 530 | 533 | 537 | 541 | 544 | 548 | 120 |
| 65 | 528 | 531 | 534 | 538 | 541 | 545 | 548 | 551 | 555 | 115 |
| 70 | 536 | 539 | 542 | 545 | 548 | 551 | 554 | 557 | 560 | 110 |
| 75 | 542 | 545 | 548 | 551 | 554 | 557 | 559 | 562 | 565 | 105 |
| 80 | 547 | 550 | 553 | 555 | 558 | 561 | 563 | 566 | 568 | 100 |
| 85 | 550 | 553 | 555 | 558 | 560 | 563 | 566 | 568 | 570 | 95 |
| 90 | 551 | 554 | 556 | 559 | 561 | 564 | 566 | 569 | 571 | 90 |

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 56° | 57° | 56° | 59° | 60° | 61° | 62° | 63° | 64° | |
| 0 | 6.80479 | 6.80486 | 6.80493 | 6.80500 | 6.80506 | 6.80513 | 6.80520 | 6.80526 | 6.80532 | 180 |
| 5 | 479 | 486 | 493 | 500 | 507 | 514 | 520 | 526 | 532 | 175 |
| 10 | 481 | 488 | 495 | 502 | 509 | 515 | 522 | 528 | 534 | 170 |
| 15 | 485 | 492 | 498 | 505 | 511 | 518 | 524 | 530 | 536 | 165 |
| 20 | 489 | 496 | 502 | 509 | 515 | 521 | 527 | 533 | 539 | 160 |
| 25 | 495 | 501 | 508 | 514 | 520 | 526 | 531 | 537 | 542 | 155 |
| 30 | 502 | 508 | 514 | 519 | 525 | 530 | 536 | 541 | 546 | 150 |
| 35 | 509 | 515 | 520 | 525 | 531 | 536 | 541 | 546 | 551 | 145 |
| 40 | 517 | 522 | 527 | 532 | 537 | 542 | 546 | 551 | 556 | 140 |
| 45 | 525 | 530 | 534 | 539 | 543 | 548 | 552 | 556 | 560 | 135 |
| 50 | 533 | 537 | 542 | 546 | 550 | 554 | 558 | 562 | 565 | 130 |
| 55 | 541 | 545 | 548 | 552 | 556 | 560 | 563 | 567 | 570 | 125 |
| 60 | 548 | 552 | 555 | 558 | 562 | 565 | 568 | 572 | 575 | 120 |
| 65 | 555 | 558 | 561 | 564 | 567 | 570 | 573 | 576 | 579 | 115 |
| 70 | 560 | 563 | 566 | 569 | 572 | 574 | 577 | 580 | 582 | 110 |
| 75 | 565 | 568 | 570 | 573 | 575 | 578 | 580 | 583 | 585 | 105 |
| 80 | 568 | 571 | 573 | 576 | 578 | 580 | 583 | 585 | 587 | 100 |
| 85 | 570 | 573 | 575 | 578 | 580 | 582 | 584 | 586 | 588 | 95 |
| 90 | 571 | 574 | 576 | 578 | 580 | 583 | 585 | 587 | 589 | 90 |

*Table IV. Logarithms of Radii of Curvature of the Earth's Surface (in Meters)*—Continued

| Azimuth (degrees) | Latitude | | | | | | | | | Azimuth (degrees) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 64° | 65° | 66° | 67° | 68° | 69° | 70° | 71° | 72° | |
| 0 | 6.80532 | 6.80538 | 6.80544 | 6.80550 | 6.80555 | 6.80560 | 6.80565 | 6.80570 | 6.80575 | 180 |
| 5 | 532 | 538 | 544 | 550 | 555 | 561 | 566 | 570 | 575 | 175 |
| 10 | 534 | 540 | 545 | 551 | 556 | 562 | 566 | 571 | 576 | 170 |
| 15 | 536 | 542 | 547 | 553 | 558 | 563 | 568 | 572 | 577 | 165 |
| 20 | 539 | 544 | 550 | 555 | 560 | 565 | 570 | 574 | 578 | 160 |
| 25 | 542 | 548 | 553 | 558 | 562 | 567 | 572 | 576 | 580 | 155 |
| 30 | 546 | 551 | 556 | 561 | 565 | 570 | 574 | 578 | 582 | 150 |
| 35 | 551 | 556 | 560 | 564 | 569 | 573 | 577 | 581 | 584 | 145 |
| 40 | 556 | 560 | 564 | 568 | 572 | 576 | 580 | 583 | 587 | 140 |
| 45 | 560 | 564 | 568 | 572 | 576 | 579 | 583 | 586 | 589 | 135 |
| 50 | 565 | 569 | 573 | 576 | 579 | 583 | 586 | 589 | 592 | 130 |
| 55 | 570 | 574 | 577 | 580 | 583 | 586 | 589 | 591 | 594 | 125 |
| 60 | 575 | 578 | 581 | 584 | 586 | 589 | 591 | 594 | 596 | 120 |
| 65 | 579 | 582 | 584 | 587 | 589 | 592 | 594 | 596 | 598 | 115 |
| 70 | 582 | 585 | 587 | 590 | 592 | 594 | 596 | 598 | 600 | 110 |
| 75 | 585 | 587 | 590 | 592 | 594 | 596 | 598 | 600 | 601 | 105 |
| 80 | 587 | 589 | 591 | 593 | 595 | 597 | 599 | 601 | 602 | 100 |
| 85 | 588 | 590 | 592 | 594 | 596 | 598 | 600 | 601 | 603 | 95 |
| 90 | 589 | 591 | 593 | 595 | 597 | 598 | 600 | 602 | 603 | 90 |

*Table V. Mean refraction, $r_m$*

[Pressure = 760 mm; Temperature = 10° C; Relative humidity = 60%]

| z | 00′ | 10′ | 20′ | 30′ | 40′ | 50′ | 60′ |
|---|---|---|---|---|---|---|---|
| ° | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 0 | 0.0 | 0.2 | 0.3 | 0.5 | 0.7 | 0.8 | 1.0 |
| 1 | 1.0 | 1.2 | 1.3 | 1.5 | 1.7 | 1.9 | 2.0 |
| 2 | 2.0 | 2.2 | 2.4 | 2.5 | 2.7 | 2.9 | 3.0 |
| 3 | 3.0 | 3.2 | 3.4 | 3.5 | 3.7 | 3.9 | 4.0 |
| 4 | 4.0 | 4.2 | 4.4 | 4.6 | 4.7 | 4.9 | 5.1 |
| 5 | 5.1 | 5.2 | 5.4 | 5.6 | 5.7 | 5.9 | 6.1 |
| 6 | 6.1 | 6.3 | 6.4 | 6.6 | 6.8 | 6.9 | 7.1 |
| 7 | 7.1 | 7.3 | 7.5 | 7.6 | 7.8 | 8.0 | 8.1 |
| 8 | 8.1 | 8.3 | 8.5 | 8.7 | 8.8 | 9.0 | 19.2 |
| 9 | 9.2 | 9.3 | 9.5 | 9.7 | 9.9 | 0.0 | 10.2 |
| 10 | 10.2 | 10.4 | 10.6 | 10.7 | 10.9 | 11.1 | 11.3 |
| 11 | 11.3 | 11.4 | 11.6 | 11.8 | 12.0 | 12.1 | 12.3 |
| 12 | 12.3 | 12.5 | 12.7 | 12.8 | 13.0 | 13.2 | 13.4 |
| 13 | 13.4 | 13.5 | 13.7 | 13.9 | 14.1 | 14.3 | 14.4 |
| 14 | 14.4 | 14.6 | 14.8 | 15.0 | 15.2 | 15.3 | 15.5 |
| 15 | 15.5 | 15.7 | 15.9 | 16.1 | 16.2 | 16.4 | 16.6 |
| 16 | 16.6 | 16.8 | 17.0 | 17.2 | 17.3 | 17.5 | 17.7 |
| 17 | 17.7 | 17.9 | 18.1 | 18.3 | 18.4 | 18.6 | 18.8 |
| 18 | 18.8 | 19.0 | 19.2 | 19.4 | 19.6 | 19.8 | 19.9 |
| 19 | 19.9 | 20.1 | 20.3 | 20.5 | 20.7 | 20.9 | 21.1 |
| 20 | 21.1 | 21.3 | 21.5 | 21.7 | 21.8 | 22.0 | 22.2 |
| 21 | 22.2 | 22.4 | 22.6 | 22.8 | 23.0 | 23.2 | 23.4 |
| 22 | 23.4 | 23.6 | 23.8 | 24.0 | 24.2 | 24.4 | 24.6 |
| 23 | 24.6 | 24.8 | 25.0 | 25.2 | 25.4 | 25.6 | 25.8 |
| 24 | 25.8 | 26.0 | 26.2 | 26.4 | 26.6 | 26.8 | 27.0 |
| 25 | 27.0 | 27.2 | 27.4 | 27.6 | 27.8 | 28.0 | 28.2 |
| 26 | 28.2 | 28.4 | 28.7 | 28.9 | 29.1 | 29.3 | 29.5 |
| 27 | 29.5 | 29.7 | 29.9 | 30.1 | 30.4 | 30.6 | 30.8 |
| 28 | 30.8 | 31.0 | 31.2 | 31.4 | 31.7 | 31.9 | 32.1 |
| 29 | 32.1 | 32.3 | 32.5 | 32.8 | 33.0 | 33.2 | 33.4 |
| 30 | 33.4 | 33.6 | 33.9 | 34.1 | 34.3 | 34.6 | 34.8 |
| 31 | 34.8 | 35.0 | 35.2 | 35.5 | 35.7 | 35.9 | 36.2 |
| 32 | 36.2 | 36.4 | 36.6 | 36.9 | 37.1 | 37.4 | 37.6 |
| 33 | 37.6 | 37.8 | 38.1 | 38.3 | 38.6 | 38.8 | 39.0 |
| 34 | 39.0 | 39.3 | 39.5 | 39.8 | 40.0 | 40.3 | 40.5 |
| 35 | 40.5 | 40.8 | 41.0 | 41.3 | 41.5 | 41.8 | 42.1 |
| 36 | 42.1 | 42.3 | 42.6 | 42.8 | 43.1 | 43.3 | 43.6 |
| 37 | 43.6 | 43.9 | 44.1 | 44.4 | 44.7 | 44.9 | 45.2 |
| 38 | 45.2 | 45.5 | 45.8 | 46.0 | 46.3 | 46.6 | 46.9 |
| 39 | 46.9 | 47.1 | 47.4 | 47.7 | 48.0 | 48.3 | 48.6 |
| 40 | 48.6 | 48.8 | 49.1 | 49.4 | 49.7 | 50.0 | 50.3 |
| 41 | 50.3 | 50.6 | 50.9 | 51.2 | 51.5 | 51.8 | 52.1 |
| 42 | 52.1 | 52.4 | 52.7 | 53.0 | 53.3 | 53.6 | 54.0 |
| 43 | 54.0 | 54.3 | 54.6 | 54.9 | 55.2 | 55.5 | 55.9 |
| 44 | 55.9 | 56.2 | 56.5 | 56.9 | 57.2 | 57.5 | 57.9 |

*Table V. Mean refraction, $r_m$*—Continued

| z | 00′ | 10′ | 20′ | 30′ | 40′ | 50′ | 60′ |
|---|---|---|---|---|---|---|---|
| ° | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 45 | 57.9 | 58.2 | 58.5 | 58.9 | 59.2 | 59.6 | 59.9 |
| 46 | 59.9 | 60.2 | 60.6 | 61.0 | 61.3 | 61.7 | 62.0 |
| 47 | 62.0 | 62.4 | 62.8 | 63.1 | 63.5 | 63.9 | 64.2 |
| 48 | 64.2 | 64.6 | 65.0 | 65.4 | 65.7 | 66.1 | 66.5 |
| 49 | 66.5 | 66.9 | 67.3 | 67.7 | 68.1 | 68.5 | 68.9 |
| 50 | 68.9 | 69.3 | 69.7 | 70.1 | 70.6 | 71.0 | 71.4 |
| 51 | 71.4 | 71.8 | 72.2 | 72.7 | 73.1 | 73.6 | 74.0 |
| 52 | 74.0 | 74.4 | 74.9 | 75.3 | 75.8 | 76.2 | 76.7 |
| 53 | 76.7 | 77.2 | 77.6 | 78.1 | 78.6 | 79.1 | 79.5 |
| 54 | 79.5 | 80.0 | 80.5 | 81.0 | 81.5 | 82.0 | 82.5 |
| 55 | 82.5 | 83.0 | 83.5 | 84.1 | 84.6 | 85.1 | 85.6 |
| 56 | 85.6 | 86.2 | 86.7 | 87.3 | 87.8 | 88.4 | 88.9 |
| 57 | 88.9 | 89.5 | 90.1 | 90.7 | 91.2 | 91.8 | 92.4 |
| 58 | 92.4 | 93.0 | 93.6 | 94.2 | 94.8 | 95.5 | 96.1 |
| 59 | 96.1 | 96.7 | 97.4 | 98.0 | 98.6 | 99.3 | 100.0 |
| 60 | 100.0 | 100.6 | 101.3 | 102.0 | 102.7 | 103.4 | 104.1 |
| 61 | 104.1 | 104.8 | 105.5 | 106.3 | 107.0 | 107.7 | 108.5 |
| 62 | 108.5 | 109.2 | 110.0 | 110.8 | 111.6 | 112.4 | 113.2 |
| 63 | 113.2 | 114.0 | 114.8 | 115.6 | 116.5 | 117.3 | 118.2 |
| 64 | 118.2 | 119.0 | 119.9 | 120.8 | 121.7 | 122.6 | 123.5 |
| 65 | 123.5 | 124.5 | 125.4 | 126.4 | 127.4 | 128.3 | 129.3 |
| 66 | 129.3 | 130.3 | 131.4 | 132.4 | 133.4 | 134.5 | 135.6 |
| 67 | 135.6 | 136.7 | 137.8 | 138.9 | 140.0 | 141.2 | 142.3 |
| 68 | 142.3 | 143.5 | 144.7 | 145.9 | 147.2 | 148.4 | 149.7 |
| 69 | 149.7 | 151.0 | 152.3 | 153.6 | 155.0 | 156.4 | 157.8 |
| 70 | 157.8 | 159.2 | 160.6 | 162.1 | 163.6 | 165.1 | 166.6 |
| 71 | 166.6 | 168.2 | 169.7 | 171.4 | 173.0 | 174.7 | 176.3 |
| 72 | 176.3 | 178.1 | 179.8 | 181.6 | 183.4 | 185.3 | 187.2 |
| 73 | 187.2 | 189.1 | 191.0 | 193.0 | 195.1 | 197.1 | 199.2 |
| 74 | 199.2 | 201.4 | 203.6 | 205.8 | 208.1 | 210.4 | 212.8 |
| 75 | 212.8 | 215.2 | 217.7 | 220.2 | 222.8 | 225.5 | 228.2 |
| 76 | 228.2 | 230.9 | 233.7 | 236.6 | 239.6 | 242.6 | 245.7 |
| 77 | 245.7 | 248.9 | 252.1 | 255.4 | 258.9 | 262.3 | 265.9 |
| 78 | 265.9 | 269.6 | 273.4 | 277.2 | 281.2 | 285.3 | 289.5 |
| 79 | 289.5 | 293.8 | 298.2 | 302.8 | 307.5 | 312.3 | 317.3 |
| 80 | 317.3 | 322.4 | 327.7 | 333.2 | 338.8 | 344.6 | 350.6 |
| 81 | 350.6 | 356.8 | 363.2 | 369.8 | 376.6 | 383.7 | 391.1 |
| 82 | 391.1 | 398.7 | 406.6 | 414.8 | 423.3 | 432.1 | 441.3 |
| 83 | 441.3 | 450.9 | 460.9 | 471.2 | 482.0 | 493.3 | 505.1 |
| 84 | 505.1 | 517.4 | 530.3 | 543.8 | 558.0 | 572.8 | 588.4 |
| 85 | 588.4 | 604.4 | 621.6 | 639.7 | 658.8 | 678.9 | 700.2 |
| 86 | 700.2 | 722.7 | 746.6 | 771.8 | 798.7 | 827.2 | 857.6 |
| 87 | 857.6 | 890.0 | 924.7 | 961.6 | 1,001.3 | 1,043.9 | 1,089.7 |
| 88 | 1,089.7 | 1,138.9 | 1,192.0 | 1,249.2 | 1,311.4 | 1,378.6 | 1,452.0 |
| 89 | 1,452.0 | 1,531.7 | 1,618.8 | 1,714.0 | 1,818.4 | 1,933.1 | 2,059.5 |

*Table V. Mean refraction, $r_m$*—Continued
*Pressure correction factor, $C_B$*
Apply to mean refraction
$[r = (r_m)\ (C_B)\ (C_T)]$

| Barometer | | $C_B$ | Barometer | | $C_B$ | Barometer | | $C_B$ |
|---|---|---|---|---|---|---|---|---|
| *Inches* | *mm* | | *Inches* | *mm* | | *Inches* | *mm* | |
| 20.0 | 508 | 0.670 | 23.9 | 607 | 0.799 | 27.8 | 706 | 0.929 |
| 20.1 | 511 | 0.673 | 24.0 | 610 | 0.803 | 27.9 | 709 | 0.933 |
| 20.2 | 513 | 0.676 | 24.1 | 612 | 0.806 | 28.0 | 711 | 0.936 |
| 20.3 | 516 | 0.679 | 24.2 | 615 | 0.809 | 28.1 | 714 | 0.939 |
| 20.4 | 518 | 0.682 | 24.3 | 617 | 0.813 | 28.2 | 716 | 0.942 |
| 20.5 | 521 | 0.685 | 24.4 | 620 | 0.816 | 28.3 | 719 | 0.946 |
| 20.6 | 523 | 0.688 | 24.5 | 622 | 0.820 | 28.4 | 721 | 0.949 |
| 20.7 | 526 | 0.692 | 24.6 | 625 | 0.823 | 28.5 | 724 | 0.953 |
| 20.8 | 528 | 0.696 | 24.7 | 627 | 0.826 | 28.6 | 726 | 0.956 |
| 20.9 | 531 | 0.699 | 24.8 | 630 | 0.829 | 28.7 | 729 | 0.959 |
| 21.0 | 533 | 0.703 | 24.9 | 632 | 0.832 | 28.8 | 732 | 0.963 |
| 21.1 | 536 | 0.706 | 25.0 | 635 | 0.835 | 28.9 | 734 | 0.966 |
| 21.2 | 538 | 0.709 | 25.1 | 637 | 0.838 | 29.0 | 737 | 0.970 |
| 21.3 | 541 | 0.712 | 25.2 | 640 | 0.842 | 29.1 | 739 | 0.973 |
| 21.4 | 544 | 0.716 | 25.3 | 643 | 0.846 | 29.2 | 742 | 0.976 |
| 21.5 | 546 | 0.719 | 25.4 | 645 | 0.849 | 29.3 | 744 | 0.979 |
| 21.6 | 549 | 0.722 | 25.5 | 648 | 0.853 | 29.4 | 747 | 0.983 |
| 21.7 | 551 | 0.725 | 25.6 | 650 | 0.856 | 29.5 | 749 | 0.986 |
| 21.8 | 554 | 0.729 | 25.7 | 653 | 0.859 | 29.6 | 752 | 0.989 |
| 21.9 | 556 | 0.732 | 25.8 | 655 | 0.862 | 29.7 | 754 | 0.992 |
| 22.0 | 559 | 0.735 | 25.9 | 658 | 0.866 | 29.8 | 757 | 0.996 |
| 22.1 | 561 | 0.739 | 26.0 | 660 | 0.869 | 29.9 | 759 | 0.999 |
| 22.2 | 564 | 0.742 | 26.1 | 663 | 0.872 | 30.0 | 762 | 1.003 |
| 22.3 | 566 | 0.746 | 26.2 | 665 | 0.875 | 30.1 | 765 | 1.007 |
| 22.4 | 569 | 0.749 | 26.3 | 668 | 0.879 | 30.2 | 767 | 1.010 |
| 22.5 | 572 | 0.752 | 26.4 | 671 | 0.882 | 30.3 | 770 | 1.013 |
| 22.6 | 574 | 0.755 | 26.5 | 673 | 0.885 | 30.4 | 772 | 1.016 |
| 22.7 | 576 | 0.759 | 26.6 | 676 | 0.889 | 30.5 | 775 | 1.020 |
| 22.8 | 579 | 0.762 | 26.7 | 678 | 0.892 | 30.6 | 777 | 1.023 |
| 22.9 | 582 | 0.766 | 26.8 | 681 | 0.896 | 30.7 | 780 | 1.026 |
| 23.0 | 584 | 0.770 | 26.9 | 683 | 0.899 | 30.8 | 782 | 1.029 |
| 23.1 | 587 | 0.773 | 27.0 | 686 | 0.902 | 30.9 | 785 | 1.033 |
| 23.2 | 589 | 0.776 | 27.1 | 688 | 0.905 | 31.0 | 787 | 1.036 |
| 23.3 | 592 | 0.779 | 27.2 | 691 | 0.909 | | | |
| 23.4 | 594 | 0.783 | 27.3 | 693 | 0.912 | | | |
| 23.5 | 597 | 0.786 | 27.4 | 696 | 0.916 | | | |
| 23.6 | 599 | 0.789 | 27.5 | 699 | 0.920 | | | |
| 23.7 | 602 | 0.792 | 27.6 | 701 | 0.923 | | | |
| 23.8 | 605 | 0.796 | 27.7 | 704 | 0.926 | | | |

*Table V. Mean refraction, $r_m$*—Continued
*Temperature correction factor, $C_T$*
Apply to mean refraction
$[r = (r_m)\ (C_B)\ (C_T)]$

| Temperature | | $C_T$ | Temperature | | $C_T$ | Temperature | | $C_T$ |
|---|---|---|---|---|---|---|---|---|
| Fahrenheit | Centigrade | | Fahrenheit | Centigrade | | Fahrenheit | Centigrade | |
| ° | ° | | ° | ° | | ° | ° | |
| −25 | −31.7 | 1.172 | 14 | −10.0 | 1.076 | 53 | 11.7 | 0.994 |
| −24 | −31.1 | 1.169 | 15 | −9.4 | 1.073 | 54 | 12.2 | 0.992 |
| −23 | −30.6 | 1.166 | 16 | −8.9 | 1.071 | 55 | 12.8 | 0.990 |
| −22 | −30.0 | 1.164 | 17 | −8.3 | 1.069 | 56 | 13.3 | 0.988 |
| −21 | −29.4 | 1.161 | 18 | −7.8 | 1.067 | 57 | 13.9 | 0.986 |
| −20 | −28.9 | 1.158 | 19 | −7.2 | 1.064 | 58 | 14.4 | 0.985 |
| −19 | −28.3 | 1.156 | 20 | −6.7 | 1.062 | 59 | 15.0 | 0.983 |
| −18 | −27.8 | 1.153 | 21 | −6.1 | 1.060 | 60 | 15.6 | 0.981 |
| −17 | −27.2 | 1.151 | 22 | −5.6 | 1.058 | 61 | 16.1 | 0.979 |
| −16 | −26.7 | 1.148 | 23 | −5.0 | 1.056 | 62 | 16.7 | 0.977 |
| −15 | −26.1 | 1.145 | 24 | −4.4 | 1.054 | 63 | 17.2 | 0.975 |
| −14 | −25.6 | 1.143 | 25 | −3.9 | 1.051 | 64 | 17.8 | 0.973 |
| −13 | −25.0 | 1.140 | 26 | −3.3 | 1.049 | 65 | 18.3 | 0.972 |
| −12 | −24.4 | 1.138 | 27 | −2.8 | 1.047 | 66 | 18.9 | 0.970 |
| −11 | −23.9 | 1.135 | 28 | −2.2 | 1.045 | 67 | 19.4 | 0.968 |
| −10 | −23.3 | 1.133 | 29 | −1.7 | 1.043 | 68 | 20.0 | 0.966 |
| −9 | −22.8 | 1.130 | 30 | −1.1 | 1.041 | 69 | 20.6 | 0.964 |
| −8 | −22.2 | 1.128 | 31 | −0.6 | 1.039 | 70 | 21.1 | 0.962 |
| −7 | −21.7 | 1.125 | 32 | 0.0 | 1.036 | 71 | 21.7 | 0.961 |
| −6 | −21.1 | 1.123 | 33 | +0.6 | 1.034 | 72 | 22.2 | 0.959 |
| −5 | −20.6 | 1.120 | 34 | 1.1 | 1.032 | 73 | 22.8 | 0.957 |
| −4 | −20.0 | 1.118 | 35 | 1.7 | 1.030 | 74 | 23.3 | 0.955 |
| −3 | −19.4 | 1.115 | 36 | 2.2 | 1.028 | 75 | 23.9 | 0.953 |
| −2 | −18.9 | 1.113 | 37 | 2.8 | 1.026 | 76 | 24.4 | 0.952 |
| −1 | −18.3 | 1.111 | 38 | 3.3 | 1.024 | 77 | 25.0 | 0.950 |
| 0 | −17.8 | 1.108 | 39 | 3.9 | 1.022 | 78 | 25.6 | 0.948 |
| +1 | −17.2 | 1.106 | 40 | 4.4 | 1.020 | 79 | 26.1 | 0.946 |
| 2 | −16.7 | 1.103 | 41 | 5.0 | 1.018 | 80 | 26.7 | 0.945 |
| 3 | −16.1 | 1.101 | 42 | 5.6 | 1.016 | 81 | 27.2 | 0.943 |
| 4 | −15.6 | 1.099 | 43 | 6.1 | 1.014 | 82 | 27.8 | 0.941 |
| 5 | −15.0 | 1.096 | 44 | 6.7 | 1.012 | 83 | 28.3 | 0.939 |
| 6 | −14.4 | 1.094 | 45 | 7.2 | 1.010 | 84 | 28.9 | 0.938 |
| 7 | −13.9 | 1.092 | 46 | 7.8 | 1.008 | 85 | 29.4 | 0.936 |
| 8 | −13.3 | 1.089 | 47 | 8.3 | 1.006 | 86 | 30.0 | 0.934 |
| 9 | −12.8 | 1.087 | 48 | 8.9 | 1.004 | 87 | 30.6 | 0.933 |
| 10 | −12.2 | 1.085 | 49 | 9.4 | 1.002 | 88 | 31.1 | 0.931 |
| 11 | −11.7 | 1.082 | 50 | 10.0 | 1.000 | 89 | 31.7 | 0.929 |
| 12 | −11.1 | 1.080 | 51 | 10.6 | 0.998 | 90 | 32.2 | 0.928 |
| 13 | −10.6 | 1.078 | 52 | 11.1 | 0.996 | 91 | 32.8 | 0.926 |

*Table V. Mean refraction, $r_m$*—Continued

| Temperature | | $C_T$ | Temperature | | $C_T$ | Temperature | | $C_T$ |
|---|---|---|---|---|---|---|---|---|
| Fahrenheit | Centigrade | | Fahrenheit | Centigrade | | Fahrenheit | Centigrade | |
| ° | ° | | ° | ° | | ° | ° | |
| 92 | 33.3 | 0.924 | 107 | 41.7 | 0.900 | 122 | 50.0 | 0.878 |
| 93 | 33.9 | 0.923 | 108 | 42.2 | 0.899 | 123 | 50.6 | 0.877 |
| 94 | 34.4 | 0.921 | 109 | 42.8 | 0.897 | 124 | 51.1 | 0.876 |
| 95 | 35.0 | 0.919 | 110 | 43.3 | 0.895 | 125 | 51.7 | 0.874 |
| 96 | 35.6 | 0.917 | 111 | 43.9 | 0.894 | 126 | 52.2 | 0.873 |
| 97 | 36.1 | 0.916 | 112 | 44.4 | 0.892 | 127 | 52.8 | 0.871 |
| 98 | 36.7 | 0.914 | 113 | 45.0 | 0.891 | 128 | 53.3 | 0.870 |
| 99 | 37.2 | 0.912 | 114 | 45.6 | 0.890 | 129 | 53.9 | 0.868 |
| 100 | 37.8 | 0.911 | 115 | 46.1 | 0.888 | 130 | 54.4 | 0.867 |
| 101 | 38.3 | 0.909 | 116 | 46.7 | 0.886 | | | |
| 102 | 38.9 | 0.908 | 117 | 47.2 | 0.885 | | | |
| 103 | 39.4 | 0.906 | 118 | 47.8 | 0.884 | | | |
| 104 | 40.0 | 0.905 | 119 | 48.3 | 0.882 | | | |
| 105 | 40.6 | 0.903 | 120 | 48.9 | 0.881 | | | |
| 106 | 41.1 | 0.902 | 121 | 49.4 | 0.880 | | | |

*Table VI. Azimuth Reduction For Polaris Observations Near Elongation*

(FOR REDUCING TO ELONGATION OBSERVATIONS MADE NEAR ELONGATION)

| *Time / Azimuth at Elong. | 0° 50′ | 1° 0′ | 1° 10′ | 1° 20′ | 1° 30′ | 1° 40′ | 1° 50′ | 2° 0′ | Azimuth at Elong. / Time* |
|---|---|---|---|---|---|---|---|---|---|
| m | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | m |
| 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 |
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | + 0.1 | + 0.1 | + 0.1 | + 0.1 | 1 |
| 2 | + 0.1 | + 0.1 | + 0.2 | + 0.2 | 0.2 | 0.2 | 0.3 | 0.3 | 2 |
| 3 | 0.3 | 0.3 | 0.4 | 0.4 | 0.5 | 0.5 | 0.6 | 0.6 | 3 |
| 4 | 0.5 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 4 |
| 5 | + 0.7 | + 0.9 | + 1.0 | + 1.1 | + 1.3 | + 1.4 | + 1.6 | + 1.7 | 5 |
| 6 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.1 | 2.3 | 2.5 | 6 |
| 7 | 1.4 | 1.7 | 2.0 | 2.2 | 2.5 | 2.8 | 3.1 | 3.4 | 7 |
| 8 | 1.8 | 2.2 | 2.6 | 2.9 | 3.3 | 3.7 | 4.0 | 4.4 | 8 |
| 9 | 2.3 | 2.8 | 3.2 | 3.7 | 4.2 | 4.6 | 5.1 | 5.6 | 9 |
| 10 | + 2.9 | + 3.4 | + 4.0 | + 4.6 | + 5.1 | + 5.7 | + 6.3 | + 6.9 | 10 |
| 11 | 3.5 | 4.1 | 4.8 | 5.5 | 6.2 | 6.9 | 7.6 | 8.3 | 11 |
| 12 | 4.1 | 4.9 | 5.8 | 6.6 | 7.4 | 8.2 | 9.0 | 9.9 | 12 |
| 13 | 4.8 | 5.8 | 6.8 | 7.7 | 8.7 | 9.7 | 10.6 | 11.6 | 13 |
| 14 | 5.6 | 6.7 | 7.8 | 9.0 | 10.1 | 11.2 | 12.3 | 13.4 | 14 |
| 15 | + 6.4 | + 7.7 | + 9.0 | +10.3 | +11.6 | +12.8 | +14.1 | +15.4 | 15 |
| 16 | 7.3 | 8.8 | 10.2 | 11.7 | 13.2 | 14.6 | 16.1 | 17.5 | 16 |
| 17 | 8.2 | 9.9 | 11.5 | 13.2 | 14.9 | 16.5 | 18.2 | 19.8 | 17 |
| 18 | 9.2 | 11.1 | 12.9 | 14.8 | 16.7 | 18.5 | 20.4 | 22.2 | 18 |
| 19 | 10.3 | 12.4 | 14.4 | 16.5 | 18.6 | 20.6 | 22.7 | 24.7 | 19 |
| 20 | +11.4 | +13.7 | +16.0 | +18.3 | +20.6 | +22.8 | +25.1 | +27.4 | 20 |
| 21 | 12.6 | 15.1 | 17.6 | 20.1 | 22.7 | 25.2 | 27.7 | 30.2 | 21 |
| 22 | 13.8 | 16.6 | 19.3 | 22.1 | 24.9 | 27.6 | 30.4 | 33.2 | 22 |
| 23 | 15.1 | 18.1 | 21.1 | 24.2 | 27.2 | 30.2 | 33.2 | 36.2 | 23 |
| 24 | 16.4 | 19.7 | 23.0 | 26.3 | 29.6 | 32.9 | 36.2 | 39.5 | 24 |
| 25 | +17.8 | +21.4 | +25.0 | +28.5 | +32.1 | +35.7 | +39.2 | +42.8 | 25 |

*Sidereal time from elongation.

*Table VI. Azimuth Reduction For Polaris Observations Near Elongation*—Continued.

| *Time / Azimuth at Elong. | 2° 0′ | 2° 10′ | 2° 20′ | 2° 30′ | 2° 40′ | 2° 50′ | 3° 0′ | 3° 10′ | Azimuth at Elong. / Time* |
|---|---|---|---|---|---|---|---|---|---|
| m | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | m |
| 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 |
| 1 | + 0.1 | + 0.1 | + 0.1 | + 0.1 | + 0.1 | + 0.1 | + 0.1 | + 0.1 | 1 |
| 2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 2 |
| 3 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.9 | 0.9 | 1.0 | 3 |
| 4 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.6 | 1.7 | 4 |
| 5 | + 1.7 | + 1.9 | + 2.0 | + 2.1 | + 2.3 | + 2.4 | + 2.6 | + 2.7 | 5 |
| 6 | 2.5 | 2.7 | 2.9 | 3.1 | 3.3 | 3.5 | 3.7 | 3.9 | 6 |
| 7 | 3.4 | 3.7 | 3.9 | 4.2 | 4.5 | 4.8 | 5.0 | 5.3 | 7 |
| 8 | 4.4 | 4.8 | 5.1 | 5.5 | 5.9 | 6.2 | 6.6 | 7.0 | 8 |
| 9 | 5.6 | 6.0 | 6.5 | 7.0 | 7.4 | 7.9 | 8.3 | 8.8 | 9 |
| 10 | + 6.9 | + 7.4 | + 8.0 | + 8.6 | + 9.2 | + 9.7 | +10.3 | +10.9 | 10 |
| 11 | 8.3 | 9.0 | 9.7 | 10.4 | 11.1 | 11.8 | 12.4 | 13.1 | 11 |
| 12 | 9.9 | 10.7 | 11.5 | 12.3 | 13.2 | 14.0 | 14.8 | 15.6 | 12 |
| 13 | 11.6 | 12.6 | 13.5 | 14.5 | 15.4 | 16.4 | 17.4 | 18.4 | 13 |
| 14 | 13.4 | 14.6 | 15.7 | 16.8 | 17.9 | 19.0 | 20.2 | 21.3 | 14 |
| 15 | +15.4 | +16.7 | +18.0 | +19.3 | +20.6 | +21.9 | +23.1 | +24.4 | 15 |
| 16 | 17.5 | 19.0 | 20.5 | 21.9 | 23.4 | 24.9 | 26.3 | 27.8 | 16 |
| 17 | 19.8 | 21.5 | 23.1 | 24.8 | 26.4 | 28.1 | 29.7 | 31.4 | 17 |
| 18 | 22.2 | 24.1 | 25.9 | 27.8 | 29.6 | 31.5 | 33.3 | 35.2 | 18 |
| 19 | 24.7 | 26.8 | 28.9 | 30.9 | 33.0 | 35.1 | 37.1 | 39.2 | 19 |
| 20 | +27.4 | +29.7 | +32.0 | +34.3 | +36.6 | +38.8 | +41.1 | +43.4 | 20 |
| 21 | 30.2 | 32.7 | 35.2 | 37.8 | 40.3 | 42.8 | 45.3 | 47.9 | 21 |
| 22 | 33.2 | 35.9 | 38.7 | 41.5 | 44.2 | 47.0 | 49.8 | 52.5 | 22 |
| 23 | 36.2 | 39.3 | 42.3 | 45.3 | 48.3 | 51.4 | 54.4 | 57.4 | 23 |
| 24 | 39.5 | 42.8 | 46.0 | 49.3 | 52.6 | 55.9 | 59.2 | 62.5 | 24 |
| 25 | +42.8 | +46.4 | +49.9 | +53.5 | +57.1 | +60.7 | +64.2 | +67.8 | 25 |

*Sidereal time from elongation.

*Table VII. Curvature Correction For Azimuth Observations On Close Circumpolar Stars*

| 2r / Azimuth of Star | 1 m | 2 m | 3 m | 4 m | 5 m | 6 m | 7 m | 8 m | 9 m | 10 m |
|---|---|---|---|---|---|---|---|---|---|---|
| ° ′ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 0 00 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 |
| 10 | .0 | .0 | .0 | .0 | .0 | .1 | .1 | .1 | .1 | .1 |
| 20 | .0 | .0 | .0 | .0 | .1 | .1 | .1 | .2 | .2 | .3 |
| 30 | .0 | .0 | .0 | .1 | .1 | .2 | .2 | .3 | .3 | .4 |
| 40 | .0 | .0 | .1 | .1 | .1 | .2 | .3 | .4 | .5 | .6 |
| 50 | .0 | .0 | .1 | .1 | .2 | .3 | .3 | .5 | .6 | .7 |
| 1 00 | .0 | .0 | .1 | .1 | .2 | .3 | .4 | .5 | .7 | .9 |
| 10 | .0 | .0 | .1 | .2 | .2 | .4 | .5 | .6 | .8 | 1.0 |
| 20 | .0 | .0 | .1 | .2 | .3 | .4 | .6 | .7 | .9 | 1.1 |
| 30 | .0 | .0 | .1 | .2 | .3 | .5 | .6 | .8 | 1.0 | 1.3 |
| 40 | .0 | .1 | .1 | .2 | .4 | .5 | .7 | .9 | 1.2 | 1.4 |
| 50 | .0 | .1 | .1 | .3 | .4 | .6 | .8 | 1.0 | 1.3 | 1.6 |
| 2 00 | .0 | .1 | .2 | .3 | .4 | .6 | .8 | 1.1 | 1.4 | 1.7 |
| 10 | .0 | .1 | .2 | .3 | .5 | .7 | .9 | 1.2 | 1.5 | 1.9 |
| 20 | .0 | .1 | .2 | .3 | .5 | .7 | 1.0 | 1.3 | 1.6 | 2.0 |
| 30 | .0 | .1 | .2 | .3 | .5 | .8 | 1.1 | 1.4 | 1.7 | 2.1 |
| 40 | .0 | .1 | .2 | .4 | .6 | .8 | 1.1 | 1.5 | 1.9 | 2.3 |
| 50 | .0 | .1 | .2 | .4 | .6 | .9 | 1.2 | 1.6 | 2.0 | 2.4 |
| 3 00 | .0 | .1 | .2 | .4 | .6 | .9 | 1.3 | 1.6 | 2.1 | 2.6 |
| 10 | .0 | .1 | .2 | .4 | .7 | 1.0 | 1.3 | 1.7 | 2.2 | 2.7 |

Note. 2r=Difference in time between direct and reverse pointings

*Table VIII. Parallax and Altitude of the Sun*

(Based on solar parallax=8″.80)

| Altitude | Jan. 1 | Feb. 1 | Mar. 1 | Apr. 1 | May 1 | June 1 | July 1 | Aug. 1 | Sept. 1 | Oct. 1 | Nov. 1 | Dec. 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0° | 8″.95 | 8″.93 | 8″.88 | 8″.81 | 8″.73 | 8″.68 | 8″.66 | 8″.67 | 8″.72 | 8″.79 | 8″.87 | 8″.92 |
| 3 | 8.94 | 8.92 | 8.87 | 8.80 | 8.72 | 8.67 | 8.65 | 8.66 | 8.71 | 8.78 | 8.86 | 8.91 |
| 6 | 8.90 | 8.88 | 8.83 | 8.76 | 8.68 | 8.63 | 8.61 | 8.62 | 8.67 | 8.74 | 8.82 | 8.87 |
| 9 | 8.84 | 8.82 | 8.77 | 8.70 | 8.62 | 8.57 | 8.55 | 8.56 | 8.61 | 8.68 | 8.76 | 8.81 |
| 12 | 8.75 | 8.73 | 8.69 | 8.62 | 8.54 | 8.49 | 8.47 | 8.48 | 8.53 | 8.60 | 8.68 | 8.72 |
| 15 | 8.64 | 8.63 | 8.58 | 8.51 | 8.43 | 8.38 | 8.36 | 8.37 | 8.42 | 8.49 | 8.57 | 8.62 |
| 18 | 8.51 | 8.49 | 8.45 | 8.38 | 8.30 | 8.26 | 8.24 | 8.25 | 8.29 | 8.36 | 8.44 | 8.48 |
| 21 | 8.36 | 8.34 | 8.29 | 8.23 | 8.15 | 8.10 | 8.08 | 8.09 | 8.14 | 8.21 | 8.28 | 8.33 |
| 24 | 8.18 | 8.16 | 8.11 | 8.05 | 7.97 | 7.93 | 7.91 | 7.92 | 7.97 | 8.03 | 8.10 | 8.15 |
| 27 | 7.97 | 7.96 | 7.91 | 7.85 | 7.78 | 7.73 | 7.72 | 7.72 | 7.77 | 7.83 | 7.90 | 7.95 |
| 30 | 7.75 | 7.73 | 7.69 | 7.63 | 7.56 | 7.52 | 7.50 | 7.51 | 7.55 | 7.61 | 7.68 | 7.72 |
| 33 | 7.51 | 7.49 | 7.45 | 7.39 | 7.32 | 7.28 | 7.26 | 7.27 | 7.31 | 7.37 | 7.44 | 7.48 |
| 36 | 7.24 | 7.22 | 7.18 | 7.13 | 7.06 | 7.02 | 7.01 | 7.01 | 7.05 | 7.11 | 7.18 | 7.22 |
| 39 | 6.96 | 6.94 | 6.90 | 6.85 | 6.78 | 6.75 | 6.73 | 6.74 | 6.78 | 6.83 | 6.89 | 6.93 |
| 42 | 6.65 | 6.64 | 6.60 | 6.55 | 6.49 | 6.45 | 6.44 | 6.44 | 6.48 | 6.53 | 6.59 | 6.63 |
| 44 | 6.44 | 6.42 | 6.39 | 6.34 | 6.28 | 6.24 | 6.23 | 6.24 | 6.27 | 6.32 | 6.38 | 6.42 |
| 46 | 6.22 | 6.20 | 6.17 | 6.12 | 6.06 | 6.03 | 6.02 | 6.02 | 6.06 | 6.11 | 6.16 | 6.20 |
| 48 | 5.99 | 5.98 | 5.94 | 5.89 | 5.84 | 5.81 | 5.79 | 5.80 | 5.83 | 5.88 | 5.93 | 5.97 |
| 50 | 5.75 | 5.74 | 5.71 | 5.66 | 5.61 | 5.58 | 5.57 | 5.57 | 5.61 | 5.65 | 5.70 | 5.73 |
| 52 | 5.51 | 5.50 | 5.47 | 5.42 | 5.38 | 5.34 | 5.33 | 5.34 | 5.37 | 5.41 | 5.46 | 5.49 |
| 54 | 5.26 | 5.25 | 5.22 | 5.18 | 5.13 | 5.10 | 5.09 | 5.10 | 5.13 | 5.17 | 5.21 | 5.24 |
| 56 | 5.00 | 4.99 | 4.97 | 4.93 | 4.88 | 4.85 | 4.84 | 4.85 | 4.88 | 4.92 | 4.96 | 4.99 |
| 58 | 4.74 | 4.73 | 4.71 | 4.67 | 4.63 | 4.60 | 4.59 | 4.59 | 4.62 | 4.66 | 4.70 | 4.73 |
| 60 | 4.48 | 4.46 | 4.44 | 4.40 | 4.36 | 4.34 | 4.33 | 4.34 | 4.36 | 4.40 | 4.44 | 4.46 |
| 62 | 4.20 | 4.19 | 4.17 | 4.14 | 4.10 | 4.08 | 4.07 | 4.07 | 4.09 | 4.13 | 4.16 | 4.19 |
| 64 | 3.92 | 3.91 | 3.89 | 3.86 | 3.83 | 3.81 | 3.80 | 3.80 | 3.82 | 3.85 | 3.89 | 3.91 |
| 66 | 3.64 | 3.63 | 3.61 | 3.58 | 3.55 | 3.53 | 3.52 | 3.53 | 3.55 | 3.57 | 3.61 | 3.63 |
| 68 | 3.35 | 3.35 | 3.33 | 3.30 | 3.27 | 3.25 | 3.24 | 3.25 | 3.27 | 3.29 | 3.32 | 3.34 |
| 70 | 3.06 | 3.05 | 3.04 | 3.01 | 2.99 | 2.97 | 2.96 | 2.97 | 2.98 | 3.01 | 3.03 | 3.05 |
| 72 | 2.77 | 2.76 | 2.74 | 2.72 | 2.70 | 2.68 | 2.68 | 2.68 | 2.69 | 2.72 | 2.74 | 2.76 |
| 74 | 2.47 | 2.46 | 2.45 | 2.43 | 2.41 | 2.39 | 2.39 | 2.39 | 2.40 | 2.42 | 2.44 | 2.46 |
| 76 | 2.17 | 2.16 | 2.15 | 2.13 | 2.11 | 2.10 | 2.09 | 2.10 | 2.11 | 2.13 | 2.15 | 2.16 |
| 78 | 1.86 | 1.86 | 1.85 | 1.83 | 1.81 | 1.80 | 1.80 | 1.80 | 1.81 | 1.83 | 1.84 | 1.85 |
| 80 | 1.55 | 1.55 | 1.54 | 1.53 | 1.52 | 1.51 | 1.50 | 1.51 | 1.51 | 1.53 | 1.54 | 1.55 |
| 82 | 1.25 | 1.24 | 1.24 | 1.23 | 1.22 | 1.21 | 1.21 | 1.21 | 1.21 | 1.22 | 1.23 | 1.24 |
| 84 | .94 | .93 | .93 | .92 | .91 | .91 | .90 | .91 | .91 | .92 | .93 | .93 |
| 86 | .62 | .62 | .62 | .61 | .61 | .61 | .60 | .61 | .61 | .61 | .62 | .62 |
| 88 | .31 | .31 | .31 | .31 | .30 | .30 | .30 | .30 | .30 | .31 | .31 | .31 |
| 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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## *Table IX. Values of* $\left(\frac{2\sin^2\frac{t}{2}}{\sin 1''}\right)$

| $t$ | 0m | 1m | 2m | 3m | 4m | 5m | 6m | 7m | 8m |
|---|---|---|---|---|---|---|---|---|---|
| s | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 0 | 0 | 1. 96 | 7. 85 | 17. 67 | 31. 42 | 49. 09 | 70. 68 | 96. 20 | 125. 65 |
| 1 | 0 | 2. 03 | 7. 98 | 17. 87 | 31. 68 | 49. 41 | 71. 07 | 96. 66 | 126. 17 |
| 2 | 0 | 2. 10 | 8. 12 | 18. 07 | 31. 94 | 49. 74 | 71. 47 | 97. 12 | 126. 70 |
| 3 | 0 | 2. 16 | 8. 25 | 18. 27 | 32. 20 | 50. 07 | 71. 86 | 97. 58 | 127. 22 |
| 4 | . 01 | 2. 23 | 8. 39 | 18. 47 | 32. 47 | 50. 40 | 72. 26 | 98. 04 | 127. 75 |
| 5 | . 01 | 2. 31 | 8. 52 | 18. 67 | 32. 74 | 50. 73 | 72. 66 | 98. 50 | 128. 28 |
| 6 | . 02 | 2. 38 | 8. 66 | 18. 87 | 33. 01 | 51. 07 | 73. 06 | 98. 97 | 128. 81 |
| 7 | . 02 | 2. 45 | 8. 80 | 19. 07 | 33. 27 | 51. 40 | 73. 46 | 99. 43 | 129. 34 |
| 8 | . 03 | 2. 52 | 8. 94 | 19. 28 | 33. 54 | 51. 74 | 73. 86 | 99. 90 | 129. 87 |
| 9 | . 04 | 2. 60 | 9. 08 | 19. 48 | 33. 81 | 52. 07 | 74. 26 | 100. 37 | 130. 40 |
| 10 | . 05 | 2. 67 | 9. 22 | 19. 69 | 34. 09 | 52. 41 | 74. 66 | 100. 84 | 130. 94 |
| 11 | . 06 | 2. 75 | 9. 36 | 19. 90 | 34. 36 | 52. 75 | 75. 06 | 101. 31 | 131. 47 |
| 12 | . 08 | 2. 83 | 9. 50 | 20. 11 | 34. 64 | 53. 09 | 75. 47 | 101. 78 | 132. 01 |
| 13 | . 09 | 2. 91 | 9. 64 | 20. 32 | 34. 91 | 53. 43 | 75. 88 | 102. 25 | 132. 55 |
| 14 | . 11 | 2. 99 | 9. 79 | 20. 53 | 35. 19 | 53. 77 | 76. 29 | 102. 72 | 133. 09 |
| 15 | . 12 | 3. 07 | 9. 94 | 20. 74 | 35. 46 | 54. 11 | 76. 69 | 103. 20 | 133. 63 |
| 16 | . 14 | 3. 15 | 10. 09 | 20. 95 | 35. 74 | 54. 46 | 77. 10 | 103. 67 | 134. 17 |
| 17 | . 16 | 3. 23 | 10. 24 | 21. 16 | 36. 02 | 54. 80 | 77. 51 | 104. 15 | 134. 71 |
| 18 | . 18 | 3. 32 | 10. 39 | 21. 38 | 36. 30 | 55. 15 | 77. 93 | 104. 63 | 135. 25 |
| 19 | . 20 | 3. 40 | 10. 54 | 21. 60 | 36. 58 | 55. 50 | 78. 34 | 105. 10 | 135. 80 |
| 20 | . 22 | 3. 49 | 10. 69 | 21. 82 | 36. 87 | 55. 84 | 78. 75 | 105. 58 | 136. 34 |
| 21 | . 24 | 3. 58 | 10. 84 | 22. 03 | 37. 15 | 56. 19 | 79. 16 | 106. 06 | 136. 88 |
| 22 | . 26 | 3. 67 | 11. 00 | 22. 25 | 37. 44 | 56. 55 | 79. 58 | 106. 55 | 137. 43 |
| 23 | . 28 | 3. 76 | 11. 15 | 22. 47 | 37. 72 | 56. 90 | 80. 00 | 107. 03 | 137. 98 |
| 24 | . 31 | 3. 85 | 11. 31 | 22. 70 | 38. 01 | 57. 25 | 80. 42 | 107. 51 | 138. 53 |
| 25 | . 34 | 3. 94 | 11. 47 | 22. 92 | 38. 30 | 57. 60 | 80. 84 | 107. 99 | 139. 08 |
| 26 | . 37 | 4. 03 | 11. 63 | 23. 14 | 38. 59 | 57. 96 | 81. 26 | 108. 48 | 139. 63 |
| 27 | . 40 | 4. 12 | 11. 79 | 23. 37 | 38. 88 | 58. 32 | 81. 68 | 108. 97 | 140. 18 |
| 28 | . 43 | 4. 22 | 11. 95 | 23. 60 | 39. 17 | 58. 68 | 82. 10 | 109. 46 | 140. 74 |
| 29 | . 46 | 4. 32 | 12. 11 | 23. 82 | 39. 46 | 59. 03 | 82. 52 | 109. 95 | 141. 29 |

*Table IX. Values of* $\left(\frac{2 \sin^2 \frac{t}{2}}{\sin 1''}\right)$—Continued

| $t$ | 0m | 1m | 2m | 3m | 4m | 5m | 6m | 7m | 8m |
|---|---|---|---|---|---|---|---|---|---|
| s | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 30 | .49 | 4.42 | 12.27 | 24.05 | 39.76 | 59.40 | 82.95 | 110.44 | 141.85 |
| 31 | .52 | 4.52 | 12.43 | 24.28 | 40.05 | 59.75 | 83.38 | 110.93 | 142.40 |
| 32 | .56 | 4.62 | 12.60 | 24.51 | 40.35 | 60.11 | 83.81 | 111.43 | 142.96 |
| 33 | .59 | 4.72 | 12.76 | 24.74 | 40.65 | 60.47 | 84.23 | 111.92 | 143.52 |
| 34 | .63 | 4.82 | 12.93 | 24.98 | 40.95 | 60.84 | 84.66 | 112.41 | 144.08 |
| 35 | .67 | 4.92 | 13.10 | 25.21 | 41.25 | 61.20 | 85.09 | 112.90 | 144.64 |
| 36 | .71 | 5.03 | 13.27 | 25.45 | 41.55 | 61.57 | 85.52 | 113.40 | 145.20 |
| 37 | .75 | 5.13 | 13.44 | 25.68 | 41.85 | 61.94 | 85.95 | 113.90 | 145.76 |
| 38 | .79 | 5.24 | 13.62 | 25.92 | 42.15 | 62.31 | 86.39 | 114.40 | 146.33 |
| 39 | .83 | 5.34 | 13.79 | 26.16 | 42.45 | 62.68 | 86.82 | 114.90 | 146.89 |
| 40 | .87 | 5.45 | 13.96 | 26.40 | 42.76 | 63.05 | 87.26 | 115.40 | 147.46 |
| 41 | .91 | 5.56 | 14.13 | 26.64 | 43.06 | 63.42 | 87.70 | 115.90 | 148.03 |
| 42 | .96 | 5.67 | 14.31 | 26.88 | 43.37 | 63.79 | 88.14 | 116.40 | 148.60 |
| 43 | 1.01 | 5.78 | 14.49 | 27.12 | 43.68 | 64.16 | 88.57 | 116.90 | 149.17 |
| 44 | 1.06 | 5.90 | 14.67 | 27.37 | 43.99 | 64.54 | 89.01 | 117.41 | 149.74 |
| 45 | 1.10 | 6.01 | 14.85 | 27.61 | 44.30 | 64.91 | 89.45 | 117.92 | 150.31 |
| 46 | 1.15 | 6.13 | 15.03 | 27.86 | 44.61 | 65.29 | 89.89 | 118.43 | 150.88 |
| 47 | 1.20 | 6.24 | 15.21 | 28.10 | 44.92 | 65.67 | 90.33 | 118.94 | 151.45 |
| 48 | 1.26 | 6.36 | 15.39 | 28.35 | 45.24 | 66.05 | 90.78 | 119.45 | 152.03 |
| 49 | 1.31 | 6.48 | 15.57 | 28.60 | 45.55 | 66.43 | 91.23 | 119.96 | 152.61 |
| 50 | 1.36 | 6.60 | 15.76 | 28.85 | 45.87 | 66.81 | 91.68 | 120.47 | 153.19 |
| 51 | 1.42 | 6.72 | 15.95 | 29.10 | 46.18 | 67.19 | 92.12 | 120.98 | 153.77 |
| 52 | 1.48 | 6.84 | 16.14 | 29.36 | 46.50 | 67.58 | 92.57 | 121.49 | 154.35 |
| 53 | 1.53 | 6.96 | 16.32 | 29.61 | 46.82 | 67.96 | 93.02 | 122.01 | 154.93 |
| 54 | 1.59 | 7.09 | 16.51 | 29.86 | 47.14 | 68.35 | 93.47 | 122.53 | 155.51 |
| 55 | 1.65 | 7.21 | 16.70 | 30.12 | 47.46 | 68.73 | 93.92 | 123.05 | 156.09 |
| 56 | 1.71 | 7.34 | 16.89 | 30.38 | 47.79 | 69.12 | 94.38 | 123.57 | 156.67 |
| 57 | 1.77 | 7.46 | 17.08 | 30.64 | 48.11 | 69.51 | 94.83 | 124.09 | 157.25 |
| 58 | 1.83 | 7.60 | 17.28 | 30.90 | 48.43 | 69.90 | 95.29 | 124.61 | 157.84 |
| 59 | 1.89 | 7.72 | 17.47 | 31.16 | 48.76 | 70.29 | 95.74 | 125.13 | 158.43 |

*Table IX. Values of* $\left(\frac{2 \sin^2 \frac{t}{2}}{\sin 1''}\right)$—Continued

| $t$ | $9^m$ | $10^m$ | $11^m$ | $12^m$ | $13^m$ | $14^m$ | $15^m$ | $16^m$ |
|---|---|---|---|---|---|---|---|---|
| $s$ | " | " | " | " | " | " | " | " |
| 0 | 159. 02 | 196. 32 | 237. 54 | 282. 68 | 331. 74 | 384. 74 | 441. 63 | 502. 46 |
| 1 | 159. 61 | 196. 97 | 238. 26 | 283. 47 | 332. 59 | 385. 65 | 442. 62 | 503. 50 |
| 2 | 160. 20 | 197. 63 | 238. 98 | 284. 26 | 333. 44 | 386. 56 | 443. 60 | 504. 55 |
| 3 | 160. 80 | 198. 28 | 239. 70 | 285. 04 | 334. 29 | 387. 48 | 444. 58 | 505. 60 |
| 4 | 161. 39 | 198. 94 | 240. 42 | 285. 83 | 335. 15 | 388. 40 | 445. 56 | 506. 65 |
| 5 | 161. 98 | 199. 60 | 241. 14 | 286. 62 | 336. 00 | 389. 32 | 446. 55 | 507. 70 |
| 6 | 162. 58 | 200. 26 | 241. 87 | 287. 41 | 336. 86 | 390. 24 | 447. 54 | 508. 76 |
| 7 | 163. 17 | 200. 92 | 242. 60 | 288. 20 | 337. 72 | 391. 16 | 448. 53 | 509. 81 |
| 8 | 163. 77 | 201. 59 | 243. 33 | 389. 00 | 338. 58 | 392. 09 | 449. 51 | 510. 86 |
| 9 | 164. 37 | 202. 25 | 244. 06 | 289. 79 | 339. 44 | 393. 01 | 450. 50 | 511. 92 |
| 10 | 164. 97 | 202. 92 | 244. 79 | 290. 58 | 340. 30 | 393. 94 | 451. 50 | 512. 98 |
| 11 | 165. 57 | 203. 58 | 245. 52 | 291. 38 | 341. 16 | 394. 86 | 452. 49 | 514. 03 |
| 12 | 166. 17 | 204. 25 | 246. 25 | 292. 18 | 342. 02 | 395. 79 | 453. 48 | 515. 09 |
| 13 | 166. 77 | 204. 92 | 246. 98 | 292. 98 | 342. 88 | 396. 72 | 454. 48 | 516. 15 |
| 14 | 167. 37 | 205. 59 | 247. 72 | 293. 78 | 343. 75 | 397. 65 | 455. 47 | 517. 21 |
| 15 | 167. 97 | 206. 26 | 248. 45 | 294. 58 | 344. 62 | 398. 58 | 456. 47 | 518. 27 |
| 16 | 168. 58 | 206. 93 | 249. 19 | 295. 38 | 345. 49 | 399. 52 | 457. 47 | 519. 34 |
| 17 | 169. 19 | 207. 60 | 249. 93 | 296. 18 | 346. 36 | 400. 45 | 458. 47 | 520. 40 |
| 18 | 169. 80 | 208. 27 | 250. 67 | 296. 99 | 347. 23 | 401. 38 | 459. 47 | 521. 47 |
| 19 | 170. 41 | 208. 94 | 251. 41 | 297. 79 | 348. 10 | 402. 32 | 460. 47 | 522. 53 |
| 20 | 171. 02 | 209. 62 | 252. 15 | 298. 60 | 348. 97 | 403. 26 | 461. 47 | 523. 60 |
| 21 | 171. 63 | 210. 30 | 252. 89 | 299. 40 | 349. 84 | 404. 20 | 462. 48 | 524. 67 |
| 22 | 172. 24 | 210. 98 | 253. 63 | 300. 21 | 350. 71 | 405. 14 | 463. 48 | 525. 74 |
| 23 | 172. 85 | 211. 66 | 254. 37 | 301. 02 | 351. 58 | 406. 08 | 464. 48 | 526. 81 |
| 24 | 173. 47 | 212. 34 | 255. 12 | 301. 83 | 352. 46 | 407. 02 | 465. 49 | 527. 89 |
| 25 | 174. 08 | 213. 02 | 255. 87 | 302. 64 | 353. 34 | 407. 96 | 466. 50 | 528. 96 |
| 26 | 174. 70 | 213. 70 | 256. 62 | 303. 46 | 354. 22 | 408. 90 | 467. 51 | 530. 03 |
| 27 | 175. 32 | 214. 38 | 257. 37 | 304. 27 | 355. 10 | 409. 84 | 468. 52 | 531. 11 |
| 28 | 175. 94 | 215. 07 | 258. 12 | 305. 09 | 355. 98 | 410. 79 | 469. 53 | 532. 18 |
| 29 | 176. 56 | 215. 75 | 258. 87 | 305. 90 | 356. 86 | 411. 73 | 470. 54 | 533. 26 |

*Table IX. Values of* $\left(\frac{2 \sin^2 \frac{t}{2}}{\sin 1''}\right)$—Continued

| $t$ | 9m | 10m | 11m | 12m | 13m | 14m | 15m | 16m |
|---|---|---|---|---|---|---|---|---|
| s | " | " | " | " | " | " | " | " |
| 30 | 177. 18 | 216. 44 | 259. 62 | 306. 72 | 357. 74 | 412. 68 | 471. 55 | 534. 33 |
| 31 | 177. 80 | 217. 12 | 260. 37 | 307. 54 | 358. 62 | 413. 63 | 472. 57 | 535. 41 |
| 32 | 178. 43 | 217. 81 | 261. 12 | 308. 36 | 359. 51 | 414. 59 | 473. 58 | 536. 50 |
| 33 | 179. 05 | 218. 50 | 261. 83 | 309. 18 | 360. 39 | 415. 54 | 474. 60 | 537. 58 |
| 34 | 179. 68 | 219. 19 | 262. 64 | 310. 00 | 361. 28 | 416. 49 | 475. 62 | 538. 67 |
| 35 | 180. 30 | 219. 88 | 263. 39 | 310. 82 | 362. 17 | 417. 44 | 476. 64 | 539. 75 |
| 36 | 180. 93 | 220. 58 | 264. 15 | 311. 65 | 363. 07 | 418. 40 | 477. 65 | 540. 83 |
| 37 | 181. 56 | 221. 27 | 264. 91 | 312. 47 | 363. 96 | 419. 35 | 478. 67 | 541. 91 |
| 38 | 182. 19 | 221. 97 | 265. 68 | 313. 30 | 364. 85 | 420. 31 | 479. 70 | 543. 00 |
| 39 | 182. 82 | 222. 66 | 266. 44 | 314. 12 | 365. 75 | 421. 27 | 480. 72 | 544. 09 |
| 40 | 183. 46 | 223. 36 | 267. 20 | 314. 95 | 366. 64 | 422. 23 | 481. 74 | 545. 18 |
| 41 | 184. 09 | 224. 06 | 267. 96 | 315. 78 | 367. 53 | 423. 19 | 482. 77 | 546. 27 |
| 42 | 184. 72 | 224. 76 | 208. 73 | 316. 61 | 368. 42 | 424. 15 | 483. 79 | 547. 36 |
| 43 | 185. 35 | 225. 46 | 269. 49 | 317. 44 | 369. 31 | 425. 11 | 484. 82 | 548. 45 |
| 44 | 185. 99 | 226. 16 | 270. 26 | 318. 27 | 370. 21 | 426. 07 | 485. 85 | 549. 55 |
| 45 | 186. 63 | 226. 86 | 271. 02 | 319. 10 | 371. 11 | 427. 04 | 486. 88 | 550. 64 |
| 46 | 187. 27 | 227. 57 | 271. 79 | 319. 94 | 372. 01 | 428. 01 | 487. 91 | 551. 73 |
| 47 | 187. 91 | 228. 27 | 272. 56 | 320. 78 | 372. 91 | 428. 97 | 488. 94 | 552. 83 |
| 48 | 188. 55 | 228. 98 | 273. 34 | 321. 62 | 373. 82 | 429. 93 | 489. 97 | 553. 93 |
| 49 | 189. 19 | 229. 68 | 274. 11 | 322. 45 | 374. 72 | 430. 90 | 491. 01 | 555. 03 |
| 50 | 189. 83 | 230. 39 | 274. 88 | 323. 29 | 375. 62 | 431. 87 | 492. 05 | 556. 13 |
| 51 | 190. 47 | 231. 10 | 275. 65 | 324. 13 | 376. 52 | 432. 84 | 493. 08 | 557. 24 |
| 52 | 191. 12 | 231. 81 | 276. 43 | 324. 97 | 377. 43 | 433. 82 | 494. 12 | 558. 34 |
| 53 | 191. 76 | 232. 52 | 277. 20 | 325. 81 | 378. 34 | 434. 79 | 495. 15 | 559. 44 |
| 54 | 192. 41 | 233. 24 | 277. 98 | 326. 66 | 379. 26 | 435. 76 | 496. 19 | 560. 55 |
| 55 | 193. 06 | 233. 95 | 278. 76 | 327. 50 | 380. 17 | 436. 73 | 497. 23 | 561. 65 |
| 56 | 193. 71 | 234. 67 | 279. 55 | 328. 35 | 381. 08 | 437. 71 | 498. 28 | 562. 76 |
| 57 | 194. 36 | 235. 38 | 280. 33 | 329. 19 | 381. 99 | 438. 69 | 499. 32 | 563. 87 |
| 58 | 195. 01 | 236. 10 | 281. 12 | 330. 04 | 382. 90 | 439. 67 | 500. 37 | 564. 98 |
| 59 | 195. 66 | 236. 82 | 281. 90 | 330. 89 | 383. 82 | 440. 65 | 501. 41 | 566. 08 |

*Table IX.* *Values of* $\left(\frac{2\sin^2\frac{t}{2}}{\sin 1''}\right)$—Continued

| $\tau$ | 17m | 18m | 19m | 20m | 21m | 22m | 23m | 24m | 25m |
|---|---|---|---|---|---|---|---|---|---|
| $s$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ |
| 0 | 567. 2 | 635. 9 | 708. 4 | 784. 9 | 865. 3 | 949. 6 | 1037. 8 | 1129. 9 | 1225. 9 |
| 1 | 568. 3 | 637. 0 | 709. 7 | 786. 2 | 866. 6 | 951. 0 | 1039. 3 | 1131. 4 | 1227. 5 |
| 2 | 569. 4 | 638. 2 | 710. 9 | 787. 5 | 868. 0 | 952. 4 | 1040. 8 | 1133. 0 | 1229. 2 |
| 3 | 570. 5 | 639. 4 | 712. 1 | 788. 8 | 869. 4 | 953. 8 | 1042. 3 | 1134. 6 | 1230. 8 |
| 4 | 571. 6 | 640. 6 | 713. 4 | 790. 1 | 870. 8 | 955. 3 | 1043. 8 | 1136. 2 | 1232. 5 |
| 5 | 572. 8 | 641. 7 | 714. 6 | 791. 4 | 872. 1 | 956. 7 | 1045. 3 | 1137. 8 | 1234. 1 |
| 6 | 573. 9 | 642. 9 | 715. 9 | 792. 7 | 873. 5 | 958. 2 | 1046. 8 | 1139. 3 | 1235. 7 |
| 7 | 575. 0 | 644. 1 | 717. 1 | 794. 0 | 874. 9 | 959. 6 | 1048. 3 | 1140. 9 | 1237. 3 |
| 8 | 576. 1 | 645. 3 | 718. 4 | 795. 4 | 876. 3 | 961. 1 | 1049. 8 | 1142. 5 | 1239. 0 |
| 9 | 577. 2 | 646. 5 | 719. 6 | 796. 7 | 877. 6 | 962. 5 | 1051. 3 | 1144. 0 | 1240. 6 |
| 10 | 578. 4 | 647. 7 | 720. 9 | 798. 0 | 879. 0 | 963. 9 | 1052. 8 | 1145. 6 | 1242. 3 |
| 11 | 579. 5 | 648. 9 | 722. 1 | 799. 3 | 880. 4 | 965. 4 | 1054. 3 | 1147. 2 | 1243. 9 |
| 12 | 580. 6 | 650. 0 | 723. 4 | 800. 7 | 881. 8 | 966. 9 | 1055. 9 | 1148. 8 | 1245. 6 |
| 13 | 581. 7 | 651. 2 | 724. 6 | 802. 0 | 883. 2 | 968. 3 | 1057. 4 | 1150. 4 | 1247. 2 |
| 14 | 582. 9 | 652. 4 | 725. 9 | 803. 3 | 884. 6 | 969. 8 | 1058. 9 | 1152. 0 | 1248. 9 |
| 15 | 584. 0 | 653. 6 | 727. 2 | 804. 6 | 886. 0 | 971. 2 | 1060. 4 | 1153. 6 | 1250. 5 |
| 16 | 585. 1 | 654. 8 | 728. 4 | 806. 0 | 887. 4 | 972. 7 | 1062. 0 | 1155. 2 | 1252. 2 |
| 17 | 586. 2 | 656. 0 | 729. 7 | 807. 3 | 888. 8 | 974. 1 | 1063. 5 | 1156. 8 | 1253. 8 |
| 18 | 587. 4 | 657. 2 | 730. 9 | 808. 6 | 890. 2 | 975. 5 | 1065. 0 | 1158. 3 | 1255. 5 |
| 19 | 588. 5 | 658. 4 | 732. 2 | 809. 9 | 891. 6 | 977. 0 | 1066. 5 | 1159. 9 | 1257. 1 |
| 20 | 589. 6 | 659. 6 | 733. 5 | 811. 3 | 893. 0 | 978. 5 | 1068. 1 | 1161. 5 | 1258. 8 |
| 21 | 590. 8 | 660. 8 | 734. 7 | 812. 6 | 894. 4 | 979. 9 | 1069. 6 | 1163. 1 | 1260. 5 |
| 22 | 591. 9 | 662. 0 | 736. 0 | 813. 9 | 895. 8 | 981. 4 | 1071. 1 | 1164. 7 | 1262. 2 |
| 23 | 593. 0 | 663. 2 | 737. 3 | 815. 2 | 897. 2 | 982. 9 | 1072. 6 | 1166. 3 | 1263. 8 |
| 24 | 594. 2 | 664. 4 | 738. 5 | 816. 6 | 898. 6 | 984. 4 | 1074. 2 | 1167. 9 | 1265. 5 |
| 25 | 595. 3 | 665. 6 | 739. 8 | 817. 9 | 900. 0 | 985. 8 | 1075. 7 | 1169. 5 | 1267. 1 |
| 26 | 596. 5 | 666. 8 | 741. 1 | 819. 2 | 901. 4 | 987. 3 | 1077. 2 | 1171. 1 | 1268. 8 |
| 27 | 597. 6 | 668. 0 | 742. 3 | 820. 5 | 902. 8 | 988. 8 | 1078. 7 | 1172. 7 | 1270. 5 |
| 28 | 598. 7 | 669. 2 | 743. 6 | 821. 9 | 904. 2 | 990. 3 | 1080. 3 | 1174. 3 | 1272. 1 |
| 29 | 599. 9 | 670. 4 | 744. 9 | 823. 2 | 905. 6 | 991. 8 | 1081. 8 | 1175. 9 | 1273. 7 |

*Table IX. Values of* $\left(\frac{2\sin^2\frac{t}{2}}{\sin 1''}\right)$—Continued

| $\tau$ | 17m | 18m | 19m | 20m | 21m | 22m | 23m | 24m | 25m |
|---|---|---|---|---|---|---|---|---|---|
| s | " | " | " | " | " | " | " | " | " |
| 30 | 601. 0 | 671. 6 | 746. 2 | 824. 6 | 907. 0 | 993. 2 | 1083. 3 | 1177. 5 | 1275. 4 |
| 31 | 602. 2 | 672. 8 | 747. 4 | 825. 9 | 908. 4 | 994. 7 | 1084. 8 | 1179. 1 | 1277. 1 |
| 32 | 603. 3 | 674. 1 | 748. 7 | 827. 3 | 909. 8 | 996. 2 | 1086. 4 | 1180. 7 | 1278. 8 |
| 33 | 604. 5 | 675. 3 | 750. 0 | 828. 6 | 911. 2 | 997. 6 | 1087. 9 | 1182. 3 | 1280. 4 |
| 34 | 605. 6 | 676. 5 | 751. 3 | 829. 9 | 912. 6 | 999. 1 | 1089. 5 | 1183. 9 | 1282. 1 |
| 35 | 606. 8 | 677. 7 | 752. 6 | 831. 2 | 914. 0 | 1000. 6 | 1091. 0 | 1185. 5 | 1283. 8 |
| 36 | 607. 9 | 678. 9 | 753. 8 | 832. 6 | 915. 5 | 1002. 1 | 1092. 6 | 1187. 1 | 1285. 5 |
| 37 | 609. 1 | 680. 1 | 755. 1 | 833. 9 | 916. 9 | 1003. 5 | 1094. 1 | 1188. 7 | 1287. 1 |
| 38 | 610. 2 | 681. 3 | 756. 4 | 835. 3 | 918. 3 | 1005. 0 | 1095. 7 | 1190. 3 | 1288. 8 |
| 39 | 611. 4 | 682. 6 | 757. 7 | 836. 6 | 919. 7 | 1006. 5 | 1097. 2 | 1191. 9 | 1290. 5 |
| 40 | 612. 5 | 683. 8 | 759. 0 | 838. 0 | 921. 1 | 1008. 0 | 1098. 8 | 1193. 5 | 1292. 2 |
| 41 | 613. 7 | 685. 0 | 760. 2 | 839. 3 | 922. 5 | 1009. 4 | 1100. 3 | 1195. 1 | 1293. 8 |
| 42 | 614. 8 | 686. 2 | 761. 5 | 840. 7 | 923. 9 | 1010. 9 | 1101. 9 | 1196. 7 | 1295. 5 |
| 43 | 616. 0 | 687. 4 | 762. 8 | 842. 0 | 925. 3 | 1012. 4 | 1103. 4 | 1198. 3 | 1297. 2 |
| 44 | 617. 2 | 688. 7 | 764. 1 | 843. 4 | 926. 8 | 1013. 9 | 1105. 0 | 1199. 9 | 1298. 9 |
| 45 | 618. 3 | 689. 9 | 765. 4 | 844. 7 | 928. 2 | 1015. 4 | 1106. 5 | 1201. 5 | 1300. 5 |
| 46 | 619. 5 | 691. 1 | 766. 7 | 846. 1 | 929. 6 | 1016. 9 | 1108. 1 | 1203. 1 | 1302. 2 |
| 47 | 620. 6 | 692. 4 | 768. 0 | 847. 5 | 931. 0 | 1018. 4 | 1109. 6 | 1204. 7 | 1303. 9 |
| 48 | 621. 8 | 693. 6 | 769. 3 | 848. 9 | 932. 4 | 1019. 9 | 1111. 2 | 1206. 4 | 1305. 6 |
| 49 | 623. 0 | 694. 8 | 770. 6 | 850. 2 | 933. 8 | 1021. 4 | 1112. 7 | 1208. 0 | 1307. 3 |
| 50 | 624. 1 | 696. 0 | 771. 9 | 851. 6 | 935. 2 | 1022. 8 | 1114. 3 | 1209. 6 | 1309. 0 |
| 51 | 625. 3 | 697. 3 | 773. 1 | 852. 9 | 936. 6 | 1024. 3 | 1115. 8 | 1211. 2 | 1310. 7 |
| 52 | 626. 5 | 698. 5 | 774. 5 | 854. 3 | 938. 1 | 1025. 8 | 1117. 4 | 1212. 9 | 1312. 4 |
| 53 | 627. 6 | 699. 7 | 775. 7 | 855. 7 | 939. 5 | 1027. 3 | 1118. 9 | 1214. 5 | 1314. 1 |
| 54 | 628. 8 | 701. 0 | 777. 1 | 857. 1 | 940. 9 | 1028. 8 | 1120. 5 | 1216. 1 | 1315. 7 |
| 55 | 630. 0 | 702. 2 | 778. 4 | 858. 4 | 942. 3 | 1030. 3 | 1122. 0 | 1217. 7 | 1317. 4 |
| 56 | 631. 2 | 703. 5 | 779. 7 | 859. 8 | 943. 8 | 1031. 8 | 1123. 6 | 1219. 4 | 1319. 1 |
| 57 | 632. 3 | 704. 7 | 781. 0 | 861. 1 | 945. 2 | 1033. 3 | 1125. 1 | 1221. 0 | 1320. 8 |
| 58 | 633. 5 | 705. 9 | 782. 3 | 862. 5 | 946. 6 | 1034. 8 | 1126. 7 | 1222. 6 | 1322. 5 |
| 59 | 634. 7 | 707. 1 | 783. 6 | 863. 9 | 948. 1 | 1036. 3 | 1128. 3 | 1224. 2 | 1324. 2 |

*Table X. Values of* $\left(\frac{2 \sin^4 \frac{t}{2}}{\sin 1''}\right)$

| $t$ | $n$ | diff. | $t$ | $n$ | diff |
|---|---|---|---|---|---|
| 01m 00s | 0. 00001 | | 05 00 | 0. 00584 | |
| | | 0. 00002 | | | 0. 00172 |
| 01 20 | . 00003 | | 05 20 | . 00756 | |
| | | . 00004 | | | . 00208 |
| 01 40 | . 00007 | | 05 40 | . 00964 | |
| | | . 00008 | | | . 00247 |
| 02 00 | . 00015 | | 06 00 | . 01211 | |
| | | . 00013 | | | . 00292 |
| 02 20 | . 00028 | | 06 20 | . 01503 | |
| | | . 00019 | | | . 00343 |
| 02 40 | . 00047 | | 06 40 | . 01846 | |
| | | . 00029 | | | . 00398 |
| 03 00 | . 00076 | | 07 00 | . 02244 | |
| | | . 00039 | | | . 00458 |
| 03 20 | . 00115 | | 07 20 | . 02702 | |
| | | . 00054 | | | . 00526 |
| 03 40 | . 00169 | | 07 40 | . 03228 | |
| | | . 00070 | | | . 00599 |
| 04 00 | . 00239 | | 08 00 | . 03827 | |
| | | . 00091 | | | . 00679 |
| 04 20 | . 00330 | | 08 20 | . 04506 | |
| | | . 00113 | | | |
| 04 40 | . 00443 | | | | |
| | | . 00141 | | | |

*Table XI. Factors for the Determination of a Rejection Limit*

The rejection factor, $t'$, is multiplied by the probable error to establish rejection limit. (Based on Table IX–A Chauvenet Practical Astronomy Vol. II)

| $n$ | $t'$ | $n$ | $t'$ | $n$ | $t'$ |
|---|---|---|---|---|---|
| 8 | 2. 76 | 23 | 3. 40 | 38 | 3. 68 |
| 9 | 2. 84 | 24 | 3. 43 | 39 | 3. 69 |
| 10 | 2. 91 | 25 | 3. 45 | 40 | 3. 70 |
| 11 | 2. 97 | 26 | 3. 47 | 41 | 3. 72 |
| 12 | 3. 02 | 27 | 3. 49 | 42 | 3. 73 |
| 13 | 3. 07 | 28 | 3. 51 | 43 | 3. 74 |
| 14 | 3. 11 | 29 | 3. 53 | 44 | 3. 76 |
| 15 | 3. 15 | 30 | 3. 55 | 45 | 3. 77 |
| 16 | 3. 19 | 31 | 3. 57 | 46 | 3. 78 |
| 17 | 3. 23 | 32 | 3. 59 | 47 | 3. 79 |
| 18 | 3. 26 | 33 | 3. 60 | 48 | 3. 80 |
| 19 | 3. 29 | 34 | 3. 62 | 49 | 3. 81 |
| 20 | 3. 32 | 35 | 3. 63 | 50 | 3. 82 |
| 21 | 3. 35 | 36 | 3. 65 | 51 | 3. 83 |
| 22 | 3. 38 | 37 | 3. 66 | 52 | 3. 84 |

$n$=No. of observations.

## *Table XII. Correction to Latitude for Reduction to Meridian*

(Star off the meridian but instrument in the meridian. The sign of the correction to the latitude is positive except for stars south of the equator and subpolars.)

| δ | 6ˢ | 7ˢ | 8ˢ | 9ˢ | 10ˢ | 15ˢ | 20ˢ | 22ˢ | 24ˢ | 26ˢ | 28ˢ | 30ˢ | 32ˢ | 34ˢ | 36ˢ | 38ˢ | 40ˢ | 42ˢ | 44ˢ | 46ˢ | 48ˢ | 50ˢ | 52ˢ | 54ˢ | 56ˢ | 58ˢ | 60ˢ | δ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ° |
| 1 | | | | | | | | | | | | | | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .02 | .02 | 89 |
| 2 | | | | | | | | | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .02 | .02 | .02 | .02 | .02 | .02 | .03 | .03 | .03 | .03 | .03 | 88 |
| 3 | | | | | | | .01 | .01 | .01 | .01 | .01 | .01 | .01 | .02 | .02 | .02 | .02 | .03 | .03 | .03 | .03 | .04 | .04 | .04 | .04 | .05 | .05 | 87 |
| 4 | | | | | | . | .01 | .01 | .01 | .01 | .01 | .02 | .02 | .02 | .02 | .03 | .03 | .03 | .04 | .04 | .04 | .05 | .05 | .06 | .06 | .06 | .07 | 86 |
| 5 | | | | | | .01 | .01 | .01 | .01 | .02 | .02 | .02 | .02 | .03 | .03 | .03 | .04 | .04 | .05 | .05 | .05 | .06 | .06 | .07 | .07 | .08 | .09 | 85 |
| 6 | | | | | | .01 | .01 | .01 | .02 | .02 | .02 | .03 | .03 | .03 | .04 | .04 | .05 | .05 | .06 | .06 | .07 | .07 | .08 | .08 | .09 | .10 | .10 | 84 |
| 7 | | | | | | .01 | .01 | .02 | .02 | .02 | .03 | .03 | .03 | .04 | .04 | .05 | .05 | .06 | .06 | .07 | .08 | .08 | .09 | .10 | .10 | .11 | .12 | 83 |
| 8 | | | | | | .01 | .02 | .02 | .02 | .03 | .03 | .03 | .04 | .04 | .05 | .05 | .06 | .07 | .07 | .08 | .09 | .09 | .10 | .11 | .12 | .13 | .14 | 82 |
| 9 | | | | | | .01 | .02 | .02 | .02 | .03 | .03 | .04 | .04 | .05 | .05 | .06 | .07 | .07 | .08 | .09 | .10 | .11 | .11 | .12 | .13 | .14 | .15 | 81 |
| 10 | | | | | | .01 | .02 | .02 | .03 | .03 | .04 | .04 | .05 | .05 | .06 | .07 | .07 | .08 | .09 | .10 | .11 | .12 | .13 | .14 | .15 | .16 | .17 | 80 |
| 12 | | | | | .01 | .01 | .02 | .03 | .03 | .04 | .05 | .05 | .06 | .06 | .07 | .08 | .09 | .10 | .11 | .12 | .13 | .14 | .15 | .16 | .17 | .19 | .20 | 78 |
| 14 | | | | .01 | .01 | .01 | .03 | .03 | .04 | .04 | .05 | .06 | .07 | .07 | .08 | .09 | .10 | .11 | .12 | .14 | .15 | .16 | .17 | .19 | .20 | .22 | .23 | 76 |
| 16 | | | | .01 | .01 | .02 | .03 | .03 | .04 | .05 | .06 | .07 | .07 | .08 | .09 | .10 | .12 | .13 | .14 | .15 | .17 | .18 | .20 | .21 | .23 | .24 | .26 | 74 |
| 18 | | | .01 | .01 | .01 | .02 | .03 | .04 | .05 | .05 | .06 | .07 | .08 | .09 | .10 | .12 | .13 | .14 | .16 | .17 | .18 | .20 | .22 | .23 | .25 | .27 | .29 | 72 |
| 20 | | | .01 | .01 | .01 | .02 | .04 | .04 | .05 | .06 | .07 | .08 | .09 | .10 | .11 | .13 | .14 | .15 | .17 | .19 | .20 | .22 | .24 | .26 | .28 | .29 | .32 | 70 |
| 22 | | | .01 | .01 | .01 | .02 | .04 | .05 | .05 | .06 | .07 | .09 | .10 | .11 | .12 | .14 | .15 | .17 | .18 | .20 | .22 | .24 | .26 | .28 | .30 | .32 | .34 | 68 |
| 24 | | | .01 | .01 | .01 | .02 | .04 | .05 | .06 | .07 | .08 | .09 | .10 | .12 | .13 | .15 | .16 | .18 | .20 | .21 | .23 | .25 | .27 | .29 | .32 | .34 | .36 | 66 |
| 26 | | | 01 | .01 | .01 | .02 | .04 | .05 | .06 | .07 | .08 | .10 | .11 | .12 | .14 | .15 | .17 | .19 | .21 | .23 | .25 | .27 | .29 | .31 | .34 | .36 | .39 | 64 |
| 28 | | .01 | .01 | .01 | .01 | .03 | .05 | .05 | .07 | .08 | .09 | .10 | .12 | .13 | .15 | .16 | .18 | .20 | .22 | .24 | .26 | .28 | .31 | .33 | .35 | .38 | .41 | 62 |
| 30 | | .01 | .01 | .01 | .01 | .03 | .05 | .06 | .07 | .08 | .09 | .11 | .12 | .14 | .15 | .17 | .19 | .21 | .23 | .25 | .27 | .30 | .32 | .34 | .37 | .40 | .42 | 60 |
| 32 | | .01 | .01 | .01 | .01 | .03 | .05 | .06 | .07 | .08 | .10 | .11 | .13 | .14 | .16 | .18 | .20 | .22 | .24 | .26 | .28 | .31 | .33 | .36 | .39 | .41 | .44 | 58 |
| 34 | | .01 | .01 | .01 | .01 | .03 | .05 | .06 | .07 | .09 | .10 | .11 | .13 | .15 | .16 | .18 | .20 | .22 | .24 | .27 | .29 | .32 | .34 | .37 | .40 | .42 | .45 | 56 |
| 36 | | .01 | .01 | .01 | .01 | .03 | .05 | .06 | .07 | .09 | .10 | .12 | .13 | .15 | .17 | .19 | .21 | .23 | .25 | .28 | .30 | .32 | .35 | .38 | .41 | .44 | .47 | 54 |
| 38 | .01 | .01 | .01 | .01 | .01 | .03 | .05 | .06 | .08 | .09 | .10 | .12 | .13 | .15 | .17 | .19 | .21 | .23 | .26 | .28 | .30 | .33 | .36 | .39 | .41 | .44 | .48 | 52 |
| 40 | .01 | .01 | .01 | .01 | .01 | .03 | .05 | .07 | .08 | .09 | .11 | .12 | .14 | .16 | .17 | .19 | .21 | .24 | .26 | .28 | .31 | .34 | .36 | .39 | .42 | .45 | .48 | 50 |
| 45 | .01 | .01 | .01 | .01 | .01 | .03 | .05 | .07 | .08 | .09 | .11 | .12 | .14 | .16 | .18 | .20 | .22 | .24 | .26 | .29 | .31 | .34 | .37 | .40 | .43 | .46 | .49 | 45 |

*Table XII. Correction to Latitude for Reduction to Meridian*—Continued

| δ | 60s | 65s | 70s | 75s | 80s | 85s | 90s | 95s | 100s | 105s | 110s | 115s | 120s | δ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ° |
| 0 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | .00 | 90 |
| 2 | .03 | .04 | .05 | .05 | .06 | .07 | .08 | .09 | .10 | .10 | .12 | .13 | .14 | 88 |
| 4 | .07 | .08 | .09 | .11 | .12 | .14 | .15 | .17 | .19 | .21 | .23 | .25 | .27 | 86 |
| 6 | .10 | .12 | .14 | .16 | .18 | .20 | .23 | .26 | .28 | .31 | .34 | .37 | .41 | 84 |
| 8 | .14 | .16 | .18 | .21 | .24 | .27 | .30 | .34 | .38 | .41 | .45 | .50 | .54 | 82 |
| 10 | .17 | .20 | .23 | .26 | .30 | .34 | .38 | .42 | .47 | .51 | .56 | .62 | .67 | 80 |
| 12 | .20 | .24 | .27 | .31 | .36 | .40 | .45 | .50 | .55 | .61 | .67 | .73 | .80 | 78 |
| 14 | .23 | .27 | .31 | .36 | .41 | .46 | .52 | .58 | .64 | .71 | .77 | .85 | .92 | 76 |
| 16 | .26 | .31 | .35 | .41 | .46 | .52 | .59 | .65 | .72 | .80 | .87 | .96 | 1.04 | 74 |
| 18 | .29 | .34 | .39 | .45 | .51 | .58 | .65 | .72 | .80 | .88 | .97 | 1.06 | 1.15 | 72 |
| 20 | .32 | .37 | .43 | .49 | .56 | .63 | .71 | .79 | .88 | .97 | 1.06 | 1.16 | 1.26 | 70 |
| 22 | .34 | .40 | .46 | .53 | .61 | .68 | .77 | .85 | .95 | 1.04 | 1.15 | 1.25 | 1.36 | 68 |
| 24 | .36 | .43 | .50 | .57 | .65 | .73 | .82 | .91 | 1.01 | 1.12 | 1.23 | 1.34 | 1.46 | 66 |
| 26 | .39 | .46 | .53 | .60 | .69 | .78 | .87 | .97 | 1.07 | 1.18 | 1.30 | 1.42 | 1.55 | 64 |
| 28 | .41 | .48 | .55 | .64 | .72 | .82 | .92 | 1.02 | 1.13 | 1.25 | 1.37 | 1.49 | 1.63 | 62 |
| 30 | .42 | .50 | .58 | .66 | .76 | .85 | .96 | 1.07 | 1.18 | 1.30 | 1.43 | 1.56 | 1.70 | 60 |
| 32 | .44 | .52 | .60 | .69 | .78 | .89 | .99 | 1.11 | 1.22 | 1.35 | 1.48 | 1.62 | 1.76 | 58 |
| 34 | .45 | .54 | .62 | .71 | .81 | .91 | 1.02 | 1.14 | 1.26 | 1.39 | 1.53 | 1.67 | 1.82 | 56 |
| 36 | .47 | .55 | .63 | .73 | .83 | .94 | 1.05 | 1.17 | 1.30 | 1.43 | 1.57 | 1.71 | 1.87 | 54 |
| 38 | .48 | .56 | .65 | .74 | .85 | .96 | 1.07 | 1.19 | 1.32 | 1.46 | 1.60 | 1.75 | 1.90 | 52 |
| 40 | .48 | .57 | .66 | .76 | .86 | .97 | 1.09 | 1.21 | 1.34 | 1.48 | 1.62 | 1.78 | 1.93 | 50 |
| 42 | .49 | .57 | .66 | .76 | .87 | .98 | 1.10 | 1.22 | 1.36 | 1.49 | 1.64 | 1.79 | 1.95 | 48 |
| 44 | .49 | .58 | .67 | .77 | .87 | .98 | 1.10 | 1.23 | 1.36 | 1.50 | 1.65 | 1.80 | 1.96 | 46 |
| 45 | .49 | .58 | .67 | .77 | .87 | .98 | 1.10 | 1.23 | 1.36 | 1.50 | 1.65 | 1.80 | 1.96 | 45 |

*Table XIII. Correction to Latitude for Differential Refraction, $\frac{1}{2}(r-r')$*

(The sign of the correction is the same as that of the micrometer difference)

| One-half diff. of zenith distances | Zenith distance | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0° | 10° | 20° | 25° | 30° | 35° | 40° | 45° |
| ′ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 0. 0 | 0. 00 | 0. 00 | 0. 00 | 0. 00 | 0. 00 | 0. 00 | 0. 00 | 0. 00 |
| . 5 | 0. 01 | 0. 01 | 0. 01 | 0. 01 | 0. 01 | 0. 01 | 0. 01 | 0. 02 |
| 1. 0 | 0. 02 | 0. 02 | 0. 02 | 0. 02 | 0. 02 | 0. 03 | 0. 03 | 0. 03 |
| 1. 5 | 0. 03 | 0. 03 | 0. 03 | 0. 03 | 0. 03 | 0. 04 | 0. 04 | 0. 05 |
| 2. 0 | 0. 03 | 0. 03 | 0. 04 | 0. 04 | 0. 04 | 0. 05 | 0. 06 | 0. 07 |
| 2. 5 | 0. 04 | 0. 04 | 0. 05 | 0. 05 | 0. 06 | 0. 06 | 0. 07 | 0. 08 |
| 3. 0 | 0. 05 | 0. 05 | 0. 06 | 0. 06 | 0. 07 | 0. 08 | 0. 09 | 0. 10 |
| 3. 5 | 0. 06 | 0. 06 | 0. 07 | 0. 07 | 0. 08 | 0. 09 | 0. 10 | 0. 12 |
| 4. 0 | 0. 07 | 0. 07 | 0. 08 | 0. 08 | 0. 09 | 0. 10 | 0. 11 | 0. 13 |
| 4. 5 | 0. 08 | 0. 08 | 0. 09 | 0. 09 | 0. 10 | 0. 11 | 0. 13 | 0. 15 |
| 5. 0 | 0. 08 | 0. 09 | 0. 10 | 0. 10 | 0. 11 | 0. 13 | 0. 14 | 0. 17 |
| 5. 5 | 0. 09 | 0. 10 | 0. 10 | 0. 11 | 0. 12 | 0. 14 | 0. 16 | 0. 18 |
| 6. 0 | 0. 10 | 0. 10 | 0. 11 | 0. 12 | 0. 13 | 0. 15 | 0. 17 | 0. 20 |
| 6. 5 | 0. 11 | 0. 11 | 0. 12 | 0. 13 | 0. 14 | 0. 16 | 0. 19 | 0. 22 |
| 7. 0 | 0. 12 | 0. 12 | 0. 13 | 0. 14 | 0. 16 | 0. 18 | 0. 20 | 0. 23 |
| 7. 5 | 0. 13 | 0. 13 | 0. 14 | 0. 15 | 0. 17 | 0. 19 | 0. 21 | 0. 25 |
| 8. 0 | 0. 13 | 0. 14 | 0. 15 | 0. 16 | 0. 18 | 0. 20 | 0. 23 | 0. 27 |
| 8. 5 | 0. 14 | 0. 15 | 0. 16 | 0. 17 | 0. 19 | 0. 21 | 0. 24 | 0. 29 |
| 9. 0 | 0. 15 | 0. 16 | 0. 17 | 0. 18 | 0. 20 | 0. 23 | 0. 26 | 0. 30 |
| 9. 5 | 0. 16 | 0. 16 | 0. 18 | 0. 19 | 0. 21 | 0. 24 | 0. 27 | 0. 32 |
| 10. 0 | 0. 17 | 0. 17 | 0. 19 | 0. 20 | 0. 22 | 0. 25 | 0. 29 | 0. 34 |
| 10. 5 | 0. 18 | 0. 18 | 0. 20 | 0. 21 | 0. 23 | 0. 26 | 0. 30 | 0. 35 |
| 11. 0 | 0. 18 | 0. 19 | 0. 21 | 0. 22 | 0. 25 | 0. 28 | 0. 31 | 0. 37 |
| 11. 5 | 0. 19 | 0. 20 | 0. 22 | 0. 23 | 0. 26 | 0. 29 | 0. 33 | 0. 39 |
| 12. 0 | 0. 20 | 0. 21 | 0. 23 | 0. 25 | 0. 27 | 0. 30 | 0. 34 | 0. 40 |
| 12. 5 | 0. 21 | 0. 22 | 0. 24 | 0. 26 | 0. 28 | 0. 31 | 0. 36 | 0. 42 |
| 13. 0 | 0. 22 | 0. 22 | 0. 25 | 0. 27 | 0. 29 | 0. 33 | 0. 37 | 0. 44 |
| 13. 5 | 0. 23 | 0. 23 | 0. 26 | 0. 28 | 0. 30 | 0. 34 | 0. 39 | 0. 45 |
| 14. 0 | 0. 23 | 0. 24 | 0. 27 | 0. 29 | 0. 31 | 0. 35 | 0. 40 | 0. 47 |
| 14. 5 | 0. 24 | 0. 25 | 0. 28 | 0. 30 | 0. 32 | 0. 36 | 0. 41 | 0. 49 |
| 15. 0 | 0. 25 | 0. 26 | 0. 29 | 0. 31 | 0. 34 | 0. 38 | 0. 43 | 0. 50 |
| 15. 5 | 0. 26 | 0. 27 | 0. 29 | 0. 32 | 0. 35 | 0. 39 | 0. 44 | 0. 52 |
| 16. 0 | 0. 27 | 0. 28 | 0. 30 | 0. 33 | 0. 36 | 0. 40 | 0. 46 | 0. 54 |
| 16. 5 | 0. 28 | 0. 29 | 0. 31 | 0. 34 | 0. 37 | 0. 41 | 0. 47 | 0. 55 |
| 17. 0 | 0. 29 | 0. 29 | 0. 32 | 0. 35 | 0. 38 | 0. 43 | 0. 49 | 0. 57 |
| 17. 5 | 0. 29 | 0. 30 | 0. 33 | 0. 36 | 0. 39 | 0. 44 | 0. 50 | 0. 59 |
| 18. 0 | 0. 30 | 0. 31 | 0. 34 | 0. 37 | 0. 40 | 0. 45 | 0. 51 | 0. 60 |
| 18. 5 | 0. 31 | 0. 32 | 0. 35 | 0. 38 | 0. 41 | 0. 46 | 0. 53 | 0. 62 |
| 19. 0 | 0. 32 | 0. 33 | 0. 36 | 0. 39 | 0. 43 | 0. 48 | 0. 54 | 0. 64 |
| 19. 5 | 0. 33 | 0. 34 | 0. 37 | 0. 40 | 0. 44 | 0. 49 | 0. 56 | 0. 65 |
| 20. 0 | 0. 34 | 0. 35 | 0. 38 | 0. 41 | 0. 45 | 0. 50 | 0. 57 | 0. 67 |

*Elevation Factor for Differential Refraction*

| For elevation | Multiply by | For elevation | Multiply by | For elevation | Multiply by | For elevation | Multiply by |
|---|---|---|---|---|---|---|---|
| *feet* | | *feet* | | *meters* | | *meters* | |
| 1, 000 | 0. 96 | 9, 000 | 0. 72 | 200 | 0. 98 | 2, 800 | 0. 71 |
| 2, 000 | . 93 | 10, 000 | . 69 | 400 | . 95 | 3, 000 | . 70 |
| 3, 000 | . 90 | 11, 000 | . 67 | 600 | . 93 | 3, 200 | . 68 |
| 4, 000 | . 86 | 12, 000 | . 64 | 800 | . 91 | 3, 400 | . 66 |
| 5, 000 | . 83 | 13, 000 | . 62 | 1, 000 | . 89 | 3, 600 | . 65 |
| 6, 000 | . 80 | 14, 000 | . 60 | 1, 200 | . 87 | 3, 800 | . 63 |
| 7, 000 | . 77 | 15, 000 | . 58 | 1, 400 | . 85 | 4, 000 | . 62 |
| 8, 000 | . 75 | | | 1, 600 | . 83 | 4, 200 | . 60 |
| | | | | 1, 800 | . 81 | 4, 400 | . 59 |
| | | | | 2, 000 | . 79 | 4, 600 | . 58 |
| | | | | 2, 200 | . 77 | 4, 800 | . 56 |
| | | | | 2, 400 | . 75 | 5, 000 | . 55 |
| | | | | 2, 600 | . 73 | | |

*Table XIV. Reduction of Latitude to Sea Level*

(The correction is negative in every case)

| $h$ \ $\phi$ | | 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 85° | 80° | 75° | 70° | 65° | 60° | 55° | 50° | 45° |
| *feet* | *meters* | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 100 | 30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 |
| 200 | 61 | .00 | .00 | .01 | .01 | .01 | .01 | .01 | .01 | .01 |
| 300 | 91 | .00 | .01 | .01 | .01 | .01 | .01 | .01 | .02 | .02 |
| 400 | 122 | .00 | .01 | .01 | .01 | .02 | .02 | .02 | .02 | .02 |
| 500 | 152 | .00 | .01 | .01 | .02 | .02 | .02 | .02 | .03 | .03 |
| 600 | 183 | .01 | .01 | .02 | .02 | .02 | .03 | .03 | .03 | .03 |
| 700 | 213 | .01 | .01 | .02 | .02 | .03 | .03 | .03 | .04 | .04 |
| 800 | 244 | .01 | .01 | .02 | .03 | .03 | .04 | .04 | .04 | .04 |
| 900 | 274 | .01 | .02 | .02 | .03 | .04 | .04 | .04 | .05 | .05 |
| 1000 | 305 | .01 | .02 | .03 | .03 | .04 | .05 | .05 | .05 | .05 |
| 1100 | 335 | .01 | .02 | .03 | .04 | .04 | .05 | .05 | .06 | .06 |
| 1200 | 366 | .01 | .02 | .03 | .04 | .05 | .05 | .06 | .06 | .06 |
| 1300 | 396 | .01 | .02 | .03 | .04 | .05 | .06 | .06 | .07 | .07 |
| 1400 | 427 | .01 | .02 | .04 | .05 | .06 | .06 | .07 | .07 | .07 |
| 1500 | 457 | .01 | .03 | .04 | .05 | .06 | .07 | .07 | .08 | .08 |
| 1600 | 488 | .01 | .03 | .04 | .05 | .06 | .07 | .08 | .08 | .08 |
| 1700 | 518 | .02 | .03 | .04 | .06 | .07 | .08 | .08 | .09 | .09 |
| 1800 | 549 | .02 | .03 | .05 | .06 | .07 | .08 | .09 | .09 | .09 |
| 1900 | 579 | .02 | .03 | .05 | .06 | .08 | .09 | .09 | .10 | .10 |
| 2000 | 610 | .02 | .04 | .05 | .07 | .08 | .09 | .10 | .10 | .10 |
| 2100 | 640 | .02 | .04 | .05 | .07 | .08 | .09 | .10 | .11 | .11 |
| 2200 | 671 | .02 | .04 | .06 | .07 | .09 | .10 | .11 | .11 | .11 |
| 2300 | 701 | .02 | .04 | .06 | .08 | .09 | .10 | .11 | .12 | .12 |
| 2400 | 732 | .02 | .04 | .06 | .08 | .10 | .11 | .12 | .12 | .13 |
| 2500 | 762 | .02 | .04 | .07 | .08 | .10 | .11 | .12 | .13 | .13 |
| 2600 | 792 | .02 | .05 | .07 | .09 | .10 | .12 | .13 | .13 | .14 |
| 2700 | 823 | .02 | .05 | .07 | .09 | .11 | .12 | .13 | .14 | .14 |
| 2800 | 853 | .03 | .05 | .07 | .09 | .11 | .13 | .14 | .14 | .15 |
| 2900 | 884 | .03 | .05 | .08 | .10 | .12 | .13 | .14 | .15 | .15 |
| 3000 | 914 | .03 | .05 | .08 | .10 | .12 | .14 | .15 | .15 | .16 |
| 3100 | 945 | .03 | .06 | .08 | .10 | .12 | .14 | .15 | .16 | .16 |
| 3200 | 975 | .03 | .06 | .08 | .11 | .13 | .14 | .16 | .16 | .17 |
| 3300 | 1006 | .03 | .06 | .09 | .11 | .13 | .15 | .16 | .17 | .17 |
| 3400 | 1036 | .03 | .06 | .09 | .11 | .14 | .15 | .17 | .17 | .18 |
| 3500 | 1067 | .03 | .06 | .09 | .12 | .14 | .16 | .17 | .18 | .18 |
| 3600 | 1097 | .03 | .06 | .09 | .12 | .14 | .16 | .18 | .18 | .19 |
| 3700 | 1128 | .03 | .07 | .10 | .12 | .15 | .17 | .18 | .19 | .19 |
| 3800 | 1158 | .03 | .07 | .10 | .13 | .15 | .17 | .19 | .20 | .20 |
| 3900 | 1189 | .04 | .07 | .10 | .13 | .16 | .18 | .19 | .20 | .20 |
| 4000 | 1219 | .04 | .07 | .10 | .13 | .16 | .18 | .20 | .21 | .21 |
| 4100 | 1250 | .04 | .07 | .11 | .14 | .16 | .19 | .20 | .21 | .21 |
| 4200 | 1280 | .04 | .07 | .11 | .14 | .17 | .19 | .21 | .22 | .22 |
| 4300 | 1311 | .04 | .08 | .11 | .14 | .17 | .19 | .21 | .22 | .22 |
| 4400 | 1341 | .04 | .08 | .11 | .15 | .18 | .20 | .22 | .23 | .23 |
| 4500 | 1372 | .04 | .08 | .12 | .15 | .18 | .20 | .22 | .23 | .23 |

*Table XIV. Reduction of Latitude to Sea Level*—Continued

| h \ φ | | 5° 85° | 10° 80° | 15° 75° | 20° 70° | 25° 65° | 30° 60° | 35° 55° | 40° 50° | 45° |
|---|---|---|---|---|---|---|---|---|---|---|
| *feet* | *meters* | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ | ″ |
| 4600 | 1402 | .04 | .08 | .12 | .15 | .18 | .21 | .23 | .24 | .24 |
| 4700 | 1433 | .04 | .08 | .12 | .16 | .19 | .21 | .23 | .24 | .24 |
| 4800 | 1463 | .04 | .09 | .13 | .16 | .19 | .22 | .24 | .25 | .25 |
| 4900 | 1494 | .04 | .09 | .13 | .16 | .20 | .22 | .24 | .25 | .26 |
| 5000 | 1524 | .05 | .09 | .13 | .17 | .20 | .23 | .24 | .26 | .26 |
| 5100 | 1554 | .05 | .09 | .13 | .17 | .20 | .23 | .25 | .26 | .27 |
| 5200 | 1585 | .05 | .09 | .14 | .17 | .21 | .23 | .25 | .27 | .27 |
| 5300 | 1615 | .05 | .09 | .14 | .18 | .21 | .24 | .26 | .27 | .28 |
| 5400 | 1646 | .05 | .10 | .14 | .18 | .22 | .24 | .26 | .28 | .28 |
| 5500 | 1676 | .05 | .10 | .14 | .18 | .22 | .25 | .27 | .28 | .29 |
| 5600 | 1707 | .05 | .10 | .15 | .19 | .22 | .25 | .27 | .29 | .29 |
| 5700 | 1737 | .05 | .10 | .15 | .19 | .23 | .26 | .28 | .29 | .30 |
| 5800 | 1768 | .05 | .10 | .15 | .19 | .23 | .26 | .28 | .30 | .30 |
| 5900 | 1798 | .05 | .11 | .15 | .20 | .24 | .27 | .29 | .30 | .31 |
| 6000 | 1829 | .05 | .11 | .16 | .20 | .24 | .27 | .29 | .31 | .31 |
| 6100 | 1859 | .06 | .11 | .16 | .20 | .24 | .28 | .30 | .31 | .32 |
| 6200 | 1890 | .06 | .11 | .16 | .21 | .25 | .28 | .30 | .32 | .32 |
| 6300 | 1920 | .06 | .11 | .16 | .21 | .25 | .28 | .31 | .32 | .33 |
| 6400 | 1951 | .06 | .11 | .17 | .21 | .26 | .29 | .31 | .33 | .33 |
| 6500 | 1981 | .06 | .12 | .17 | .22 | .26 | .29 | .32 | .33 | .34 |
| 6600 | 2012 | .06 | .12 | .17 | .22 | .26 | .30 | .32 | .34 | .34 |
| 6700 | 2042 | .06 | .12 | .17 | .22 | .27 | .30 | .33 | .34 | .35 |
| 6800 | 2073 | .06 | .12 | .18 | .23 | .27 | .31 | .33 | .35 | .35 |
| 6900 | 2103 | .06 | .12 | .18 | .23 | .28 | .31 | .34 | .35 | .36 |
| 7000 | 2134 | .06 | .12 | .18 | .23 | .28 | .32 | .34 | .36 | .36 |
| 7100 | 2164 | .06 | .13 | .19 | .24 | .28 | .32 | .35 | .36 | .37 |
| 7200 | 2195 | .07 | .13 | .19 | .24 | .29 | .33 | .35 | .37 | .38 |
| 7300 | 2225 | .07 | .13 | .19 | .24 | .29 | .33 | .36 | .37 | .38 |
| 7400 | 2256 | .07 | .13 | .19 | .25 | .30 | .33 | .36 | .38 | .39 |
| 7500 | 2286 | .07 | .13 | .20 | .25 | .30 | .34 | .37 | .38 | .39 |
| 7600 | 2316 | .07 | .14 | .20 | .25 | .30 | .34 | .37 | .39 | .40 |
| 7700 | 2347 | .07 | .14 | .20 | .26 | .31 | .35 | .38 | .40 | .40 |
| 7800 | 2377 | .07 | .14 | .20 | .26 | .31 | .35 | .38 | .40 | .41 |
| 7900 | 2408 | .07 | .14 | .21 | .26 | .32 | .36 | .39 | .41 | .41 |
| 8000 | 2438 | .07 | .14 | .21 | .27 | .32 | .36 | .39 | .41 | .42 |
| 8100 | 2469 | .07 | .14 | .21 | .27 | .32 | .37 | .40 | .42 | .42 |
| 8200 | 2499 | .07 | .15 | .21 | .27 | .33 | .37 | .40 | .42 | .43 |
| 8300 | 2530 | .08 | .15 | .22 | .28 | .33 | .37 | .41 | .43 | .43 |
| 8400 | 2560 | .08 | .15 | .22 | .28 | .34 | .38 | .41 | .43 | .44 |
| 8500 | 2591 | .08 | .15 | .22 | .28 | .34 | .38 | .42 | .44 | .44 |

*Table XV. Temperature Conversion Table*

(To use the table, look in the middle column for the temperature reading you have. If the reading you have is in degrees Celsius, read the Fahrenheit equivalent in the right hand column. If the reading you have is in degrees Fahrenheit, read the Celsius equivalent in the left hand column.)

| −80 to 34 | | | 35 to 77 | | | 78 to 290 | | |
|---|---|---|---|---|---|---|---|---|
| C | | F | C | | F | C | | F |
| −62 | −80 | −112 | 1. 7 | 35 | 95. 0 | 25. 6 | 78 | 172. 4 |
| −57 | −70 | −94 | 2. 2 | 36 | 96. 8 | 26. 1 | 79 | 174. 2 |
| −51 | −60 | −76 | 2. 8 | 37 | 98. 6 | 26. 7 | 80 | 176. 0 |
| −46 | −50 | −58 | 3. 3 | 38 | 100. 4 | 27. 2 | 81 | 177. 8 |
| −40 | −40 | −40 | 3. 9 | 39 | 102. 2 | 27. 8 | 82 | 179. 6 |
| −34 | −30 | −22 | 4. 4 | 40 | 104. 0 | 28. 3 | 83 | 181. 4 |
| −29 | −20 | −4 | 5. 0 | 41 | 105. 8 | 28. 9 | 84 | 183. 2 |
| −23 | −10 | 14 | 5. 6 | 42 | 107. 6 | 29. 4 | 85 | 185. 0 |
| −17. 8 | 0 | 32 | 6. 1 | 43 | 109. 4 | 30. 0 | 86 | 186. 8 |
| −17. 2 | 1 | 33. 8 | 6. 7 | 44 | 111. 2 | 30. 6 | 87 | 188. 6 |
| −16. 7 | 2 | 35. 6 | 7. 2 | 45 | 113. 0 | 31. 1 | 88 | 190. 4 |
| −16. 1 | 3 | 37. 4 | 7. 8 | 46 | 114. 8 | 31. 7 | 89 | 192. 2 |
| −15. 6 | 4 | 39. 2 | 8. 3 | 47 | 116. 6 | 32. 2 | 90 | 194. 0 |
| −15. 0 | 5 | 41. 0 | 8. 9 | 48 | 118. 4 | 32. 8 | 91 | 195. 8 |
| −14. 4 | 6 | 42. 8 | 9. 4 | 49 | 120. 2 | 33. 3 | 92 | 197. 6 |
| −13. 9 | 7 | 44. 6 | 10. 0 | 50 | 122. 0 | 33. 9 | 93 | 199. 4 |
| −13. 3 | 8 | 46. 4 | 10. 6 | 51 | 123. 8 | 34. 4 | 94 | 201. 2 |
| −12. 8 | 9 | 48. 2 | 11. 1 | 52 | 125. 6 | 35. 0 | 95 | 203. 0 |
| −12. 2 | 10 | 50. 0 | 11. 7 | 53 | 127. 4 | 35. 6 | 96 | 204. 8 |
| −11. 7 | 11 | 51. 8 | 12. 2 | 54 | 129. 2 | 36. 1 | 97 | 206. 6 |
| −11. 1 | 12 | 53. 6 | 12. 8 | 55 | 131. 0 | 36. 7 | 98 | 208. 4 |
| −10. 6 | 13 | 55. 4 | 13. 3 | 56 | 132. 8 | 37. 2 | 99 | 210. 2 |
| −10. 0 | 14 | 57. 2 | 13. 9 | 57 | 134. 6 | 37. 8 | 100 | 212. 0 |
| −9. 4 | 15 | 59. 0 | 14. 4 | 58 | 136. 4 | 43 | 110 | 230 |
| −8. 9 | 16 | 60. 8 | 15. 0 | 59 | 138. 2 | 49 | 120 | 248 |
| −8. 3 | 17 | 62. 6 | 15. 6 | 60 | 140. 0 | 54 | 130 | 266 |
| −7. 8 | 18 | 64. 4 | 16. 1 | 61 | 141. 8 | 60 | 140 | 284 |
| −7. 2 | 19 | 66. 2 | 16. 7 | 62 | 143. 6 | 66 | 150 | 302 |
| −6. 7 | 20 | 68. 0 | 17. 2 | 63 | 145. 4 | 71 | 160 | 320 |
| −6. 1 | 21 | 69. 8 | 17. 8 | 64 | 147. 2 | 77 | 170 | 338 |
| −5. 6 | 22 | 71. 6 | 18. 3 | 65 | 149. 0 | 82 | 180 | 356 |
| −5. 0 | 23 | 73. 4 | 18. 9 | 66 | 150. 8 | 88 | 190 | 374 |
| −4. 4 | 24 | 75. 2 | 19. 4 | 67 | 152. 6 | 93 | 200 | 392 |
| −3. 9 | 25 | 77. 0 | 20. 0 | 68 | 154. 4 | 99 | 210 | 410 |
| −3. 3 | 26 | 78. 8 | 20. 6 | 69 | 156. 2 | 100 | 212 | 413. 6 |
| −2. 8 | 27 | 80. 6 | 21. 1 | 70 | 158. 0 | 104 | 220 | 428 |
| −2. 2 | 28 | 82. 4 | 21. 7 | 71 | 159. 8 | 110 | 230 | 446 |
| −1. 7 | 29 | 84. 2 | 22. 2 | 72 | 161. 6 | 116 | 240 | 464 |
| −1. 1 | 30 | 86. 0 | 22. 8 | 73 | 163. 4 | 121 | 250 | 482 |
| −0. 6 | 31 | 87. 8 | 23. 3 | 74 | 165. 2 | 127 | 260 | 500 |
| 0. 0 | 32 | 89. 6 | 23. 9 | 75 | 167. 0 | 132 | 270 | 518 |
| 0. 6 | 33 | 91. 4 | 24. 4 | 76 | 168. 8 | 138 | 280 | 536 |
| 1. 1 | 34 | 93. 2 | 25. 0 | 77 | 170. 6 | 143 | 290 | 554 |

Formulas: $C = \frac{5}{9}(F-32)$ or $F = \frac{9}{5}C+32$

*Table XVI. Conversion of Barometric Pressure (from Feet to Inches and Millimeters of Mercury)*

$$P=29.9212\left(\frac{288-0.00198h}{288}\right)^{5.256}$$

| Altitude (h) (feet) | Barometric pressure (P) | | Altitude (h) (feet) | Barometric pressure (P) | | Altitude (h) (feet) | Barometric pressure (P) | |
|---|---|---|---|---|---|---|---|---|
| | in. Hg | mm Hg | | in. Hg | mm Hg | | in. Hg | mm Hg |
| −1000 | 31. 02 | 787. 87 | 3500 | 26. 33 | 668. 68 | 8000 | 22. 22 | 564. 52 |
| −900 | 30. 91 | 785. 04 | 3600 | 26. 23 | 666. 20 | 8100 | 22. 14 | 562. 37 |
| −800 | 30. 80 | 782. 23 | 3700 | 26. 13 | 663. 74 | 8200 | 22. 06 | 560. 22 |
| −700 | 30. 69 | 779. 42 | 3800 | 26. 04 | 661. 28 | 8300 | 21. 97 | 558. 08 |
| −600 | 30. 58 | 776. 62 | 3900 | 25. 94 | 658. 83 | 8400 | 21. 89 | 555. 94 |
| −500 | 30. 47 | 773. 83 | 4000 | 25. 84 | 656. 39 | 8500 | 21. 80 | 553. 82 |
| −400 | 30. 36 | 771. 05 | 4100 | 25. 75 | 653. 95 | 8600 | 21. 72 | 551. 69 |
| −300 | 30. 25 | 768. 28 | 4200 | 25. 65 | 651. 53 | 8700 | 21. 64 | 549. 58 |
| −200 | 30. 14 | 765. 51 | 4300 | 25. 56 | 649. 10 | 8800 | 21. 55 | 547. 47 |
| −100 | 30. 03 | 762. 75 | 4400 | 25. 46 | 646. 69 | 8900 | 21. 47 | 545. 37 |
| 0 | 29. 92 | 760. 00 | 4500 | 25. 37 | 644. 29 | 9000 | 21. 39 | 543. 27 |
| 100 | 29. 81 | 757. 26 | 4600 | 25. 27 | 641. 89 | 9100 | 21. 31 | 541. 18 |
| 200 | 29. 71 | 754. 52 | 4700 | 25. 18 | 639. 50 | 9200 | 21. 22 | 539. 10 |
| 300 | 29. 60 | 751. 80 | 4800 | 25. 08 | 637. 11 | 9300 | 21. 14 | 537. 02 |
| 400 | 29. 49 | 749. 08 | 4900 | 24. 99 | 634. 73 | 9400 | 21. 06 | 534. 95 |
| 500 | 29. 38 | 746. 37 | 5000 | 24. 90 | 632. 36 | 9500 | 20. 98 | 532. 89 |
| 600 | 29. 28 | 743. 67 | 5100 | 24. 80 | 630. 00 | 9600 | 20. 90 | 530. 83 |
| 700 | 29. 17 | 740. 97 | 5200 | 24. 71 | 627. 64 | 9700 | 20. 82 | 528. 78 |
| 800 | 29. 07 | 738. 29 | 5300 | 24. 62 | 625. 30 | 9800 | 20. 74 | 526. 74 |
| 900 | 28. 96 | 735. 61 | 5400 | 24. 53 | 622. 96 | 9900 | 20. 66 | 524. 70 |
| 1000 | 28. 86 | 732. 94 | 5500 | 24. 43 | 620. 62 | 10000 | 20. 58 | 522. 67 |
| 1100 | 28. 75 | 730. 27 | 5600 | 24. 34 | 618. 29 | 10100 | 20. 50 | 520. 64 |
| 1200 | 28. 65 | 727. 62 | 5700 | 24. 25 | 615. 97 | 10200 | 20. 42 | 518. 62 |
| 1300 | 28. 54 | 724. 97 | 5800 | 24. 16 | 613. 66 | 10300 | 20. 34 | 516. 61 |
| 1400 | 28. 44 | 722. 33 | 5900 | 24. 07 | 611. 36 | 10400 | 20. 26 | 514. 61 |
| 1500 | 28. 34 | 719. 70 | 6000 | 23. 98 | 609. 06 | 10500 | 20. 18 | 512. 61 |
| 1600 | 28. 23 | 717. 08 | 6100 | 23. 89 | 606. 76 | 10600 | 20. 10 | 510. 61 |
| 1700 | 28. 13 | 714. 46 | 6200 | 23. 80 | 604. 48 | 10700 | 20. 02 | 508. 63 |
| 1800 | 28. 03 | 711. 85 | 6300 | 23. 71 | 602. 20 | 10800 | 19. 95 | 506. 65 |
| 1900 | 27. 92 | 709. 25 | 6400 | 23. 62 | 599. 93 | 10900 | 19. 87 | 504. 67 |
| 2000 | 27. 82 | 706. 66 | 6500 | 23. 53 | 597. 66 | 11000 | 19. 79 | 502. 70 |
| 2100 | 27. 72 | 704. 07 | 6600 | 23. 44 | 595. 41 | 11100 | 19. 71 | 500. 74 |
| 2200 | 27. 62 | 701. 50 | 6700 | 23. 35 | 593. 16 | 11200 | 19. 64 | 498. 78 |
| 2300 | 27. 52 | 698. 92 | 6800 | 23. 26 | 590. 91 | 11300 | 19. 56 | 496. 84 |
| 2400 | 27. 42 | 696. 36 | 6900 | 23. 18 | 588. 68 | 11400 | 19. 48 | 494. 89 |
| 2500 | 27. 32 | 693. 81 | 7000 | 23. 09 | 586. 45 | 11500 | 19. 41 | 492. 96 |
| 2600 | 27. 22 | 691. 26 | 7100 | 23. 00 | 584. 22 | 11600 | 19. 33 | 491. 02 |
| 2700 | 27. 12 | 688. 72 | 7200 | 22. 91 | 582. 01 | 11700 | 19. 26 | 489. 10 |
| 2800 | 27. 02 | 686. 19 | 7300 | 22. 83 | 579. 80 | 11800 | 19. 18 | 487. 18 |
| 2900 | 26. 92 | 683. 66 | 7400 | 22. 74 | 577. 60 | 11900 | 19. 10 | 485. 27 |
| 3000 | 26. 82 | 681. 15 | 7500 | 22. 65 | 575. 40 | 12000 | 19. 03 | 483. 36 |
| 3100 | 26. 72 | 678. 64 | 7600 | 22. 57 | 573. 21 | 12100 | 18. 96 | 481. 46 |
| 3200 | 26. 62 | 676. 14 | 7700 | 22. 48 | 571. 03 | 12200 | 18. 88 | 479. 56 |
| 3300 | 26. 52 | 673. 64 | 7800 | 22. 40 | 568. 86 | 12300 | 18. 81 | 477. 68 |
| 3400 | 26. 42 | 671. 16 | 7900 | 22. 31 | 566. 69 | 12400 | 18. 73 | 475. 79 |

*Table XVI. Conversion of Barometric Pressure (from Feet to Inches and Millimeters of Mercury)*—Continued

$$P=29.9212\left(\frac{288-0.00198h}{288}\right)^{5.256}$$

| Altitude (h) (feet) | Barometric pressure (P) | | Altitude (h) (feet) | Barometric pressure (P) | | Altitude (h) (feet) | Barometric pressure (P) | |
|---|---|---|---|---|---|---|---|---|
| | in. Hg | mm Hg | | in. Hg | mm Hg | | in. Hg | mm Hg |
| 12500 | 18. 66 | 473. 92 | 15000 | 16. 89 | 428. 92 | 17500 | 15. 25 | 387. 44 |
| 12600 | 18. 58 | 472. 05 | 15100 | 16. 82 | 427. 19 | 17600 | 15. 19 | 385. 85 |
| 12700 | 18. 51 | 470. 18 | 15200 | 16. 75 | 425. 47 | 17700 | 15. 13 | 384. 27 |
| 12800 | 18. 44 | 468. 32 | 15300 | 16. 68 | 423. 76 | 17800 | 15. 07 | 382. 69 |
| 12900 | 18. 36 | 466. 47 | 15400 | 16. 62 | 422. 05 | 17900 | 15. 00 | 381. 12 |
| 13000 | 18. 29 | 464. 62 | 15500 | 16. 55 | 420. 35 | 18000 | 14. 94 | 379. 55 |
| 13100 | 18. 22 | 462. 78 | 15600 | 16. 48 | 418. 65 | 18100 | 14. 88 | 377. 98 |
| 13200 | 18. 15 | 460. 95 | 15700 | 16. 42 | 416. 96 | 18200 | 14. 82 | 376. 43 |
| 13300 | 18. 08 | 459. 12 | 15800 | 16. 35 | 415. 27 | 18300 | 14. 76 | 374. 88 |
| 13400 | 18. 00 | 457. 30 | 15900 | 16. 28 | 413. 59 | 18400 | 14. 70 | 373. 33 |
| 13500 | 17. 93 | 455. 48 | 16000 | 16. 22 | 411. 92 | 18500 | 14. 64 | 371. 79 |
| 13600 | 17. 86 | 453. 67 | 16100 | 16. 15 | 410. 24 | 18600 | 14. 58 | 370. 25 |
| 13700 | 17. 79 | 451. 86 | 16200 | 16. 09 | 408. 58 | 18700 | 14. 52 | 368. 72 |
| 13800 | 17. 72 | 450. 06 | 16300 | 16. 02 | 406. 92 | 18800 | 14. 46 | 367. 19 |
| 13900 | 17. 65 | 448. 27 | 16400 | 15. 96 | 405. 27 | 18900 | 14. 40 | 365. 67 |
| 14000 | 17. 58 | 446. 48 | 16500 | 15. 89 | 403. 62 | 19000 | 14. 34 | 364. 16 |
| 14100 | 17. 51 | 444. 70 | 16600 | 15. 83 | 401. 98 | 19100 | 14. 28 | 362. 64 |
| 14200 | 17. 44 | 442. 92 | 16700 | 15. 76 | 400. 34 | 19200 | 14. 22 | 361. 14 |
| 14300 | 17. 37 | 441. 15 | 16800 | 15. 70 | 398. 71 | 19300 | 14. 16 | 359. 64 |
| 14400 | 17. 30 | 439. 39 | 16900 | 15. 63 | 397. 08 | 19400 | 14. 10 | 358. 14 |
| 14500 | 17. 23 | 437. 63 | 17000 | 15. 57 | 395. 46 | 19500 | 14. 04 | 356. 65 |
| 14600 | 17. 16 | 435. 87 | 17100 | 15. 51 | 393. 85 | 19600 | 13. 98 | 355. 16 |
| 14700 | 17. 09 | 434. 13 | 17200 | 15. 44 | 392. 24 | 19700 | 13. 92 | 353. 68 |
| 14800 | 17. 02 | 432. 38 | 17300 | 15. 38 | 390. 63 | 19800 | 13. 87 | 352. 21 |
| 14900 | 16. 96 | 430. 65 | 17400 | 15. 32 | 389. 03 | 19900 | 13. 81 | 350. 74 |
| | | | | | | 20000 | 13. 75 | 349. 27 |

*Table XVII. Refractive Index for Group Velocity ($N_g$)*

[Values of the Wavelength of Light (λ) in Microns]

$$N_g = 1 + \left[2876.4 + 3\left(\frac{16.288}{\lambda^2}\right) + 5\left(\frac{0.136}{\lambda^4}\right)\right] 10^{-7}$$

| λ | Ng | λ | Ng | λ | Ng | λ | Ng | λ | Ng | λ | Ng | λ | Ng | λ | Ng |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0. 450 | 1. 00031342 | 0. 475 | 1. 00031063 | 0. 500 | 1. 00030827 | 0. 525 | 1. 00030626 | 0. 550 | 1. 00030453 | 0. 575 | 1. 00030304 | 0. 600 | 1. 00030173 | 0. 625 | 1. 00030059 |
| . 451 | 1. 00031330 | . 476 | 1. 00031053 | . 501 | 1. 00030818 | . 526 | 1. 00030618 | . 551 | 1. 00030447 | . 576 | 1. 00030298 | . 601 | 1. 00030168 | . 626 | 1. 00030055 |
| . 452 | 1. 00031318 | . 477 | 1. 00031042 | . 502 | 1. 00030810 | . 527 | 1. 00030611 | . 552 | 1. 00030440 | . 577 | 1. 00030293 | . 602 | 1. 00030164 | . 627 | 1. 00030050 |
| . 453 | 1. 00031306 | . 478 | 1. 00031032 | . 503 | 1. 00030801 | . 528 | 1. 00030604 | . 553 | 1. 00030434 | . 578 | 1. 00030287 | . 603 | 1. 00030159 | . 628 | 1. 00030046 |
| . 454 | 1. 00031294 | . 479 | 1. 00031022 | . 504 | 1. 00030793 | . 529 | 1. 00030596 | . 554 | 1. 00030428 | . 579 | 1. 00030282 | . 604 | 1. 00030154 | . 629 | 1. 00030042 |
| . 455 | 1. 00031282 | . 480 | 1. 00031012 | . 505 | 1. 00030784 | . 530 | 1. 00030589 | . 555 | 1. 00030422 | . 580 | 1. 00030276 | . 605 | 1. 00030149 | . 630 | 1. 00030038 |
| . 456 | 1. 00031271 | . 481 | 1. 00031003 | . 506 | 1. 00030776 | . 531 | 1. 00030582 | . 556 | 1. 00030415 | . 581 | 1. 00030271 | . 606 | 1. 00030145 | . 631 | 1. 00030034 |
| . 457 | 1. 00031259 | . 482 | 1. 00030993 | . 507 | 1. 00030767 | . 532 | 1. 00030575 | . 557 | 1. 00030409 | . 582 | 1. 00030265 | . 607 | 1. 00030140 | . 632 | 1. 00030029 |
| . 458 | 1. 00031248 | . 483 | 1. 00030983 | . 508 | 1. 00030759 | . 533 | 1. 00030568 | . 558 | 1. 00030403 | . 583 | 1. 00030260 | . 608 | 1. 00030135 | . 633 | 1. 00030025 |
| . 459 | 1. 00031236 | . 484 | 1. 00030973 | . 509 | 1. 00030751 | . 534 | 1. 00030561 | . 559 | 1. 00030397 | . 584 | 1. 00030255 | . 609 | 1. 00030130 | . 634 | 1. 00030021 |
| . 460 | 1. 00031225 | . 485 | 1. 00030964 | . 510 | 1. 00030743 | . 535 | 1. 00030554 | . 560 | 1. 00030391 | . 585 | 1. 00030249 | . 610 | 1. 00030126 | . 635 | 1. 00030017 |
| . 461 | 1. 00031213 | . 486 | 1. 00030954 | . 511 | 1. 00030735 | . 536 | 1. 00030547 | . 561 | 1. 00030385 | . 586 | 1. 00030244 | . 611 | 1. 00030121 | . 636 | 1. 00030013 |
| . 462 | 1. 00031202 | . 487 | 1. 00030945 | . 512 | 1. 00030726 | . 537 | 1. 00030540 | . 562 | 1. 00030379 | . 587 | 1. 00030239 | . 612 | 1. 00030117 | . 637 | 1. 00030009 |
| . 463 | 1. 00031191 | . 488 | 1. 00030935 | . 513 | 1. 00030718 | . 538 | 1. 00030533 | . 563 | 1. 00030373 | . 588 | 1. 00030234 | . 613 | 1. 00030112 | . 638 | 1. 00030005 |
| . 464 | 1. 00031180 | . 489 | 1. 00030926 | . 514 | 1. 00030710 | . 539 | 1. 00030526 | . 564 | 1. 00030367 | . 589 | 1. 00030229 | . 614 | 1. 00030107 | . 639 | 1. 00030001 |
| . 465 | 1. 00031169 | . 490 | 1. 00030917 | . 515 | 1. 00030703 | . 540 | 1. 00030519 | . 565 | 1. 00030361 | . 590 | 1. 00030223 | . 615 | 1. 00030103 | . 640 | 1. 00029997 |
| . 466 | 1. 00031158 | . 491 | 1. 00030907 | . 516 | 1. 00030695 | . 541 | 1. 00030512 | . 566 | 1. 00030355 | . 591 | 1. 00030218 | . 616 | 1. 00030098 | . 641 | 1. 00029993 |
| . 467 | 1. 00031147 | . 492 | 1. 00030898 | . 517 | 1. 00030687 | . 542 | 1. 00030506 | . 567 | 1. 00030349 | . 592 | 1. 00030213 | . 617 | 1. 00030094 | . 642 | 1. 00029989 |
| . 468 | 1. 00031136 | . 493 | 1. 00030889 | . 518 | 1. 00030679 | . 543 | 1. 00030499 | . 568 | 1. 00030343 | . 593 | 1. 00030208 | . 618 | 1. 00030090 | . 643 | 1. 00029985 |
| . 469 | 1. 00031126 | . 494 | 1. 00030880 | . 519 | 1. 00030671 | . 544 | 1. 00030492 | . 569 | 1. 00030338 | . 594 | 1. 00030203 | . 619 | 1. 00030085 | . 644 | 1. 00029981 |
| . 470 | 1. 00031115 | . 495 | 1. 00030871 | . 520 | 1. 00030664 | . 545 | 1. 00030486 | . 570 | 1. 00030332 | . 595 | 1. 00030198 | . 620 | 1. 00030081 | . 645 | 1. 00029977 |
| . 471 | 1. 00031104 | . 496 | 1. 00030862 | . 521 | 1. 00030656 | . 546 | 1. 00030479 | . 571 | 1. 00030326 | . 596 | 1. 00030193 | . 621 | 1. 00030076 | . 646 | 1. 00029973 |
| . 472 | 1. 00031094 | . 497 | 1. 00030853 | . 522 | 1. 00030648 | . 547 | 1. 00030473 | . 572 | 1. 00030320 | . 597 | 1. 00030188 | . 622 | 1. 00030072 | . 647 | 1. 00029970 |
| . 473 | 1. 00031083 | . 498 | 1. 00030844 | . 523 | 1. 00030641 | . 548 | 1. 00030466 | . 573 | 1. 00030315 | . 598 | 1. 00030183 | . 623 | 1. 00030068 | . 648 | 1. 00029966 |
| . 474 | 1. 00031073 | . 499 | 1. 00030836 | . 524 | 1. 00030633 | . 549 | 1. 00030460 | . 574 | 1. 00030309 | . 599 | 1. 00030178 | . 624 | 1. 00030063 | . 649 | 1. 00029962 |

*Table XVIII. Factors for Computing the Refractive Index of Radio Waves*

| Dry bulb $t$ (°F) | A | B | Dry bulb $t$ (°F) | A | B | Dry bulb $t$ (°F) | A | B |
|---|---|---|---|---|---|---|---|---|
| 0 | 191. 158 | 10. 290 | 40 | 161. 779 | 9. 466 | 80 | 138. 686 | 8. 764 |
| 1 | 190. 329 | 10. 267 | 41 | 161. 133 | 9. 447 | 81 | 138. 174 | 8. 748 |
| 2 | 189. 506 | 10. 245 | 42 | 160. 491 | 9. 428 | 82 | 137. 664 | 8. 732 |
| 3 | 188. 687 | 10. 223 | 43 | 159. 853 | 9. 409 | 83 | 137. 157 | 8. 716 |
| 4 | 187. 874 | 10. 201 | 44 | 159. 219 | 9. 391 | 84 | 136. 653 | 8. 700 |
| 5 | 187. 067 | 10. 179 | 45 | 158. 589 | 9. 372 | 85 | 136. 152 | 8. 684 |
| 6 | 186. 264 | 10. 157 | 46 | 157. 962 | 9. 354 | 86 | 135. 653 | 8. 668 |
| 7 | 185. 467 | 10. 135 | 47 | 157. 340 | 9. 335 | 87 | 135. 157 | 8. 652 |
| 8 | 184. 674 | 10. 114 | 48 | 156. 720 | 9. 317 | 88 | 134. 664 | 8. 636 |
| 9 | 183. 887 | 10. 092 | 49 | 156. 105 | 9. 298 | 89 | 134. 174 | 8. 621 |
| 10 | 183. 105 | 10. 071 | 50 | 155. 493 | 9. 280 | 90 | 133. 686 | 8. 605 |
| 11 | 182. 328 | 10. 049 | 51 | 154. 884 | 9. 262 | 91 | 133. 201 | 8. 589 |
| 12 | 181. 555 | 10. 028 | 52 | 154. 280 | 9. 244 | 92 | 132. 719 | 8. 574 |
| 13 | 180. 788 | 10. 007 | 53 | 153. 678 | 9. 226 | 93 | 132. 239 | 8. 558 |
| 14 | 180. 026 | 9. 985 | 54 | 153. 081 | 9. 208 | 94 | 131. 762 | 8. 543 |
| 15 | 179. 268 | 9. 964 | 55 | 152. 486 | 9. 190 | 95 | 131. 287 | 8. 527 |
| 16 | 178. 515 | 9. 943 | 56 | 151. 896 | 9. 172 | 96 | 130. 815 | 8. 512 |
| 17 | 177. 767 | 9. 923 | 57 | 151. 308 | 9. 154 | 97 | 130. 345 | 8. 497 |
| 18 | 177. 023 | 9. 902 | 58 | 150. 724 | 9. 137 | 98 | 129. 878 | 8. 481 |
| 19 | 176. 284 | 9. 881 | 59 | 150. 144 | 9. 119 | 99 | 129. 414 | 8. 466 |
| 20 | 175. 550 | 9. 861 | 60 | 149. 566 | 9. 102 | 100 | 128. 952 | 8. 451 |
| 21 | 174. 821 | 9. 840 | 61 | 148. 992 | 9. 084 | 101 | 128. 492 | 8. 436 |
| 22 | 174. 095 | 9. 820 | 62 | 148. 422 | 9. 067 | 102 | 128. 035 | 8. 421 |
| 23 | 173. 375 | 9. 799 | 63 | 147. 854 | 9. 049 | 103 | 127. 580 | 8. 406 |
| 24 | 172. 659 | 9. 779 | 64 | 147. 290 | 9. 032 | 104 | 127. 128 | 8. 391 |
| 25 | 171. 947 | 9. 759 | 65 | 146. 729 | 9. 015 | 105 | 126. 678 | 8. 376 |
| 26 | 171. 240 | 9. 739 | 66 | 146. 172 | 8. 998 | 106 | 126. 231 | 8. 361 |
| 27 | 170. 537 | 9. 719 | 67 | 145. 617 | 8. 981 | 107 | 125. 786 | 8. 347 |
| 28 | 169. 838 | 9. 699 | 68 | 145. 066 | 8. 964 | 108 | 125. 343 | 8. 332 |
| 29 | 169. 144 | 9. 679 | 69 | 144. 517 | 8. 947 | 109 | 124. 902 | 8. 317 |
| 30 | 168. 454 | 9. 659 | 70 | 143. 972 | 8. 930 | 110 | 124. 464 | 8. 303 |
| 31 | 167. 768 | 9. 640 | 71 | 143. 430 | 8. 913 | 111 | 124. 029 | 8. 288 |
| 32 | 167. 086 | 9. 620 | 72 | 142. 891 | 8. 896 | 112 | 123. 595 | 8. 274 |
| 33 | 166. 408 | 9. 600 | 73 | 142. 355 | 8. 879 | 113 | 123. 164 | 8. 259 |
| 34 | 165. 735 | 9. 581 | 74 | 141. 822 | 8. 863 | 114 | 122. 735 | 8. 245 |
| 35 | 165. 065 | 9. 562 | 75 | 141. 292 | 8. 846 | 115 | 122. 308 | 8. 231 |
| 36 | 164. 400 | 9. 542 | 76 | 140. 765 | 8. 830 | 116 | 121. 883 | 8. 216 |
| 37 | 163. 739 | 9. 523 | 77 | 140. 241 | 8. 813 | 117 | 121. 461 | 8. 202 |
| 38 | 163. 081 | 9. 504 | 78 | 139. 720 | 8. 797 | 118 | 121. 041 | 8. 188 |
| 39 | 162. 428 | 9. 485 | 79 | 139. 202 | 8. 781 | 119 | 120. 623 | 8. 174 |

$$A = \frac{8540}{459.688+t} \times \frac{4730}{459.688+t} \qquad B = \frac{4730}{459.688+t}$$

*Table XIX. Saturation Vapor Pressure ($e'$ and $\Delta e$) for Reduction of Psychrometric Observations (Values in Inches of Mercury)*

($\Delta e$ tabulated for P=1 inch of Hg. and $t-t'=1°$ F.)

To obtain total $\Delta e$ multiply $\Delta e$ by $P(t-t')$

| Wet bulb $t'$ °F | $e'$ (in. Hg) | $\Delta e$ per °F per inch Hg | Wet bulb $t'$ °F | $e'$ (in. Hg) | $\Delta e$ per °F per inch Hg | Wet bulb $t'$ °F | $e'$ (in. Hg) | $\Delta e$ per °F per inch Hg |
|---|---|---|---|---|---|---|---|---|
| 0 | 0. 0448 | 0. 000360 | 40 | 0. 2477 | 0. 000369 | 80 | 1. 0323 | 0. 000378 |
| 1 | . 0469 | | 41 | . 2575 | | 81 | 1. 0665 | |
| 2 | . 0492 | | 42 | . 2676 | | 82 | 1. 1017 | . 000379 |
| 3 | . 0515 | | 43 | . 2781 | . 000370 | 83 | 1. 1380 | |
| 4 | . 0539 | | 44 | . 2890 | | 84 | 1. 1752 | |
| 5 | . 0565 | . 000361 | 45 | . 3002 | | 85 | 1. 2136 | |
| 6 | . 0591 | | 46 | . 3118 | | 86 | 1. 2530 | . 000380 |
| 7 | . 0618 | | 47 | . 3239 | . 000371 | 87 | 1. 2935 | |
| 8 | . 0647 | | 48 | . 3363 | | 88 | 1. 3351 | |
| 9 | . 0677 | . 000362 | 49 | . 3491 | | 89 | 1. 3779 | |
| 10 | . 0708 | | 50 | . 3624 | | 90 | 1. 4219 | . 000381 |
| 11 | . 0740 | | 51 | . 3761 | | 91 | 1. 4671 | |
| 12 | . 0774 | | 52 | . 3903 | . 000372 | 92 | 1. 5136 | |
| 13 | . 0809 | . 000363 | 53 | . 4049 | | 93 | 1. 5613 | |
| 14 | . 0845 | | 54 | . 4200 | | 94 | 1. 6103 | |
| 15 | . 0883 | | 55 | . 4356 | | 95 | 1. 6607 | . 000382 |
| 16 | . 0923 | | 56 | . 4518 | . 000373 | 96 | 1. 7124 | |
| 17 | . 0963 | | 57 | . 4684 | | 97 | 1. 7655 | |
| 18 | . 1006 | . 000364 | 58 | . 4856 | | 98 | 1. 8200 | |
| 19 | . 1050 | | 59 | . 5033 | | 99 | 1. 8759 | . 000383 |
| 20 | . 1096 | | 60 | . 5216 | . 000374 | 100 | 1. 9334 | |
| 21 | . 1144 | | 61 | . 5405 | | 101 | 1. 9923 | |
| 22 | . 1193 | . 000365 | 62 | . 5599 | | 102 | 2. 0529 | |
| 23 | . 1245 | | 63 | . 5800 | | 103 | 2. 1149 | . 000384 |
| 24 | . 1298 | | 64 | . 6007 | | 104 | 2. 1786 | |
| 25 | . 1353 | | 65 | . 6221 | . 000375 | 105 | 2. 2440 | |
| 26 | . 1410 | . 000366 | 66 | . 6441 | | 106 | 2. 3110 | |
| 27 | . 1470 | | 67 | . 6668 | | 107 | 2. 3798 | . 000385 |
| 28 | . 1532 | | 68 | . 6902 | | 108 | 2. 4503 | |
| 29 | . 1597 | | 69 | . 7143 | . 000376 | 109 | 2. 5226 | |
| 30 | . 1663 | . 000367 | 70 | . 7392 | | 110 | 2. 5968 | |
| 31 | . 1732 | | 71 | . 7648 | | 111 | 2. 6728 | |
| 32 | . 1804 | | 72 | . 7911 | | 112 | 2. 7507 | . 000386 |
| 33 | . 1878 | | 73 | . 8183 | . 000377 | 113 | 2. 8306 | |
| 34 | . 1955 | | 74 | . 8463 | | 114 | 2. 9125 | |
| 35 | . 2034 | . 000368 | 75 | . 8751 | | 115 | 2. 9963 | |
| 36 | . 2117 | | 76 | . 9047 | | 116 | 3. 0823 | . 000387 |
| 37 | . 2202 | | 77 | . 9352 | . 000378 | 117 | 3. 1703 | |
| 38 | . 2290 | | 78 | . 9667 | | 118 | 3. 2606 | |
| 39 | . 2382 | . 000369 | 79 | . 9990 | . 000378 | 119 | 3. 3530 | . 000387 |

*Table XX. Radii of Curvature of Earth's Surface ($\rho$) (in Meters)*

(Based upon Clarke's Spheroid of 1866 as Expressed in Meters)

| Azimuth (degrees) | Latitude | | | | | |
|---|---|---|---|---|---|---|
| | 0° | 1° | 2° | 3° | 4° | 5° |
| 0 | 6335034. 50223 | 6335054. 09314 | 6335112. 84261 | 6335210. 68086 | 6335347. 49171 | 6335523. 11268 |
| 5 | 6335360. 23833 | 6335379. 73170 | 6335438. 18865 | 6335535. 53976 | 6335671. 66940 | 6335846. 41588 |
| 10 | 6336327. 74680 | 6336346. 95038 | 6336404. 53833 | 6336500. 44221 | 6336634. 54809 | 6336806. 69660 |
| 15 | 6337908. 19923 | 6337926. 92926 | 6337983. 09709 | 6338076. 63594 | 6338207. 43461 | 6338375. 33756 |
| 20 | 6340054. 44686 | 6340072. 53342 | 6340126. 77160 | 6340217. 09683 | 6340343. 40168 | 6340505. 53581 |
| 25 | 6342702. 34890 | 6342719. 64100 | 6342771. 49666 | 6342857. 85415 | 6342978. 61060 | 6343133. 62215 |
| 30 | 6345772. 59332 | 6345788. 96338 | 6345838. 05402 | 6345919. 80672 | 6346034. 12399 | 6346180. 86950 |
| 35 | 6349172. 96953 | 6349188. 31736 | 6349234. 34253 | 6349310. 99008 | 6349418. 16850 | 6349555. 74984 |
| 40 | 6352801. 04061 | 6352815. 29654 | 6352858. 04731 | 6352929. 24180 | 6353028. 79491 | 6353156. 58765 |
| 45 | 6356547. 14906 | 6356560. 27626 | 6356599. 64215 | 6356665. 19961 | 6356756. 87019 | 6356874. 54418 |
| 50 | 6360297. 67814 | 6360309. 67392 | 6360345. 64691 | 6360405. 55401 | 6360489. 32345 | 6360596. 85486 |
| 55 | 6363938. 47784 | 6363949. 37403 | 6363982. 04955 | 6364036. 46521 | 6364112. 55575 | 6364210. 22996 |
| 60 | 6367358. 35551 | 6367368. 21768 | 6367397. 79235 | 6367447. 04405 | 6367515. 91368 | 6367604. 31859 |
| 65 | 6370452. 52208 | 6370461. 44775 | 6370488. 21403 | 6370532. 78880 | 6370595. 11855 | 6370675. 12846 |
| 70 | 6373125. 88166 | 6373133. 99745 | 6373158. 33507 | 6373198. 86531 | 6373255. 53951 | 6373328. 28963 |
| 75 | 6375296. 05163 | 6375303. 50948 | 6375325. 87407 | 6375363. 11856 | 6375415. 19824 | 6375482. 05059 |
| 80 | 6376896. 00581 | 6376902. 97831 | 6376923. 88743 | 6376958. 70806 | 6377007. 39845 | 6377069. 90013 |
| 85 | 6377876. 24318 | 6377882. 91819 | 6377902. 93522 | 6377936. 27026 | 6377982. 88330 | 6378042. 71839 |
| 90 | 6378206. 40000 | 6378212. 97479 | 6378232. 69130 | 6378265. 52586 | 6378311. 43908 | 6378370. 37585 |
| | 6° | 7° | 8° | 9° | 10° | 11° |
| 0 | 6335737. 33515 | 6335989. 90465 | 6336280. 52105 | 6336608. 83896 | 6336974. 46809 | 6337376. 97367 |
| 5 | 6336059. 57160 | 6336310. 88330 | 6336600. 05231 | 6336926. 73496 | 6337290. 54281 | 6337691. 04321 |
| 10 | 6337016. 68315 | 6337264. 25817 | 6337549. 12733 | 6337870. 95189 | 6338229. 34909 | 6338623. 89255 |
| 15 | 6338580. 14515 | 6338821. 61378 | 6339099. 45621 | 6339413. 34188 | 6339762. 89722 | 6340147. 70615 |
| 20 | 6340703. 30626 | 6340936. 47764 | 6341204. 77235 | 6341507. 87090 | 6341845. 41231 | 6342216. 99445 |
| 25 | 6343322. 70415 | 6343545. 63131 | 6343802. 13795 | 6344091. 91835 | 6344414. 62703 | 6344769. 87916 |
| 30 | 6346359. 86822 | 6346570. 90665 | 6346813. 73300 | 6347088. 05755 | 6347393. 55287 | 6347729. 85428 |
| 35 | 6349723. 56980 | 6349921. 42795 | 6350149. 08798 | 6350406. 27791 | 6350692. 69043 | 6351007. 98324 |
| 40 | 6353312. 46721 | 6353496. 24722 | 6353707. 70791 | 6353946. 59635 | 6354212. 62675 | 6354505. 48083 |
| 45 | 6357018. 08072 | 6357187. 30800 | 6357382. 02339 | 6357601. 99375 | 6357846. 95565 | 6358116. 61565 |
| 50 | 6360728. 01940 | 6360882. 65989 | 6361060. 59100 | 6361261. 59946 | 6361485. 44428 | 6361731 85706 |
| 55 | 6364329. 37067 | 6364469. 83501 | 6364631. 45448 | 6364814. 03519 | 6365017. 35808 | 6365241. 17915 |
| 60 | 6367712. 15270 | 6367839. 28660 | 6367985. 56769 | 6368150. 82038 | 6368334. 84625 | 6368537. 42434 |
| 65 | 6370772. 72250 | 6370887. 78346 | 6371020. 17322 | 6371169. 73278 | 6371336. 28252 | 6371519. 62237 |
| 70 | 6373417. 02833 | 6373521. 64904 | 6373642. 02615 | 6373778. 01505 | 6373929. 45241 | 6374096. 15630 |
| 75 | 6375563. 59535 | 6375659. 73461 | 6375770. 35293 | 6375895. 31745 | 6376034. 47810 | 6376187. 66768 |
| 80 | 6377146. 13808 | 6377236. 02078 | 6377339. 44033 | 6377456. 27253 | 6377586. 37710 | 6377729. 59776 |
| 85 | 6378115. 70375 | 6378201. 75175 | 6378300. 75912 | 6378412. 60700 | 6378537. 16110 | 6378674. 27182 |
| 90 | 6378442. 26546 | 6378527. 02163 | 6378624. 54263 | 6378734. 71140 | 6378857. 39565 | 6378992. 44808 |

*Table XX. Radii of Curvature of Earth's Surface ($\rho$) (in Meters)*—Continued

(Based upon Clarke's Spheroid of 1866 as Expressed in Meters)

| Azimuth (degrees) | Latitude | | | | | |
|---|---|---|---|---|---|---|
| | 12° | 13° | 14° | 15° | 16° | 17° |
| 0 | 6337815. 87693 | 6338290. 65562 | 6338800. 74463 | 6339345. 53656 | 6339924. 38239 | 6340536. 59226 |
| 5 | 6338127. 75974 | 6338600. 17267 | 6339107. 71964 | 6339649. 79620 | 6340225. 75651 | 6340834. 91408 |
| 10 | 6339054. 11275 | 6339519. 49753 | 6340019. 49268 | 6340553. 50256 | 6341120. 89072 | 6341720. 98065 |
| 15 | 6340567. 31045 | 6341021. 21031 | 6341508. 86490 | 6342029. 69295 | 6342583. 07338 | 6343168. 34602 |
| 20 | 6342622. 17451 | 6343060. 46951 | 6343531. 35680 | 6344034. 27466 | 6344568. 62295 | 6345133. 76375 |
| 25 | 6345157. 25103 | 6345576. 28041 | 6346026. 46718 | 6346507. 27386 | 6347018. 12615 | 6347558. 41366 |
| 30 | 6348096. 56021 | 6348493. 23267 | 6348919. 39772 | 6349374. 54603 | 6349858. 13342 | 6350369. 58151 |
| 35 | 6351351. 77943 | 6351723. 66791 | 6352123. 20387 | 6352549. 90928 | 6353003. 27343 | 6353482. 75353 |
| 40 | 6354824. 80808 | 6355170. 22625 | 6355541. 32171 | 6355937. 65000 | 6356358. 73623 | 6356807. 07573 |
| 45 | 6358410. 65066 | 6358728. 70831 | 6359070. 40737 | 6359435. 33811 | 6359823. 06285 | 6360233. 11640 |
| 50 | 6362000. 54229 | 6362291. 17765 | 6362603. 41444 | 6362936. 87791 | 6363291. 16775 | 6363665. 85851 |
| 55 | 6365485. 22975 | 6365749. 21689 | 6366032. 82356 | 6366335. 70914 | 6366657. 50973 | 6366997. 83860 |
| 60 | 6368758. 31135 | 6368997. 24193 | 6369253. 92901 | 6369528. 06410 | 6369819. 31765 | 6370127. 33948 |
| 65 | 6371719. 53205 | 6371935. 77134 | 6372168. 08033 | 6372416. 17973 | 6372679. 77120 | 6272958. 53769 |
| 70 | 6374277. 92640 | 6374474 54425 | 6374685. 77349 | 6374911. 36015 | 6375151. 03295 | 6375404. 50355 |
| 75 | 6376354. 70214 | 6376535. 38075 | 6376729. 48631 | 6376936. 78548 | 6377157. 02894 | 6377389. 95178 |
| 80 | 6377885. 76246 | 6378054. 68356 | 6378236. 15805 | 6378429. 96778 | 6378635. 87972 | 6378853. 64621 |
| 85 | 6378823. 77450 | 6378985. 48953 | 6349159. 22258 | 6379344. 76486 | 6379541. 89331 | 6379750. 37086 |
| 90 | 6379139. 70647 | 6349298. 99394 | 6379470. 11911 | 6379652. 87630 | 6379847. 04581 | 6380052. 39420 |
| | 18° | 19° | 20° | 21° | 22° | 23° |
| 0 | 6341181. 43620 | 6341858. 14492 | 6342565. 91069 | 6343303. 88826 | 6344071. 19580 | 6344866. 91585 |
| 5 | 6341476. 54246 | 6342149. 87613 | 6342854. 11133 | 6343588. 40691 | 6344351. 88536 | 6345143. 63372 |
| 10 | 6342353. 05650 | 6343016. 36391 | 6343710. 11081 | 6344433. 46838 | 6345185. 57188 | 6345965. 52173 |
| 15 | 6343784. 81234 | 6344431. 73623 | 6345108. 34481 | 6345813. 82931 | 6346547. 34599 | 6347308. 01708 |
| 20 | 6345729. 02208 | 6346353. 68666 | 6347007. 01070 | 6347688. 21275 | 6348396. 47762 | 6349130. 95722 |
| 25 | 6348127. 49055 | 6348724. 67625 | 6349349. 25625 | 6350000. 48291 | 6350677. 57628 | 6351379. 72503 |
| 30 | 6350908. 27833 | 6351473. 57907 | 6352064. 80675 | 6352681. 25300 | 6353322. 17890 | 6353986. 81582 |
| 35 | 6353987. 77526 | 6354517. 73351 | 6355071. 99303 | 6355649. 88912 | 6356250. 72842 | 6356873. 78973 |
| 40 | 6357273. 13455 | 6357765. 35009 | 6358280. 13174 | 6358816. 86158 | 6359374. 89504 | 6359953. 56170 |
| 45 | 6360665. 00665 | 6361118. 21508 | 6361592. 19738 | 6362086. 38408 | 6362600. 18117 | 6363132. 97084 |
| 50 | 6364060. 50010 | 6364474. 61831 | 6364907. 71538 | 6365359. 27050 | 6365828. 74050 | 6366315. 56041 |
| 55 | 6367356. 28665 | 6367732. 42284 | 6368125. 79473 | 6368535. 92898 | 6368962. 33192 | 6369404. 49006 |
| 60 | 6370451. 75908 | 6370792. 18616 | 6371148. 21100 | 6371519. 40500 | 6371905. 32111 | 6372305. 49441 |
| 65 | 6373252. 14380 | 6373560. 23617 | 6373882. 44390 | 6374218. 37895 | 6374567. 63666 | 6374929. 79612 |
| 70 | 6375671. 46695 | 6375951. 60183 | 6376244. 57090 | 6376550. 02131 | 6376867. 58507 | 6377196. 87944 |
| 75 | 6377635. 27375 | 6377892. 69955 | 6378161. 91930 | 6378442. 60874 | 6378734. 42973 | 6379037. 03060 |
| 80 | 6379083. 00530 | 6379323. 68094 | 6379575. 38345 | 6379837. 80973 | 6380110. 64368 | 6380393. 55653 |
| 85 | 6379969. 94678 | 6380200. 35685 | 6380441. 32375 | 6380692. 55738 | 6380953. 75515 | 6381224. 60238 |
| 90 | 6380268. 67446 | 6380495. 62637 | 6380732. 97681 | 6380980. 44003 | 6381237. 71799 | 6381504. 50075 |

*Table XX. Radii of Curvature of Earth's Surface ($\rho$) (in Meters)*—Continued

(Based upon Clarke's Spheroid of 1866 as Expressed in Meters)

| Azimuth (degrees) | Latitude | | | | | |
|---|---|---|---|---|---|---|
| | 24° | 25° | 26° | 27° | 28° | 29° |
| 0 | 6345690. 09640 | 6346539. 75196 | 6347414. 86468 | 6348314. 38546 | 6349237. 23520 | 6350182. 30601 |
| 5 | 6345962. 70465 | 6346808. 11748 | 6347678. 85935 | 6348573. 88633 | 6349492. 12461 | 6350432. 47178 |
| 10 | 6346772. 38443 | 6347605. 19369 | 6348462. 95149 | 6349344. 62921 | 6350249. 16883 | 6351175. 48410 |
| 15 | 6348094. 93176 | 6348907. 14723 | 6349743. 68973 | 6350603. 55571 | 6351485. 71292 | 6352389. 10163 |
| 20 | 6349890. 77158 | 6350675. 00983 | 6351482. 73126 | 6352312. 96636 | 6353164. 71796 | 6354036. 96236 |
| 25 | 6352106. 08730 | 6352855. 79172 | 6353627. 93843 | 6354421. 60000 | 6355235. 82263 | 6356069. 62713 |
| 30 | 6354674. 36624 | 6355384. 00471 | 6356114. 87880 | 6356866. 11005 | 6357636. 79500 | 6358426. 00625 |
| 35 | 6357518. 32478 | 6358183. 55913 | 6358868. 69304 | 6359572. 90240 | 6360295. 33969 | 6361035. 13495 |
| 40 | 6360552. 16600 | 6361169. 98807 | 6361806. 28460 | 6362460. 28961 | 6363131. 21544 | 6363818. 25356 |
| 45 | 6363684. 11215 | 6364252. 94178 | 6364838. 77482 | 6365440. 90554 | 6366058. 60821 | 6366691. 13797 |
| 50 | 6366819. 14417 | 6367338. 88525 | 6367874. 15743 | 6368424. 31542 | 6368988. 69575 | 6369566. 61742 |
| 55 | 6369861. 87080 | 6370333. 92290 | 6370820. 07726 | 6371319. 74753 | 6371832. 33076 | 6372357. 20817 |
| 60 | 6372719. 44261 | 6373146. 66664 | 6373586. 65120 | 6374038. 86541 | 6374502. 76338 | 6374977. 78487 |
| 65 | 6375304. 42075 | 6375691. 05875 | 6376089. 24369 | 6376498. 49497 | 6376918. 31846 | 6377348. 20705 |
| 70 | 6377537. 50741 | 6377889. 05816 | 6378251. 10753 | 6378623. 21850 | 6379004. 94176 | 6379395. 81618 |
| 75 | 6379350. 04651 | 6379673. 10000 | 6380005. 80125 | 6380347. 74875 | 6380698. 52960 | 6381057. 72005 |
| 80 | 6380686. 20726 | 6380988. 24298 | 6381299. 29932 | 6381619. 00090 | 6381946. 96172 | 6382282. 78567 |
| 85 | 6381504. 77260 | 6381793. 92802 | 6382091. 71986 | 6382397. 78878 | 6382711. 76531 | 6383033. 27023 |
| 90 | 6381780. 46678 | 6382065. 28337 | 6382358. 60697 | 6382660. 08367 | 6382969. 34953 | 6383286. 03108 |
| | 30° | 31° | 32° | 33° | 34° | 35° |
| 0 | 6351148. 46250 | 6352134. 54301 | 6353139. 36108 | 6354161. 70670 | 6355200. 34779 | 6356254. 03160 |
| 5 | 6351393. 79800 | 6352374. 94738 | 6353374. 73930 | 6354391. 96974 | 6355425. 41270 | 6356473. 82166 |
| 10 | 6352122. 46185 | 6353088. 96321 | 6354073. 82494 | 6355075. 86081 | 6356093. 86295 | 6357126. 60325 |
| 15 | 6353312. 63581 | 6354255. 20446 | 6355215. 67280 | 6356192. 88362 | 6357185. 65868 | 6358192. 80000 |
| 20 | 6354928. 65050 | 6355838. 70922 | 6356766. 04242 | 6357709. 53241 | 6358668. 04119 | 6359640. 41176 |
| 25 | 6356922. 01015 | 6357791. 94527 | 6358678. 38422 | 6359580. 25810 | 6360496. 47864 | 6361425. 93943 |
| 30 | 6359232. 79348 | 6360056. 18463 | 6360895. 18695 | 6361748. 78823 | 6362615. 95794 | 6363495. 64845 |
| 35 | 6361791. 39679 | 6362563. 21343 | 6363349. 65378 | 6364149. 76851 | 6364962. 59115 | 6365787. 13928 |
| 40 | 6364520. 57563 | 6365237. 33435 | 6365967. 66452 | 6366710. 68406 | 6367465. 49501 | 6368231. 18461 |
| 45 | 6367337. 73168 | 6367997. 60883 | 6368669. 97240 | 6369354. 00991 | 6370048. 89425 | 6370753. 78475 |
| 50 | 6370157. 38278 | 6370760. 27831 | 6371374. 57548 | 6371999. 53156 | 6372634. 39060 | 6373278. 38419 |
| 55 | 6372893. 74582 | 6373441. 29540 | 6373999. 19496 | 6374566. 76973 | 6375143. 33287 | 6375728. 18631 |
| 60 | 6375463. 35598 | 6375958. 88978 | 6376463. 78701 | 6376977. 43683 | 6377499. 21747 | 6378028. 49706 |
| 65 | 6377787. 64123 | 6378236. 08975 | 6378693. 01018 | 6379157. 84962 | 6379630. 04531 | 6380109. 02533 |
| 70 | 6379795. 36936 | 6380203. 11822 | 6380618. 56956 | 6381041. 22060 | 6381470. 55963 | 6381906. 06662 |
| 75 | 6381424. 88604 | 6381799. 58365 | 6382181. 35967 | 6382569. 75210 | 6382964. 29073 | 6383364. 49766 |
| 80 | 6382626. 06695 | 6382976. 39056 | 6383333. 33281 | 6383696. 46180 | 6384065. 33791 | 6384439. 51440 |
| 85 | 6383361. 91510 | 6383697. 30265 | 6384039. 02730 | 6384386. 67556 | 6384739. 82665 | 6385098. 05284 |
| 90 | 6383609. 74569 | 6383940. 10208 | 6384276. 70075 | 6384619. 13446 | 6384966. 98871 | 6385319. 84225 |

*Table XX. Radii of Curvature of Earth's Surface ($\rho$) (in Meters)*—Continued

(Based upon Clarke's Spheroid of 1866 as Expressed in Meters)

| Azimuth (degrees) | Latitude | | | | | |
|---|---|---|---|---|---|---|
| | 36° | 37° | 38° | 39° | 40° | 41° |
| 0 | 6357321.48618 | 6358401.42190 | 6359492.53291 | 6360593.49875 | 6361702.98586 | 6362819.64920 |
| 5 | 6357535.93096 | 6358610.45738 | 6359696.10156 | 6360791.54960 | 6361895.47460 | 6363006.53820 |
| 10 | 6358172.83480 | 6359231.29340 | 6360300.69898 | 6361379.75717 | 6362467.16078 | 6363561.59142 |
| 15 | 6359213.09134 | 6360245.29955 | 6361288.17609 | 6362340.45848 | 6363400.87181 | 6364448.13023 |
| 20 | 6360625.46949 | 6361622.02350 | 6362628.86810 | 6363644.78413 | 6364668.54051 | 6365698.89566 |
| 25 | 6362367.51728 | 6363320.07350 | 6364282.45528 | 6365253.49699 | 6366232.02166 | 6367216.84232 |
| 30 | 6364386.79626 | 6365288.32328 | 6366199.13806 | 6367118.13713 | 6368044.20631 | 6368976.22198 |
| 35 | 6366622.41561 | 6367467.40925 | 6368321.09683 | 6369182.44376 | 6370050.40546 | 6370923.92858 |
| 40 | 6369006.82638 | 6369791.48118 | 6370584.19841 | 6371384.01706 | 6372189.96686 | 6373001.06953 |
| 45 | 6371467.82812 | 6372190.15951 | 6372919.90348 | 6373656.17511 | 6374398.08098 | 6375144.72034 |
| 50 | 6373930.73248 | 6374590.64506 | 6375257.32191 | 6375929.95433 | 6376607.72596 | 6377289.81373 |
| 55 | 6376320.62160 | 6376919.92070 | 6377525.35688 | 6378136.19558 | 6378751.69529 | 6379371.10841 |
| 60 | 6378564.63428 | 6379106.97921 | 6379654.87406 | 6380207.65396 | 6380764.64780 | 6381325.17893 |
| 65 | 6380594.20922 | 6381085.00876 | 6381580.82862 | 6382081.06705 | 6382585.11665 | 6383092.36510 |
| 70 | 6382347.21376 | 6382793.46621 | 6383244.28263 | 6383699.11588 | 6384157.41369 | 6384618.61928 |
| 75 | 6383769.88791 | 6384197.96998 | 6384594.24644 | 6385012.21450 | 6385433.36664 | 6385857.19123 |
| 80 | 6384818.53784 | 6385201.94872 | 6385589.28200 | 6385980.06760 | 6386373.83105 | 6386770.09400 |
| 85 | 6385460.92009 | 6385827.98850 | 6386198.81287 | 6386572.94319 | 6386949.92521 | 6387329.30100 |
| 90 | 6385677.26751 | 6386038.83123 | 6386404.09489 | 6386772.61523 | 6387143.94483 | 6387517.63261 |
| | 42° | 43° | 44° | 45° | 46° | 47° |
| 0 | 6363942.13381 | 6365069.07652 | 6366199.10750 | 6367330.85194 | 6368462.93175 | 6369593.96718 |
| 5 | 6364123.39225 | 6365244.68037 | 6366369.03958 | 6367495.10198 | 6368621.49637 | 6369746.84996 |
| 10 | 6364661.72100 | 6365766.21340 | 6366873.72601 | 6367982.91140 | 6369092.41891 | 6370200.89631 |
| 15 | 6365540.93852 | 6366617.99361 | 6367697.98619 | 6368779.60224 | 6369861.52466 | 6370942.43487 |
| 20 | 6366734.59894 | 6367774.39225 | 6368817.01146 | 6369861.18796 | 6370905.65022 | 6371949.12531 |
| 25 | 6368206.76343 | 6369200.58231 | 6370197.09064 | 6371195.07585 | 6372193.32260 | 6373190.61432 |
| 30 | 6369913.05250 | 6370853.55958 | 6371796.59953 | 6372741.02479 | 6373685.68523 | 6374629.42959 |
| 35 | 6371801.95229 | 6372683.40956 | 6373567.22843 | 6374452.33331 | 6375337.64630 | 6376222.08843 |
| 40 | 6373816.33981 | 6374634.78683 | 6375455.41510 | 6376277.22588 | 6377099.21831 | 6377920.39065 |
| 45 | 6375895.18608 | 6376648.56591 | 6377403.94345 | 6378160.39928 | 6378917.01210 | 6379672.85986 |
| 50 | 6377975.38883 | 6378663.61773 | 6379353.66322 | 6380044.68535 | 6380735.84250 | 6381426.29239 |
| 55 | 6379993.68221 | 6380618.65967 | 6381245.28044 | 6381872.78175 | 6382500.39935 | 6383127.36837 |
| 60 | 6381888.56613 | 6382454.12427 | 6383021.16528 | 6383588.99886 | 6384156.93343 | 6384724.27689 |
| 65 | 6383602.19584 | 6384113.98888 | 6384627.12151 | 6385140.96908 | 6385654.90572 | 6386168.30515 |
| 70 | 6385082.17206 | 6385547.50831 | 6386014.06183 | 6386481.26467 | 6386948.54777 | 6387415.34171 |
| 75 | 6386283.17314 | 6386710.79431 | 6387139.53447 | 6387568.87167 | 6387998.28300 | 6388427.24516 |
| 80 | 6387168.37481 | 6387568.18916 | 6387969.05059 | 6388370.47111 | 6388771.96180 | 6389173.03341 |
| 85 | 6387710.60946 | 6388093.38691 | 6388477.16762 | 6388861.48441 | 6389245.86920 | 6389629.85356 |
| 90 | 6387893.22442 | 6388270.26351 | 6388648.29115 | 6389026.84715 | 6389405.47046 | 6389783.69968 |

*Table XX. Radii of Curvature of Earth's Surface* ($\rho$) *(in Meters)*—Continued

(Based upon Clarke's Spheriod of 1866 as Expressed in Meters)

| Azimuth (degrees) | Latitude | | | | | |
|---|---|---|---|---|---|---|
| | 48° | 49° | 50° | 51° | 52° | 53° |
| 0 | 6370722.57855 | 6371847.38790 | 6372967.02071 | 6374080.10758 | 6375185.28591 | 6376281.20166 |
| 5 | 6370869.78998 | 6371988.94543 | 6373102.94871 | 6374210.43733 | 6375310.05559 | 6376400.45626 |
| 10 | 6371306.99147 | 6372409.35398 | 6373506.63682 | 6374597.49803 | 6375680.60236 | 6376754.62292 |
| 15 | 6372021.01437 | 6373095.94642 | 6374165.91762 | 6375229.61951 | 6376285.75025 | 6377333.01616 |
| 20 | 6372990.34047 | 6374028.02465 | 6375060.91005 | 6376087.73375 | 6377107.23922 | 6378118.17787 |
| 25 | 6374185.73462 | 6375177.46882 | 6376164.60541 | 6377145.93760 | 6378120.26470 | 6379086.39375 |
| 30 | 6375571.10685 | 6376509.56765 | 6377443.66575 | 6378372.25934 | 6379294.21253 | 6380208.39675 |
| 35 | 6377104.58112 | 6377984.04733 | 6378859.41302 | 6379729.60837 | 6380593.50917 | 6381450.23810 |
| 40 | 6378739.74147 | 6379556.27095 | 6380368.98200 | 6381176.88159 | 6381978.98190 | 6382774.30158 |
| 45 | 6380427.02087 | 6381178.57492 | 6381926.60441 | 6382670.19548 | 6383408.43915 | 6384140.43242 |
| 50 | 6382115.19313 | 6382801.70419 | 6383484.98751 | 6384164.20846 | 6384838.53690 | 6385507.14819 |
| 55 | 6383752.92437 | 6384376.30418 | 6384996.74685 | 6385613.49463 | 6386225.79385 | 6386832.89588 |
| 60 | 6385290.33748 | 6385854.42466 | 6386415.84993 | 6386973.92768 | 6387527.97600 | 6388077.31759 |
| 65 | 6386680.54140 | 6387190.98962 | 6387699.02680 | 6388204.03253 | 6388705.38985 | 6389202.48589 |
| 70 | 6387881.07735 | 6388345.18655 | 6388807.10288 | 6389266.26228 | 6389722.10379 | 6390174.07020 |
| 75 | 6388855.23517 | 6389281.73090 | 6389706.21184 | 6390128.15964 | 6390547.05878 | 6390962.39721 |
| 80 | 6389573.19691 | 6389971.96417 | 6390368.84846 | 6390763.36515 | 6391155.03220 | 6391543.37083 |
| 85 | 6390012.96932 | 6390394.74911 | 6390774.72695 | 6391152.43881 | 6391527.42321 | 6391899.22171 |
| 90 | 6390161.07364 | 6390537.13197 | 6390911.41568 | 6391283.46768 | 6391652.83340 | 6392019.06126 |
| | 54° | 55° | 56° | 57° | 58° | 59° |
| 0 | 6377366.51091 | 6378439.88167 | 6379499.99547 | 6380545.54903 | 6381575.25595 | 6382587.84825 |
| 5 | 6377480.30228 | 6378548.26837 | 6379603.04277 | 6380643.32883 | 6381667.84666 | 6382675.33478 |
| 10 | 6377818.24284 | 6378870.15693 | 6379909.07327 | 6380933.71489 | 6381942.82134 | 6382935.15028 |
| 15 | 6378370.13338 | 6379395.82945 | 6380408.84490 | 6381407.93487 | 6382391.87062 | 6383359.44112 |
| 20 | 6379119.31062 | 6380109.40946 | 6381087.25896 | 6382051.65778 | 6383001.42026 | 6383935.37781 |
| 25 | 6380043.14087 | 6380989.33283 | 6381923.80846 | 6382845.42013 | 6383753.03518 | 6384645.53737 |
| 30 | 6381113.69210 | 6382008.98876 | 6382893.18846 | 6383765.20571 | 6384623.96928 | 6385468.42348 |
| 35 | 6382298.56603 | 6383137.51336 | 6383966.05130 | 6384783.16315 | 6385587.84556 | 6386379.10984 |
| 40 | 6383561.86692 | 6384340.71313 | 6385109.88548 | 6385868.44051 | 6386615.44723 | 6387349.98826 |
| 45 | 6384865.27938 | 6385582.09237 | 6386289.99301 | 6386988.11337 | 6387675.59700 | 6388351.60001 |
| 50 | 6386169.22422 | 6386823.95445 | 6387470.53684 | 6388108.17893 | 6388736.09877 | 6389353.52594 |
| 55 | 6387434.05803 | 6388028.54451 | 6388615.62729 | 6389194.58705 | 6389764.71404 | 6390325.30900 |
| 60 | 6388621.28053 | 6389159.19916 | 6389690.41485 | 6390214.27689 | 6390730.14324 | 6391237.38135 |
| 65 | 6389694.71270 | 6390181.46800 | 6390662.15588 | 6391136.18760 | 6391602.98228 | 6392061.96763 |
| 70 | 6390621.60878 | 6391064.17196 | 6391501.21794 | 6391932.21146 | 6392356.62440 | 6392773.93647 |
| 75 | 6391373.66698 | 6391780.36488 | 6392181.99303 | 6392578.05950 | 6392968.07898 | 6393351.57333 |
| 80 | 6391927.90610 | 6392308.16746 | 6392683.68939 | 6393054.01190 | 6393418.68120 | 6393777.25016 |
| 85 | 6392267.37960 | 6392631.44633 | 6392990.97618 | 6393345.52873 | 6393694.66948 | 6394037.97032 |
| 90 | 6392381.70336 | 6392740.31589 | 6393094.45980 | 6393443.70125 | 6393787.61223 | 6394125.77105 |

*Table XX. Radii of Curvature of Earth's Surface ($\rho$) (in Meters)*—Continued

(Based upon Clarke's Spheroid of 1866 as Expressed in Meters)

| Azimuth (degrees) | Latitude | | | | | |
|---|---|---|---|---|---|---|
| | 60° | 61° | 62° | 63° | 64° | 65° |
| 0 | 6383582. 07809 | 6384556. 71925 | 6385510. 56880 | 6386442. 44854 | 6387351. 20660 | 6388235. 71886 |
| 5 | 6383664. 55168 | 6384634. 27742 | 6385583. 31519 | 6386510. 49283 | 6387414. 66436 | 6388294. 71143 |
| 10 | 6383909. 47912 | 6384864. 60648 | 6385799. 35379 | 6386712. 56678 | 6387603. 11697 | 6388469. 90311 |
| 15 | 6384309. 45459 | 6385240. 73998 | 6386152. 14851 | 6387042. 55510 | 6387910. 85980 | 6388755. 98928 |
| 20 | 6384852. 38046 | 6385751. 29833 | 6386631. 02298 | 6387490. 46895 | 6388328. 57503 | 6389144. 30571 |
| 25 | 6385521. 82825 | 6386380. 82859 | 6387221. 47975 | 6388042. 74498 | 6388843. 61083 | 6389623. 08836 |
| 30 | 6386297. 52953 | 6387110. 26686 | 6387905. 63439 | 6388682. 65185 | 6389440. 36095 | 6390177. 82668 |
| 35 | 6387155. 98314 | 6387917. 50974 | 6388662. 75220 | 6389390. 79261 | 6390100. 73371 | 6390791. 70004 |
| 40 | 6388071. 16098 | 6388778. 07870 | 6389469. 87175 | 6390145. 68859 | 6390804. 69689 | 6391446. 08459 |
| 45 | 6389015. 29218 | 6389665. 85792 | 6390302. 49736 | 6390924. 42734 | 6391530. 88239 | 6392121. 11574 |
| 50 | 6389959. 70250 | 6390553. 88390 | 6391135. 34000 | 6391703. 35591 | 6392257. 23295 | 6392796. 28949 |
| 55 | 6390875. 68403 | 6391415. 16339 | 6391943. 08445 | 6392458. 79846 | 6392961. 67137 | 6393451. 08464 |
| 60 | 6391735. 36896 | 6392223. 49486 | 6392701. 15969 | 6393167. 77665 | 6393622. 77225 | 6394065. 58708 |
| 65 | 6392512. 58069 | 6392954. 26849 | 6393386. 48880 | 6393808. 71075 | 6394220. 41553 | 6394621. 09708 |
| 70 | 6393183. 63585 | 6393585. 21981 | 6393978. 19539 | 6394362. 07995 | 6394736. 40183 | 6395100. 70094 |
| 75 | 6393728. 07217 | 6394097. 11351 | 6394458. 24431 | 6394811. 02103 | 6395155. 01023 | 6395489. 78906 |
| 80 | 6394129. 27897 | 6394474. 33562 | 6394811. 99647 | 6395141. 84676 | 6395463. 48118 | 6395776. 50432 |
| 85 | 6394375. 01012 | 6394705. 37523 | 6395028. 66000 | 6395344. 46731 | 6395652. 40906 | 6395952. 10662 |
| 90 | 6394457. 76285 | 6394783. 18017 | 6395101. 62344 | 6395412. 70145 | 6395716. 03189 | 6396011. 24180 |
| | 66° | 67° | 68° | 69° | 70° | 71° |
| 0 | 6389094. 89046 | 6389927. 65718 | 6390732. 98680 | 6391509. 88050 | 6392257. 37415 | 6392974. 53953 |
| 5 | 6389149. 54478 | 6389978. 10563 | 6390779. 36710 | 6391552. 33550 | 6392296. 05164 | 6393009. 59209 |
| 10 | 6389311. 85259 | 6390127. 92286 | 6390917. 10277 | 6391678. 41386 | 6392410. 91168 | 6393113. 68698 |
| 15 | 6389576. 89814 | 6390372. 57027 | 6391142. 02020 | 6391884. 29434 | 6392598. 47226 | 6393283. 66787 |
| 20 | 6389936. 65247 | 6390704. 63509 | 6391447. 30289 | 6392163. 73603 | 6392853. 04663 | 6393514. 37996 |
| 25 | 6390380. 21447 | 6391114. 05310 | 6391823. 69646 | 6392508. 26621 | 6393166. 91458 | 6393798. 82545 |
| 30 | 6390894. 13850 | 6391588. 41142 | 6392259. 78726 | 6392907. 43569 | 6393530. 55530 | 6394128. 37465 |
| 35 | 6391462. 83909 | 6392113. 32234 | 6392742. 34638 | 6393349. 13390 | 6393932. 93470 | 6394493. 02666 |
| 40 | 6392069. 06093 | 6392672. 85747 | 6393256. 72905 | 6393819. 95476 | 6394361. 83888 | 6394881. 71172 |
| 45 | 6392694. 40021 | 6393250. 02917 | 6393787. 31746 | 6394305. 60221 | 6394804. 24375 | 6395282. 62637 |
| 50 | 6393319. 86185 | 6393827. 30510 | 6394317. 99394 | 6394791. 32343 | 6395246. 70984 | 6395683. 59128 |
| 55 | 6393926. 43603 | 6394387. 14035 | 6394832. 63021 | 6395262. 35673 | 6395675. 79028 | 6396072. 42109 |
| 60 | 6394495. 67643 | 6394912. 51108 | 6395315. 57787 | 6395704. 38043 | 6396078. 43976 | 6396437. 29488 |
| 65 | 6395010. 26264 | 6395387. 43344 | 6395752. 14530 | 6396103. 94917 | 6396442. 41178 | 6396767. 11613 |
| 70 | 6395454. 52931 | 6395797. 45170 | 6396129. 04608 | 6396448. 90425 | 6396756. 63230 | 6397051. 85113 |
| 75 | 6395814. 94583 | 6396130. 08056 | 6396434. 80540 | 6396728. 74520 | 6397011. 53795 | 6397282. 83526 |
| 80 | 6396080. 53121 | 6396375. 18778 | 6396660. 11138 | 6396934. 95116 | 6397199. 36862 | 6397453. 03793 |
| 85 | 6396243. 19137 | 6396525. 30515 | 6396798. 10066 | 6397061. 24198 | 6397314. 40495 | 6397557. 27759 |
| 90 | 6396297. 96806 | 6396575. 85785 | 6396844. 56906 | 6397103. 77078 | 6397353. 14368 | 6397592. 38043 |

*Table XX. Radii of Curfature of Earth's Surface (ρ) (in meters)*—Continued

(Based upon Clarke's Spheroid of 1866 as Expressed in Meters)

| Azimuth (degrees) | Latitude | | | | | |
|---|---|---|---|---|---|---|
| | 72° | 73° | 74° | 75° | 76° | 77° |
| 0 | 6393660. 48560 | 6394314. 35963 | 6394935. 34835 | 6395522. 67900 | 6396075. 62036 | 6396593. 48375 |
| 5 | 6393692. 07040 | 6394342. 63822 | 6394960. 48650 | 6395544. 84645 | 6396094. 99065 | 6396610. 23396 |
| 10 | 6393785. 86693 | 6394426. 61625 | 6395035. 13831 | 6395610. 67619 | 6396152. 51366 | 6396659. 97616 |
| 15 | 6393939. 03055 | 6394563. 74633 | 6395157. 03888 | 6395718. 17062 | 6396246. 44360 | 6396741. 20048 |
| 20 | 6394146. 91557 | 6394749. 86833 | 6395322. 48947 | 6395864. 06756 | 6396373. 92947 | 6396851. 44121 |
| 25 | 6394403. 21546 | 6394979. 33502 | 6395526. 46925 | 6396043. 93893 | 6396531. 10145 | 6396987. 35155 |
| 30 | 6394700. 15328 | 6395245. 18266 | 6395762. 78709 | 6396252. 32463 | 6396713. 18791 | 6397144. 80491 |
| 35 | 6395028. 71669 | 6395539. 34158 | 6396024. 26892 | 6396482. 89788 | 6396914. 66000 | 6397319. 01995 |
| 40 | 6395378. 93057 | 6395852. 88043 | 6396302. 97489 | 6396728. 65682 | 6397129. 39915 | 6397504. 70551 |
| 45 | 6395740. 15916 | 6396176. 27674 | 6396590. 44000 | 6396982. 13680 | 6397350. 88260 | 6397696. 22113 |
| 50 | 6396101. 42856 | 6396499. 70575 | 6396877. 93096 | 6397235. 63687 | 6397572. 38138 | 6397887. 74821 |
| 55 | 6396451. 75993 | 6396813. 33878 | 6397156. 71133 | 6397481. 45365 | 6397787. 16469 | 6398073. 46679 |
| 60 | 6396780. 50336 | 6397107. 64198 | 6397418. 30715 | 6397712. 11552 | 6397988. 70444 | 6398247. 73241 |
| 65 | 6397077. 66203 | 6397373. 66660 | 6397654. 76483 | 6397920. 60993 | 6398170. 87390 | 6398405. 24783 |
| 70 | 6397334. 19692 | 6397603. 32163 | 6397858. 89340 | 6398100. 59700 | 6398328. 13420 | 6398541. 22420 |
| 75 | 6397542. 30278 | 6397789. 62063 | 6398024. 48383 | 6398246. 60266 | 6398455. 70307 | 6398651. 52698 |
| 80 | 6397695. 64643 | 6397926. 89500 | 6398146. 49840 | 6398354. 18572 | 6398549. 70066 | 6398732. 80188 |
| 85 | 6397789. 56048 | 6398010. 96720 | 6398221. 22463 | 6398420. 07331 | 6398607. 26785 | 6398782. 57712 |
| 90 | 6397821. 18610 | 6398039. 27851 | 6398246. 38863 | 6398442. 26088 | 6398626. 65350 | 6398799. 33881 |
| | 78° | 79° | 80° | 81° | 82° | 83° |
| 0 | 6397075. 62388 | 6397521. 43976 | 6397930. 37547 | 6398301. 92091 | 6398635. 61250 | 6398931. 03375 |
| 5 | 6397089. 93443 | 6397533. 49418 | 6397940. 36017 | 6398310. 02495 | 6398642. 02733 | 6398935. 95297 |
| 10 | 6397132. 43165 | 6397569. 29146 | 6397970. 01111 | 6398334. 09097 | 6398661. 07698 | 6398950. 56122 |
| 15 | 6397201. 82538 | 6397627. 74470 | 6398018. 42788 | 6398373. 38808 | 6398692. 18287 | 6398974. 41478 |
| 20 | 6397296. 00878 | 6397707. 07900 | 6398084. 14017 | 6398426. 72280 | 6398734. 40019 | 6399006. 78905 |
| 25 | 6397412. 12219 | 6397804. 88528 | 6398165. 15234 | 6398492. 47522 | 6398786. 44660 | 6399046. 70059 |
| 30 | 6397546. 63972 | 6397918. 19327 | 6398259. 00394 | 6398568. 64820 | 6398846. 74113 | 6399092. 93698 |
| 35 | 6397695. 47618 | 6398043. 56163 | 6398362. 84432 | 6398652. 92790 | 6398913. 45217 | 6399144. 09360 |
| 40 | 6397854. 11090 | 6398177. 18229 | 6398473. 51916 | 6398742. 75406 | 6398984. 55308 | 6399198. 61625 |
| 45 | 6398017. 72495 | 6398314. 99601 | 6398587. 66619 | 6398835. 39773 | 6399057. 88371 | 6399254. 84843 |
| 50 | 6398181. 34736 | 6398452. 81568 | 6398701. 81729 | 6398928. 04408 | 6399131. 21601 | 6399311. 08160 |
| 55 | 6398340. 00618 | 6398586. 45342 | 6398812. 50386 | 6399017. 87796 | 6399202. 32176 | 6399365. 60710 |
| 60 | 6398488. 87955 | 6398711. 84797 | 6398916. 36220 | 6399102. 16950 | 6399269. 04022 | 6399416. 76806 |
| 65 | 6398623. 44237 | 6398825. 18810 | 6399010. 23584 | 6399178. 35700 | 6399329. 34385 | 6399463. 00981 |
| 70 | 6398739. 60396 | 6398923. 02856 | 6399091. 27146 | 6399244. 12486 | 6399381. 39993 | 6399502. 92705 |
| 75 | 6398833. 83265 | 6399002. 39498 | 6399157. 00580 | 6399297. 47410 | 6399423. 62635 | 6399535. 30666 |
| 80 | 6398903. 26331 | 6399060. 87442 | 6399205. 44053 | 6399336. 78305 | 6399454. 73966 | 6399559. 16458 |
| 85 | 6398945. 78463 | 6399096. 68881 | 6399235. 10321 | 6399360. 85680 | 6399473. 79416 | 6399573. 77569 |
| 90 | 6398960. 10356 | 6399108. 74918 | 6399245. 09200 | 6399368. 96354 | 6399480. 21068 | 6399578. 69591 |

*Table XX. Radii of Curvature of Earth's Surface ($\rho$) in Meters*)—Continued

(Based upon Clarke's Spheriod of 1866 as Expressed in Meters)

| Azimuth (degrees) | Latitude | | | | | |
|---|---|---|---|---|---|---|
| | 84° | 85° | 86° | 87° | 88° | 89° |
| 0 | 6399187. 81586 | 6399405. 63820 | 6399584. 22873 | 6399723. 36439 | 6399822. 87135 | 6399882. 62530 |
| 5 | 6399191. 43500 | 6399408. 15444 | 6399585. 84067 | 6399724. 27178 | 6399823. 27484 | 6399882. 72620 |
| 10 | 6399202. 18248 | 6399415. 62673 | 6399590. 62753 | 6399726. 96639 | 6399824. 47308 | 6399883. 02585 |
| 15 | 6399219. 73184 | 6399427. 82808 | 6399598. 44388 | 6399731. 36637 | 6399826. 42967 | 6399883. 51515 |
| 20 | 6399243. 54994 | 6399444. 38779 | 6399609. 05225 | 6399737. 33802 | 6399829. 08515 | 6399884. 17923 |
| 25 | 6399272. 91321 | 6399464. 80278 | 6399622. 13033 | 6399744. 69991 | 6399832. 35883 | 6399884. 99791 |
| 30 | 6399306. 92960 | 6399488. 45280 | 6399637. 28076 | 6399753. 22835 | 6399836. 15126 | 6399885. 94632 |
| 35 | 6399344. 56566 | 6399514. 61932 | 6399654. 04326 | 6399762. 66421 | 6399840. 34719 | 6399886. 99564 |
| 40 | 6399384. 67797 | 6399542. 50735 | 6399671. 90851 | 6399772. 72083 | 6399844. 81915 | 6399888. 11398 |
| 45 | 6399426. 04778 | 6399571. 26955 | 6399690. 33370 | 6399783. 09260 | 6399849. 43125 | 6399889. 26737 |
| 50 | 6399467. 41813 | 6399600. 03200 | 6399708. 75899 | 6399793. 46441 | 6399854. 04335 | 6399890. 42075 |
| 55 | 6399507. 53197 | 6399627. 92078 | 6399726. 62455 | 6399803. 52111 | 6399858. 51531 | 6399891. 53910 |
| 60 | 6399545. 17040 | 6399654. 08844 | 6399743. 38750 | 6399812. 95713 | 6399862. 71128 | 6399892. 58841 |
| 65 | 6399579. 18968 | 6399677. 73986 | 6399758. 53851 | 6399821. 48575 | 6399866. 50375 | 6399893. 53682 |
| 70 | 6399608. 55603 | 6399698. 15633 | 6399771. 61720 | 6399828. 84783 | 6399869. 77746 | 6399894. 35550 |
| 75 | 6399632. 37702 | 6399714. 71744 | 6399782. 22614 | 6399834. 81966 | 6399872. 43298 | 6399895. 01959 |
| 80 | 6399649. 92875 | 6399726. 91993 | 6399790. 04296 | 6399839. 21979 | 6399874. 38960 | 6399895. 50890 |
| 85 | 6399660. 67778 | 6399734. 39298 | 6399794. 83015 | 6399841. 91451 | 6399875. 58788 | 6399895. 80856 |
| 90 | 6399664. 29747 | 6399736. 90950 | 6399796. 44220 | 6399842. 82196 | 6399875. 99139 | 6399895. 90948 |

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*Table XXI. Table for Inverse Position Computation (Sodano Method)*

SPHEROID

| | International | Bessel | Everest | Clarke 1880 | Clarke 1866 |
|---|---|---|---|---|---|
| $a_o$ | 6378388. | 6377397. 155 | 6377276. 34518 | 6378249. 145 | 6378206. 4 |
| $b_o$ | 6356911. 94613 | 6356078. 96282 | 6356075. 41332 | 6356514. 86955 | 6356583. 8 |
| $1/f$ | 297. | 299. 1528128 | 300. 8017 | 293. 465 | |
| $e^2$ | . 006722670022 | . 006674372232 | . 006637846630 | . 006803511283 | . 006768657997 |
| $e'^2$ | . 006768170197 | . 006719218799 | . 006682202063 | . 006850116125 | . 006814784946 |
| $\sqrt{1-e^2}$ | . 996632996633 | . 996657226818 | . 996675550703 | . 996592438621 | . 996609924696 |
| $f$ | . 003367003367 | . 003342773182 | . 003324449297 | . 003407561379 | . 003390075304 |
| N | . 500843170320 | . 500837092400 | . 500832496120 | . 500853344256 | . 500848957841 |
| $K_1$ | 237. 238891820 | 238. 961150061 | 240. 280265873 | 234. 410878424 | 235. 621842771 |
| $K_2$ | . 341888239296 | . 346864269369 | . 350699834189 | . 333795392740 | . 337248908462 |
| $K_3$ | 4. 98652064985 | 4. 98661773317 | 4. 98669115088 | 4. 98635814305 | 4. 98642820621 |
| $K_4$ | . 401081262956 | . 401073454397 | . 401067549500 | . 401094334307 | . 401088698622 |
| $K_5$ | . 799459368522 | . 799463272801 | . 799466225250 | . 799452832847 | . 799455650689 |
| $K_6$ | 3. 98652064985 | 3. 98661773317 | 3. 98669115088 | 3. 98635814305 | 3. 98642820621 |
| $K_7$ | 10756. 1654950 | 10676. 9713140 | 10618. 1450593 | 10885. 7162520 | 10829. 6878969 |
| $K_8$ | 2. 27498621062 | 2. 24190332411 | 2. 21726846302 | 2. 33026313850 | 2. 30631231403 |
| $K_9$ | 13. 6499172637 | 13. 4514199446 | 13. 3036107781 | 13. 9815788310 | 13. 8378738842 |
| $K_{10}$ | 18. 1998896850 | 17. 9352265929 | 17. 7381477042 | 18. 6421051080 | 18. 4504985122 |

$N=f/e^2$ $\quad K_3=d/e'^2$ $\quad K_6=c/e'^2$ $\quad K_9=6K_8$ $\quad c=16e^2N^2$

$K_1=(16N-e'^2)/d$ $\quad K_4=2e'^2/d$ $\quad K_7=b_o e'^2/4$ $\quad K_{10}=8K_8$

$K_2=16\sin 1''/e^2 d$ $\quad K_5=c/d$ $\quad K_8=b_o e'^4/128$ $\quad d=16e^2N^2+e'^2$

# GLOSSARY

*Aberration of light—(astronomic)*—The apparent displacement in position of a stellar body due to the velocity of light combined with the motion of the earth itself.

*Accidental error*—Any small error accidentally incurred in a measurement. Unlike systematic errors, accidental errors are not governed by fixed laws. The theory of probability is based on the occurrence of these errors, which are as likely to be positive as negative.

*Accuracy*—The degree of conformity with a standard, or the degree of perfection attained in a measurement. Accuracy relates to the quality of a result and is distinguished from precision which relates to the quality of the operation by which the result is obtained.

*Adjusted position*—An adjusted value for the horizontal or vertical position of a survey station, in which discrepancies due to errors in the observed data are removed. This adjustment forms a coordinated and correlated system of stations.

*Altimeter*—An aneroid barometer used for the measurement of approximate elevations or approximate differences of elevation.

*Altitude*—The vertical angle measured between the plane of the observer's true horizon and a line to the object. See zenith distance.

*Arithmetical mean*—The value obtained by dividing the sum of a series of values by the number of values in the series.

*Astronomical latitude*—The angle between the direction of the plumb line and the plane of the celestial equator.

*Astronomical triangle*—A spherical triangle using the celestial pole, the zenith, and the celestial object to define its vertices.

*Azimuth (astronomic)*—At the point of observation, the angle measured from the vertical plane through the celestial pole to the vertical plane through the observed object.

*Azimuth (general)*—The horizontal direction reckoned clockwise from the meridian plane.

*Azimuth (geodetic)*—For the geodesic line from A to B, the angle between the tangent to the meridian at A and the tangent to the geodesic line at A. This angle is measured clockwise from south.

*Azimuth (grid)*—The angle in the plane of projection between a straight line and the central meridian (axis of Y) of a plane-rectangular coordinate system.

*Base line (general)*—A surveyed line established with more than usual care, to which surveys are referred for coordination and correlation.

*Base line (triangulation)*—The side of one of a series of connected triangles, the length of which is measured with prescribed accuracy and precision, and from which the lengths of the other triangle sides are obtained by computation.

*Base net*—A small net of geometric figures used to expand from a base line to a line of the main scheme of a triangulation net.

*Basic control*—Horizontal or vertical control, the positions of whose stations have been accurately coordinated and correlated, forming a framework from which other surveys may be extended and adjusted.

*Bearing*—The direction of a line within a quadrant, with respect to the meridian. Bearings are measured clockwise or counterclockwise from north or south, depending on the quadrant.

*Celestial equator*—A great circle on the celestial sphere on which any point is equidistant from the celestial poles. The plane of the earth's equator, if extended, would coincide with that of the celestial equator.

*Celestial meridian*—A vertical circle, passing through both celestial poles, the plane of which is perpendicular to the celestial equator.

*Celestial pole*—A reference point located at the point of intersection of an indefinite extension of the earth's axis of rotation and the apparent celestial sphere.

*Celestial sphere*—An imaginary sphere of infinite radius with the earth as a center. It rotates from east to west on a prolongation of the earth's axis.

*Chronograph*—An instrument for producing a graphical record of time as shown by a clock or other device. Also capable of recording the occurrence of other events simultaneously upon the proper signal so that these events can be scaled with relation to the time base.

*Chronometer*—A portable timekeeper with compensated balance, capable of showing time with extreme precision and accuracy.

*Circle position*—A prescribed setting (reading) of the horizontal circle of a direction theodolite, to be used for the observation on the initial station of a series of stations that are to be observed.

*Circuit closure*—In leveling, the amount by which the algebraic sum of the measured difference of elevation around a circuit fails to equal zero.

*Circumpolar star*—A star in any given latitude which never goes below the horizon; hence its polar distance must be less than the given latitude. In astronomy only those stars with a polar distance of less than 10° are considered in practical problems.

*Closed traverse*—A traverse that starts and ends at the same point, or at stations whose positions have been determined by other surveys.

*Collimation*—The line of sight or aiming line of the instrument when coincident with the physical alinement of the instrument; thus collimation error is the angle between the line of collimation (line of sight) of a telescope and the collimation axis of the instrument.

*Control*—A system of points whose relative positions have been determined from survey data. See: basic control, horizontal control, vertical control, and picture point.

*Control survey*—A survey which provides positions (horizontal or vertical) of points to which supplementary surveys are adjusted.

*Coordinates*—Linear or angular quantities, or both, which designate the position of a point in relation to a given reference frame. There are two general divisions of coordinates used in surveying: polar coordinates and rectangular coordinates. These may be subdivided into three classes: plane coordinates, spherical coordinates, and space coordinates.

*Culmination, transit*—The instant when any point on the celestial sphere is on the meridian of an observer. When it is on that half of the meridian containing the zenith, it is called the upper transit; when it is on the other half, it is called the lower transit.

*Datum*—A numerical or geometric quantity or set of such quantities to which the positions of other quantities are determined. See: geodetic datum and level datum.

*Declination*—Celestial object: The angular distance from the celestial equator to an object measured along the hour circle passing through the object.

*Deflection of the vertical*—The angle at a point on the geoid between the vertical (plumb line) and the normal to the spheriod of reference through the point, also known as the deflection of the plumb line or station error.

*Ecliptic*—The great circle on the celestial sphere which the sun appears to describe in its annual motion among the stars. It is inclined to the celestial equator at an angle of about 23°27′.

*Elevation*—The vertical distance of a point above or below a reference surface, or level datum.

*Elongation*—That point in the apparent movement of a star when it has reached the extreme position east or west of the meridian.

*Equation: angle*—A condition equation which expresses the relationship between the sum of the measured angles of a closed figure and the theoretical value of that sum, the unknowns being the corrections to the observed directions or angles, depending upon which are used in the adjustment.

Sometimes called a triangle equation, an angle equation is used to make the sum of the three observed angles of a triangle, with corrections applied, equal to 180° plus the spherical excess of the triangle.

*Equation, azimuth*—A condition equation which expresses the relationship between the fixed azimuths of two lines which are connected by triangulation or traverse.

When a survey (triangulation or traverse) connects two lines whose azimuths are fixed by direct observation or by previous surveys, an azimuth equation is used to make the azimuth of either line as computed through the adjusted survey from the other line agree with its azimuth as previously fixed.

*Equation, condition*—An equation which expresses exactly certain relationships that must exist

among related quantities, which are not independent of one another, exist a priori, and are separate from relationships demanded by observation.

*Example:* in measuring the angles of a triangle, no condition exists until all three angles are measured. The condition equation will then express the condition that the three measured angles plus certain corrections must equal 180° plus the spherical excess of the triangle. The various condition equations set up in survey work are defined under terms which are descriptive of the conditions: as, angle equation, side equation, length equation, latitude equation, longitude equation, azimuth equation.

*Equation, correlate*—An equation derived from an observation or condition equation, employing undetermined multipliers, and expressing the condition that the sum of the squares of the residuals (or corrections) resulting from the application of these multipliers to the observation or condition equations shall be a minimum.

In the least-squares adjustment of triangulation, correlate equations are formed directly from the observation or condition equations, there being as many correlate equations as there are corrections to be determined, but only as many undetermined multipliers (correlates or correlatives) as there are observation or condition equations. From these correlate equations, the normal equations are formed, equal in number to the undetermined multipliers which constitute the unknowns in the normal equations. The solution of the normal equations determines values for the multipliers which, when substituted in the correlate equations, give values for the corrections which will satisfy the observation or condition equations, make the observations and their functions consistent among themselves, and at the same time make the adjusted values the most probable that can be derived from the given observations.

*Equation, error*—The probability equation which expresses the laws of the occurrence of accidental errors.

The error equation is the basis of the method of least squares, used in the adjustment of observations for determining the most probable value of a result from those observations.

*Equation, Laplace*—The equation which expresses the relationship between astronomic and geodetic azimuths in terms of astronomic and geodetic longitudes and geodetic latitude. A usual form is $\alpha_A - \alpha_G = -(\lambda_A - \lambda_G) \sin \phi_G$ in which $\alpha_A$ and $\lambda_A$ are astronomic azimuth and longitude, $\alpha_G$, $\lambda_G$ and $\phi_G$ are geodetic azimuth, longitude, and latitude respectively. The signs depend upon convention. As written above, north latitudes and west longitudes are considered postive.

The Laplace condition (expressed by the Laplace equation) arises from the fact that a deflection of the vertical in the plane of the prime vertical will give a difference between astronomic and geodetic longitude and between astronomic and geodetic azimuth; or, conversely, that the observed differences between astronomic and geodetic values of the longitude and of the azimuth may both be used to determine the deflection in the plane of the prime vertical. This deflection cannot have two values at the same place, and the imposition of the Laplace condition forces a consistency between them. Since longitudes can be carried through triangulation with very good accuracy and azimuths cannot, the practice is to compute the value of the deflection for azimuth from the deflection in the prime vertical obtained from the longitude. The value of the azimuth thus obtained is known as a Laplace azimuth.

*Equation, latitude*—A condition equation which expresses the relationship between the fixed latitudes of two points which are connected by triangulation or traverse.

When a survey (triangulation or traverse) connects two points whose latitudes have been fixed by direct observation or by previous surveys, a latitude equation is used to make the latitude of either point as computed through the survey from the other point agree with its latitude as previously fixed.

*Equation, length*—A condition equation which expresses the relationship between the fixed lengths of two lines which are connected by triangulation.

When a section of triangulation connects two lines whose lengths are fixed by direct measurement or by previous triangulation, a length equation is used to make the length of either line as computed through the adjusted triangulation from the other line agree with its length as previously fixed.

*Equation: longitude*—A condition equation which expresses the relationship between the fixed

longitudes of two points which are connected by triangulation of traverse.

When a survey (triangulation or traverse) connects two points whose longitudes have been fixed by direct observation or by previous surveys, a longitude equation is used to make the longitude of either point as computed through the survey from the other point agree with its longitude as previously fixed.

*Equation: normal*—An equation derived form observation or condition equations or from correlate equations, expressing the condition that the sum of the squares of the residuals (or corrections) resulting from the substitution in the observation or condition equations of factors obtained from the normal equations either directly or through the correlate equations shall be a minumum.

In a least-squares adjustment, corrections are desired to observed values which are connected together by a series of observation or condition equations, the number of such equations being smaller than the number of observed values on which they depend. The basic equations are transformed into normal equations, either directly or through the medium of correlate equations, which contain the same number of unknowns as there are equations. Factors obtained from the solution of normal equations, either directly or through the correlate equations, are applied to the observation or condition equations to obtain the desired corrections.

*Equation: observation*—A condition equation which connects interrelated unknowns by means of an observed function.

*Example:* An angle equation: the unknowns are the corrections to the observed angles; the function is the sum of those observed angles expressed as a closing error.

Alternate definition: A condition equation connecting the function observed and the unknown quantity whose value is sought.

*Equation: side*—A condition which expresses the relationship between the various sides in a triangulation figure as they may be derived by computation from one another.

A side equation is used to make the computed length of a triangle side the same for all routes through the triangulation whereby it may be derived.

*Fixed elevation*—An elevation obtained, either as a result of tide observations or previous adjustment of leveling, which is held as its accepted value in any subsequent adjustment.

*Frequency*—The number of complete cycles per second existing in any form of wave motion.

*Geodesy*—The science which treats mathematically of the figure and size of the earth. The term geodesy is often used to include both the science which must depend upon determinations of the figure and size of the earth from direct measurements made on its surface (triangulation, leveling, astronomic and gravity determinations), and the art which utilizes the scientific determinations in a practical way and is usually termed geodetic surveying or geodetic engineering.

*Geodetic control*—That control which takes into account the size and shape of the earth; implies a reference spheroid which represents the geoid and the horizontal and vertical control datum planes.

*Geodetic datum*—A datum consisting of five quantities; the latitude and the longitude of an initial point, the azimuth of a line from this point, and two constants necessary to define the terrestrial spheroid.

It forms the basis for the computation of horizontal control surveys in which the curvature of the earth is considered.

*Geodetic latitude*—The angle which the normal to the spheroid at a point makes with the plane of the geodetic equator. The geodetic latitude differs from the astronomic latitude by the meridional component of the deflection of the vertical (station error).

*Geographic latitude*—A general term applying to both astronomic and geodetic latitudes.

*Geoid*—The figure of the earth defined by the mean sea level surface of the oceans assumed to extend continuously through all continents.

*Horizon*—A great circle on the celestial sphere whose plane is perpendicular to the direction of the plumb line.

*Horizontal control*—Control which determines horizontal positions only, with respect to parallels and meridians or to other lines of reference.

*Hour angle*—The angle between the plane of the hour circle passing through a celestial body and the plane of the celestial meridian. The hour angle is reckoned from the meridian (0 hour over 0°) westward through 24 hours (360°). It may be measured by the angle at

the pole between an hour circle and the meridian, or by the arc of the equator which is intercepted by those circles.

*Hour circle*—Any great circle on the celestial sphere whose plane is perpendicular to the plane of the celestial equator. Hour circles are also called circles of declination. The hour circle which contains the zenith is identical with the celestial meridian.

*Isogonic chart*—A chart showing by means of isogonic lines, the magnetic declinations in an area of the earth's surface.

*Latitude, astronomic*—The angle between the plumb line and the plane of the celestial equator. Also defined as the angle between the plane of the horizon and the axis of rotation of the earth. Astronomic latitude is the latitude which results directly from observations on celestial bodies, uncorrected for deflection of the vertical (station error) which, in the United States, may amount to as much as 25″. Astronomic latitude applies only to positions on the earth, and is reckoned from the astronomic equator (0°), north and south through 90°.

*Latitude, geodetic*—The angle which the normal to the spheroid at a point makes with the plane of the geodetic equator. Geodetic latitudes are reckoned from the equator, but in the horizontal control survey of the United States they are computed from the latitude of station Meades Ranch as prescribed in the North American datum of 1927. In recording a geodetic position, it is essential that the geodetic datum on which it is based be also stated. A geodetic latitude differs from the corresponding astronomic latitude by the amount of the meridional component of the local deflection of the vertical (station error), which in this country may amount to more than 25″.

*Level datum*—A level surface to which elevations are referred. The generally adopted level datum for leveling in the United States is mean sea level. For local surveys, an arbitrary level datum is often adopted and defined in terms of an assumed elevation for some physical mark (bench mark).

*Level net*—Lines of spirit leveling connected together to form a system of loops or circuits extending over an area. Also called a vertical control net.

*Local hour angle*—Angular distance measured on the celestial equator between the celestial meridian and the hour circle that passes through the object. The local hour angle represents physically the amount of rotation of the celestial sphere, since the object was last on the observer's celestial meridian, and is always measured westward 0° to 360° from the celestial meridian.

*Longitude, astronomic* The angle between the plane of the celestial meridian and the plane of an initial meridian, arbitrarily chosen.

*Longitude, geodetic* The angle between the plane of the geodetic meridian and the plane of an initial meridian, arbitrarily chosen.

*Main-scheme station*—A station through which the basic survey computations are carried, also called a principal station. The main-scheme stations serve for the continued extension of the survey.

*Mean sea level*—A weighted average height of the surface of the sea for all stages of the tide.

*Meridian*—A north-south line from which longitudes and azimuths are reckoned; or a plane, normal to the geoid or shperoid, defining such a line.

*Meridian angle*—Same as *local hour angle* except that it may be measured 180° from the celestial meridian either eastward or westward.

*Nadir* That point diametrically opposite the zenith, located on the celestial sphere.

*Observer's meridian*—A celestial meridian passing through the zenith at the point of observation and the celestial poles.

*Occupied station*—A traverse or triangulation station over which a theodolite or an engineer transit is set up for the measurement of angles at this station. Also, a station at which angles have been so measured.

*Order of accuracy*—A mathematical ratio defining the general accuracy of the measurements made in a survey. The orders of accuracy for surveys are divided into four classes labeled: first order, second order, third order, and fourth or lower order.

*Picture point*—In surveying, a terrain feature that is easily identified on an aerial photograph, and whose horizontal and vertical position or both have been determined by survey measurements. Picture points are marked on the aerial photographs by the surveyor, and are used by the photomapper.

*Polar distance*—The complement of the declination, the angular distance from the pole to the celestial body measured along an hour circle.

*Position, geodetic*—The position of a point on the surface of the earth expressed in terms of geodetic latitude and geodetic longitude. A geodetic position implies an adopted geodetic datum. In a complete record of a geodetic position, the datum must be stated.

*Position, geographic*—The position of a point on the surface of the earth expressed in terms of latitude and longitude, either geodetic or astronomic.

*Prime meridian*—That meridian chosen as the zero of longitude, for most purposes it is the meridian near Greenwich, England.

*Prime vertical*—The vertical circle whose plane is perpendicular to the plane of the observer's celestial meridian.

*Right ascension*—The angle between the plane of the hour circle passing through a celestial body and the plane of the hour circle passing through the vernal equinox. Right ascension may be measured by the angle at the celestial pole between the tangents to the hour circles of the body and of the vernal equinox, or by the arc of the equator intercepted by those hour circles. Right ascension is measured eastward from the vernal equinox through 24 hours (360°).

*Spheroid*—In general any figure differing but little from a sphere. In geodesy, a mathematical figure closely approaching the geoid in form and size, and used as a surface of reference for geodetic surveys.

*Time*—The hour angle of a selected celestial object (mean sun, apparent sun, or vernal equinox), the meridian of which marks the beginning of day (civil, apparent solar, or sidereal, respectively) at a selected place.

*Vernal equinox* (first point of Aries)—The point among the stars where the sun in its apparent travel northward crosses the celestial equator.

*Vertical circles*—Great circles perpendicular to the plane of the true horizon.

*Vertical control*—Control which determines elevations above or below a plane of reference.

*Zenith*—The point where the direction of the plumb line produced above the horizon meets the celestial sphere.

*Zenith distance*—The complement of the altitude, the angular distance from the zenith of the celestial body measured along a vertical circle.

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